HOMEWORK #3
MATH 185–4
FALL 2009

DUE FRIDAY SEPTEMBER 25TH

(1) In each of the following case, find a δ such that $|f(x) - L| < \epsilon$ for all x satisfying $0 < |x - a| < \delta$.
  (i) $f(x) = x^4$, $L = a^4$.
  (ii) $f(x) = 1/x$, $a = 2, L = \frac{1}{2}$
  (iii) $f(x) = \sqrt{x}$, $a = 1, L = 1$.

(2) (a) If $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ does not exist, is it possible that
$$\lim_{x\to a}(f(x) + g(x))$$
exists?
  (b) If $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ does not exist, is it possible that
$$\lim_{x\to a}(f(x) \cdot g(x))$$
exists?

(3) Prove that
$$\lim_{x\to a} f(x) = \lim_{h\to 0} f(a + h)$$
assuming that the left side exists.

(4) (a) Suppose that $f(x) \leq g(x)$ for all x. Prove that $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$ (assuming both limits exist).
  (b) Can you replace both the $\leq$ by $<$ signs and get the same statement? Prove or give a counter-example.

(5) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then
$$\lim_{x\to a} \max(f(x), g(x)) = \max(L, M).$$

(6) Consider a fuction f with the following property
  – if g is any other function for which $\lim_{x\to 0} g(x)$ does not exist, then $\lim_{x\to 0}(f(x)+g(x))$ also does not exist.

Prove that this happens if and only if $\lim_{x\to 0} f(x)$ exists.
Hint: The words "if and only if" means you should prove that the two statements are equivalent. That is, first assume that $\lim_{x\to 0} f(x)$ exists and then prove that f satisfies the stated property. Conversely, assume that f satisfies the stated property and then prove that $\lim_{x\to 0} f(x)$ exists.

(7) Prove that $\lim_{x\to a} f(x) = L$ *if and only if* the following condition is satisfied.
  – For every $\epsilon' > 0$,there is a $\delta' > 0$ such that if x satisfies $0 < |x - a| \leq \delta'$, then $|f(x) - L| \leq \epsilon'$.