HOMEWORK #10
MATH 185–4
FALL 2009

DUE FRIDAY, NOVEMBER 20TH

(1) Suppose that f is an injective (ie, one-to-one) function. Prove that if f^{-1} is strictly increasing, then f is also strictly increasing.
(2) Suppose that $(f(x))^3 + x^2 = \cos(f(x))$ for all x and further suppose that $f(x)$ is differentiable. Find $f'(x)$ (your answer will involve $f(x)$).
(3) Suppose that h is an injective function such that $h'(x) = \cos^2(x^2+1)/(x^4+1)$. Further suppose that $h(0) = 1$. Find $(h^{-1})'(1)$.
(4) Suppose that f is a continuous injective functon on $\mathbb{R}$. Further suppose that $f = f^{-1}$. Prove that there exists at least one c such that $f(c) = c$.

1. Details on the exam

There will be 4 problems.

(i) There will be one page of definitions and short answers.
(ii) There will be a question asking you to compute a derivative from the definition (ie, $\lim_{h\to 0}\ldots$).
(iii) There will be a proof question that I won't tell you anything about (although it might involve vectors, linear transformations, the dot product, linear independence, the definition of a basis, etc.)
(iv) One of the following 4 questions will also be asked.
   (a) Prove the chain rule (see Theorem 9 in chapter #10 of your book).
   (b) Suppose that f is differentiable on $(0, 2)$. Further suppose that $f'(1) = 0$ and also that $f'(x) > 0$ for all $x \in (0, 2)$ as long as $x \neq 1$. Prove that f is strictly increasing on $(0, 2)$. (I reserve the right to slightly change the numbers on this problem).
   (c) Prove Rolle's theorem (see Theorem 3 in chapter #11 of your book).
   (d) Let $f_1, f_2, \ldots, f_n$ be n distinct functions. Use induction to prove that $(f_1 + f_2 + \cdots + f_n)' = f_1' + f_2' + \cdots + f_n'$.
(v) I will ask one extra credit question about *uniform continuity*. To prepare yourself for this question, read the appendix to chapter #8 in the book. Make sure you understand the statements and try a couple of the exercises if you want.