(1) Let \( f(x) \in \mathbb{C}[x] \) be a monic polynomial of degree \( N \leq 3 \). Suppose that \( f(x) \) shares a root with each of its \( N - 1 \) derivatives. Show that there exists \( w \in \mathbb{C} \) so that 
\[ f(x) = (x - w)^N. \]

(2) Let \( f(x), g(x) \in \mathbb{C}[x] \) be polynomials. The greatest common divisor of \( f(x), g(x) \) is denoted by \( \gcd(f(x), g(x)) \) and is defined to be the monic polynomial of largest degree which divides both \( f(x) \) and \( g(x) \). Using the Euclidean algorithm for polynomials, prove that there exists polynomials \( a(x), b(x) \) so that 
\[ \gcd(f(x), g(x)) = a(x)f(x) + b(x)g(x). \]

(3) Find the greatest common divisor of the following the polynomials \( f(x), g(x) \) in the following problems and then find polynomials \( a(x), b(x) \in \mathbb{C}[x] \) so that \( \gcd(f(x), g(x)) = a(x)f(x) + b(x)g(x) \).

(a) \( f(x) = x^2 - 1 \) and \( g(x) = x^2 + x + 1 \).
(b) \( f(x) = x^4 + 1 \) and \( g(x) = x^6 + 1 \).
(c) \( f(x) = x^m - 1 \) and \( g(x) = x^n - 1 \) where \( m, n \in \mathbb{N} \) and \( m \leq n \).

The following exercises are from Chapter 1 from Prasolov’s Polynomials
(4) Exercise 1.2.
(5) Exercise 1.3.
(6) Exercises 1.7.
(7) Exercise 1.10.