Math 3220-3 Take Home Midterm 3, April 6, 2017

Show all work!

Name:

Problem 1. If F is a differentiable real function defined in a convex open set $E \subset \mathbb{R}^n$, such that $\partial_1 F(x) = 0$ for every $x \in E$, prove that F depends only on $x_2, \ldots, x_n$.

Problem 2. Define

$$F(x, y) = (e^x \cos y - 1, e^x \sin y)$$

for all $(x, y) \in \mathbb{R}^2$. Prove that $F = G_2 \circ G_1$, where

$$G_1(x, y) = (e^x \cos y - 1, y) \text{ and } G_2(u, v) = (u, (1 + u) \tan v)$$

are primitive in some neighborhood of $(0, 0)$.

Problem 3. Let $E = \mathbb{R}^2 - \{0\}$. Let ω be a 1-form on E given by

$$\omega = \frac{x\, dy - y\, dx}{x^2 + y^2}.$$

Show that $d\omega = 0$.

Problem 4. Let ω be the 1-form from Problem 3. Let $\gamma : [0, 2\pi] \longrightarrow \mathbb{R}^2$ be the curve given by

$$\gamma(t) = (\cos t, \sin t).$$

Show that

$$\int_\gamma \omega = 2\pi.$$

Explain why this implies that $\omega \neq df$ for any continuously differentiable function f on E.