4.1 Introduction to Area

Ex 1: Write in sigma (summation) notation. (a) $a_1 - a_2 + a_3 - a_4 + a_5 - \ldots$	(b) $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \ldots$
Ex 2: Find the sum (using the special sum formula(s)) $\sum_{k=1}^{20} \left(2k^2 - 3\right)$	Special Sum Formulas: 1. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ 2. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ 3. $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Ex 3: Find the sum (using the special sum formula(s))

$$\sum_{j=1}^{n}(3j+1)^2$$

4.2 Definite Integral

Ex 1: Using the definition of the definite integral, calculate $\int_0^1 (2x+5)\,dx$.

<u>(One) Definition of the definite integral</u>:

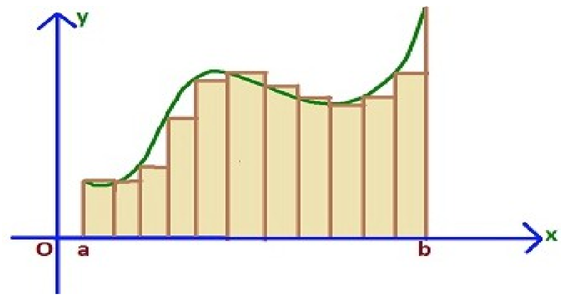
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

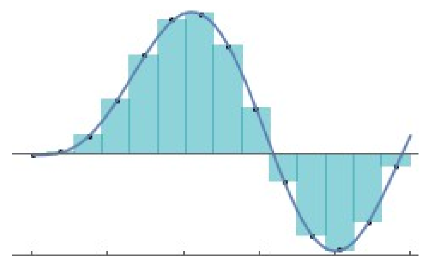
If we choose right-hand x-values for each rectangle, of uniform width, then we have the following formulas.

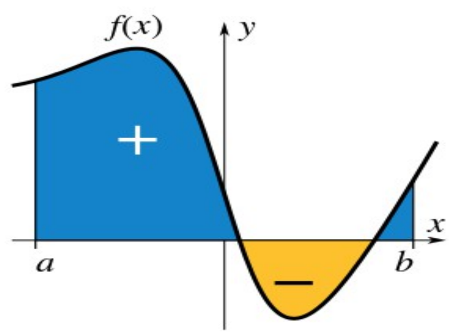
$$\Delta x = \frac{b-a}{n}$$

and

$$x_i = a + i(\Delta x)$$







<u>4.2 (continued)</u>

Ex 2: Using the definition of the definite integral, calculate $\int_{-1}^{2} (3x^2+4)\,dx$.

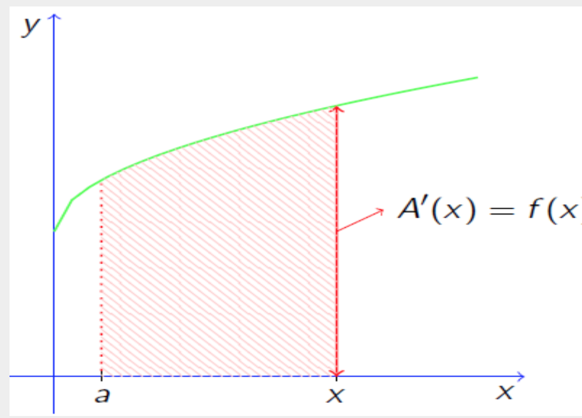
4.3 First Fundamental Theorem of Calculus

Ex 1: Find the formula for the accumulation function A(x) to represent the area as shown.

First FTC:

Given that $f(x)$ is continuous on the interval [c, d] such that a , $x \in (c, d)$,

$$D_x\left(\int_a^x f(t)\,dt\right) = f(x) \quad \text{where } a = \text{any constant.}$$

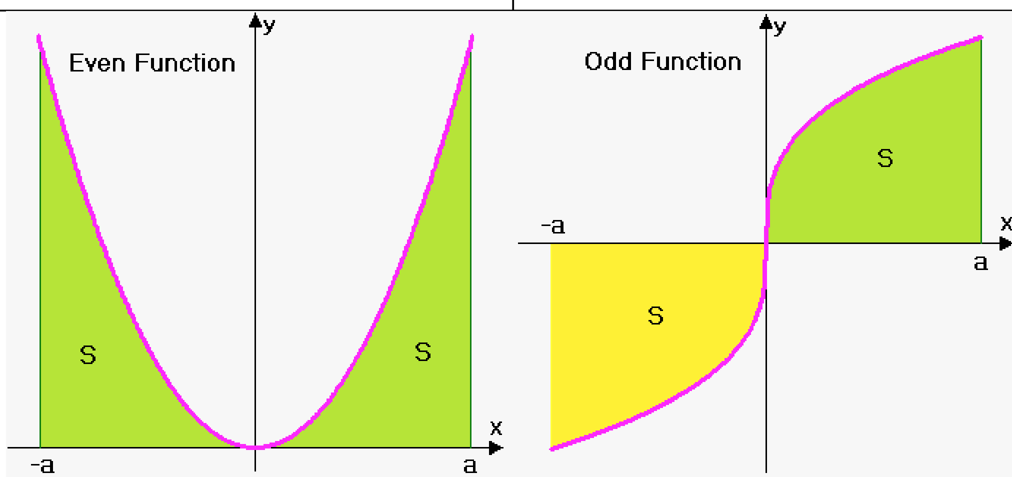


Ex 2: Find $G'(x)$.

(a) $G(x) = \int_x^1 \sqrt{t^2+1}\,dt$

(b) $G(x) = \int_2^{\tan x} e^{-t^2}\,dt$

Ex 3: Find $G'(x)$. (a) $G(x)=\int_{2}^{x^2} x\sin(t^2)\,dt$	(b) $G(x)=\int_{x^4}^{x} \sec t\,dt$



Ex 4: Given f is an odd function, g is an even function and $\int_0^1 |f(x)|\,dx=\int_0^1 g(x)\,dx=3$, use geometric reasoning to evaluate the following integrals.

(a) $\int_{-1}^{1} f(x)\,dx$	(b) $\int_{-1}^{1} g(x)\,dx$
(c) $\int_{-1}^{1} \lvert f(x)\rvert\,dx$	(d) $\int_{-1}^{1} f^3(x)\,g(x)\,dx$

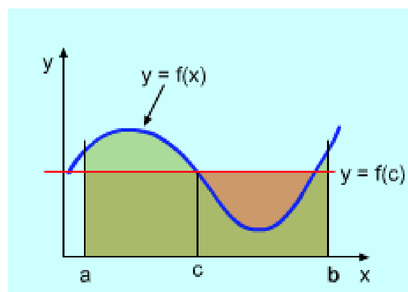
4.4 Second Fundamental Theorem of Calculus

Ex 1: Evaluate the following integrals, using the Second FTC. (a) $\int_1^3 \frac{x^4-5}{x^2}\,dx$	Second Fundamental Theorem of Calculus: If $f(x)$ is continuous on [a, b], and $F(x)$ is any antiderivative, then $\int_a^b f(x)\,dx = F(b) - F(a)$.
(b) $\int_0^1 (x^{4/3} - 2x^{1/3})\,dx$	The definite integral is also a linear operator. You list the two properties it must meet then. (a) $\int_a^b (f(x)+g(x))\,dx$ = ______________________________ AND (b) $\int_a^b k\,f(x)\,dx$ = ____________________ for any constant k.

Ex 2: Evaluate the following integrals (some are indefinite, and others are definite integrals).

(a) $\int x^3 \cos(x^4+1)\,dx$	(b) $\int x^{-3} \sec(x^{-2}-3)\tan(x^{-2}-3)\sqrt[6]{\sec(x^{-2}-3)}\,dx$
(c) $\int_1^2 \frac{x^3+2}{\sqrt{x^4+8x}}\,dx$	(d) $\int_1^4 \frac{(\sqrt{x}-1)^3}{\sqrt{x}}\,dx$

4.5 Mean Value Theorem for Integrals

Ex 1: Find the average value of the function $f(x) = \sin^2 x \cos x$ on the interval $[0, \frac{\pi}{2}]$.	<u>MVTI</u>: If $f(x)$ is continuous on [a, b], then there is some $c \in (a, b)$ such that $f(c) = \frac{1}{b-a} \int_a^b f(t)dt$. In other words, there is some x-value in (*a, b*) (which we call c) such that the area of the rectangle with height *f(c)* and width of (*b* - *a*) is the same as the area under the curve from *a* to *b*. *f(c)* is called the average value of the function on that interval. And, c is the x-value where that average value occurs.
Ex 2: Find all values of c guaranteed by the MVTI for $f(x) = x^3$ on the interval [0, 2].	

4.5 (continued)

Ex 2: Use symmetry and geometrical reasoning to help evaluate the following definite integrals.

(a) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\lvert x\rvert \sin^3 x + x^2 \tan x\right) dx$	(b) $\int_{-3}^{3} (\sin x - \cos x)^2\, dx$