Derivatives
\[ D_x e^x = e^x \]
\[ D_x \sin(x) = \cos(x) \]
\[ D_x \cos(x) = -\sin(x) \]
\[ D_x \tan(x) = \sec^2(x) \]
\[ D_x \sec(x) = \sec(x) \tan(x) \]
\[ D_x \csc(x) = -\csc(x) \cot(x) \]
\[ D_x \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \]
\[ D_x \cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}} \]
\[ D_x \tan^{-1}(x) = \frac{1}{1+x^2} \]

Integrals
\[ \int e^x \, dx = e^x + C \]
\[ \int \sin(x) \, dx = -\cos(x) + C \]
\[ \int \cos(x) \, dx = \sin(x) + C \]
\[ \int \tan(x) \, dx = \ln|\cos(x)| + C \]
\[ \int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C \]
\[ \int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)| + C \]

Calculus 3 Concepts

Cartesian coors in 3D
Given two points: \( \mathbf{a} = (x_1, y_1, z_1) \) and \( \mathbf{b} = (x_2, y_2, z_2) \), distance between them is:
\[ \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \]

Vectors
\[ \mathbf{u} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \]

U-Substitution
\[ \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} \]

Fns and Identities
\[ \sin(-x) = -\sin(x) \]
\[ \cos(-x) = \cos(x) \]

Integrals
\[ \int e^x \, dx = e^x + C \]
\[ \int \sin(x) \, dx = -\cos(x) + C \]
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Calculus 3 Concepts

Cartesian coordinates in 3D
Given two points: \( \mathbf{a} = (x_1, y_1, z_1) \) and \( \mathbf{b} = (x_2, y_2, z_2) \), distance between them is:
\[ \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \]

Vectors
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\[ \int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)| + C \]
Directional Derivatives
Let z = f(x,y) be a function, (a,b) ap point in the domain (a valid input point) and u a unit vector (2D).
The Directional Derivative is then the derivative at the point (a,b) in the direction of u or:
\[ D_u f(a,b) = u \cdot \nabla f(a,b) \]
This will return a scalar. 4-D version:
\[ D_u f(a,b,c) = u \cdot \nabla f(a,b,c) \]

Tangent Planes
Let \( F(x,y) = k \) be a surface and \( P = (x_0, y_0, z_0) \) be a point on that surface. Equation of a Tangent Plane:
\[ \nabla F(x_0, y_0, z_0) \cdot (x-x_0, y-y_0, z-z_0) = 0 \]

Approximations
Let \( z = f(x,y) \) be a differentiable function total differential of \( f = dz \)
\[ dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \]
This is the approximate change in z.
The actual change in z is the difference in z values:
\[ \Delta z = z - z_0 \]

Maxima and Minima
Internal Points
1. Take the Partial Derivatives with respect to X and Y and find the critical points (can use gradient)
2. Set derivatives equal to 0 and use to solve equations for x and y
3. Plug back into original equation for z. Use Second Derivative Test for whether points are local max, min, or saddle

Second Partial Derivative Test
1. Find all \((x,y,z)\) points such that:
\[ \nabla f(x,y) = 0 \]
2. Let \( D = f_{xx}(x,y)f_{yy}(x,y) - f_{xy}(x,y)^2 \)
   (a) \( D > 0 \) AND \( f_{xy}(x,y) < 0 \) \( f(x,y) \) is local max value
   (b) \( D > 0 \) AND \( f_{xy}(x,y) > 0 \) \( f(x,y) \) is local min value
   (c) \( D < 0 \) \( (x,y,f(x,y)) \) is a saddle point
   (d) \( D = 0 \), test is inconclusive
3. Determine if any boundary point gives min or max.
   Typically, we have to parametrize boundary and then reduce to a Calc 1 type of min/max problem to solve.

The following only apply only if a boundary is given
1. check the corner points
2. Check each line \((0 \leq x \leq 5, 0 \leq y \leq 5)\) give x=0 and x=5 on Bound Equations, this is the global min and max...second derivative test is not needed.

Lagrange Multipliers
Given a function \( f(x,y) \) with a constraint \( g(x,y) \), solve the following system of equations to find the max and min points on the constraint (NOTE: may need to also find internal points):
\[ \nabla f = \lambda \nabla g \]
\[ g(x,y) = 0 \]

Double Integrals
With Respect to the x-axis, if taking an integral,
\[ \int \int f(x) dydx \]
This will be cutting in vertical rectangles.

Polar Coordinates
When using polar coordinates, \( dA = r dr d\theta \)

Surface Area of a Curve
Let \( z = f(x,y) \) be a function, \((a,b)\) a point on the curve. The actual change in z is the difference in z values:
\[ \Delta z = z - z_0 \]

Independence of Path
Fund Thm of Line Integrals
Let \( F \) be a vector field with \( F(x,y,z) = 0 \)
\[ \oint_C F \cdot ds = \int_{a}^{b} F \cdot \vec{T} \]
where \( \vec{T} = unit \text{ vector} \)

Curv of \( \vec{F} \) across \( G \)
\[ \int_{C} \vec{F} \cdot ndS = \int_{\Gamma} \int_{D} M dx - N dy + P dz \]

Unit Circle
(cos, sin)

Green’s Theorem
(method of changing line integral for double integral) - Use for Flux and Circulation across 2D curve and line integrals over a closed boundary
\[ \oint_C \vec{F} \cdot ds = \int_{R} \int_{D} \nabla \times \vec{F} = 0 \]
Let \( R \) be a region in xy-plane
\[ \nabla \times \vec{F} \]
In Spherical Coordinates, \( r \), \( \theta \), \( \phi \)

Other Information
\[ \frac{s}{r} = \frac{k}{r^2} \]
Where a Cone is defined as \( z = \sqrt{a^2 + r^2} \).
In Spherical Coordinates, \( \phi = cos^{-1}( \frac{z}{r} ) \)

Right Circular Cylinder:
\[ V = \pi r^2 h, SA = 2\pi r^2 + 2\pi rh \]
Law of Cosines:
\[ a^2 = b^2 + c^2 - 2bc \cos(\theta) \]

Stokes Theorem
Let:
\( \vec{S} \) be a 3D surface
\[ \vec{F}(x,y,z) = M(x,y,z) \]
\( \vec{N} = N(x,y,z) \), \( P(x,y,z) \)
\( M,N,P \) have continuous 1st order partial derivatives
\( \vec{C} \) is piece-wise smooth, simple, closed, curve, positively oriented
\( \vec{T} \) is unit tangent vector to C.

Remember:
\[ \oint_C \vec{F} \cdot dS = \int_{\Gamma} \int_{D} \vec{F} \cdot ndS \]

Gauss’ Divergence Thm
(3D Analog of Green’s Theorem - Use for Flux over a 3D surface) Let:
\[ \vec{F}(x,y,z) \] be vector field continuously differentiable in solid \( S \)
\( S \) is a 3D solid -DS boundary of \( S \) (A Surface)
\( \vec{n} \) - unit outer normal to \( S \)
Then,
\[ \int_{\partial S} \vec{F} \cdot ds = \int_{S} \int_{D} \vec{F} \cdot dV = \int_{S} \int_{D} dV \]