$\boxed{9.4 \ \#35}$

Let $a_n > 0$ and suppose that $\sum a_n$ converges. Prove that $\sum a_n^2$ also converges

Pf Since $\sum a_n$ converges, then we know

$\lim_{n\to\infty} a_n = 0$ by n^{th} term test for divergence.

(which states clearly that $\sum a_n$ converges $\Rightarrow \lim_{n\to\infty} a_n = 0$).

Then there exists an N large enough so that $\forall\ n > N$, $a_n^2 < a_n$ (since $\forall\ x \in (0,1)$ $x^2 < x$) because for those n-values, $a_n << 1$.

By OCT, since $\sum a_n$ converges and $\sum a_n^2 < \sum a_n$, then so does $\sum a_n^2$ converge.

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$\boxed{9.5 \ \#31}$ Claim $\sum a_n$ diverges $\Rightarrow \sum |a_n|$ diverges

Pf Cleverest way to do this is to invoke the Absolute Convergence Test (pg 476). The contrapositive statement for that test is:

If $\sum a_n$ doesn't converge, then $\sum |a_n|$ doesn't converge, which is exactly our claim. //

$\boxed{9.5 \ \#33}$ ① Show that positive terms of alternating harmonic series form a diverged series.
② Show the same for the series of negative terms.

Pf alt. harmonic series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -1 + \frac{1}{2} + \frac{-1}{3} + \frac{1}{4} + \frac{-1}{5} + \cdots$$

① $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots = \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$ (p-series, $p=1$)

diverges

② $-1 - \frac{1}{3} - \frac{1}{5} - \frac{1}{7} - \cdots = -\sum_{n=1}^{\infty} \frac{1}{2n-1}$ by LCT, w/ $b_n = \frac{1}{n}$

So $\sum b_n$ diverges

we get $\lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} < \infty \Rightarrow \sum_{n=1}^{\infty} \frac{1}{2n-1}$ also

diverges

$\Rightarrow -\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges #