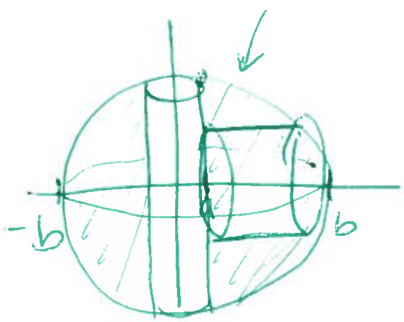
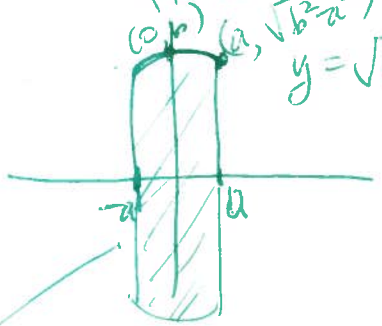
Ex 4 (#19) A round hole of radius a is drilled through the center of a solid sphere of radius b ($b>a$). Find the volume of the remaining solid.

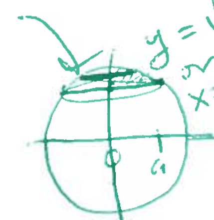


find this volume



$=$  $+ \; 2$ 

cylinder



$V_c = 2\left(\pi a^2 \sqrt{b^2-a^2}\right)$

$$V = \pi \int_{\sqrt{b^2-a^2}}^{b} r^2 \, dy$$

$$V = \pi \int_{\sqrt{b^2-a^2}}^{b} (b^2-y^2)\, dy$$

$$= \pi \left(b^2 y - \frac{1}{3}y^3\right)\Big|_{\sqrt{b^2-a^2}}^{b}$$

$$= \pi\Big(\left(b^3 - \frac{1}{3}b^3\right) - \left(b^2\sqrt{b^2-a^2} - \frac{1}{3}(b^2-a^2)\sqrt{b^2-a^2}\right)\Big)$$

$$= \pi\left(\frac{2}{3}b^3 - \frac{2}{3}b^2\sqrt{b^2-a^2} - \frac{1}{3}a^2\sqrt{b^2-a^2}\right)$$

$$V = 2\pi a^2 \sqrt{b^2-a^2} + 2\left(\frac{2\pi}{3}b^3 - \frac{2\pi}{3}b^2\sqrt{b^2-a^2} - \frac{\pi}{3}a^2\sqrt{b^2-a^2}\right)$$

$$V = \frac{4\pi}{3}\left[b^3 - b^2\sqrt{b^2-a^2} - a^2\sqrt{b^2-a^2}\right]$$

total volume left

$$= \frac{4}{3}\pi b^3 - \frac{4\pi}{3}\left[b^3 - b^2\sqrt{b^2-a^2} + a^2\sqrt{b^2-a^2}\right)$$

$$= \frac{4\pi}{3}(b^2-a^2)\sqrt{b^2-a^2} = \frac{4\pi}{3}(b^2-a^2)^{3/2}$$

Surface Area

$$SA = \int_a^b 2\pi f(x)\, ds$$

$$ds = \sqrt{1+(f'(x))^2}\, dx$$

only for $y=f(x)$ and rotated about x-axis

Ex 2 Find surface area.

revolve $y = \dfrac{x^6+2}{8x^2}$ $\quad x \in [1,3]$

about x-axis $\qquad y = \frac{1}{8}x^4 + \frac{1}{4}x^{-2}$

$$y' = \frac{1}{2}x^3 - \frac{1}{2}x^{-3} = \frac{1}{2}\left(x^3 - \frac{1}{x^3}\right) = \frac{1}{2}\left(\frac{x^6-1}{x^3}\right)$$

$$SA = \int_1^3 2\pi\left(\frac{1}{8}x^4 + \frac{1}{4}x^{-2}\right)\sqrt{1+\left(\frac{x^6-1}{2x^3}\right)^2}\, dx$$

$$= 2\pi\int_1^3 \left(\frac{1}{8}x^4+\frac{1}{4}x^{-2}\right)\left(\frac{x^6+1}{2x^3}\right) dx$$

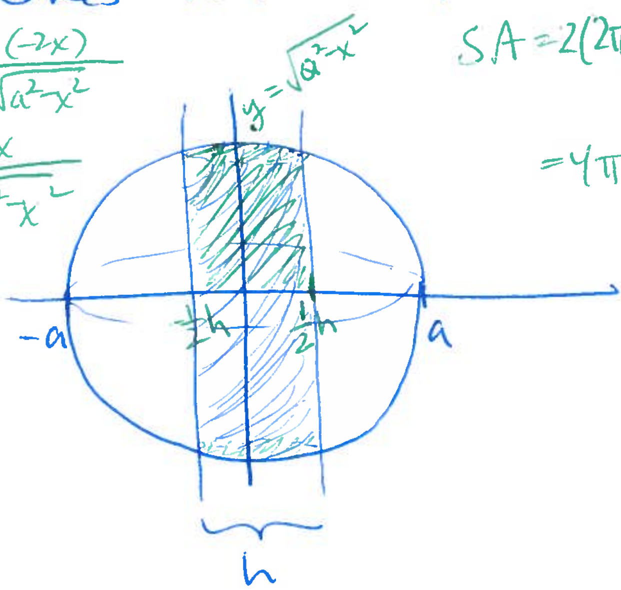
$$= 2\pi\int_1^3 \left(\frac{1}{16}x^7 + \frac{3}{16}x + \frac{1}{8}x^{-5}\right) dx$$

$$= 2\pi\left(\frac{x^8}{16(8)} + \frac{3x^2\cdot 4}{32\cdot 4} + \frac{x^{-4}\cdot 4}{-32\cdot 4}\right)\Bigg|_1^3$$

$$= \frac{2\pi}{32(4)}\left(\left(3^8 + 12(9) - 4\left(\frac{1}{81}\right)\right) - (1+12-4)\right)$$

Ex 3 Show that the area of the part of the surface of a sphere of radius a between two parallel planes h units apart $(h<2a)$ is $2\pi a h$.

$$y' = \frac{(-2x)}{2\sqrt{a^2-x^2}} = \frac{-x}{\sqrt{a^2-x^2}}$$



$$SA = 2(2\pi)\int_0^{\frac{1}{2}h} \sqrt{a^2-x^2}\left(\sqrt{1+\frac{x^2}{a^2-x^2}}\right) dx$$

$$= 4\pi\int_0^{h/2} \cancel{\sqrt{a^2-x^2}}\left(\frac{\sqrt{a^2-\cancel{x^2}+\cancel{x^2}}}{\cancel{\sqrt{a^2-x^2}}}\right) dx$$

$$= 4\pi a\int_0^{h/2} dx = 4\pi a\left(x\Big|_0^{h/2}\right)$$

$$= 4\pi a\left(h/2 - 0\right)$$

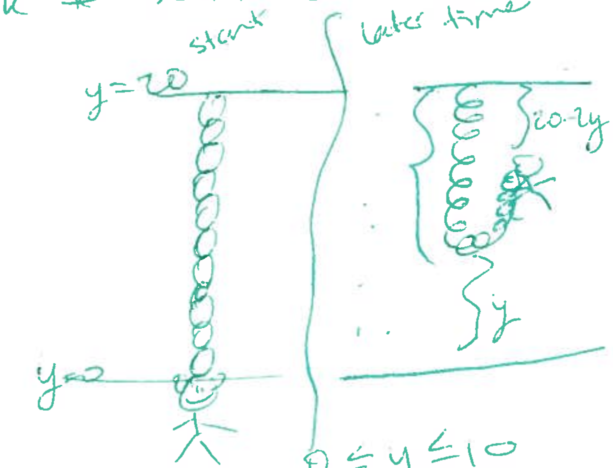
$$= 2\pi a h.$$

__Ex2__ A 10-pound monkey hangs at the end of a 20-foot chain that weighs $\frac{1}{2}$ pound/foot. How much work does it do in climbing the chain to the top? (Assume the end of the chain is attached to the monkey.)

total work $= W_1 + W_2$

W_1 = work to lift monkey

W_2 = work to lift chain



$0 \le y \le 10$

$$W_1 = \underbrace{10}_{\text{wt of mky}}\underbrace{(20)}_{\text{length}} = 200 \text{ ft-lb}$$

$$W_2 = \int_0^{10} (10-y)\,dy$$

$$= \left(10y - \frac{y^2}{2}\right)\Big|_0^{10}$$

$$= (100 - 50)$$

$$= 50$$

$$\Delta W_2 \simeq \frac{1}{2}(20-2y)\,dy$$

$$\left(\frac{lb}{ft}(ft)(ft) = ft\text{-}lb\right)$$

$$= (10-y)\,\Delta y$$

ΔW_2 = wt density of chain · length its travels (Δy)

$$W_1 + W_2 = 200 + 50 = 250 \text{ ft-lbs}$$