3.5 Graphing Polynomial Fns

Division Algorithm

If P + D are polynomials $(D(x) \neq 0)$, $\exists!$ polynomials Q & R $\ni$ $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$

$\ni$ Q (quotient) is unique polynomial, R(x) (remainder) is polynomial w/ degree < D(x).

① If $R(x) = 0$, then $D(x)$ is factor of $P(x)$.

② If degree of $D(x)$ is > degree of $P(x)$, then $Q(x) = 0$.

③ we can also write it as $P(x) = Q(x)D(x) + R(x)$.

Ex 1 Do long division.

$$\frac{3x^4 + 2x^3 - 5x + 1}{x^2 + 1}$$

- must use long division if dividing by polynomial w/ degree > 1.
- can use synthetic division only if dividing by linear polynomial

Remainder Thm

when f(x) polynomial is divided by $(x-r)$, the remainder is $f(r)$.

3.5 (cont)

Ex 2 Do synthetic Division.

(a) $\dfrac{x^5-3x^4+2x^2-5}{x+2}$

(b) $\dfrac{4x^3-x^2+5x+1}{2x-1}$

Ex 3 Use synthetic division to calculate $f\left(\frac{1}{2}\right)$ for $f(x)=8x^4-6x^3+5x^2+4x-3$

3.5 (cont)

Graphs of Polynomials

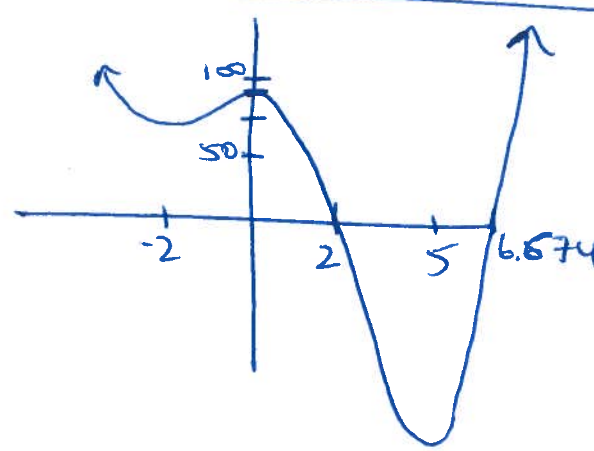
- always continuous
- no "pointy points"
- graph of n^{th}-degree polynomial has one y-intercept (where $x=0$) and at most n x-intercepts
- graph of n^{th}-degree polynomial has at most $n-1$ turning pts.

Vocab <u>increasing</u>: a fn is increasing if $a<b$ $\Rightarrow f(a)<f(b)$.

<u>decreasing</u>: a fn is decreasing if $a<b \Rightarrow f(a)>f(b)$.

<u>inflection pt</u>: the pt on a graph of $f(x)$ where it changes from being concave up to concave down (or vice versa)

Ex 4 $f(x)=x^4-4x^3-20x^2+96$



(a) Find critical values + where f is positive or negative.

$\longleftrightarrow f(x)$

3.5 (cont)

Ex 4 (cont)

(b) find turning pts and state where $f(x)$ is increasing and decreasing

(c) Find inflection pts and state where $f(x)$ is concave up / down.

Ex 5 Sketch graph of $f(x) = (x-1)(x-4)(x+3)$

(plot pts)

3.6 Polynomial Eqns

Goal: To graph polynomial fns with more skill than just plotting points!

Factor Thm

If $P(x) = (x-r)(Q(x)) + R$ and $R=0$, then $x-r$ is factor of $P(x)$.

i.e. if r is zero or root of $P(x)$, then $x-r$ is factor

Rational Root Thm (A1)

If $P(x)$ is polynomial w/ integer coefficients, then if there are any rational roots of $P(x)$, they are in the form of factors of constant term over factors of leading coefficient

ex $P(x) = 3x^5 + 4x^3 - 2x + 10$

if there are rational roots, they are from these choices: $\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{2}{1}, \pm\frac{2}{3}, \pm\frac{5}{1}, \pm\frac{5}{3}, \pm\frac{10}{1}, \pm\frac{10}{3}$

Fundamental Thm of Algebra

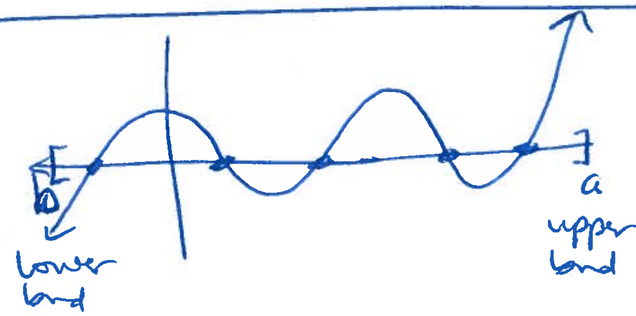
Every polynomial of degree n has n roots.

this assumes complex roots are okay!

multiplicity: $x=a$ is a root of multiplicity k if, in factored form, the polynomial has $(x-a)^k$ as a factor.

3.6 (cont)

Ex) For $y = x(x-1)^2(x+3)^3(x+5)$, name all the roots/zeros along with their multiplicity. So how many zeros does this polynomial have? What is its degree? How many distinct roots are there?



(☆2)

Upper/lower Bound Thm

$P(x)$ is polynomial w/ real coefficients + positive leading coefficient.

- If $a > 0$ + the last row of synthetic division for $(x-a)$ has all nonnegative #s, then a is an upper bound for all real roots of $P(x)$.
- If $b < 0$ + the last row of synthetic division for $(x-b)$ has #s that alternate in sign, then b is lower bound for all real roots of $P(x)$.

3.6 (cont)

Ex 2 Show this eqn has no rational roots.

$$2x^3+5x^2-3x+1=0$$

(use ☆1)

(☆3) **Conjugate Pair Thm**

① If $P(x)=0$ is polynomial eqn w/ real coefficients, then when $a+bi$ is a root, so is $a-bi$.

② If $P(x)=0$ is polynomial eqn w/ rational coefficients, then when $m+\sqrt{n}$ is root so is $m-\sqrt{n}$

(☆4) **Descartes Rule of Signs**

$P(x)$ is polynomial w/ $\mathbb{R}$ coefficients. (in descending order)

Count # of sign changes of $P(x)$ and $P(-x)$.

① The # of positive $\mathbb{R}$ zeros = # sign changes in $P(x)$ (or # decreased by even integer)

② The # of negative $\mathbb{R}$ zeros = # sign changes in $P(-x)$ (or that # minus an even integer)

3.6 (cont)

Ex 3 Is $2+\sqrt{5}$ a root of $x^4-4x^3-5x^2+16x+4=0$? If so, what is another root? (use ☆3)

Ex 4 Solve this polynomial eqn (over $\mathbb{R}$) (use ☆1, ☆2, ☆3, ☆4)

$x^4-12x^3-13=6(3-2x-5x^2)$

use ☆1, ☆2, ☆3, ☆4

3.6 (cont)

Ex 5 Solve polynomial eqn over $\mathbb{C}$.

$$x^4 - 20x^2 + 125 = 0$$

Ex 6 Solve $p(x) = (x^2+2x+5)(x^2-3x+5)$