Checklist and Assignment

Checklist:
- Approximate Doubling Time
- Approximate Half-Life Time
- Exact Formulas for Doubling and Half-Life

Assignment:
1. p 487 - 488 Quick Quiz
2. p 488 - 489 Exercises 1, 3, 4, 6, 7, 11, 25, 28, 31, 33, 38, 43, 44, 46, 51

Key Words

- **Doubling Time** The time required for a quantity to double in exponential growth.
- **Half-Life** The time required for a quantity to decrease in half (by percentage).

Remember: Doubling (×2) is increasing by 100%. Half decay is decreasing by 50%.

**Doubling Time**

The **doubling time** is the time it takes a quantity that grows exponentially to double. This time is written $T_{\text{double}}$.

Calculations knowing the doubling time can be made to find any value of a quantity growing exponentially after any amount of time.

Consider an initial population of 10,000 that grows with a doubling time of 10 years. What is the population after 20 years? After 25 years?
Doubling Time

The formula

\[
\text{new value} = \text{initial value} \times 2^{t/T_{\text{double}}}
\]

gives the new value of a quantity after a time \( t \) when the initial value and doubling time are known.

Using the formula, a population of initial value of 10,000 will have a new value of 40,000 in 20 years and 56568.54 (\( \approx 56569 \)) in 25 years.

Doubling Time

The world’s population went from 3 billion in 1960 to 6 billion in 2000. Suppose the world population continued to grow from 2000 on with a doubling time of \( T_{\text{double}} = 40 \) years.

What would the population be in 2030? in 2200?

Ans: (a) 10.1 billion (b) 192 billion. Is it possible for the human population on Earth to reach 192 billion?

Approximate the Doubling Time

Consider a short term measure of time, like month or year. One can measure growth during that short time in terms of percent. For instance, with world population increases about 1.4% per year after 2000.

The rule of 70 approximates \( T_{\text{double}} \) by the formula

\[
T_{\text{double}} \approx \frac{70}{P}
\]

with \( P \%) as the growth rate per time period. This approximation works best with small rates, and works poorly for rates above 15%.
Growth of a Rabbit Community

A rabbit population starts with a population of 100 and grows at 7% per month.

<table>
<thead>
<tr>
<th>Month</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>$1.07^1 \times 100 = 107$</td>
</tr>
<tr>
<td>2</td>
<td>$1.07^2 \times 100 = 114$</td>
</tr>
<tr>
<td>3</td>
<td>$1.07^3 \times 100 = 123$</td>
</tr>
<tr>
<td>4</td>
<td>$1.07^4 \times 100 = 131$</td>
</tr>
<tr>
<td>5</td>
<td>$1.07^5 \times 100 = 140$</td>
</tr>
<tr>
<td>6</td>
<td>$1.07^6 \times 100 = 150$</td>
</tr>
<tr>
<td>7</td>
<td>$1.07^7 \times 100 = 161$</td>
</tr>
<tr>
<td>8</td>
<td>$1.07^8 \times 100 = 172$</td>
</tr>
<tr>
<td>9</td>
<td>$1.07^9 \times 100 = 184$</td>
</tr>
<tr>
<td>10</td>
<td>$1.07^{10} \times 100 = 197$</td>
</tr>
<tr>
<td>11</td>
<td>$1.07^{11} \times 100 = 210$</td>
</tr>
<tr>
<td>12</td>
<td>$1.07^{12} \times 100 = 225$</td>
</tr>
<tr>
<td>13</td>
<td>$1.07^{13} \times 100 = 241$</td>
</tr>
<tr>
<td>14</td>
<td>$1.07^{14} \times 100 = 258$</td>
</tr>
<tr>
<td>15</td>
<td>$1.07^{15} \times 100 = 276$</td>
</tr>
</tbody>
</table>

Rule of 70

Approximately, $T_{\text{double}} \approx 10$ months. Using this and

\[
\text{new value} = \text{initial value} \times 2^{t/T_{\text{double}}}
\]

test how well these values match the table values. Test for $t = 5, 10, 15$.

Example: Prices rise at a rate of 0.3% per month. What is their doubling time? By what factor will prices increase in 1 year? In 8 years?

Ans: (a) About 233 months. (b) 1.036 (c) 1.331

Half-Life

The formula

\[
\text{new value} = \text{initial value} \times \left(\frac{1}{2}\right)^{t/T_{\text{half}}}
\]
gives the new value of a quantity after a time $t$ when the initial value and the half-life are known.

A half-life is the period of time it takes a quantity to decay, or decrease, by 50%.

Plutonium-239 has a half life of about 24,000 years. 100 lbs of Pu-239 will weight 50 lbs after 24,000 years, 25 lbs after 48,000 years, etc.
Approximate the Half-Life Time

Again, knowing the decay rate of a quantity over a small period (like 1 month or 1 year), we can approximate the half-life.

\[ T_{\text{half}} \approx \frac{70}{P} \]

with \( P \% \) as the decay rate per time period. This approximation works best with small rates, and works poorly for decay rates above 15%.

Examples of Half-Life

**Example:** Carbon-14 has a half-life of about 5700 years. This builds up in living organisms when they are alive. Once the organisms die, the carbon-14 decays. What fraction of the carbon-14 remains in an animal bone still remains 1000 years after the animal died?

Ans: The fractional amount is \((1/2)^{1000 \text{ yr}/5700 \text{ yr}} \approx 0.885.\) About 88.5%.

Examples of Half-Life

**Example:** The area of forest is reduced each year because of urban encroachment. If the rate of the area decreases at 2.6% each year, what is the half-life of the forest? What fraction of the forest remains after 30 years?

Ans: About 26.9 years. 46.2%