Biology in Time *and* Space: A Partial Differential Equation Approach

James P. Keener
University of Utah

26 December, 2015

Contents

Part I

The Text

Chapter 1

Introduction

Mathematical biology is, broadly speaking, the study of how biological objects move and interact. The interaction part of biology is often studied using dynamical systems including discrete time maps and ordinary differential equations. Indeed, there is a lot of insight that can be gained from the study of these dynamical descriptions of biological processes. And, there are many excellent books that present this point of view. However, this perspective is limited when it comes to understanding real biological situations, because biological objects are essentially *never* homogeneously distributed in space, or well-mixed, even in a test tube or on a Petrie dish, let alone in the human body or on a mountain slope. In fact, it is a primary feature of biological objects that there are spatial differences and correspondingly movement of objects from one place to another. So, whether one is studying the spread of an infectious disease in Africa, the spread of an invasive species in the agricultural regions of California, or the movement of an action potential along a nerve axon, it is crucially important to include the effects of spatial differences.

This book is about the dynamics of biological objects in time *and* space. Consequently, it is about partial differential equations. It is intended for an undergraduate audience, and no previous background in partial differential equations is required. In fact, in the Appendices, you will find summaries of the background that is needed from calculus, from ordinary differential equations, and from probability theory and stochastic processes, because these are used a lot. Also, this material is presented from a heavily computational perspective, using Matlab to compute solutions and make plots. So, in the appendix you will find a primer for Matlab as well as Matlab codes for many of the calculations done in this text. The hope is that these codes can be readily modified to be used for Exercises, and that this will facilitate the process of learning how to understand partial differential equations.

The content of the book is organized pedagogically around mathematical material. However, all of the models described in this book are derived with a specific biological question in mind. Give a summary of the topics covered.