COPYRIGHT & LICENSE TOPOLOGY	DEFINITION *metric space* TOPOLOGY
DEFINITION *subspace* TOPOLOGY	DEFINITION *isometry* TOPOLOGY
DEFINITION *open set* TOPOLOGY	PROPOSITION *open balls are open* TOPOLOGY
THEOREM *unions and intersections of open sets* TOPOLOGY	DEFINITION *closed set* TOPOLOGY
DEFINITION *closed ball* TOPOLOGY	PROPOSITION *closed balls are closed sets* TOPOLOGY

A **metric space** (X, d) is a set X and a function $d : X \times X \to \mathbb{R}$ satisfying $\forall\, x, y, z \in X$

1. $d(x, y) \geq 0$
2. $d(x, y) = 0 \Leftrightarrow x = y$
3. $d(x, y) = d(y, x)$
4. $d(x, z) \leq d(x, y) + d(y, z)$

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Suppose (X_1, d_1) and (X_2, d_2) are metric spaces. A function $f : X_1 \to X_2$ is called an **isometry** if f is one–to–one, onto and

$$d_2(f(x), f(y)) = d_1(x, y) \quad \forall\, x, y \in X_1$$

If (X, d) is a metric space, and $A \subset X$ then $(A, d|_{A\times A})$ is a metric space and is called a **subspace** of (X, d).

If (X, d) is a metric space, then for each $x \in X$ and for each $r > 0$, $B(x, r)$ is open in X.

Supposing (X, d) is a metric space, then a subset $U \subset X$ is **open** iff

$$\forall\, x \in U, \exists\, r > 0 \text{ such that } B(x, r) \subset U$$

Let (X, d) be a metric space, $F \subset X$ is **closed** iff $X - F$ is open.

Let (X, d) be a metric space and let $\{U_\alpha\}_{\alpha\in A}$ be any collection of open sets in (X, d), then

1. $X, \varnothing$ are open.
2. $\bigcup_{\alpha\in A} U_\alpha$ is open.
3. Let $\{U_1, \ldots, U_n\}$ be a finite collection of open sets, then $\bigcap_{i=1}^{n} U_i$ is open.

A closed ball $\overline{B}(x, r)$, is a closed set.

A **closed ball** centered at x of radius r is denoted $\overline{B}(x, r)$, and defined to be:

$$\overline{B}(x, r) = \{y \in X \mid d(x, y) \leq r\}$$

THEOREM *unions and intersections of closed sets* TOPOLOGY	DEFINITION *interior* TOPOLOGY
DEFINITION *closure* TOPOLOGY	DEFINITION *exterior & frontier* TOPOLOGY
DEFINITION *distance from a point to a set* TOPOLOGY	DEFINITION *limit of a sequence* TOPOLOGY
DEFINITION *Cauchy Sequence* TOPOLOGY	DEFINITION *convergent sequence* TOPOLOGY
THEOREM *convergence implies Cauchy* TOPOLOGY	DEFINITION *complete metric space* TOPOLOGY

Let (X, d) be a metric space with $A \subset X$. The **interior** of A denoted A° is defined to be:

$$A^\circ = \{x \in A \mid \exists\, r > 0 \text{ such that } B(x, r) \subset A\}$$

Let (X, d) be a metric space and let $\{F_\alpha\}_{\alpha \in A}$ be any collection of closed sets in (X, d), then

1. $X, \varnothing$ are closed.
2. $\bigcap_{\alpha \in A} F_\alpha$ is closed.
3. Let $\{F_1, \ldots, F_n\}$ be a finite collection of closed sets, then $\bigcup_{i=1}^{n} F_i$ is closed.

Let (X, d) be a metric space with $A \subset X$.

The **exterior** of a set A is defined to be $(X - A)^\circ$.

The **frontier** of a set A is defined to be $\overline{A} - A^\circ$.

Let (X, d) be a metric space with $A \subset X$. The **closure** of A denoted $\overline{A}$ is defined to be:

$$\overline{A} = \{x \in X \mid \forall\, r > 0, B(x, r) \cap A \neq \varnothing\}$$

Suppose (X, d) is a metric space. A sequence $\{x_n\} \subset X$ has **limit** x, denoted $\lim_{n\to\infty} \{x_n\} = x$ iff

$$\forall\, \varepsilon > 0,\ \exists\, N \in \mathbb{N} \text{ such that}$$

$$n \geq N \Rightarrow x_n \in B(x, \varepsilon)$$

Suppose (X, d) is a metric space with $A \subset X$ and $x \in X$. We define **the distance from x to A** by

$$d(x, A) = \inf\{d(x, y) \mid y \in A\}$$

A sequence $\{x_n\}$ **converges** iff $\lim \{x_n\}$ exits.

Suppose (X, d) is a metric space. A sequence $\{x_n\} \subset X$ is called a **Cauchy sequence** iff

$$\forall\, \varepsilon > 0,\ \exists\, N \in \mathbb{N} \text{ such that}$$

$$m, n \geq N \Rightarrow d(x_m, x_n) < \varepsilon$$

A metric space (X, d) is **complete** iff every Cauchy sequence in X is convergent.

If a sequence $\{x_n\}$ is convergent then it is Cauchy.

Theorem *limits are unique* Topology	Theorem *distinct points have a radius of separation* Topology
Definition *continuous function* Topology	Definition *continuous function (alternate definition)* Topology
Definition *Lipschitz function* Topology	Theorem *Lipschitz functions are uniformly continuous* Topology
Definition *bi-Lipschitz* Topology	Theorem f *continuous iff* *the preimage of every open set is open* Topology
Theorem *continuous functions and sequences* Topology	Definition *homeomorphism* Topology

Suppose (X, d) is a metric space, and $x, y \in X$ with $x \neq y$, then $\exists\, r > 0$ such that $B(x, r) \cap B(y, r) = \varnothing$

If the limit of $\{x_n\}$ exists, then that limit is unique.

Suppose $(X_1, d_1), (X_2, d_2)$ are metric spaces. A function $f : X_1 \to X_2$ is **continuous** on X_1 iff

$$\forall\, x \in X_1,\ \forall\, \varepsilon > 0,\ \exists\, \delta > 0 \text{ such that}$$

$$f(B(x, \delta)) \subset B(f(x), \varepsilon)$$

Suppose $(X_1, d_1), (X_2, d_2)$ are metric spaces. A function $f : X_1 \to X_2$ is **continuous** at $x \in X_1$ iff

$$\forall\, \varepsilon > 0,\ \exists\, \delta(x, \varepsilon) > 0 \text{ such that}$$

$$d_1(x, y) < \delta \Rightarrow d_2(f(x), f(y)) < \varepsilon$$

If $f : X_1 \to X_2$ is Lipschitz on X_1, then f is uniformly continuous on X_1.

Suppose $(X_1, d_1), (X_2, d_2)$ are metric spaces. A function $f : X_1 \to X_2$ is called **Lipschitz** iff

$$\forall\, x, y \in X_1\ \exists\, c > 0 \text{ such that}$$

$$d_2(f(x), f(y)) \leq c d_1(x, y)$$

A Lipschitz function can be thought of as a "bounded distortion."

A function $f : X_1 \to X_2$ is continuous iff

$$\forall\, U \text{ open } \subset X_2 \Rightarrow f^{-1}(U) \text{ open } \subset X_1$$

Or equivalently:

$$\forall\, U \text{ closed } \subset X_2 \Rightarrow f^{-1}(U) \text{ closed } \subset X_1$$

Suppose $(X_1, d_1), (X_2, d_2)$ are metric spaces. A function $f : X_1 \to X_2$ is called **bi-Lipschitz** iff

$$\forall\, x, y \in X_1\ \exists\, c_1, c_2 > 0 \text{ such that}$$

$$c_1 d_1(x, y) \leq d_2(f(x), f(y)) \leq c_2 d_1(x, y)$$

A function $f : (X_1, d_1) \to (X_2, d_2)$ is called a **homeomorphism** iff

1. f is continuous
2. f is 1-1 and onto
3. f^{-1} is continous

A function $f : (X_1, d_1) \to (X_2, d_2)$ is continuous iff

$$\forall \text{ convergent sequences } \{x_n\} \subset X_1,$$

$$\lim_{n \to \infty} f(x_n) = f(\lim_{n \to \infty} \{x_n\})$$

DEFINITION *equivalent metrics* TOPOLOGY	REMARK *two metrics are equivalent iff* *the identity map is a homeomorphism* TOPOLOGY
THEOREM *composition of continuous functions* *preserves continuity* TOPOLOGY	DEFINITION *homeomorphic spaces* TOPOLOGY
DEFINITION *topology* TOPOLOGY	DEFINITION *topological space* TOPOLOGY
TOPOLOGY	TOPOLOGY
TOPOLOGY	TOPOLOGY

Two metrics, d_1, d_2 are equivalent iff $id : (X, d_1) \to (X, d_2)$ is a homeomorphism.

Two metrics d_1, d_2 are called **equivalent** iff they have the same open sets.

Two metric spaces are **homeomorphic** iff there exists a homeomorphism between them.

Suppose $f : X_1 \to X_2$ and $g : X_2 \to X_3$. If f and g are continuous then $g \circ f$ is continuous.

A **topological space** (X, τ) is a set X and a topology τ on X.

Suppose X is a set. A collection τ of subsets of X is called a **topology** on X iff

1. $X \in \tau$ and $\varnothing \in \tau$
2. $U_\alpha \in \tau$ for $\alpha \in A \Rightarrow \bigcup_{\alpha \in A} U_\alpha \in \tau$
3. $U_1, U_2, \ldots, U_n \in \tau \Rightarrow \bigcap_{i=1}^{\infty} U_i \in \tau$