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Probability

Definition

choose notation

Probability

Theorem

binomial theorem

Probability

Definition

n distinct items divided into
r distinct groups

Probability

Axioms

axioms of probability

Probability

Proposition

probability of the complement

Probability

Proposition

probability of the union of two events

Probability

Definition

conditional probability

Probability

Theorem

the multiplication rule

Probability

Theorem

Bayes' formula

Probability

n choose k is a brief way of saying how many ways can you choose k objects from a set of n objects, when the order of selection is not relevant.

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Obviously, this implies $0 \leq k \leq n$.

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Suppose you want to divide n distinct items in to r distinct groups each with size $n_1, n_2, \ldots, n_r$, how do you count the possible outcomes?
If $n_1 + n_2 + \ldots + n_r = n$, then the number of possible divisions can be counted by the following formula:

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1!\; n_2! \ldots n_r!}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

If E^c denotes the complement of event E, then

$$P(E^c) = 1 - P(E)$$

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. For any sequence of mutually exclusive events $E_1, E_2, \ldots$
   (i.e. events where $E_i E_j = \emptyset$ when $i \neq j$)

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

If $P(F) > 0$, then

$$P(E \mid F) = \frac{P(EF)}{P(F)}$$

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

$$\begin{aligned} P(E) &= P(EF) + P(EF^c) \\ &= P(E \mid F)P(F) + P(E \mid F^c)P(F^c) \\ &= P(E \mid F)P(F) + P(E \mid F^c)[1 - P(F)] \end{aligned}$$

$$P(E_1 E_2 E_3 \ldots E_n) = P(E_1)P(E_2 \mid E_1)P(E_3 \mid E_2 E_1) \ldots P(E_n \mid E_1 \ldots E_{n-1})$$

Definition

independent events

Probability

Definition

probability mass function of a discrete random variable

Probability

Definition

cumulative distribution function F

Probability

Theorem

properties of the cumulative distribution function

Probability

Definition

expected value
(discrete case)

Probability

Proposition

expected value of a function of X
(discrete case)

Probability

Corollary

linearity of expectation

Probability

Definition/Theorem

variance

Probability

Definition

probability mass function of a Bernoulli random variable

Probability

Definition

probability mass function of a binomial random variable

Probability

For a discrete random variable X, we define the *probability mass function* $p(a)$ of X by

$$p(a) = P\{X = a\}$$

Probability mass functions are often written as a table.

Two events E and F are said to be *independent* iff

$$P(EF) = P(E)P(F)$$

Otherwise they are said to be *dependent*.

The cumulative distribution function satisfies the following properties:

1. F is a nondecreasing function
2. $\lim_{a\to\infty} F(a) = 1$
3. $\lim_{a\to-\infty} F(a) = 0$

The *cumulative distribution function (F)* is defined to be

$$F(a) = \sum_{all\ x \le a} p(x)$$

The cumulative distribution function $F(a)$ denotes the probability that the random variable X has a value less than or equal to a.

If X is a discrete random variable that takes on the values denoted by x_i $(i = 1 \ldots n)$ with respective probabilities $p(x_i)$, then for any real–valued function f

$$E[f(X)] = \sum_{i=1}^{n} f(x_i)p(x)$$

$$E[X] = \sum_{x:p(x)>0} xp(x)$$

If X is a random variable with mean μ, then we define the *variance* of X to be

$$\begin{aligned} \mathrm{var}(X) &= E[(X-\mu)^2] \\ &= E[X^2] - (E[X])^2 \\ &= E[X^2] - \mu^2 \end{aligned}$$

The first line is the actual definition, but the second and third equations are often more useful and can be shown to be equivalent by some algebraic manipulation.

If α and β are constants, then

$$E[\alpha X + \beta] = \alpha E[X] + \beta$$

Suppose n independent Bernoulli trials are performed. If the probability of success is p and the probability of failure is $1-p$, then X is said to be a *binomial random variable* with parameters (n, p).
The probability mass function is given by:

$$p(i) = \binom{n}{i} p^i (1-p)^{n-i}$$

where $i = 0, 1, \ldots, n$

If an experiment can be classified as either success or failure, and if we denote success by $X = 1$ and failure by $X = 0$ then, X is a *Bernoulli random variable* with probability mass function:

$$\begin{aligned} p(0) &= P\{X = 0\} = 1 - p \\ p(1) &= P\{X = 1\} = p \end{aligned}$$

where p is the probability of success and $0 \le p \le 1$.

Theorem

properties of binomial random variables

Probability

Definition

probability mass function of a Poisson random variable

Probability

Theorem

properties of Poisson random variables

Probability

Definition

probability mass function of a geometric random variable

Probability

Theorem

properties of geometric random variables

Probability

Definition

probability mass function of a negative binomial random variable

Probability

Theorem

properties of negative binomial random variables

Probability

Definition

probability density function of a continuous random variable

Probability

Definition

probability density function of a uniform random variable

Probability

Theorem

properties of uniform random variables

Probability

A random variable X that takes on one of the values 0,1,..., is said to be a *Poisson random variable* with parameter λ if for some $\lambda > 0$

$$p(i) = P\{X = i\} = \frac{\lambda^i}{i!}e^{-\lambda}$$

where $i = 0, 1, 2, \ldots$

If X is a binomial random variable with parameters n and p, then

$$\begin{aligned} E[X] &= np \\ \text{var}(X) &= np(1-p) \end{aligned}$$

Suppose independent Bernoulli trials, are repeated until success occurs. If we let X equal the number of trials required to achieve success, then X is a *geometric random variable* with probability mass function:

$$p(n) = P\{X = n\} = (1-p)^{n-1}p$$

where $n = 1, 2, \ldots$

If X is a Poisson random variable with parameter λ, then

$$\begin{aligned} E[X] &= \lambda \\ \text{var}(X) &= \lambda \end{aligned}$$

Suppose that independent Bernoulli trials (with probability of succes p) are performed until r successes occur. If we let X equal the number of trials required, then X is a *negative binomial random variable* with probability mass function:

$$p(n) = P\{X = n\} = \binom{n-1}{r-1}p^r(1-p)^{n-r}$$

where $n = r, r+1, \ldots$

If X is a geometric random variable with parameter p, then

$$\begin{aligned} E[X] &= \frac{1}{p} \\ \text{var}(X) &= \frac{1-p}{p^2} \end{aligned}$$

We define X to be a *continuous* random variable if there exists a function f, such that for any set B of real numbers

$$P\{X \in B\} = \int_B f(x)dx$$

The function f is called the *probability density function* of the random variable X.

If X is a negative binomial random variable with parameters (p, r), then

$$\begin{aligned} E[X] &= \frac{r}{p} \\ \text{var}(X) &= \frac{r(1-p)}{p^2} \end{aligned}$$

If X is a uniform random variable with parameters (α, β), then

$$\begin{aligned} E[X] &= \frac{\alpha+\beta}{2} \\ \text{var}(X) &= \frac{(\beta-\alpha)^2}{12} \end{aligned}$$

If X is a *uniform* random variable on the interval (α, β), then its probability density function is given by

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$