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Electrodynamics

Definition

the vector operator ∇

Electrodynamics

Definition

curl

Electrodynamics

Theorem

curl–less or irrotational fields

Electrodynamics

Theorem

gradient theorem

Electrodynamics

Definition

gradient

Electrodynamics

Definition

divergence

Electrodynamics

Definition

5 species of second derivatives

Electrodynamics

Theorem

divergence–less or solenoidal fields

Electrodynamics

Theorem

Green's theorem

Electrodynamics

The gradient ∇T points in the direction of maximum increase of the function T.

$$\nabla T \equiv \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z}$$

The magnitude $|\nabla T|$ is the slope along this direction.

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$$\begin{aligned} \nabla \cdot \mathbf{v} &= (\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}) \cdot (v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}) \\ &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \end{aligned}$$

The *divergence* is a measure of how much the vector function $\mathbf{v}$ spreads out from the point in question.

$$\nabla \equiv \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

By applying ∇ twice we can construct five species of second derivatives.

1. divergence of a gradient $\nabla \cdot (\nabla T) = \nabla^2$ (Laplacian)
2. curl of a gradient $\nabla \times (\nabla T) = 0$ (always)
3. gradient of a divergence $\nabla(\nabla \cdot \mathbf{v})$ (seldom occurs)
4. divergence of a curl $\nabla \cdot (\nabla \times \mathbf{v}) = 0$ (always)
5. curl of a curl $\nabla \times (\nabla \times \mathbf{v}) = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$

$$\nabla \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

The *curl* is a measure of how much the vector field "curls around" the point in question.

For a given vector field $\mathbf{F}$ the following statements are equivalent, i.e. each implies the others.

1. $\nabla \cdot \mathbf{F} = 0$ everywhere
2. $\int \mathbf{F} \cdot d\mathbf{a}$ is independent of surface
3. $\oint \mathbf{F} \cdot d\mathbf{a} = 0$ over any closed surface
4. $\mathbf{F} = \nabla \times \mathbf{A}$ for some vector potential $\mathbf{A}$

For a given vector field $\mathbf{F}$ the following statements are equivalent, i.e. each implies the others.

1. $\nabla \times \mathbf{F} = 0$ everywhere
2. $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot d\mathbf{l}$ is path independent
3. $\oint \mathbf{F} \cdot d\mathbf{l} = 0$ on any closed loop
4. $\mathbf{F} = -\nabla V$ for some scalar potential V

$$\int (\nabla \cdot \mathbf{A}) dV = \oint \mathbf{A} \cdot d\mathbf{a}$$

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$$

THEOREM

Stokes' theorem

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$$\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$$