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Calculus I

Formula

quadratic formula

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Definition

absolute value

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Theorem

properties of absolute values

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Definition

equation of a line in various forms

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Definition

equation of a circle

Calculus I

Definition

$\sin, \cos, \tan$

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Definition

$\sec, \csc, \tan, \cot$

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Definition

midpoint formula

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Definition

function

Calculus I

The solutions or roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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File last updated on Sunday 8th July, 2007, at 17:15

1. $|ab| = |a||b|$
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
3. $|a + b| \leq |a| + |b|$
4. $|a - b| \geq ||a| - |b||$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

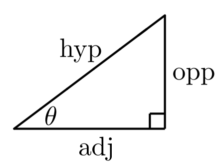
The equation of a circle centered at (h, k) with radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Form	Equation
point–slope	$y - y_1 = m(x - x_1)$
slope–intercept	$y = mx + b$
two point	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
standard	$Ax + By + C = 0$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

A **function** is a mapping that associates with each object x in one set, which we call the **domain**, a single value $f(x)$ from a second set which we call the **range**.

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points, then the midpoint of the line segment that joins these two points is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Definition

even and odd functions

Calculus I

Definition

limit

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Definition

one–sided limit

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Theorem

limit exists iff both the right–handed and left–handed limits exist and are equal

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Theorem

main limit theorem (part 1)

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Theorem

main limit theorem (part 2)

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Theorem

squeeze theorem

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Theorem

two special trigonometric limits

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Definition

point-wise continuity

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Theorem

composition limit theorem

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If a function $f(x)$ is defined on an open interval containing c, except possibly at c, then the **limit of $f(x)$ as x approaches c equals L** is denoted

$$\lim_{x\to c} f(x) = L$$

The above equality holds if and only if for any $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

even	$f(-x) = f(x)$	for all x	e.g. $x^2, \cos(x)$
odd	$f(-x) = -f(x)$	for all x	e.g. $x, \sin(x)$

$$\lim_{x\to c} f(x) = L \Leftrightarrow \lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = L$$

right–handed limit

$$\lim_{x\to c^+} f(x) = L$$

iff for any $\varepsilon > 0$ there exists a δ such that

$$0 < x - c < \delta \Rightarrow |f(x) - L| < \varepsilon$$

Let f, g be functions that have limits at c, and let n be a positive integer.

7. $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ if $\lim_{x\to c} g(x) \neq 0$
8. $\lim_{x\to c}[f(x)]^n = [\lim_{x\to c} f(x)]^n$
9. $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$ provided that $\lim_{x\to c} f(x) > 0$ when n is even.

Let k be a constant, and f, g be functions that have limits at c.

1. $\lim_{x\to c} k = k$
2. $\lim_{x\to c} x = c$
3. $\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$
4. $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
5. $\lim_{x\to c}[f(x) - g(x)] = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$
6. $\lim_{x\to c}[f(x) \cdot g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1 - \cos x}{x} = 0$$

Suppose f, g and h are functions which satisfy the inequality $f(x) \le g(x) \le h(x)$ for all x near c, (except possibly at c). Then

$$\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L \Rightarrow \lim_{x\to c} g(x) = L$$

If $\lim_{x\to c} g(x) = L$ and f is continuous at L, then

$$\lim_{x\to c} f(g(x)) = f(\lim_{x\to c} g(x)) = f(L)$$

Let f be defined on an open interval containing c, then we say that f is **point-wise continuous** at c if

$$\lim_{x\to c} f(x) = f(c)$$

Definition

continuity on an interval

Calculus I

Definition

derivative

Calculus I

Definition

equivalent form for the derivative

Calculus I

Theorem

differentiability and continuity

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Theorem

constant and power rules

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Theorem

differentiation rules

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Theorem

derivatives of trig functions

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Theorem

chain rule

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Theorem

generalized power rule

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Definition

notation for higher-order derivatives

Calculus I

The **derivative** of a function f is another function f' (read "f prime") whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists and is not ∞ or $-\infty$.

A function f is said to be **continuous on an open inteval** iff f is continuous at every point of the open interval.
A function f is said to be **continuous on a closed interval** $[a, b]$ iff

1. f is continuous on (a, b) and
2. $\lim_{x \to a^+} f(x) = f(a)$ and
3. $\lim_{x \to b^-} f(x) = f(b)$

If the function f is differentiable at c, then f is continuous at c.

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Let f and g be functions of x and k a constant.

1. **scalar product rule** $(kf)' = kf'$
2. **sum rule** $(f + g)' = f' + g'$
3. **difference rule** $(f - g)' = f' - g'$
4. **product rule** $(fg)' = f'g + fg'$
5. **quotient rule** $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$f(x) = k$	$f'(x) = 0$
$f(x) = x$	$f'(x) = 1$
$f(x) = x^n$	$f'(x) = nx^{n-1}$

Let $u = g(x)$ and $y = f(u)$. If g is differentiable at x, and f is differentiable at $u = g(x)$, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x and

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

In Leibniz notation

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(\tan x)' = \sec^2 x$$
$$(\cot x)' = -\csc^2 x$$
$$(\sec x)' = \sec x \tan x$$
$$(\csc x)' = -\csc x \cot x$$

Derivative	$f'(x)$	y'	D	Leibniz
first	$f'(x)$	y'	$D_x y$	$\frac{dy}{dx}$
second	$f''(x)$	y''	$D_x^2 y$	$\frac{d^2y}{dx^2}$
third	$f'''(x)$	y'''	$D_x^3 y$	$\frac{d^3y}{dx^3}$
fourth	$f^{(4)}(x)$	$y^{(4)}$	$D_x^4 y$	$\frac{d^4y}{dx^4}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
nth	$f^{(n)}(x)$	$y^{(n)}$	$D_x^n y$	$\frac{d^ny}{dx^n}$

If f is a differentiable function and n is an integer, then the power of the function

$$y = [f(x)]^n$$

is differentiable and

$$\frac{dy}{dx} = n\,[f(x)]^{n-1} f'(x)$$

Theorem

extreme value theorem

Calculus I

Theorem

intermediate value theorem

Calculus I

Definition

critical point
stationary point
singular point

Calculus I

Definition

increasing
decreasing
monotonic

Calculus I

Theorem

monotonicity theorem

Calculus I

Definition

concave up
concave down

Calculus I

Theorem

concavity theorem

Calculus I

Definition

inflection point

Calculus I

definition

local maximum
local minimum
local extremum

Calculus I

Theorem

first derivative test

Calculus I

If the function f is continuous on the closed interval $[a, b]$ and v is any value between the minimum and maximum of f on $[a, b]$, then f takes on the value v.

If the function f is continuous on the closed interval $[a, b]$, then f has a maximum value and a minimum value on the interval $[a, b]$.

A function f defined on the interval I is

- **increasing** on $I \Leftrightarrow$ for every $x_1, x_2 \in I$ $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
- **decreasing** on $I \Leftrightarrow$ for every $x_1, x_2 \in I$ $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

The function f is said to be **monotonic** on I if f is either increasing or decreasing on I.

If f is a function defined on an open interval containing the point c, we call c a **critical point** of f iff either

- $f'(c) = 0$ or
- $f'(c)$ does not exist

Furthermore when $f'(c) = 0$ we call c a **stationary point** of f, and when $f'(c)$ does not exist we call c a **singular point** of f.

Suppose f is differentiable on an open interval I, then if f' is increasing on I we say that f is **concave up** on I.

If f' is decreasing on I we say that f is **concave down** on I.

Suppose f is differentiable on an open interval I, then

- $f'(x) > 0$ for each $x \in I \Rightarrow f$ is increasing on I
- $f'(x) < 0$ for each $x \in I \Rightarrow f$ is decreasing on I

Let f be continuous at c, then the ordered pair $(c, f(c))$ is called an **inflection point** of f if f is concave up on one side of c and concave down on the other side of c.

Let f be twice differentiable on the open interval I.

- $f''(x) > 0$ for each $x \in I \Rightarrow$ f is concave up on I
- $f''(x) < 0$ for each $x \in I \Rightarrow$ f is concave down on I

Let f be differentiable on an open interval (a, b) that contains c.

1. $f'(x) > 0 \ \forall x \in (a, c)$ and $f'(x) < 0 \ \forall x \in (c, b) \Rightarrow f(c)$ is a **local maximum** of f.
2. $f'(x) < 0 \ \forall x \in (a, c)$ and $f'(x) > 0 \ \forall x \in (c, b) \Rightarrow f(c)$ is a **local minimum** of f.
3. If $f'(x)$ has the same sign on both sides of c, then $f(c)$ is **not** a **local extremum**.

Let the function f be defined on an interval I containing c. We say f has a **local maximum** at c iff there exists an interval (a, b) containing c such that $f(x) \leq f(c)$ for all $x \in (a, b)$.

We say f has a **local minimum** at c iff there exists an interval (a, b) containing c such that $f(x) \geq f(c)$ for all $x \in (a, b)$.

A **local extremum** is either a local maximum or a local minimum.

Theorem

second derivative test

Calculus I

Theorem

mean value theorem

Calculus I

Calculus I

Calculus I

Calculus I

Calculus I

Calculus I

Calculus I

Calculus I

Calculus I

If f is continuous on a closed interval $[a, b]$ and differentiable on its interior (a, b), then there is at least one point c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

or equivalently

$$f(b) - f(a) = f'(c)(b - a)$$

Let f be twice differentiable on an open interval containing c, and suppose $f'(c) = 0$.

1. If $f''(c) < 0$, then f has a **local maximum** at c.
2. If $f''(c) > 0$, then f has a **local minimum** at c.
3. If $f''(c) = 0$, then the test fails.