Very Short Biographies of Some Mathematicians Who Made Important Contributions to Algebra

Jason Underdown

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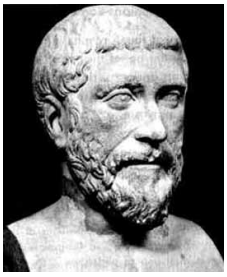
1 Ancient Mathematicians

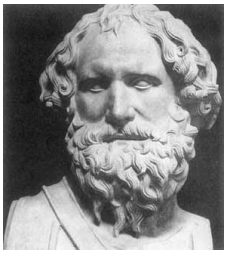
Pythagoras [about 569 BC – about 475 BC]



- A Greek mathematician.
- Pythagoras had the idea to found all mathematics, music, and astronomy on number but the discovery of irrational numbers so disturbed the Pythagoreans, that they turned away from arithmetic, which seemed to contain numbers that could not be written, to geometry, where such numbers could be represented infallibly by the lengths of line segments. (Derbyshire, 32)
- The theorem now known as Pythagoras's theorem was known to the Babylonians 1000 years earlier but he may have been the first to prove it.

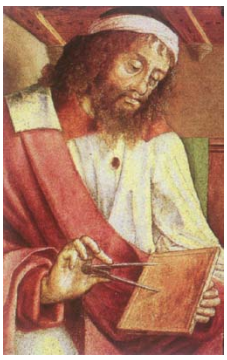
- None of Pythagoras's writings have survived.
- Pythagoras founded a philosophical and religious school in Croton (now Crotone, on the east of the heel of southern Italy) that had many followers. Pythagoras was the head of the society with an inner circle of followers known as mathematikoi. The mathematikoi lived permanently with the Society, had no personal possessions and were vegetarians. They were taught by Pythagoras himself and obeyed strict rules.

Archimedes [287 BC – 212 BC]



- A native of Syracuse, Sicily.
- Visited Egypt and while there invented a water pumping device now known as Archimede's screw. He also invented many more mechanical devices such as the double pulley and several wartime siege weapons.
- Studied many geometric problems especially ones related to conic sections, and used the method of exhaustion (a precursor to modern day integration) to calculate several areas and volumes.
- Widely considered to be the greatest mathematician of his time.

Euclid [about 325 BC – about 265 BC]



- A Greek Mathematician who lived in Alexandria Egypt.
- A pupil of Plato.
- Wrote "The Elements", a book in thirteen volumes which presented his ideas as a series of definitions, axioms, theorems and proofs. His writings are arguably the most influential of all math books. In addition, "The Elements" has been in print at one press or another pretty much since the development of the printing press. It is second only to the Bible in number of copies printed.
- Biographies of many famous mathematicians indicate that Euclid's work provide their initial introduction into mathematics. It provided them with a model of how "pure mathematics" should be written, with well–thought–out axioms, precise definitions, carefully stated theorems, and logically coherent proofs.

Diophantus [about 200 AD – about 284 AD]

- A Greek mathematician who lived in Alexandria Egypt.
- Best known for his book *Arithmetica*.
- Sometimes referred to as the "father of algebra" because he was the first mathematican to express algebraic problems via symbolic equations, although the symbols he used are very different from modern notation.

- He studied what are today called "Diophantine problems", which have fewer equations than unknown variables and involve finding integers that work correctly for all equations. In other words, they define an algebraic curve or algebraic surface and ask about the lattice points on it. An example is finding all rational solutions for $x^2 + y^2 = a^2$ when a is fixed.

Hypatia [about 370 AD – 415 AD]

- A Greek mathematician who lived in Alexandria, Egypt.
- She is the first woman to make a substantial contribution to the development of mathematics.
- She taught at the Museion in Alexandria.
- She was killed by a fanatical Christian sect in the streets of Alexandria.

2 Medieval Mathematicians

Abu Ja'far Muhammad ibn Musa Al-Khwarizmi [790 – 850]

- An Islamic mathematician who lived in Baghdad.

- Al–Khwarizmi's main algebraic achievement, in fact, was to bring forward the idea of equations as objects of interest, classifying all equations of the first and second degrees in one unknown and giving rules for manipulating them. (Derbyshire, 50)
- He used both algebraic and geometric techniques for solving equations, however he lacked a good symbolic notation.
- He used the terms *al–jabr* "completion", and *al–muqabala* "reduction", to describe his algebraic methods.

Omar Khayyam [1048 – 1122]

- An Islamic mathematician from Persia.
- "Best known in the West as the author of the Rubaiyat, a collection of four–line poems offering a highly personal view of life–a sort of death-haunted hedonism with an alcoholic thread..." (Derbyshire, 52)
- Khayyam's main importance for the development of algebra is that he opened the first serious assault on the cubic equation.
- His ingenius solution method involved intersecting a parabola with a circle.

Leonardo of Pisa (Fibonacci) [1170 – 1240]

- Leonardo's father was an official of the independent republic of Pisa, and was appointed to represent its merchants in the trading colony of Bugia on the North African coast in 1192.
- Leonardo accompanied his father and was exposed to the works of many Muslim scholars.
- Wrote: *Liber abacci*, "Book of Calculation" which introduced Arabic (actually Indian) numerals including zero to the West. The book borrowed many problems verbatim from al–Khwarizmi, Abu Kamil and others.
- Later known as *Fibonacci*, literally son of Bonacci.

3 Renaissance Mathematicians

Luca Pacioli [1445 – 1517]

- Invented double–entry bookkeeping.
- Friend of Leonardo da Vinci
- Coined the term million
- Wrote: *Summa* in 1494. This book did not break any new ground, but standardized the current notation of the day. He used co for *cosa* ("thing"), ce for *census* ("property"), and cu for *cubus* ("cube"). Classified the following types of cubic equations as unsolvable:
  1. $n = ax + bx^3$
  2. $n = ax^2 + bx^3$

3. $ax + n = bx^3$

- The third classification above was not actually listed in his book, but we'll refer to this classification system below.

Scipione del Ferro [1456 – 1526]

- The first to solve the depressed cubic for the case where the discriminant is negative.
- A professor at the University of Bologna.
- Shared his secret solution of the cubic with his student Antonio Maria Fiore, who subsequently challenged Tartaglia.
- Never published his solution of the cubic.

Niccolo Fontana (Tartaglia) [1499 – 1557]

- Mathematician, engineeer, surveyor and bookkeeper
- "Tartaglia" means stutterer in Italian.
- Translated Euclid's *Elements* into Italian.
- Independently discovered how to solve cubic equations with negative discriminant.
- Confided his solution of the cubic to Cardano in the form of a poem, but later regretted it.

Girolamo Cardano [1501 – 1576]

- A physician by trade.
- Avid gambler and caster of horoscopes.
- Wrote at least 131 publications including an autobiography. Several of his books were bestsellers in Europe.
- Cajoled Tartaglia into divulging his secret solution to the cubic.
- Wrote *Ars Magna* [1545] in which he published a general method of solving cubic equations as well as quartic equations.

Lodovico Ferrari [1522 – 1565]

- Was a secretary to Cardano beginning at age 14.
- Discovered a way to solve the general quartic equation by transforming it to a cubic equation.
- Allowed Cardano to publish his solution to the quartic in *Ars Magna*.

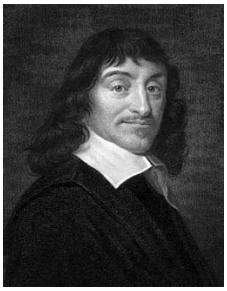
Rafael Bombelli [1526 – 1572]

- A civil engineer who was responsible for draining marshland in central Italy.
- His friend Antonio Maria Pazzi introduced him to Diophantus' writings which were at the university in Rome.
- Wrote *l'Algebra* [1572] with the goal of making an easier to understand version of Cardano's *Ars Magna*.
    1. First clear usage of negative and complex numbers.
    2. Included 143 problems from Diophantus' writings.
    3. Was the first introduction to Diophantus' writings for most Europeans.
    4. Still lacked good symbolism.

François Viète [1540 – 1603]

- Born into a Huguenot family (French protestant).
- A lawyer by trade.
- Privy councillor to both Henry III and Henry IV.
- Wrote *In artem analyticem isagoge* or "Introduction to the Analytic Art".
- This book had the first systematic use of letters to represent numbers. Unknowns were represented by uppercase vowels (A,E,I,O,U,Y) and data such as coefficients were represented by uppercase consonants.
- Discovered that polynomial coefficents can always be represented via symmetric polynomials in the roots. This led Lagrange and later Galois on to important discoveries.

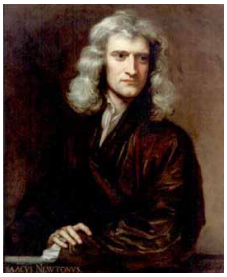
René Descartes [1596 – 1650]



- Although he was French, he lived most of his life in Holland.
- Famous philosopher, best known for saying, "Cogito ergo sum." (I think therefore I am.).
- Wrote *La géométrie*.
    1. Introduced the xy coordinate system which is now named for him, i.e. Cartesian coordinates.
    2. Borrowed the $+, -, \sqrt{}$ symbols from German mathematicians.
    3. Used superscripts for exponentiation.
    4. Used lowercase letters such as $a, b, c, \ldots$ to represent data such as polynomial coefficients, and letters such as x, y, z to represent unknowns.

4 Enlightenment Mathematicians

Isaac Newton [1642 – 1727]



- An English mathematician widely regarded as the inventor of Calculus.

- Arguably the most influential scientist of all time.
- Wrote *Principia*.
    1. Formulated the laws of motion.
    2. Formulated the law of universal gravitation.
    3. Derived Kepler's laws of planetary motion from his laws of motion and universal gravitation.
- Built the first practical reflecting telescope.
- Taught algebra at Cambridge University for ten years.
- His algebra lecture notes were published by William Whiston under the title *Arithmetica universalis* "Universal Arithmetic".
- Some of his notes from his early years show that he was very interested in symmetric polynomials and although his notes neither prove nor even state it, the theorem which states: Any symmetric polynomial in any number of unknowns can be written in terms of elementary symmetric polynomials is often called Newton's Theorem.
- Invented a very efficient numerical method for computing the real roots of polynomial equations known as *Newton's Method*.

Gottfried Wilhelm von Leibniz [1646 – 1716]

- A German mathematician.
- Independently invented calculus and introduced the widely used notations: $\frac{df}{dx}$ and $\int f(x)dx$.
- Made major contributions to the study of *determinants* which are important in solving systems of linear equations.

Leonhard Euler [1707 – 1783]

- A Swiss mathematician who made enormous contributions to a wide range of mathematics.
- Proved Fermat's last theorem for the case $n = 3$.
- Introduced the formula $e^{ix} = \cos x + i \sin x$.

Alexandre Théophile Vandermonde [1735 – 1796]

- A French mathematician.
- Wrote *Mémoire sur la résolution des equations*, "Memoir on the solution of equations" in 1771.
- Key insight was that each solution of a polynomial equation could be written in terms of all the solutions no matter the degree of the equation. This is useful because it allows one to then rewrite the solutions in terms of elementary symmetric polynomials which are known expressions in the coefficients of the original equation.
- The *Vandermonde determinant* is named after him although the expression only occurs in his writings as part of a calculation to rewrite a polynomial in terms of elmentary symmetric polynomials, not as an actual determinant of a matrix.
- He loved music and wrote a controversial paper proposing that musicians should ignore all theory of music and rely solely on their trained ears when judging music.

Giuseppe Lodovico Lagrangia (Joseph – Louis Lagrange) [1736 – 1813]

- Born in Turin, Italy. Lived most of his life in France, and spent some time in Berlin Germany.
- Was Euler's successor in the court of Frederick the Great in Berlin.
- Wrote *Reflexions sur la Résolution Algébrique des Equations*, "Reflections on the Solution of Algebraic Equations", which introduced the notion of using permutations of the solutions to study the solutions of equations. This was a long work of about 220 pages that examines from all angles the solution of equations of degree 2, 3, and 4, and seeks to deal with the algebraic solutions of higher degree.
- There was much overlap between Lagrange's *Réflexions* and Vandermonde's earlier work though Lagrange was not aware of his work.
- Pioneered the use of a *resolvent* or special equation of degree $n!$, with useful properties which Galois later built upon.
- Proved that the order of a subgroup must divide the order of its parent group. This is called *Lagrange's Theorem* in abstract algebra.
- Made very important contributions to the field of mechanics, or the study of motion.

Paolo Ruffini [1765 – 1822]

- An Italian mathematician.
- Wrote: *General theory of equations in which it is shown that the algebraic solution of the general equation of degree greater than four is impossible.*
- Was the first to give a proof that the general quintic equation is unsolvable, but there was one minor gap in the proof.
- His work was largely ignored by the mathematical community probably because of his very novel techniques and verbose writing sytle.

5 19th Century Mathematicians

Carl Friedrich Gauss [1777 – 1855]

- A German mathematician.

- At the young age of 19, he proved that the regular 17 sided polygon known as the heptadecagon was constructible by ruler and compass. This is fundamentally an algebraic problem.
- Wrote *Disquistiones arithmeticae* at age 21, although it wasn't published until 1801.
- A good portion of his book (mentioned above) is devoted to the study of a certain subclass of polynomial equations known as *cyclotomic equations*, that is equations derived from $\frac{z^n-1}{z-1} = 0$. These equations are important because they are always solvable in radicals. Also, their solutions correspond to the points on the unit circle which correspond to the vertices of regular polygons, hence *cyclo* (circle) *tomic* (cut).
- He wrote in his thesis of 1801 that the algebraic solution of an equation was no better than devising a symbol for the root of the equation and then saying that the equation had a root equal to the symbol.
- Computed the orbit of the asteroid Ceres, based upon just three orbital positions.
- Made significant contributions to algebra, number theory, differential geometry and physics.
- Was the first person to prove the Fundamental Theorem of Algebra.
- Widely considered as the greatest mathematician to have ever lived.

Augustin–Louis Cauchy [1789 – 1857]

- A French mathematician.

- Along with Ruffini, Cauchy was one of the first to investigate what he referred to as *compounding* of permutations, and what we today call composition of permutations.
- Also contributed greatly to the theory of determinants.
- Was also a pioneer in Analysis.

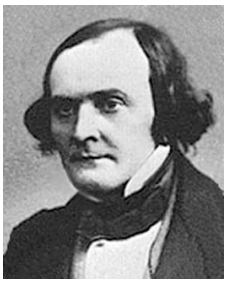
Niels Henrik Abel [1802 – 1829]

- A Norwegian mathematician.
- Was the first to rigorously prove the insolvability of the general quintic.
- Struggled financially his whole adult life due to not being able to procure employment as a mathematician until after his death.
- Was friends with August Crelle, who published several of his papers in his newly created journal nicknamed Crelle's Journal.
- Gauss refused to even look at his proof of the unsolvability of the quintic.
- Died at the young age of 26 from tuberculosis.

Évariste Galois [1811 – 1832]

- A French mathematician.
- Was the first person to develop a complete explanation of which polynomial equations are solvable in radicals. He did this by associating a group to each equation. The properties of the associated group completely determine whether the equation is solvable via radicals or not.
- Was rejected for admittance into the top schools of the day.
- Was arrested for allegedly making threats on the life of the king.
- While he was imprisoned he fell in love with the daughter of the jailer, Stephanie du Motel, but his love was unrequited.
- Was shot in a duel, and died the next day. The circumstances under which the duel occurred are not certain. He was only 23 at the time of his death.
- Rewrote his earlier paper which explained the exact circumstances under which a polynomial equation is solvable via radicals, the night before his duel while imprisoned in jail.
- Galois theory effectively finished the theory of equations which had puzzled mankind for so many centuries, but it also gave birth to several new areas of research which fall under the umbrella of abstract algebra.

Arthur Cayley [1821 – 1895]



- An English mathematician.
- Coined the terms "matrix" and "matrices" for rectangular grids of numbers.
- Axiomatized group theory.
- Showed that all finite groups are isomorphic to a subgroup of S_n for some n.

Sophus Lie [1842 – 1899]

- A Norwegian mathematician.
- His last name rhymes with "key".
- His program of study involved applying the ideas of Galois Theory to the study of differential equations. His goal was to exploit symmetries in nonlinear differential equations to find solutions. This resulted in the notion of Lie groups which are groups with an additional topological structure.