1. (5 points) Suppose that $X$ is a random variable with probability mass function given by

$$f_X(k) = \begin{cases} 
1/4 & k = -1 \\
1/4 & k = 1 \\
1/2 & k = 2 \\
0 & \text{otherwise}
\end{cases}$$

and let $Y = X^2$. Find the probability mass function of $Y$.

Solution:

$$f_Y(k) = \begin{cases} 
1/2 & k = 1 \\
1/2 & k = 4 \\
0 & \text{otherwise}
\end{cases}$$

2. (5 points) Suppose that $X$ is a random variable with probability density function given by

$$f_X(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

and let $Y = e^{-X}$. Find the probability density function of $Y$.

Solution: The possible values of $Y$ are $(0, \infty)$. The inverse function if $g(x) = e^{-x}$ is $h(y) = -\ln(y)$, valid when $y > 0$. For this function and $y > 0$, we have $h'(y) = -1/y$.

$$f_Y(t) = \begin{cases} 
\frac{1}{\sqrt{2\pi}} e^{-\ln^2(y)/2} \frac{1}{y} & y > 0 \\
0 & y \leq 0
\end{cases}$$