Please inform your instructor if you find any errors in the quiz solutions.

1. (5 points) Suppose that A, B, and C are events with $P(B \cap C) > 0$. Show that $P(A \cap B | C) = P(A | B \cap C)P(B | C)$. Remember to explain why $P(C) > 0$.

**Solution:** $B \cap C$ is a subset of C and so $P(B \cap C) \leq P(C)$. Because of this, the fact that $P(B \cap C) > 0$ implies that $P(C) > 0$. We then have by definition

$$P(A | B \cap C)P(B | C) = \frac{P(A \cap B \cap C)P(B | C)}{P(B \cap C)P(C)} = \frac{P(A \cap B \cap C)}{P(C)} = P(A \cap B | C).$$

2. (5 points) Suppose that we roll a fair die. If the die roll is between 1 and 3, then we flip one fair coin. If the die roll is between 4 and 6, then we flip two fair coins.

1. What is the probability that we flip exactly one heads?

2. Given that we flip exactly one heads (and some number of tails) what is the probability that the die roll was between 1 and 3?

**Solution:** Let $H$ denote the event that we flip exactly one heads and let $D$ denote the event that the result of our die roll lies in $\{1, 2, 3\}$.

1. Notice that $P(H | D) = 1/2$ (this is the probability of flipping exactly one heads from one coin flip) and $P(H | D^c) = 1/2$ (this is the probability of flipping exactly one heads from two coin flips).

$$P(H) = P(H | D)P(D) + P(H | D^c)P(D^c)$$
$$= (1/2)(1/2) + (1/2)(1/2).$$

2. $P(D | H) = \frac{P(H | D)P(D)}{P(H | D)P(D) + P(H | D^c)P(D^c)} = \frac{(1/2)(1/2)}{(1/2)(1/2) + (1/2)(1/2)} = 1/2.$