Please inform your instructor if you find any errors in the quiz solutions.

1. (5 points) Suppose that you draw 5 cards randomly from a standard deck of 52 cards, without replacement. What is the probability of drawing five cards which all have different ranks? (There are 4 suits, each consisting of 13 ranks in a standard deck of cards.)

Solution: You can solve this problem by assuming that either order matters or it does not. If order does not matter, then since there are 13 possible ranks and we need to choose five of those ranks where order does not matter and then one card from each rank, the probability is

\[
\frac{\binom{13}{5} \cdot 4^5}{\binom{52}{5}}.
\]

If order does matter, then there are 13!/(13 − 5)! = 13!/8! possible permutations of different ranks and 52!/(52 − 5)! = 52!/47! possible hands. The probability is then

\[
\frac{\frac{13!}{8!} \cdot 4^5}{\frac{52!}{47!}}.
\]

Multiply by 1 = 5!/5! to see that these two answers are equal.

2. (5 points) Suppose that each person is assigned a integer number between 1 and 365 uniformly at random on the day that they are born. If 5 randomly chosen people are in a room, what is the probability that no two people in the room were assigned the same number?

Solution: You can do this problem either by assuming that order matters or by assuming that it does not.

First, we assume order matters. Then there are 365^5 possible assignments of numbers among the 5 people in the room. To pick 5 unique numbers is the same as sampling from \{1, 2, \ldots, 365\} 5 times without replacement, where order matters. There are then 365!/(365 − 5)! = 365!/360! possible ways to choose 5 distinct numbers for the different people in the room. The probability is then

\[
\frac{365!}{365^5 \cdot 360!}.
\]

If we assume that the order of the people does not matter then there are 365^5/5! possible collections of numbers that the people can have been assigned. Again picking 5 different numbers is the same as sampling 7 times from \{1, 2, \ldots, 365\} 5 times without replacement, but now where order does not matter. There are \(\binom{365}{5}\) ways to do this. The probability is then

\[
\frac{\binom{365}{5}}{365^5 / 5!} = \frac{365!}{365^5 \cdot 360!}.
\]