1. (10 points) Suppose that $X$ and $Y$ are jointly continuous with joint pdf

$$f_{X,Y}(x, y) = \begin{cases} 
8xy & 0 < x < y < 1 \\
0 & \text{otherwise}
\end{cases}.$$

Find $P(Y/2 < X)$.

Solution:

$$P(Y/2 < X) = \int_{0}^{1} \int_{y/2}^{y} 8xy \, dx \, dy$$

$$= \int_{0}^{1} 8y \left( \frac{y^2}{2} - \frac{y^2}{8} \right) \, dy$$

$$= \int_{0}^{1} 4y^3 - y^3 \, dy$$

$$= 1 - \frac{1}{4} = \frac{3}{4}.$$