Solve the following problems.

1. Solve the following.
   (a) Let $f$ be the probability density function of a continuous random variable $X$. Find the probability density function of $Y = X^2$.
   (b) Let $X$ be a standard normal random variable. Find the distribution of $Y = X^2$.

2. Let $X$ and $Y$ be two continuous independent random variables with densities given by
   
   $$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the probability density function of $Z = X + Y$.
   (b) Find the probability density function of $W = Y/X$.

3. Let $X$ and $Y$ be two independent random variables both with distribution $N(0,1)$. Find the probability density function of $Z = Y/X$.

4. Let $X, Y,$ and $Z$ be three continuous independent random variables with densities given by
   
   $$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   Compute $P(X \geq 2Y \geq 3Z)$.

5. Let $X$ and $Y$ be two independent random variables, both with distribution $N(0, \sigma^2)$ for some $\sigma > 0$. Let $R$ and $\Theta$ be two random variables defined by
   
   $$X = R \cos(\Theta), \quad Y = R \sin(\Theta),$$

   where $R \geq 0$. Prove that $R$ and $\Theta$ are independent and find their density functions.

6. In Las Vegas, a roulette table is made of 38 boxes, namely 18 black boxes, 18 red boxes, a '0' box and a box '00'. If you bet $1 on 'black' you get $2 if the ball stops in a black box and $0 otherwise. Let $X$ be your profit. Compute $E[X]$. 