Solve the following problems.

1. We toss a fair coin $n$ times, what is the probability that we get at least 3 heads given that we get at least one.

2. A fair die is rolled. If the outcome is odd, a fair coin is tossed repeatedly. If the outcome is even, a bias coin is tossed repeatedly. If the first $n$ throws result in heads, what is the probability that the fair coin is being used?

3. In a certain village, 20% of the population has some disease. A test is administered which has the property that if a person is sick, the test will be positive 90% of the time and if the person is not sick, then the test will still be positive 30% of the time. All people tested positive are prescribed a drug which always cures the disease but produces a rash 25% of the time. Given that a random person has the rash, what is the probability that this person had the disease to start with?

4. A single card is drawn from a standard 52-card deck. Give examples of events $A$ and $B$ that are:
   (a) Disjoint but not independent;
   (b) Independent but not disjoint;
   (c) Independent and disjoint;
   (d) Neither independent nor disjoint.

5. Prove the following statements.
   (a) If an event $A$ is independent of itself then $P(A) = 0$ or 1.
   (b) If $P(A) = 0$ or 1 then $A$ is independent of any event $B$.

6. We assume that the gender of a child is independent of the gender of a child is independent of the gender of the other children of the same couple and that the probability to get a boy is .52. Compute, for a 4-child family, the probabilities of the following events:
   (a) all children have the same gender;
   (b) the three oldest children are boys and the youngest is a girl;
   (c) there are exactly three boys;
   (d) the two oldest are boys;
   (e) there is at least a girl.