1. Let $\mathbf{F} = (1yz)\mathbf{i} + (3xz)\mathbf{j} + (9xy)\mathbf{k}$. Compute the following:

A. div $\mathbf{F}$

$$\nabla \cdot \mathbf{F} = \frac{\partial(1yz)}{\partial x} + \frac{\partial(3xz)}{\partial y} + \frac{\partial(9xy)}{\partial z} = 0 + 0 + 0 = \boxed{0}$$

B. curl $\mathbf{F}$

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1yz & 3xz & 9xy \end{vmatrix} = \boxed{6x\mathbf{i} - 8y\mathbf{j} + 2z\mathbf{k}}$$

C. div curl $\mathbf{F}$

$$\nabla \cdot \nabla \times \mathbf{F} = \frac{\partial(6x)}{\partial x} + \frac{\partial(-8y)}{\partial y} + \frac{\partial(2z)}{\partial z} = 6 - 8 + 2 = \boxed{0}$$

2. Let $F(x,y,z)=\left(-5xz^2,\ 6xyz,\ -6xy^3z\right)$ be a vector field and $f(x,y,z)=x^3y^2z$.

Find ∇f

$$\left(f_x,f_y,f_z\right)=\left(3x^2y^2z,\ 2x^3yz,\ x^3y^2\right)$$

Find $\nabla\times\mathbf{F}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} & \frac{\partial y}{\partial z} \\ -5xz^2 & 6xyz & -6xy^3z \end{vmatrix} = \boxed{\left[(-6x)\left(3y^2\right)(z)-6xy\right]\mathbf{i}+\left[6y^3z-10xz\right]\mathbf{j}+\left[yz-0\right]\mathbf{k}}$$

Find $\mathbf{F}\times\nabla f$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5xz^2 & 6xyz & -6xy^3z \\ 3x^2y^2z & 2x^3yz & x^3y^2 \end{vmatrix} = \boxed{\left(6x^4y^3z+12x^4y^4z^2\right)\mathbf{i}+\left(5x^4y^2z^2-18x^3y^5z^2\right)\mathbf{j}+\left(-10x^4yz^3-18x^3y^3z^2\right)\mathbf{k}}$$

Find $\mathbf{F}\cdot\nabla f$

$$\left(-5xz^2,\ 6xyz,\ -6xy^3z\right)\cdot\left(3x^2y^2z,\ 2x^3yz,\ x^3y^2\right)$$

$$\boxed{-15x^3y^2z^3+12x^4y^2z^2-6x^4y^5z}$$

3. Let $\mathbf{F}=\left(10xyz+3\sin x,\ 5x^2z,\ 5x^2y\right)$. Find a function f so that $\mathbf{F}=\nabla f$, and $f(0,0,0)=0$.

$$\mathbf{F}=\left(10xyz+\sin x,\ 5x^2z,\ 5x^2y\right)$$

$$f_x=10xyz+3\sin x \ \Rightarrow\ f=5x^2yz-3\cos x$$
$$f_y=5x^2z \qquad \Rightarrow\ f=5x^2yz+C_2(x,z)$$
$$f_z=5x^2y \qquad \Rightarrow\ f=5x^2yz+C_3(x,y)$$

$$f=5x^2yz-3\cos x+C$$
$$f(0,0,0)=-3+C=0\ \Rightarrow C=3$$
$$\boxed{f=5x^2yz-3\cos x+3}$$

4. For each of the following vector fields $\mathbf{F}$, decide whether it is conservative or not by computing the curl $\mathbf{F}$. Type in a potential function f (that is, $\nabla f = \mathbf{F}$). If not conservative, type N.

A. $\mathbf{F}(x,y) = (-16x+4y)\mathbf{i} + (4x+2y)\mathbf{j}$

$M = \text{-}16x+4y$ and $N = 4x+2y$

Take the partial derivative in terms of x and y.

$(-16x+4y)_y = 4 = (4x+2y)_x$

The field is conservative.

$\int M\,dx = -8x^2 + 4yx + k(y)$

$(-8x^2+4yx+k(y))_y = 4x + k'(y) = 4x+2y \quad k'(y) = 2y$

$$\boxed{f(x,y) = -8x^2 + 4yx + y^2}$$

B. $\mathbf{F}(x,y) = -8y\mathbf{i} - 7x\mathbf{j}$

$(-8y)_y = -8 \neq (-7x)_x = -7$

Not conservative.

C. $\mathbf{F}(x,y,z) = -8x\mathbf{i} - 7y\mathbf{j} + \mathbf{k}$

$$\frac{\partial(-7y)}{\partial x} - \frac{\partial(-8x)}{\partial y} = 0$$

It is conservative.

$\mathbf{F}=(-8x,\ -7y,\ 0)+(0,0,1)$

$$\boxed{f(x,y,z) = -4x^2 - \frac{7}{2}y^2 + z}$$

D. $\mathbf{F} = (-8\sin y)\mathbf{i} + (8y - 8x\cos y)\mathbf{j}$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -8\cos y - (-8\cos y) = 0$$

$$\boxed{f = -8x\sin y + 4y^2}$$

E. $\mathbf{F}(x,y,z) = -8x^2\mathbf{i} + 4y^2\mathbf{j} + 1z^2\mathbf{k}$

$$f = \frac{-8}{3}x^3 + \frac{4}{3}y^3 + \frac{z^3}{3}$$

5. Let C be the curve which is the union of two line segments, the first going from $(0,0)$ to $(3,4)$ and the second going from $(3,4)$ to $(6,0)$.

Compute the integral $\int_C 3\,dy - 4\,dx$.

$$\int_C (-4,3)\cdot(dx,dy) = -4x + 3y\Big|_{(0,0)}^{(6,0)} = \boxed{-24}$$

6. Let C be the counter-clockwise planar circle with center at the origin and radius $r > 0$. Without computing them, determine for the following vector field $\mathbf{F}$ whether the line integrals $\int_C \mathbf{F} \cdot d\mathbf{r}$ are positive, negative, or zero and type P, N, or Z as appropriate.

$x = r\cos\theta \quad y = r\sin\theta$

$dr = (dx, dy) = (-y, x)\,d\theta$

A. $\mathbf{F}$ = the radial vector field $= x\mathbf{i} + y\mathbf{j}$:

$(x, y) \cdot (-y, x)\,d\theta = \boxed{0}$

B. $\mathbf{F}$ = the circulating vector field$= -y\mathbf{i} + x\mathbf{j}$

$(-y, x) \cdot (-y, x)\,d\theta = r^2 d\theta$ therefore it is $\boxed{\text{positive}}$

C. $\mathbf{F}$ = the circulating vector field $= y\mathbf{i} - x\mathbf{j}$

$(y, -x) \cdot (-y, x)\,d\theta = -r^2 d\theta$ which is $\boxed{\text{negative}}$

D. $\mathbf{F}$ = the constant vector field=$\mathbf{i}+\mathbf{j}$

The field is constant and a constant field is always conservative.

7. Suppose that $\nabla f(x,y,z) = 2xyze^{x^2}\,\mathbf{i} + ze^{x^2}\,\mathbf{j} + ye^{x^2}\,\mathbf{k}$. If $f(0,0,0) = -3$, find $f(2,2,6)$.

$$f_x = 2xye^{x^2} \quad \Rightarrow \quad f = yze^{x^2} + C_1(y,z)$$
$$f_y = ze^{x^2} \quad \Rightarrow \quad f = yze^{x^2} + C_2(x,z)$$
$$f_z = ye^{x^2} \quad \Rightarrow \quad f = yze^{x^2} + C_3(x,y)$$

$$f(x,y,z) = yze^{x^2} + C$$
$$f(0,0,0) = C = -3$$
$$\boxed{f(2,2,6) = 12e^4 - 3}$$

8. Evaluate the line integral $\int_C x^4 z\,ds$, where C is the line segment from $(0,2,5)$ to $(7,8,1)$.

$$L:(0,2,5)+t(7,6,-4)=(x,y,z)$$

$$ds=\sqrt{(dx)^2+(dy)^2+(dz)^2}=\|(7,6,-4)\|dt$$

$$=\sqrt{101}dt$$

$$\int_{t=0}^{1}(7t)^4(5-4t)\sqrt{101}\,dt$$

$$\sqrt{101}\left[7^4t^5-7^3\frac{28}{6}t^6\right]_0^1=\boxed{\sqrt{101}\left(7^4-7^3\frac{14}{3}\right)}$$

9. Evaluate the line integral $\int_C 5xy^2\,ds$, where C is the right half of the circle $x^2+y^2=36$.

$x=6\cos\theta \quad y=6\sin\theta$, θ from $\frac{-\pi}{2}$ to $\frac{-\pi}{2}$

$\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} 5\left(6^3\right)\sin^2\theta\cos\theta(6d\theta) \qquad ds=6\,d\theta$ (arc length)

$5\left(6^4\right)\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\theta d(\sin\theta)$

$5\left(6^4\right)\frac{1}{3}\sin^3\theta\Big|_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = \boxed{(5)\left(6^3\right)(4)}$

10. Suppose C is any curve from $(0,0,0)$ to $(1,1,1)$ and $\mathbf{F}(x,y,z)=(4z+5y)\mathbf{i}+(3z+5x)\mathbf{j}+(3y+4x)\mathbf{k}$. Compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

$\mathbf{F}$ is conservative.

$\nabla f = \mathbf{F}$

$f = 4zx + 5xy + 3yz$

$4zx + 5xy + 3yz\big|_{(0,0,0)}^{(1,1,1)} = \boxed{12}$

11. Let C be the positively oriented square with vertices $(0,0),\ (1,0),\ (1,1),(0,1)$.
Use Green's Theorem to evaluate the line integral $\int_C 4y^2x\,dx + 8x^2y\,dy$.

$$\oint \left(4y^2x,\ 8x^2y\right)\cdot\left(dx,dy\right)$$

$$\int_{x=0}^{1}\int_{y=0}^{1}\left(16xy - 8xy\right)dx\,dy$$

$$8\int_{x=0}^{1} x\,dx\int_{y=0}^{1} y\,dy = (8)\left(\frac{1}{4}\right) = \boxed{2}$$

12. Let C be the positively oriented circle $x^2 + y^2 = 1$. Use Green's Theorem to evaluate the line integral $\int_C 18y\,dx + 1x\,dy$.

$\oint_C (18y, x) \cdot d\mathbf{r}$

$\int_{r=0}^{1} \int_{\theta=0}^{2\pi} (1-18) r\,dr\,d\theta$

$\boxed{-17\pi}$

13. Let $\mathbf{F} = 2x\mathbf{i} + y\mathbf{j}$ and let $\mathbf{n}$ be the outward uni normal vector to the positively oriented circle $x^2 + y^2 = 1$. Compute the flux integral $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$.

Method 1

You can use Gauss' Divergence Theorem $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds = \iint_S \nabla \cdot \mathbf{F}\, dA$.

$$\oint \mathbf{F} \cdot \mathbf{n}\, ds = \iint_S (2+1)\, dA = \boxed{3\pi}$$

Method 2

$$\int (2x, y) \cdot (x, y)\, ds = \int 2x\, dy + y(-dx)$$

$x = \cos\theta \quad y = \sin\theta \quad ds = r d\theta \ \Rightarrow \ ds = d\theta$ because the radius is 1.

$dx = -y d\theta \quad dy = \cos\theta d\theta$

$$\oint (-y, 2x) \cdot d\mathbf{r}$$

$$\iint_S \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 3(\text{the area}) = 3\left(\pi (1)^2\right) = \boxed{3\pi}$$

14. Use Green's Theorem to compute the area inside the ellipse $\frac{x^2}{3^2}+\frac{y^2}{19^2}=1$.

Use the fact that the area can be written as

$$\iint_D dx\,dy = \iint_D \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right) = \int_{\partial D} P\,dx + Q\,dy.$$

We need P, Q $\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=1$

We choose $P=0$ and $Q=x$.

$$\oint (P,Q)\cdot d\mathbf{r} = \int_{\theta=0}^{2\pi} x\,dy$$

$$\frac{x^2}{3^2}+\frac{y^2}{19^2}=1$$

$$x = 3\cos\theta$$

$$y = 19\sin\theta \implies dy = 19\cos\theta d\theta$$

$$\int_{\theta=0}^{2\pi} 3(19)\cos^2\theta\,d\theta$$

$$\int_{\theta=0}^{2\pi} 3(19)\left(\frac{\cos 2\theta+1}{2}\right)d\theta$$

$$3(19)\frac{1}{2}\theta\Big|_0^{2\pi} = (3)(19)\pi = \boxed{57\pi}$$

15. Let $\mathbf{F} = -3y\mathbf{i} + 2x\mathbf{j}$. Use the tangential vector form of Green's Theorem to compute the circulation integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the positively oriented circle $x^2 + y^2 = 1$

$$\oint \mathbf{F} \cdot d\mathbf{r} = \oint \mathbf{F} \cdot \mathbf{T}\, ds = \iint_{x^2+y^2 \le 1} 2 - (-3)\, dA = \boxed{5\pi}$$

16. Evaluate $\iint_M \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = \left(3xy^2,\ 3x^2y,\ z^3\right)$ and M is the surface of the sphere of radius 6 centered at the origin.

$$\iint\limits_M \mathbf{F}\cdot ds = \int \mathbf{F}\cdot\mathbf{n}\,d\mathbf{S} = \iiint\limits_{B_6} \nabla\cdot\mathbf{F}\,dV$$

$$\iiint\limits_{B_6} 3y^2 + 3x^2 + 3z^2\,dV$$

$$\int_{\theta=0}^{2\pi} 1\,d\theta \int_{\phi=0}^{\pi} \sin\phi\,d\phi \int_{\rho=0}^{6} 3\rho^4\,d\rho$$

$$2\pi(2)\left(\frac{3}{5}\right)\left(6^5\right) = \boxed{\frac{12\pi}{5}6^5}$$

17. Find the outward flux of the vector field $\mathbf{F}=\left(x^3,\ y^3,\ z^2\right)$ across the surface of the region that is enclosed by the circular cylinder $x^2+y^2=9$ and the planes $z=0$ and $z=4$.

$$\iint_{\partial y} \mathbf{F}\cdot d\mathbf{S} = \iint_{x^2+y^2=9} 3x^2+3y^2+2z\,dV$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{3} 3r^3 \int_{z=0}^{4} dz\,dr\,d\theta$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{3} r \int_{z=0}^{4} 2z\,dz\,dr\,d\theta$$

$$2\pi\left(\frac{3}{4}\right)\left(3^4\right)(4)+(2\pi)\left(\frac{1}{2}\right)\left(3^2\right)\left(4^2\right)=\boxed{630\pi}$$

18. Let $\mathbf{F}=\left(y^2+z^3,\ x^3+z^2,\ xz\right)$. Evaluate $\iint_{\partial w} \mathbf{F}\cdot d\mathbf{S}$ for each of the following regions W :

$\nabla \cdot \mathbf{F} = 0 + 0 + x = x$

A. $x^2 + y^2 \le z \le 6$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{6}}\int_{z=r^2}^{6} r\cos\theta\, dzr\, dr\, d\theta$$

$$\sin\theta\Big|_0^{2\pi} = \boxed{0}$$

B. $x^2 + y^2 \le z \le 6,\ x \ge 0$

$$\int_{\theta=\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_{r=0}^{\sqrt{6}}\int_{z=r^2}^{6} r^2\cos\theta\, dz\, dr\, d\theta$$

$$\sin\theta\Big|_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^{\sqrt{6}} \left(6-r^2\right) r^2\, dr$$

$$2\left(\frac{6}{3}r^3 - \frac{1}{5}r^5\right)\Big|_0^{\sqrt{6}} = \boxed{2\left(\frac{24\sqrt{6}}{5}\right)}$$

C. $x^2 + y^2 \le z \le 6,\ x \le 0$

$A = B + C$

$C = -B$

$$\boxed{-2\left(\frac{24\sqrt{6}}{5}\right)}$$

19. Use the divergence theorem to find the outward flux of the vector field $\mathbf{F}(x,y,z) = 4x^2\mathbf{i} + 3y^2\mathbf{j} + 5z^2\mathbf{k}$ across the boundary of the rectangular prism: $0 \le x \le 1,\ 0 \le y \le 5,\ 0 \le z \le 2.$

$\nabla \cdot \mathbf{F} = 8x + 6y + 10z$

$\int_{x=0}^{1}\int_{y=0}^{5}\int_{z=0}^{2} 8x + 6y + 10z\,dz\,dy\,dx$

$\int_{x=0}^{1}\int_{y=0}^{5} 20 + 16x + 12y\,dy\,dx$

$\int_{x=0}^{1} 250 + 80x dx$

$\boxed{290}$

20. Let S be the part of the plane $4x+1y+z=3$ which lies in the first octant, oriented upward. Find teh flux of the vector field $\mathbf{F}=1\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ across the surface S.

The surface given by $z=-4x-y+3$ is bounded by $\left(\frac{3}{4},0,0\right)$, $(0,3,0)$, and $(0,0,3)$.

$$n=(4,1,1) \quad n=\frac{(4,1,1)}{\sqrt{18}}$$

$$dS=\sqrt{(z_x)^2+(z_y)^2+1}dA$$

$$dS=\sqrt{(-4)^2+(-1)^2+1}dA=\sqrt{18}dA$$

$$\int_{x=0}^{\frac{3}{4}}\int_{y=0}^{3-4x}(1,4,2)\cdot\frac{(4,1,1)}{\sqrt{18}}\sqrt{18}\,dA$$

$$\int_{x=0}^{\frac{3}{4}}\int_{y=0}^{3-4x}10\,dy\,dx$$

$$10\int_{x=0}^{\frac{3}{4}}3-4x\,dx$$

$$10\left(3x-2x^2\right)\Big|_0^{\frac{3}{4}}=\boxed{\frac{45}{4}}$$

21. Find directly the flux of $\mathbf{F}(x,y,z)=\left(3xy^2,\ 3x^2y,\ z^3\right)$ out of the sphere of radius 3 centered at the origin, without the aid of Gauss' Divergence Theorem.

The flux is given by the integral: $3^5\int_a^b\int_c^d f(\theta,\phi)\,d\theta\,d\phi$.

$$\nabla\cdot\mathbf{F}=3y^2+3x^2+3z^2=3\rho^2$$

$$\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi}\int_{\rho=0}^{3}3\rho^2\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\left.\frac{3\rho^5}{5}\right|_0^3\int_{a=0}^{\pi}\int_{c=0}^{\pi}\sin\phi\,d\theta\,d\phi$$

$$3^5\int_{a=0}^{\pi}\int_{c=0}^{2\pi}\frac{3}{5}\sin\phi\,d\theta\,d\phi$$

$$\boxed{a=0,\ b=\pi,\ c=0,\ d=2\pi}$$

Find $f(\theta,\phi)$

$$\iint\mathbf{F}\cdot\mathbf{n}\,d\mathbf{S}$$

$$\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\frac{\pi}{2}}\left(3xy^2,\ 3x^2y,\ z^3\right)\cdot\frac{(3,3,3)}{3}$$

$$3^2\sin\phi d\phi d\theta$$

$$\boxed{f(\theta,\phi)=6\cos^2\theta\sin^2\theta\sin^5\phi+\cos^4\phi\sin\phi}$$

$$\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\frac{\pi}{2}}3^5\left(3\sin^4\phi\cos^2\phi\sin^2\theta+3\sin^4\phi\cos^2\phi\sin^2\theta\right)+\cos^4\phi\sin\phi\,d\phi\,d\theta$$

$$\boxed{\frac{729\pi^2}{16}}$$

22. Let $\mathbf{F} = (2x,\ 2y,\ 2x+2z)$. Use Stokes' Theorem to evaluate the integral of $\mathbf{F}$ around the curve consisting of the straight lines joining the points $(1,0,1)$, $(0,1,0)$ and $(0,0,1)$. In particular, compute the unit normal vector of the plane spanned by the points $(1,0,1)$, $(0,1,0)$, and $(0,0,1)$ and the curl of $\mathbf{F}$ as well as the value of the integral:

Find the normal vector of the plane.

$$n = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (0,1,1)$$

$$n = \frac{(0,1,1)}{\sqrt{2}}$$

Find $\nabla \times \mathbf{F}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & 2y & 2x+2z \end{vmatrix} = (0,-2,0)$$

$$\iint \nabla \times \mathbf{F} d\mathbf{S} = \iint (0,-2,0)(0,1,1)\,dA \quad \text{where } dS = \sqrt{2}dA$$

$$-2\int_{x=0}^{1}\int_{y=0}^{1-x} dy\,dx$$

$$-2\int_{x=0}^{1}(1-x)\,dx$$

$$-2\left(x - \frac{1}{2}x^2\right)\Big|_0^1 = \boxed{-1}$$

23. Use Stoke's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + 3\left(x^2 + y^2\right)\mathbf{k}$ and C is the boundary of the part of the paraboloid where $z = 64 - x^2 - y^2$ which lies above the xy-plane and C is oriented counterclockwise when viewed from above.

$$\iint \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\mathbf{S}$$

$$\iint \nabla \times \mathbf{F} \cdot \mathbf{n}\, dA$$

$$\iint_D \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} dx\, dy$$

$$\iint_{x^2+y^2 \le 64} (-2x)(6y) + 2y(6x) + 0\, dx\, dy$$

$$\iint_{x^2+y^2 \le 64} 0\, dA = \boxed{0}$$

24. Use Stoke's Theorem to evalute $\iint_S curl\mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = -1yz\mathbf{i} + 1xz\mathbf{j} + 18\left(x^2+y^2\right)z\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies the cylinder $x^2 + y^2 = 1$, oriented upward.

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_{z=x^2+y^2=1} \left(-yz,\ xz,\ 18r^2z\right) \cdot \left(dx, dy, 0\right)$$

$$\iint_{z=x^2+y^2=1} \left(-yz,\ xz,\ 18r^2z\right)\left(-yd\theta,\ xd\theta,\ 0\right)$$

$$\int_{\theta=0}^{2\pi} y^2 + x^2\, d\theta = \int_0^{2\pi} \cos^2\theta + \sin^2\theta d\theta = \int_0^{2\pi} 1\, d\theta = \boxed{2\pi}$$

25. Let M be the capped cylindrical surface which is the union of two surfaces, a cylinder given by $x^2 + y^2 = 49, 0 \le z \le 1,$ and a hemispherical cap defined by $x^2 + y^2 + (z-1)^2 = 49,\ z \ge 1$. For the vector field $\mathbf{F}=\left(zx + z^2 y + 5y,\ z^3 yx + 5x,\ z^4 x^2\right),$ compute $\iint_M (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ in any way you like.

$$\int_M \nabla \times \mathbf{F}\, d\mathbf{S} = \int_{\partial M} \mathbf{F} \cdot d\mathbf{r}$$

$\int_{x^2+y^2=49} \left(zx + z^2 y + 5y,\ z^3 yx\right) \cdot (-y, x)\, d\theta$ where $z = 0$

$\boxed{0}$