(3) Find the mult inverse of $2+5i$ in $\mathbb{Z}[i]/(31)$.

(4) similar.

Study these.

Recall: How to compute $\frac{1}{2+5i}$ in $\mathbb{C}$.

Get rid of i in the denominator!

$$\frac{1}{2+5i} = \frac{(2-5i)}{(2-5i)} \frac{1}{2+5i} = \frac{2-5i}{29}.$$

made with ziteboard

$$= \frac{2}{29} + \frac{-5}{29} i.$$

Can do the same in $\mathbb{Z}[i]/(3 i)$

$$\frac{1}{2+5i} = \frac{(2-5i)}{(2-5i)} \cdot \frac{1}{2+5i} = \frac{2-5i}{29}$$

$$= \frac{2}{29} + \frac{-5}{29} i$$

$\frac{2}{29}$ in $\mathbb{Z}/3\mathbb{Z}$

in $\mathbb{Z}/3\mathbb{Z}$

So compute $\frac{1}{29}$ in $\mathbb{Z}/31\mathbb{Z}$ then simplify.

Get $\frac{1}{29} = 15$ and 31.

$$\frac{1}{2+5i} = \frac{2}{29} + \frac{-5}{29} i$$

$$= 15 \cdot 2 + 15 \cdot (-5)\, i$$

$$= 30 + 18\, i.$$

To find mult. inverse of $a+b\sqrt{D}$

in $\mathbb{Z}[\sqrt{D}]/(n)$

if it exists.

$$\frac{1}{a+b\sqrt{D}} = \frac{a-b\sqrt{D}}{a^2-b^2D} = \frac{a}{a^2-b^2D} + \frac{-b}{a^2-b^2D}\sqrt{D}$$

Can compute this as long as

$\gcd(a^2-b^2D, n) = 1.$

(5) and (6) ~ Is $a+b\sqrt{D}$ a quadratic integer?

$\underline{a, b \in \mathbb{Q}}$

$$\Updownarrow$$

$$x^2 - 2a\,x + a^2 - b^2 D$$

has integer coefficients

i.e. $2a$ is an integer and $a^2 - b^2 D$ is an integer.

(5) Is $\dfrac{1+\sqrt{5}}{2}$ a quadratic integer?

$$\frac{1}{2} + \frac{1}{2}\sqrt{5}$$

$a = \frac{1}{2} \quad b = \frac{1}{2} \quad D = 5$

$\underline{2a} = 2\,\frac{1}{2} = 1$ ✓

$\underline{a^2 - b^2 D} = \frac{1}{4} - \frac{5}{4} = -1$ ✓

Should know how to check, but don't need to be able to do (6).

$\frac{1}{29} \equiv 15 \mod 31$ just means

15 is the mult.

inverse of 29 mod 31.

Euclidean alg + unravel.

$31 = 29 \cdot 1 + 2$

$29 = 14 \cdot 2 + 1$

$1 = 29 - 14 \cdot (2)$

$1 = 29 - 14(31 - 1 \cdot 29)$

$1 = 15 \cdot 29 - 14 \cdot 31$.

so $1 \equiv 15 \cdot 29 \mod 31$.

so 15 is the mult. inverse of 29 mod 31!