If \( f(x) \in K[x] \), \( K \) a field, then in video I claimed
\[
f(a) = 0 \iff (x-a) \mid f(x)
\]
i.e. \( f(x) = q(x)(x-a) \).

Start with Exercise 2-1 to justify this.

(Note: one way when \( K = \mathbb{R} \) is to write down the Taylor expansion of \( f(x) \) at \( x=a \).)
\[ n = 2 \]

\[ x - a \begin{pmatrix} \frac{c_2 x + (c_1 + ac_2)}{c_2 x^2 + c_1 x + c_0} \\ -(c_2 x^2 - ac_2 x) \\ (c_1 + ac_2) x + c_0 \\ -(c_1 + ac_2) x - (c_1 + ac_2) a \end{pmatrix} \]

\[ c_0 + c_1 a + c_2 a^2 \]

Remainder = \( f(a) \).
2-(2) Describe an algorithm to find the prime factorization of any polynomial in \( \mathbb{F}_p[x] \).

Prime in \( \mathbb{K}[x] = \) a monic polynomial \( ax \) such that if

Any non-zero polynomial \( bx \)

has a unique prime factorization,

\[
\text{Fix} := c \cdot a_1(x)^{n_1} \cdot a_2(x)^{n_2} \cdot \ldots \cdot a_k(x)^{n_k} \quad (c \in \mathbb{K} \quad a \text{ are prime}, \quad a > 1)
\]
Exercise 1: Pascal's triangle is binomial coefficients.
\( \binom{n}{k} \) means the number of ways to choose \( k \) flavors of ice cream from a menu with \( n \) flavors.