

Reminders/Announcements

- Final 2 weeks from today. 8:00am - 10:00am in this room
if there will be a zoom option like midterm
- will post final exam information on website by next Tuesday, will go over at the start of Tuesday's class.
- Week 13 HW is last one required.
Week 13 quizzes were last required quizzes
- Week 14 HW/quizzes on Canvas.

In the lecture - showed that there are always positive integer solutions (x, y) to $x^2 - Dy^2 = 1$

D positive squarefree integer

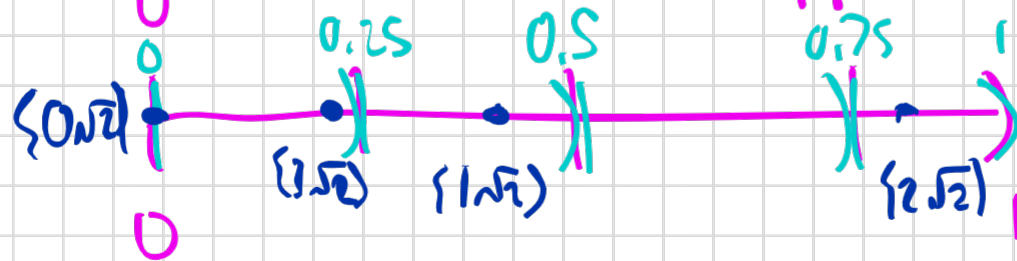

related it to rational approximation of $\sqrt{D}$.

Pirichlet's principle - gave a way to find good approximations of any irrational number by rational numbers.

Pigeonhole principle was also important. Already used in proof of Dirichlet.

Ex 2. - (1) - Use Dirichlet's principle to give a good rational approximation of π .

Example: Use Dirichlet's principle with $n=4$ to give a rational approximation of $\sqrt{2}$.



$\{0\sqrt{2}\}$

$\{1\sqrt{2}\}$

$\{2\sqrt{2}\}$

$\{3\sqrt{2}\}$

$\{4\sqrt{2}\}$

.4142...

.8284...

.2426...

.4142...

$\{1\sqrt{2}\}$ $\{0\sqrt{2}\}$ is box

$$3\sqrt{2} - 0\sqrt{2} = (3\sqrt{2}) + \{1\sqrt{2}\} - (\{0\sqrt{2}\} + \{0\sqrt{2}\})$$

$$3\sqrt{2} = 4 + .24...$$

$$\sqrt{2} = \frac{4}{3} + \frac{.24...}{3}$$

$$\sqrt{2} = \frac{4}{3} + .08...$$

So $\frac{4}{3}$ is a good rational approx of $\sqrt{2}$.

Exercise 2-(1) - $n=10$, approximate π

Exercise 2-(2) Hint: I, II, III, IIII, ...
pigeons.

So what are the pigeonholes?

↓
Congruence class mod 691

Put 2 pigeons in same pigeon hole
if they're $\equiv \text{mod } 691$.
(there are 691 holes $= \mathbb{Z}/691\mathbb{Z}$).

