- Midterm on Thursday:

  10 T/F questions.

  5 very-short answer questions

  a positive integer.

  1 hour.

  + extra time to upload.

- Office hours Tuesday (tonight) at 8:30pm – 10:00pm

- Next week Class optional

  Tuesday 3/9 go over the midterm.

  Thursday 3/11 class = office hours

  office hours = don't exist.

Exam:

From the syllabus / bulletin:

Subgroups, normal subgroups, quotient groups, homomorphisms, isomorphism theorems, groups acting on sets, orbits, stabilizers, orbit decomp. formula, Lagrange's Theorem, (structure of G-sets).

Examples of groups + actions

Cayley's Theorem, Sylow theorems, permutation groups, symmetric & alternating groups, simplicity of $A_n, n \geq 5$, direct & semidirect products, exact & split exact sequences, commutator subgroups, solvable groups, nilpotent groups

solvability of p-groups, classical matrix groups,
(nilpotence)
automorphism groups, Jordan-Hölder theorem,
~~Free groups, presentations of groups~~

Week 4, Exercise 4 - (1).

Suppose K is a cyclic group
($\mathbb{Z}/n\mathbb{Z}$ or $\mathbb{Z}$)

and $\phi_i : K \to \mathrm{Aut}(H)$ (H a group).

Suppose the subgroups $\phi_1(K)$ & $\phi_2(K)$

are conjugate in $\mathrm{Aut}(H)$.

(If K is infinite, assume both are injective).

Construct an explicit isomorphism

$$H \rtimes_{\phi_1} K \cong H \rtimes_{\phi_2} K.$$

$$\mathbb{Z} \twoheadrightarrow (\mathbb{Z}/11\mathbb{Z})^\times = \mathrm{Aut}(\mathbb{Z}/11\mathbb{Z})$$

$\phi_1 : 1 \mapsto 2$

$\phi_2 : 1 \to 6$

$5^2 = 24 = 3 \nearrow$ 5^3: $15 = 4 \nearrow$ 5^4: $20 = 9 \nearrow$ 5^5: $45 = 1$

~~Can't precompose with an aut of $\mathbb{Z}$ to go from $\phi_1 \to \phi_2$.~~

$6^2 = 36 = 3$

$\mathrm{Aut}(\mathbb{Z}) = \{\pm 1\} \; (= \mathbb{Z}^\times).$

$$\left(\phi_1 \circ (\cdot\,-1)\right)(1) = \phi_1(-1) = 2^{-1} = 6.$$

$$\phi_1 \circ (\cdot\,-1) = 2.$$

Can use this to show $\mathbb{Z}/11\mathbb{Z} \rtimes_{\phi_1} \mathbb{Z} \cong \mathbb{Z}/11\mathbb{Z} \rtimes_{\phi_2} \mathbb{Z}$

$\phi(10) = 4.$ ← # of generators of $(\mathbb{Z}/11\mathbb{Z})^\times \cong \mathbb{Z}/10\mathbb{Z}.$

$3 \in (\mathbb{Z}/11\mathbb{Z})^\times$

$3^2 = 9 \to 27 \to 5 = 3^3 \to 15 \to 4 = 3^4 \to 12 \to 1$

$3^5 = 1$

$-3 \in (\mathbb{Z}/11\mathbb{Z})^\times$ has order 10

$-3 = 8$

$\mathbb{Z} \quad (\mathbb{Z}/11\mathbb{Z})^\times$

should take $\phi_1 : 1 \mapsto 2$

$\psi_2' : 1 \mapsto 8$