6320 - Modern Alg II.

Group, Galois theory, Representation theory.

Rough schedule:

Field extensions.

Week 1 - Motivation, first concepts.
Weeks 2-5 - The structure of finite groups.
Week 6 - Character theory of abelian group
<br>\[ \text{Hom}(G, \mathbb{C}^*) \].

Week 7 - Review, midterm.
Week 8 - Spring Break.
Weeks 9-12 - Galois theory.

D\text{2n}: dishedal group of order 2n.

Good teacher \hspace{1cm} Bad teacher

Rigid symmetries of the regular \n-gon in the plane.

 Regular n-gon

0°, 90°, 180°, 270° rotation

Reflect about any axis of

\[ \langle r, \tau \mid r^n = 1, \tau^2 = 1, \tau r \tau = r^{-1} \rangle \]
Autopmorphisms \[ \text{Symmetries} \] \[ \text{Structure} \]

Fewer symmetries. \[ \rightarrow \] More structure

\[ \text{GL}_2(\mathbb{R}) \leftrightarrow \mathbb{R}^2 \]

2x2 invertible matrices

All maps from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \) as a set

\[ \text{Defn} \] A group is a set \( G \) equipped with

- Multiplication \( m : G \times G \rightarrow G \)
  \[ (a, b) \mapsto ab \]
- Inversion \( i : G \rightarrow G \)
  \[ a \mapsto a^{-1} \]
- Identity \( e : \ast \rightarrow G \)
  \[ * \mapsto e \]

satisfying some compatibilities:

- Associativity
- Existence of inverses
Examples: \( \mathbb{Z} / n\mathbb{Z} \)  
\( \text{GL}_n(\mathbb{R}) \)  
\( \text{Sn} \)  
\( \text{SL}_n(\mathbb{R}) \)  
\( \text{Aut}_\mathbb{R}(\mathbb{R}^n) \)  
\( (x \mapsto x) \)  
\( \cong \)  
\( \text{Fundamental group} \)  
\( \text{D}_2n \)  
\( \text{Mapping class group} \)
$R^x$ for $R$ any ring.

$S_n = \text{Aut}_{\text{set}}(\{1, 2, \ldots, n\})$

$GL_n(R) = \text{Aut}_{R\text{-vector space}}(R^n)$

$GL_1(R) = \text{Aut}_{R\text{-module}}(R)$

$O(n) = \text{Aut}_{\text{inner product spaces}}(R^n, \langle \cdot, \cdot \rangle)$

$SL_n(R) = \text{Aut}_{\text{vector space with multilinear form}}(R^n, e_1^* \wedge e_2^* \wedge \ldots \wedge e_n^*)$

$= \text{Aut}_{R\text{-vector space with volume and orientation}}(R^n)$
$S^1 = \int_0^1 \circ \circ = 1$

$\mathbb{R} \to S^1$

$\pi$

$t \mapsto e^{2\pi i t}$

$\mathbb{C} = \text{Aut}(\mathbb{R} \to S^1)$

acting by translation

$\text{Aut}_S^* \text{ polynomial space } R$

$s i t \mapsto \pi G(t) \to \pi(+)$. 

$n = t \mapsto t + n$,

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$G$ a group. $G$ is a right $G$-set.

$x \cdot g = xg$.

$\text{Aut}_{\text{Right}}(G) = G$

$G$ acts by multiplication on the left.