

6320-001 - SPRING 2021 - MIDTERM (3/04)

1. INSTRUCTIONS

The midterm is open-book (Dummit and Foote) and open-notes, but no other sources are allowed. The exam must be taken individually and during the regular class period while logged on to Zoom. You will be put in individual breakout rooms so that you can call me in to ask any questions.

You have **1 hour** to work on the exam; after that period, you will upload your **ANSWERS ONLY** to Gradescope. After the hour, no more work is to be done, but you can copy your answers onto a new piece of paper to take a picture of, or type them on your computer in a word processor or TeX, etc.; whatever works best for you to upload. Please stay in the Zoom session while doing this so that we can troubleshoot if you have any difficulties.

NOTE: All group orders are given with their prime factorization below; the factorization may or may not be useful depending on the problem!

2. TRUE/FALSE

For each of the following, please answer TRUE or FALSE (1 pt each).

1. Every group of order $121 = 11^2$ is abelian. **T**
2. Every group of order $57 = 3 \cdot 19$ is abelian. **F**
3. Every group of order $91 = 7 \cdot 13$ is cyclic. **T**
4. Every subgroup of the quaternion group with $8 = 2^3$ elements, Q_8 , is normal. **T**
5. The dihedral group with $30 = 2 \cdot 3 \cdot 5$ elements, D_{30} , is solvable. **T**
6. There is a simple group of order $60 = 2^2 \cdot 3 \cdot 5$. **T**
7. There is a simple group of order $168 = 2^3 \cdot 3 \cdot 7$. **T**
8. There is a simple group of order $88 = 2^3 \cdot 11$. **F**
9. There is a simple group of order $360 = 2^3 \cdot 3^2 \cdot 5$. **T**
10. There is a simple group of order $256 = 2^8$. **F**

3. SHORT ANSWER

The answer to each of the following is a single positive integer (1 pt each).

1. How many Sylow 7-subgroups are there in a simple group of order $168 = 2^3 \cdot 3 \cdot 7$? **8**
2. How many conjugacy classes are there in S_5 ? **7**
3. How many elements are in the center of the dihedral group with $14 = 2 \cdot 7$ elements, D_{14} ? **1**
4. How many elements are in the conjugacy class of

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \in \text{GL}_2(\mathbb{F}_5)?$$

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5. How many orbits are there for the action of $\text{GL}_2(\mathbb{F}_3)$ on the set of ordered pairs (\vec{v}_1, \vec{v}_2) of vectors $\vec{v}_i \in \mathbb{F}_3^2$? Here the action is the diagonal action, that is,

$$A \cdot (\vec{v}_1, \vec{v}_2) = (A\vec{v}_1, A\vec{v}_2).$$

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