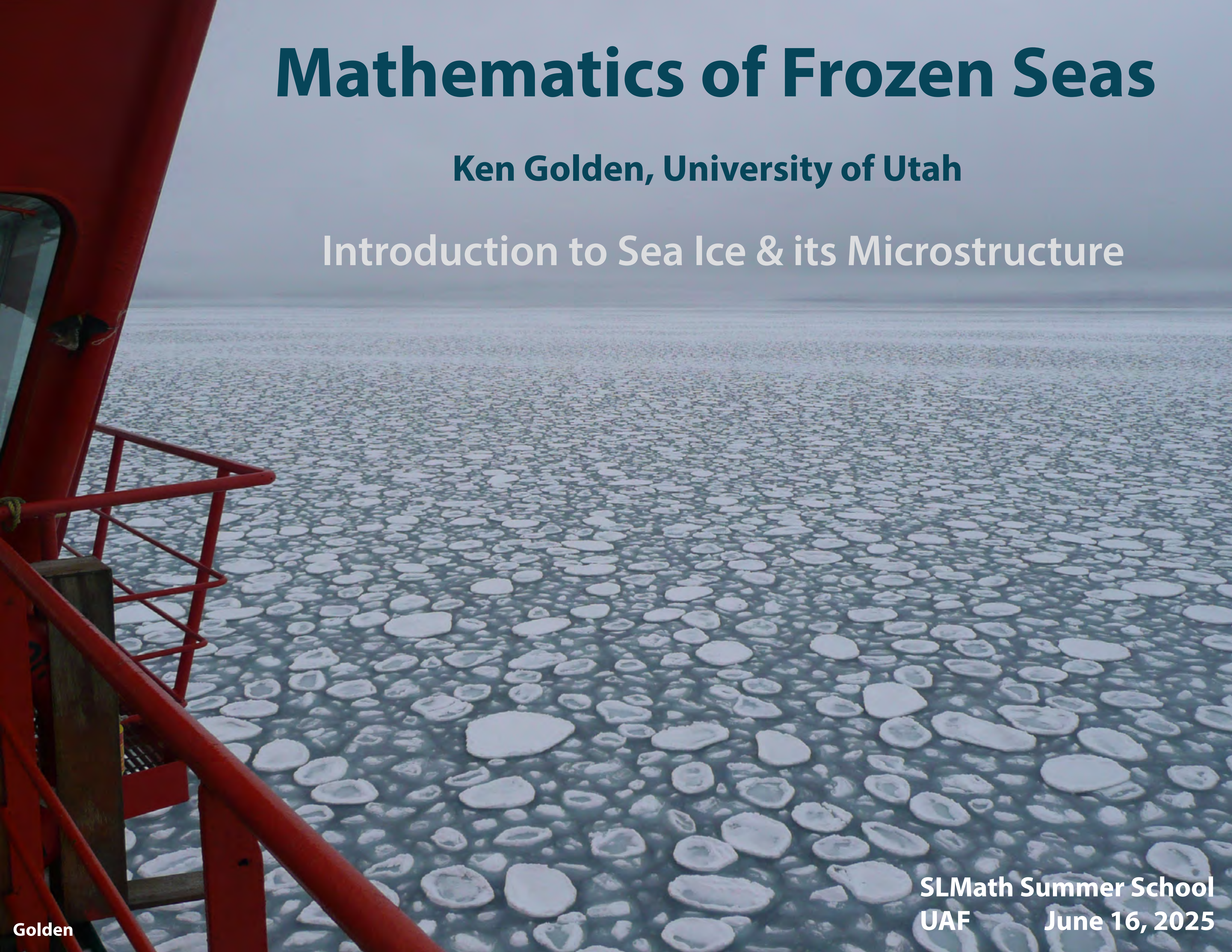
Mathematics of Frozen Seas

Ken Golden, University of Utah

Introduction to Sea Ice & its Microstructure



Golden

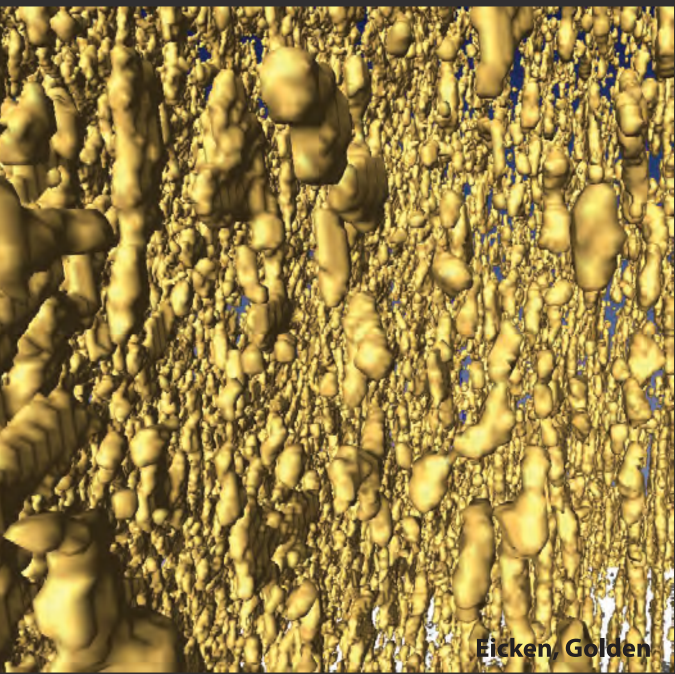
SLMath Summer School
UAF June 16, 2025

sea ice

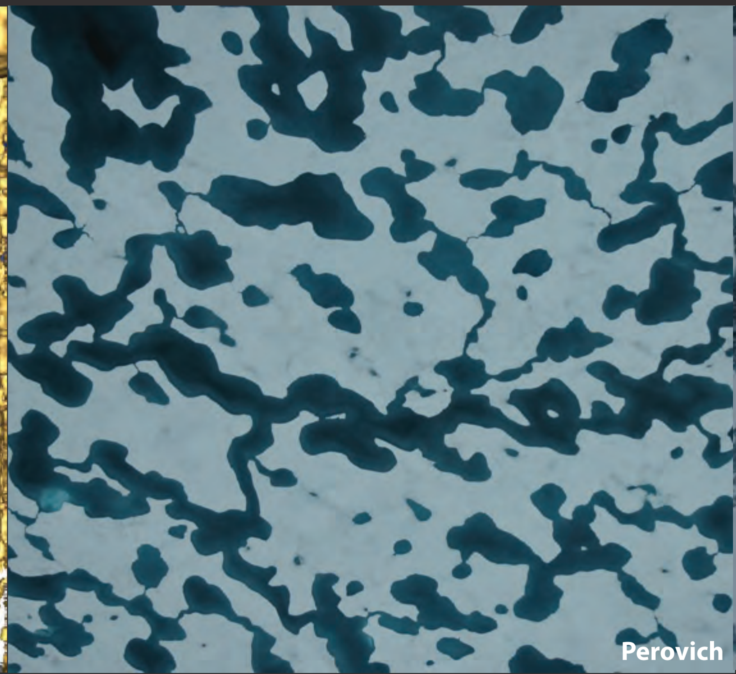
semiconductor

invisibility cloak

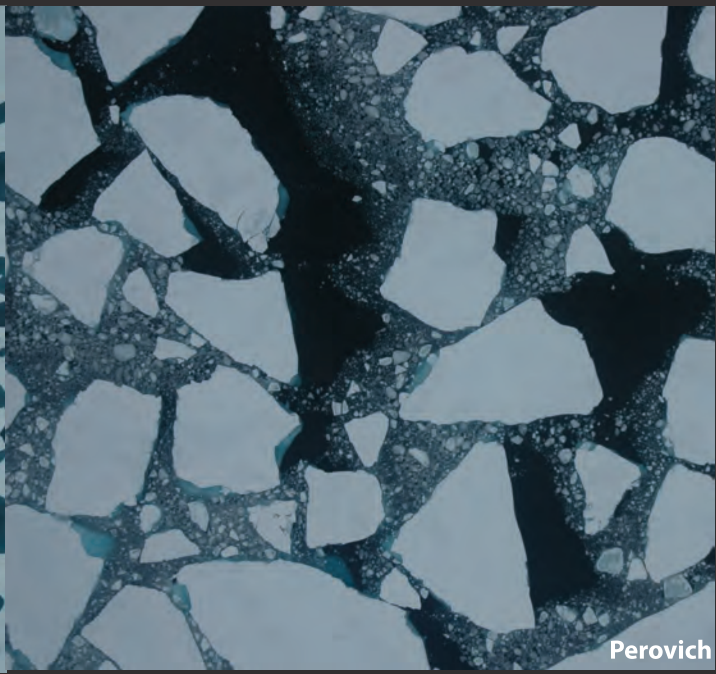
Sea ice is a multiscale composite material.



micro

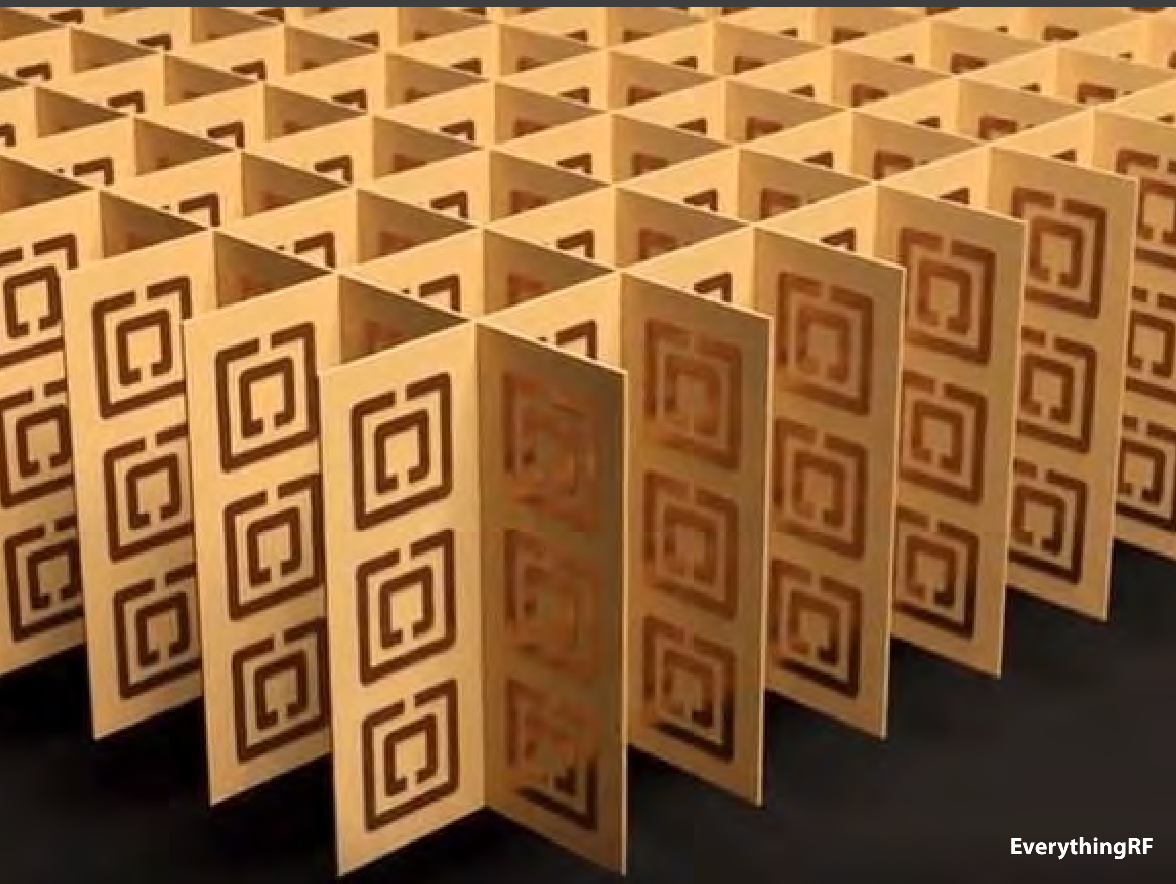


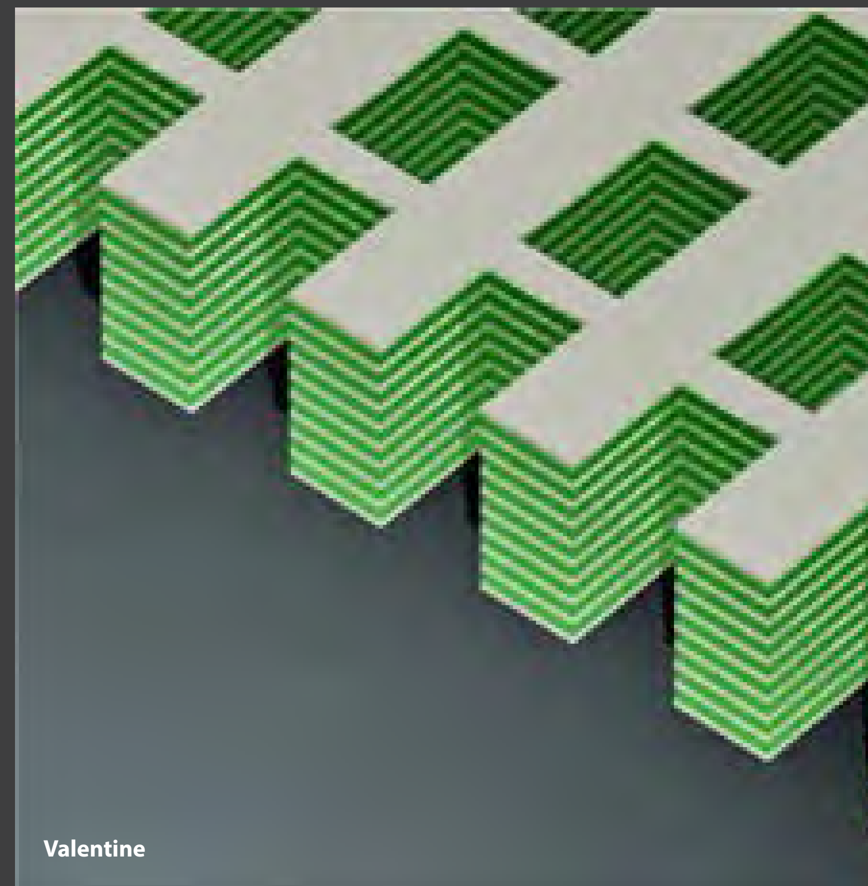
meso



macro

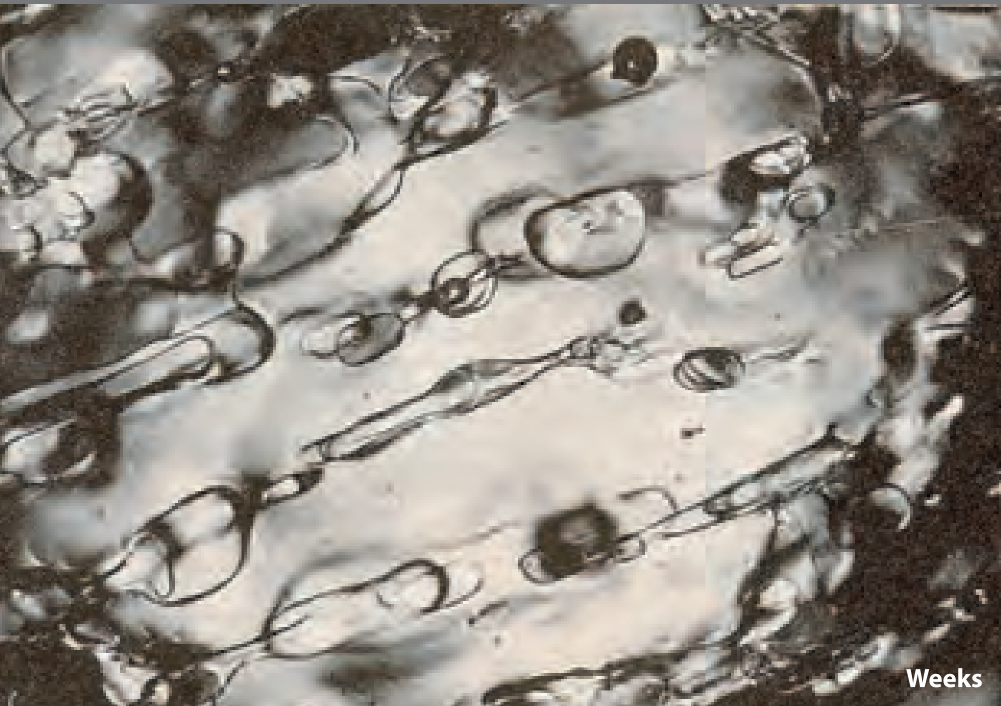
metamaterials

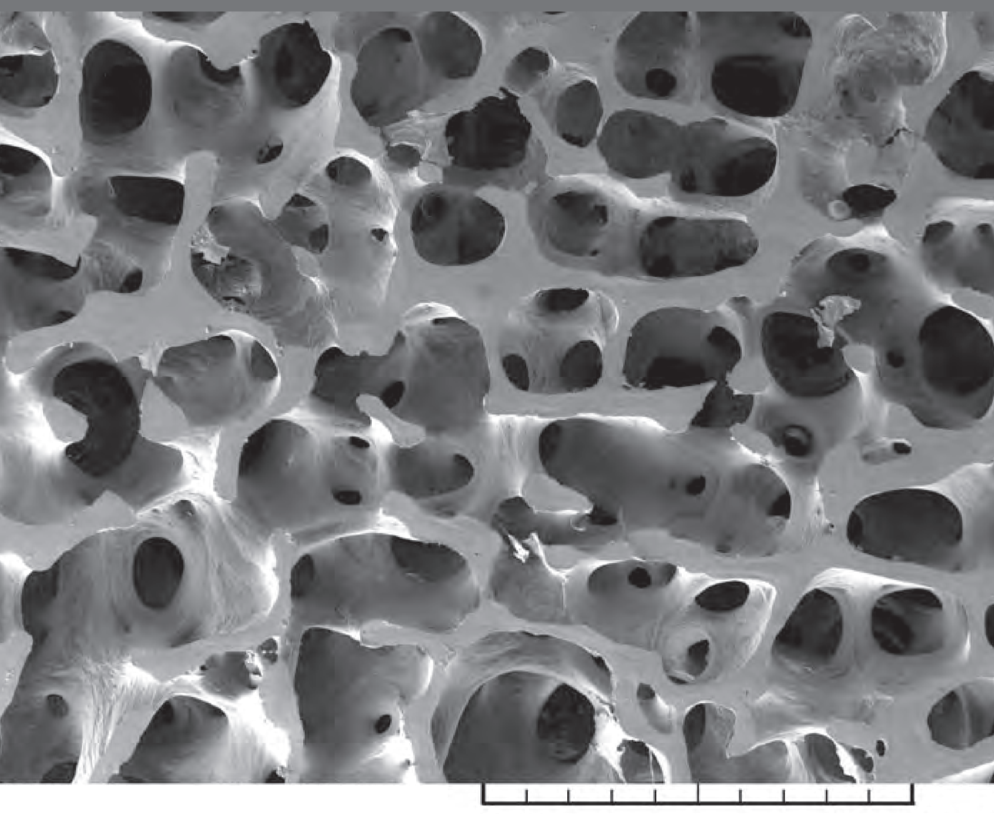


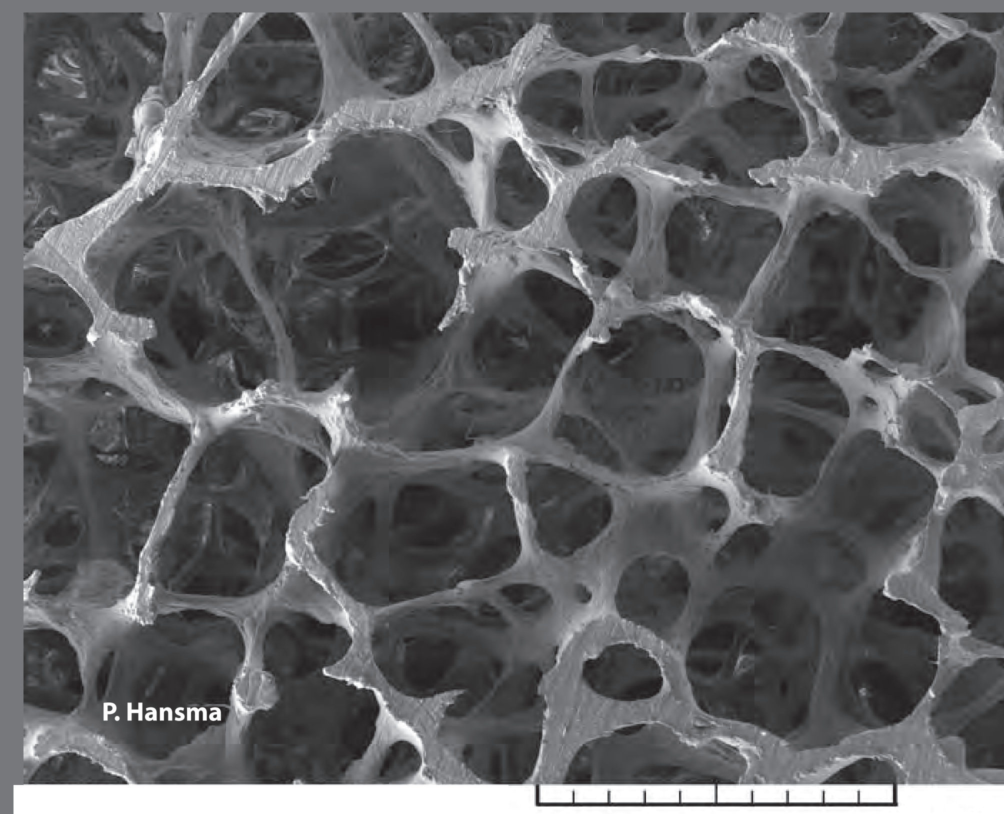


sea ice

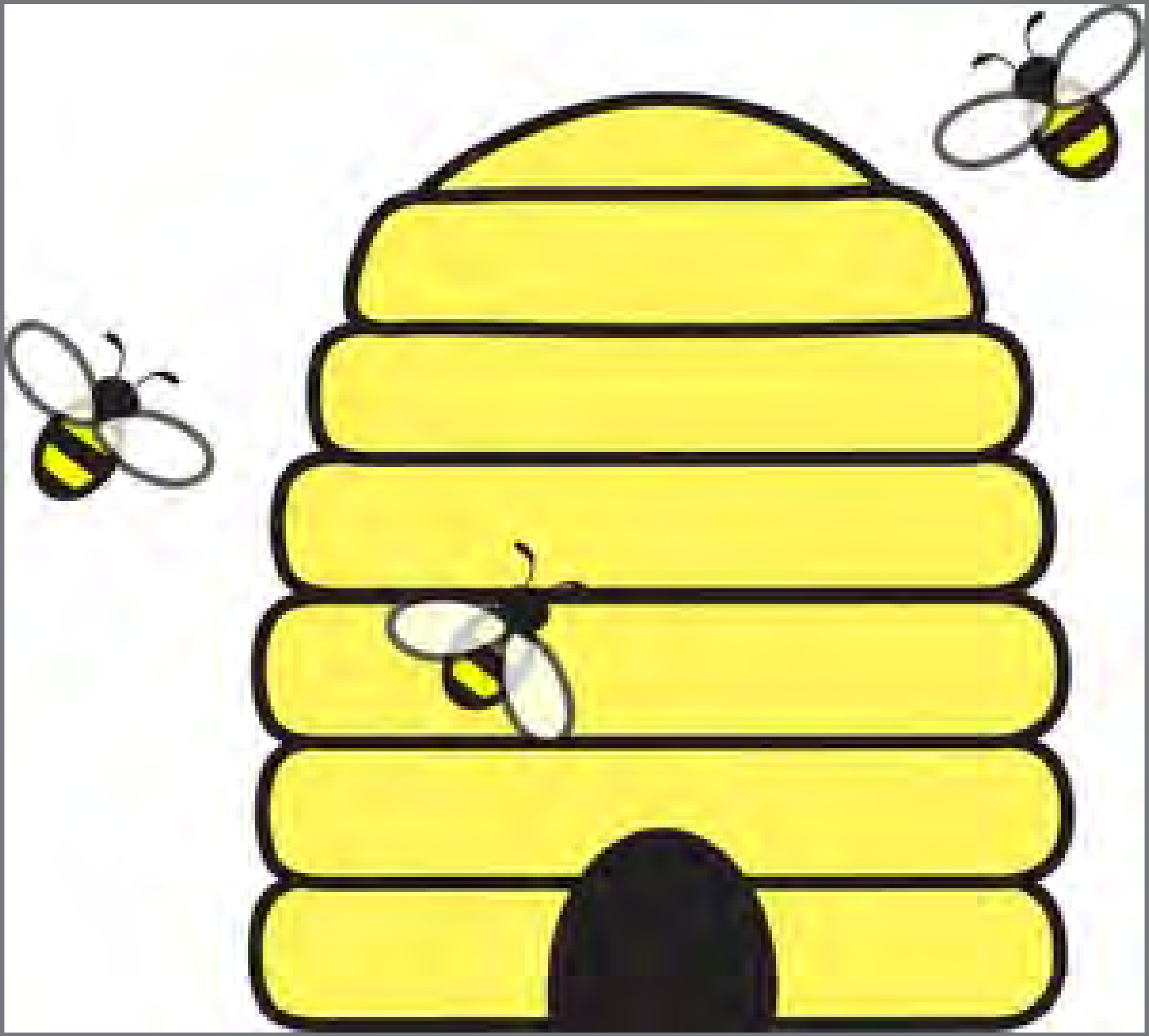
human bone

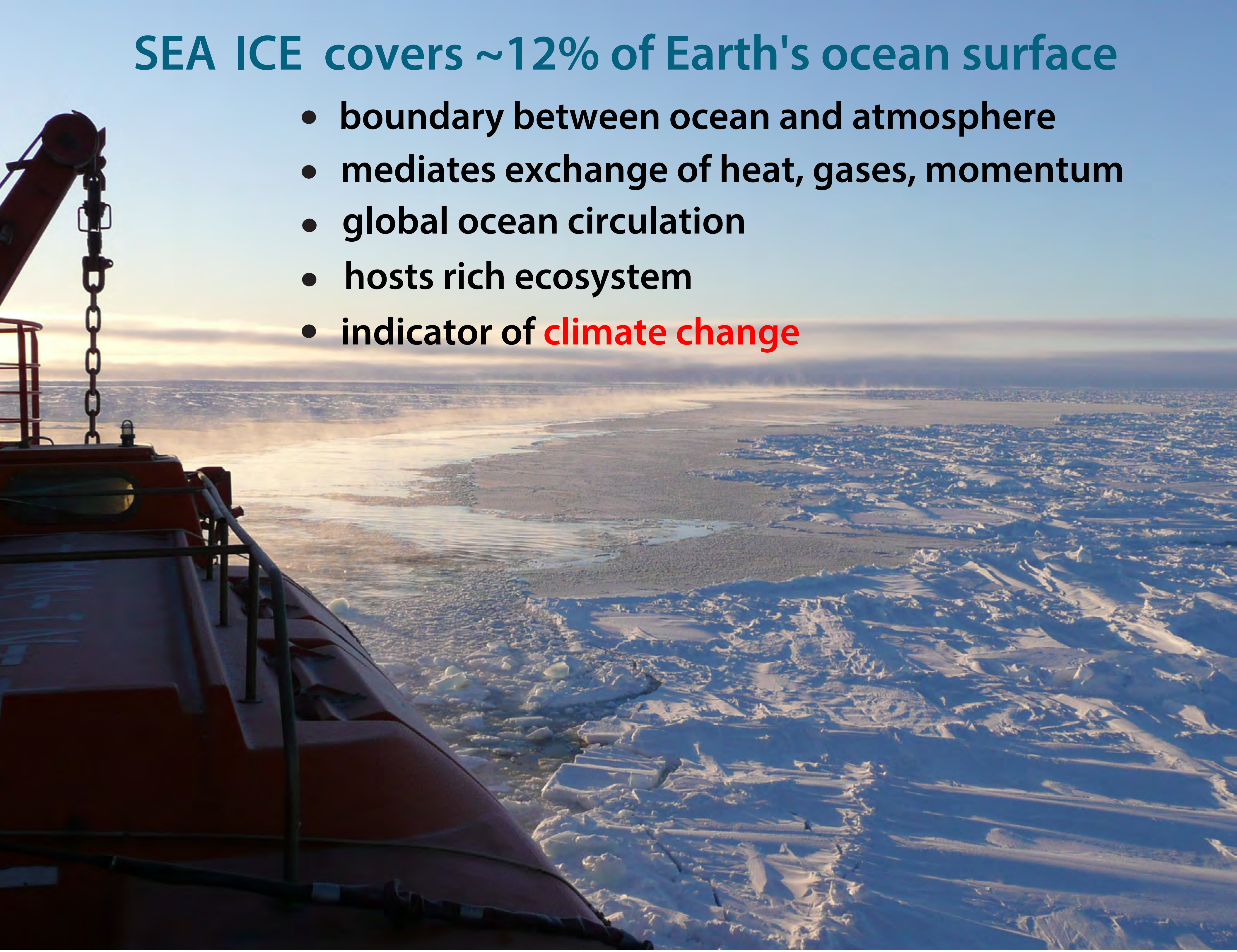






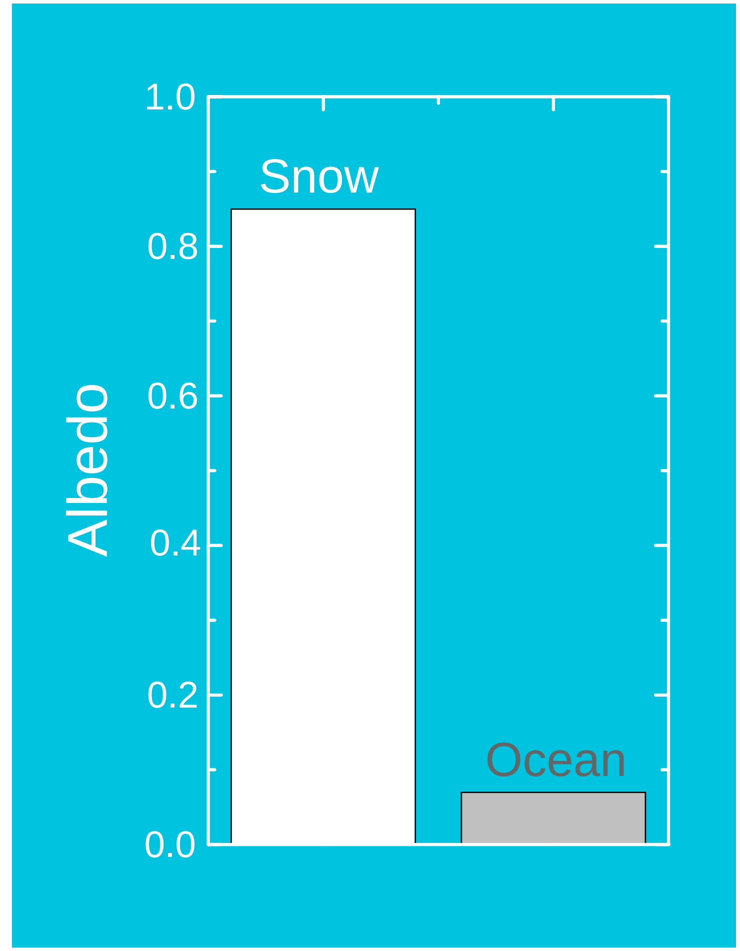
cross-pollination




SEA ICE covers ~12% of Earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• hosts rich ecosystem
• indicator of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

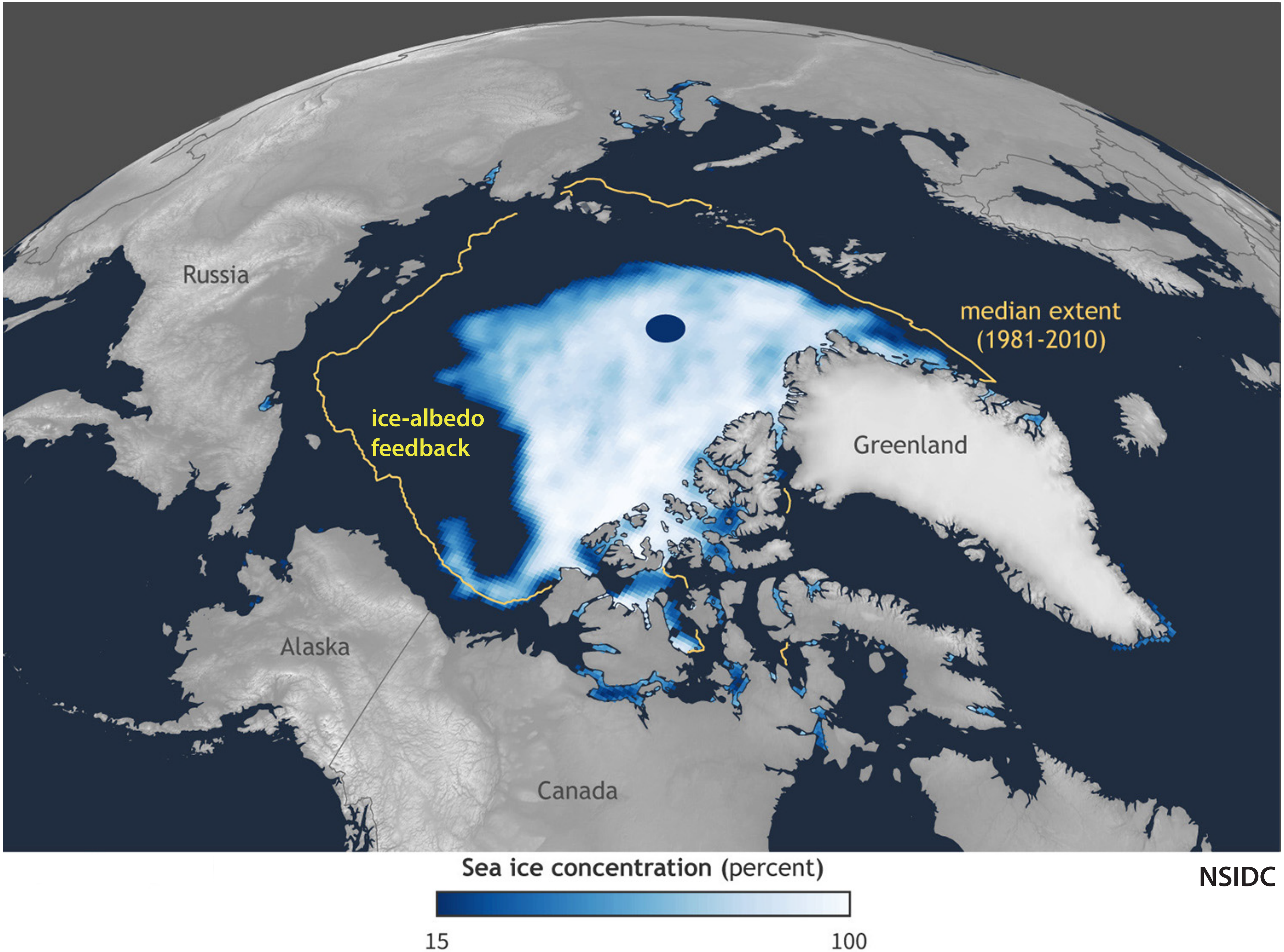
white snow and ice reflect



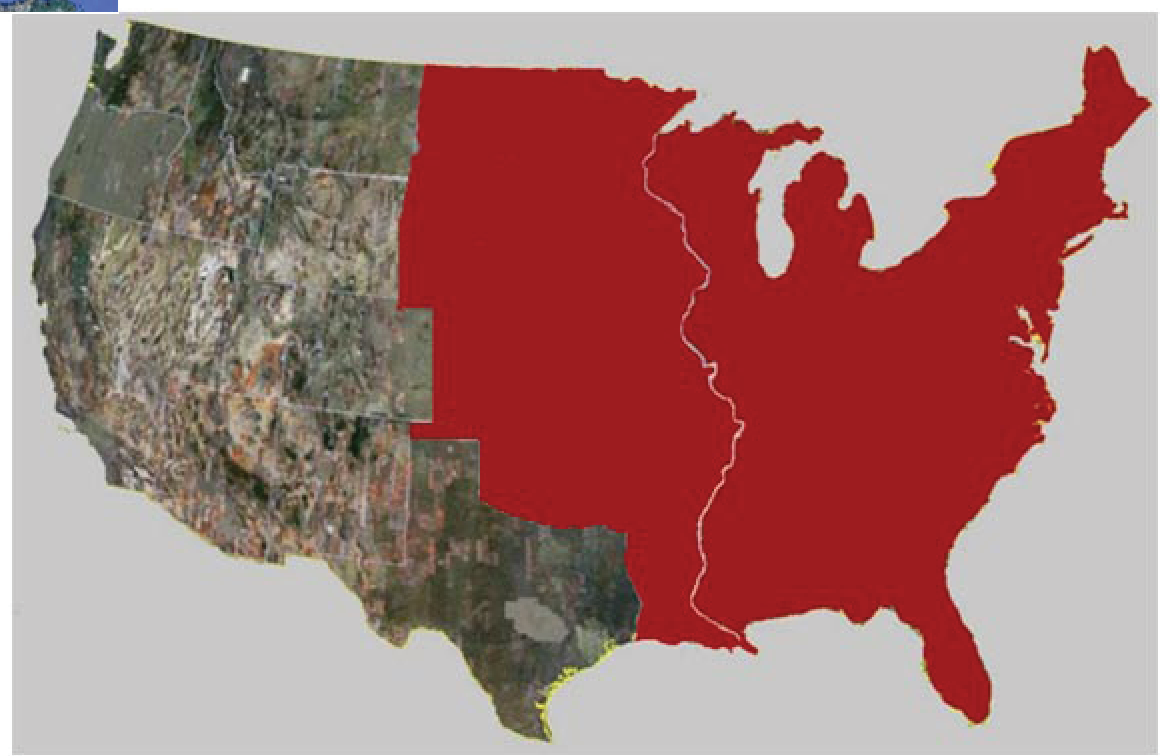
dark water and land absorb

$$\textit{\textbf{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Arctic sea ice extent September 15, 2020

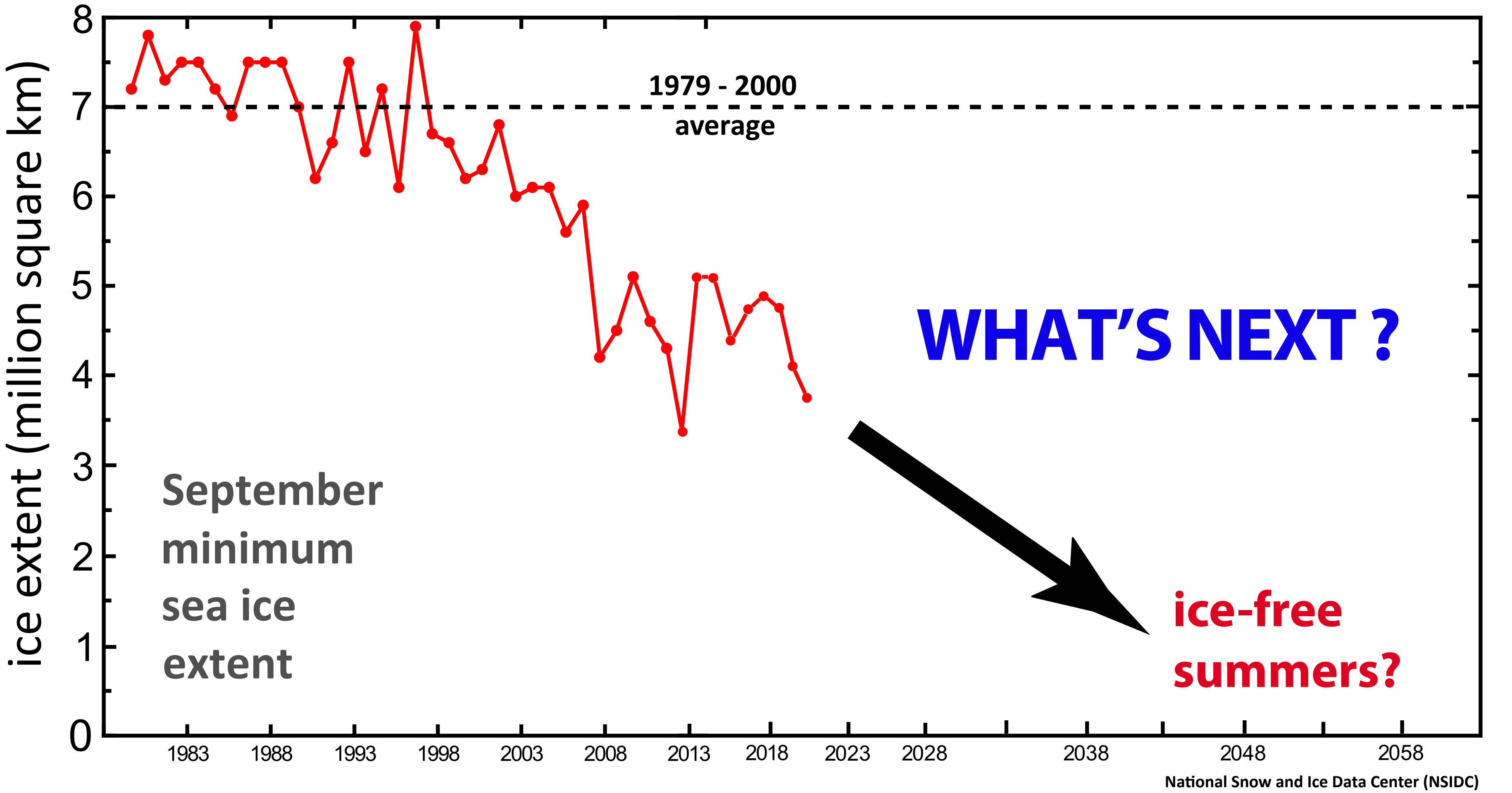


recent losses
in comparison to
the United States



Perovich

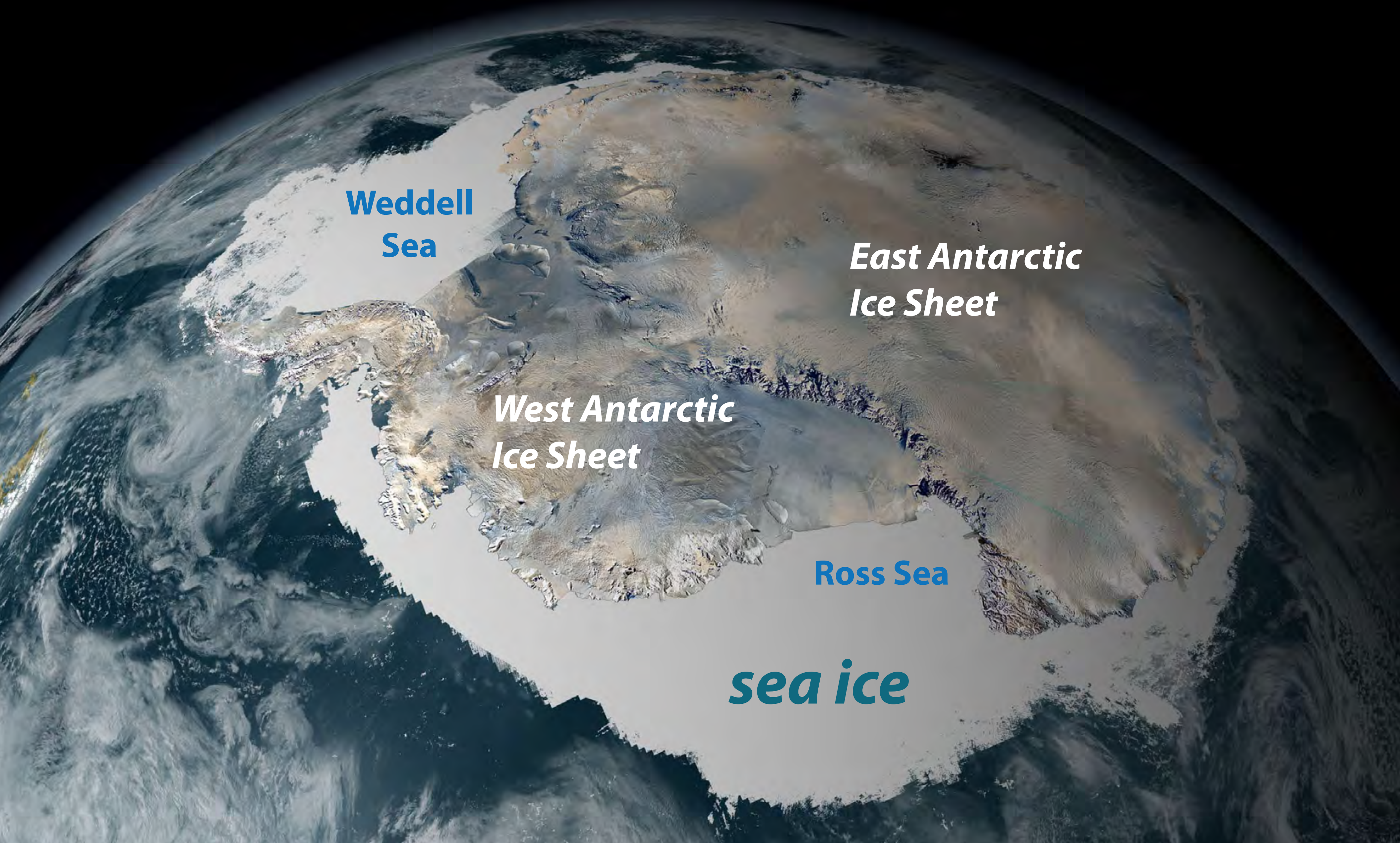
ARCTIC summer sea ice loss



predictions require lots of math modeling

ANTARCTICA

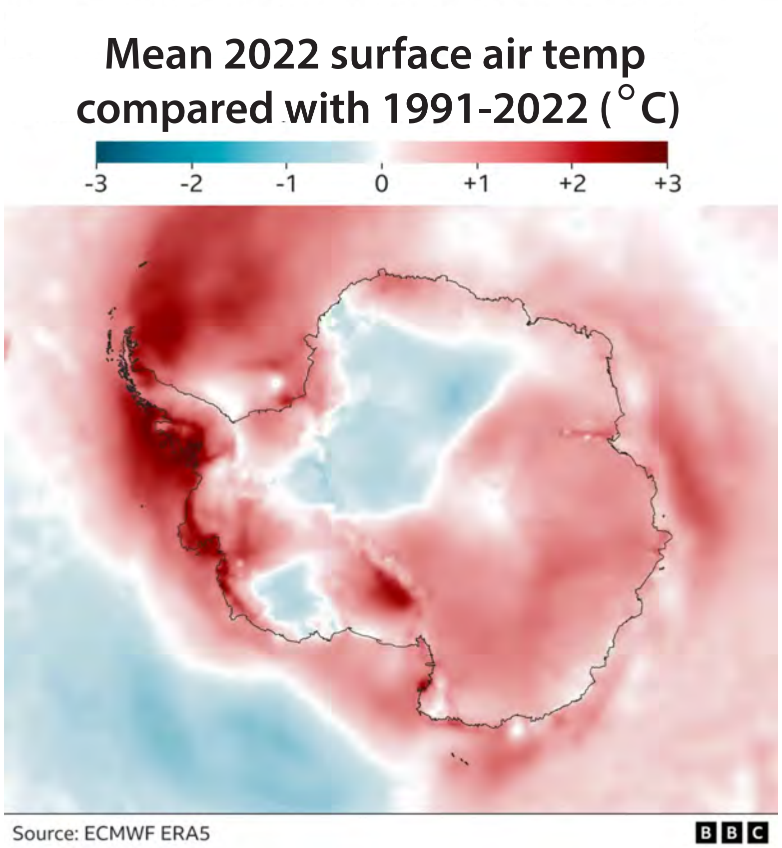
southern cryosphere

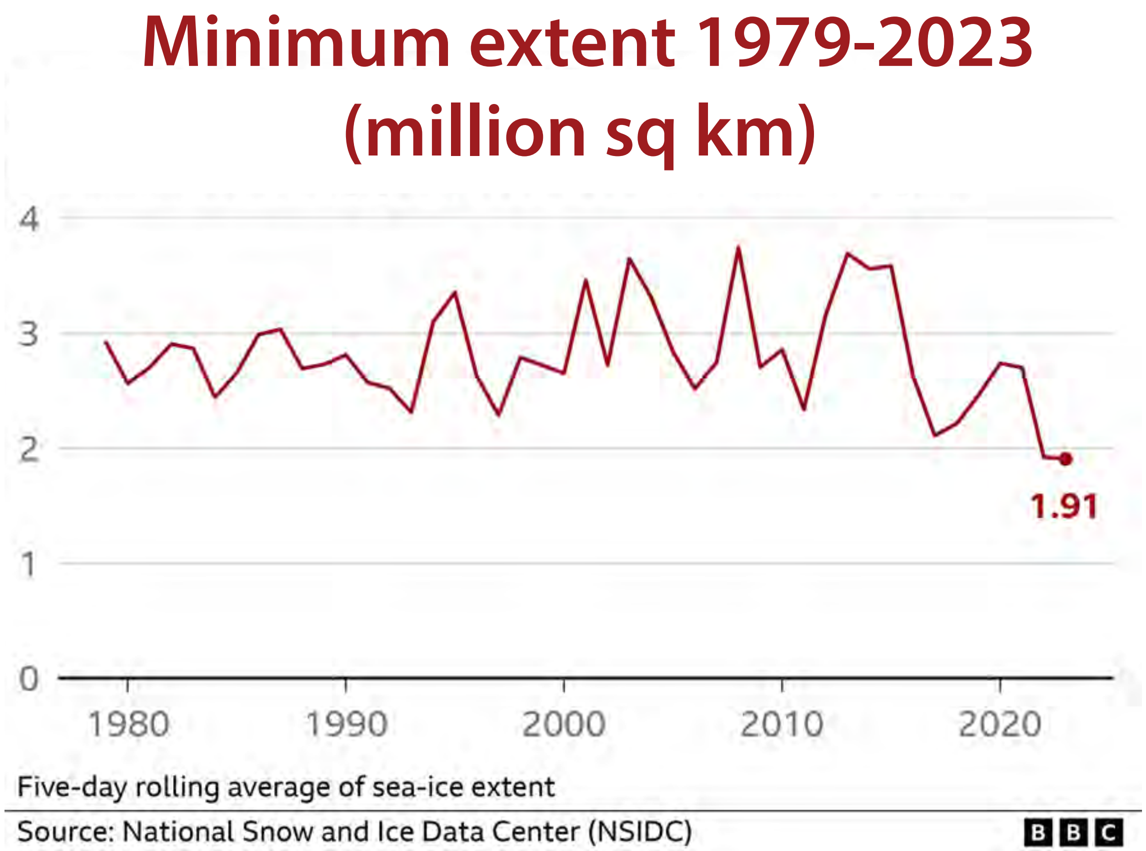


New Record Low for Antarctic Sea Ice

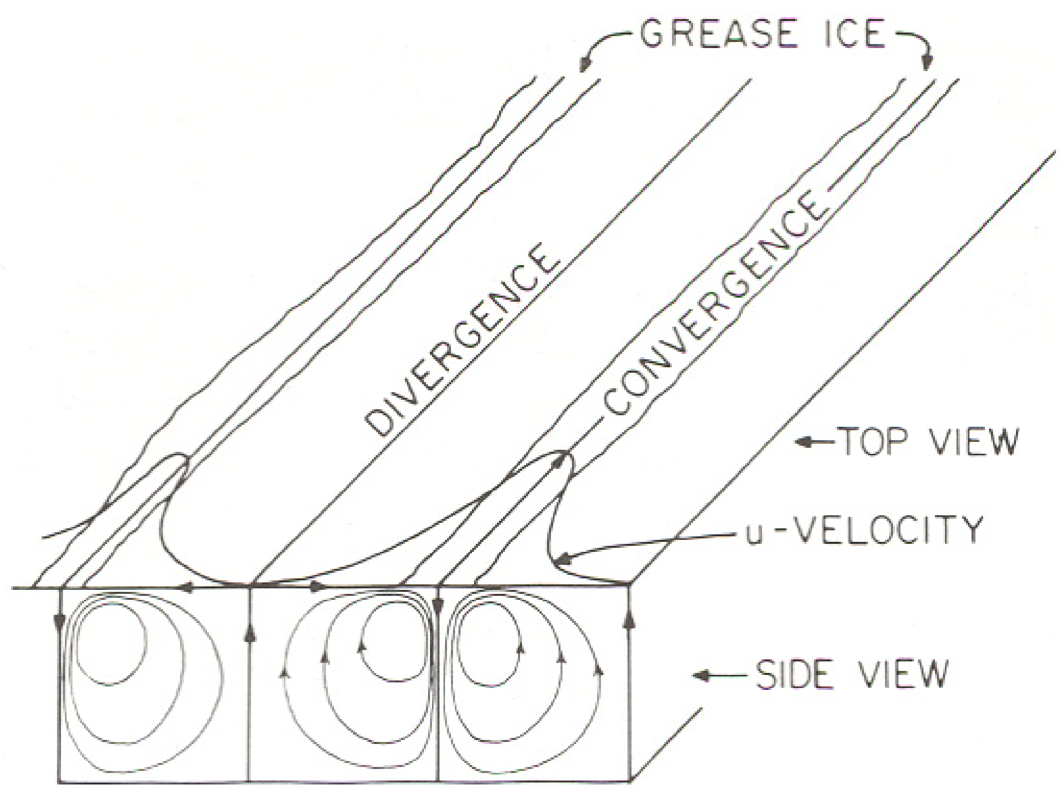
February 13, 2023

Much of Antarctica warmer than average



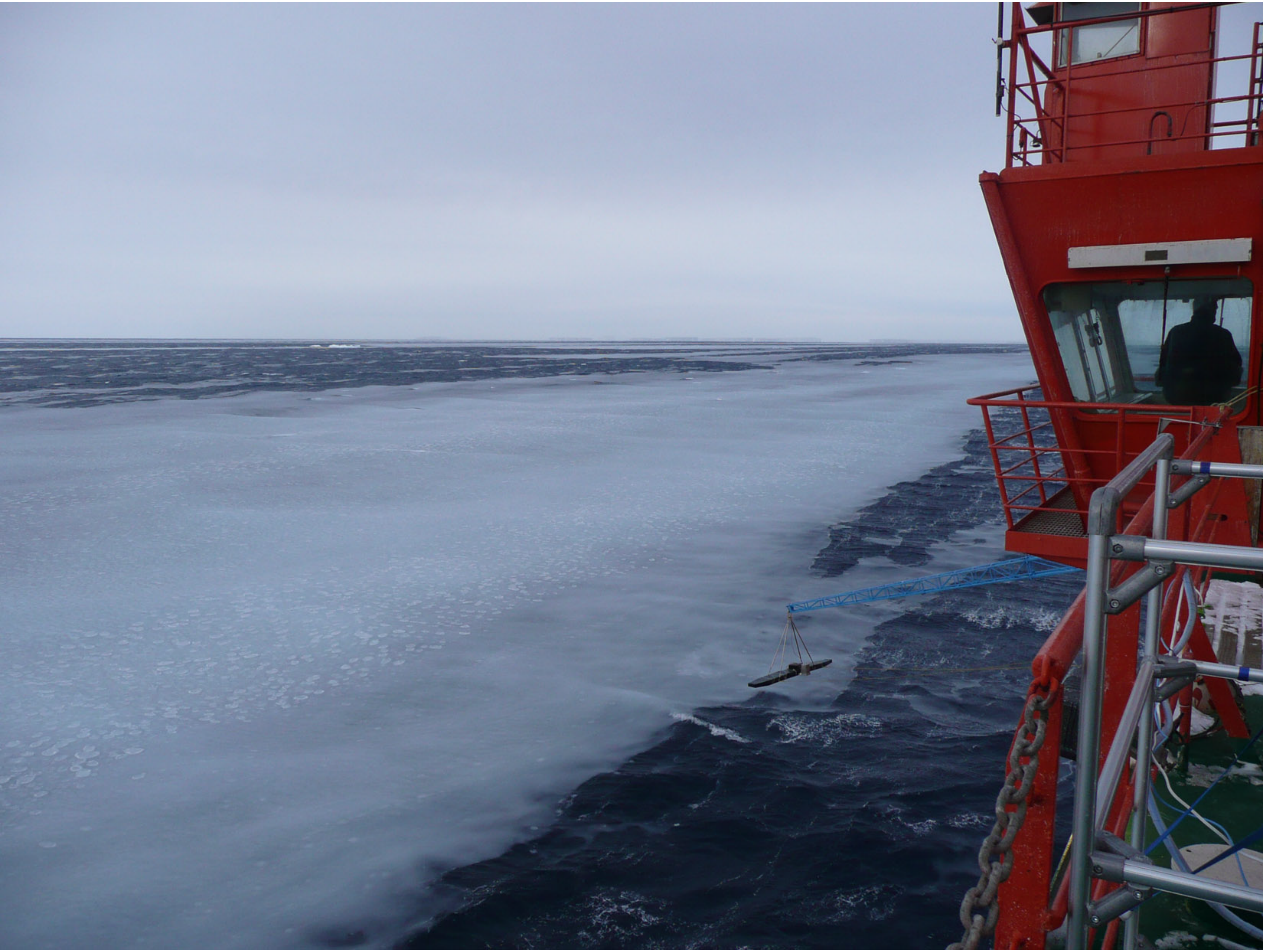


sea ice formation



effect of Langmuir circulation
on grease and pancake ice

Martin and Kauffman, 1981



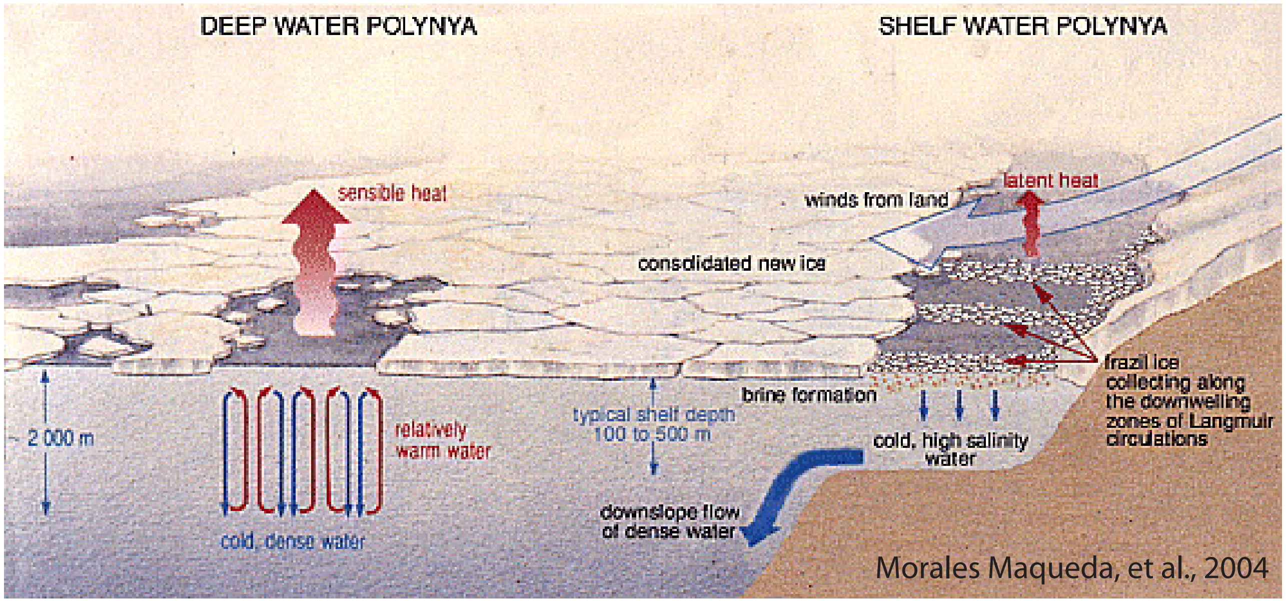
Polynyas

Size: 100 m - 1000 km

Two mechanisms can contribute to keeping polynyas open:

1. **Latent heat (or coastal) polynyas:** Mertz Glacier Polynya
   Sea ice grows in open-water and is continually removed by winds and currents (e.g. katabatic winds)
   - latent heat released to the ocean during ice formation perpetuates the process

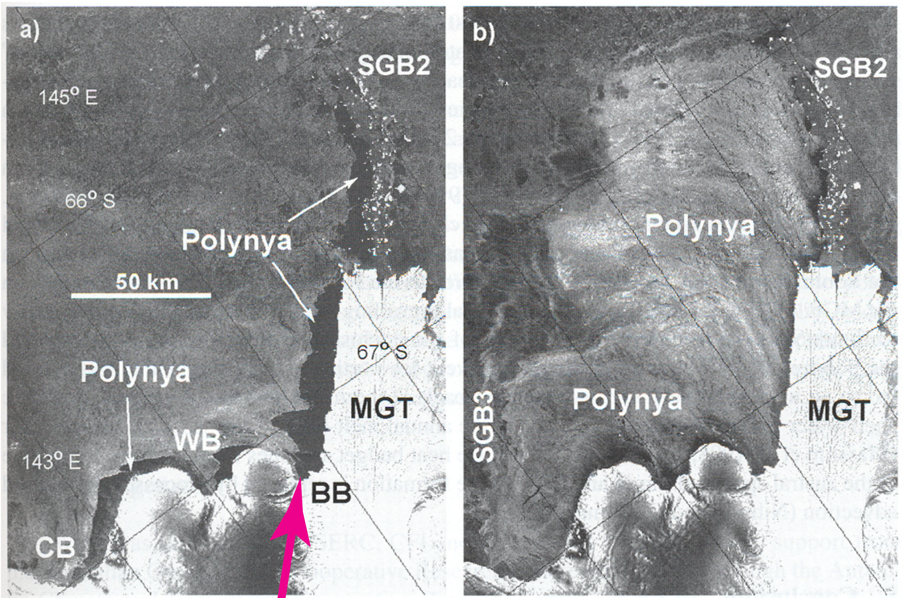
2. **Sensible heat (or open-ocean) polynyas:** Weddell Polynya
   Upwelling warm waters, vertical heat diffusion, or convection may provide enough oceanic heat flux to maintain ice-free region



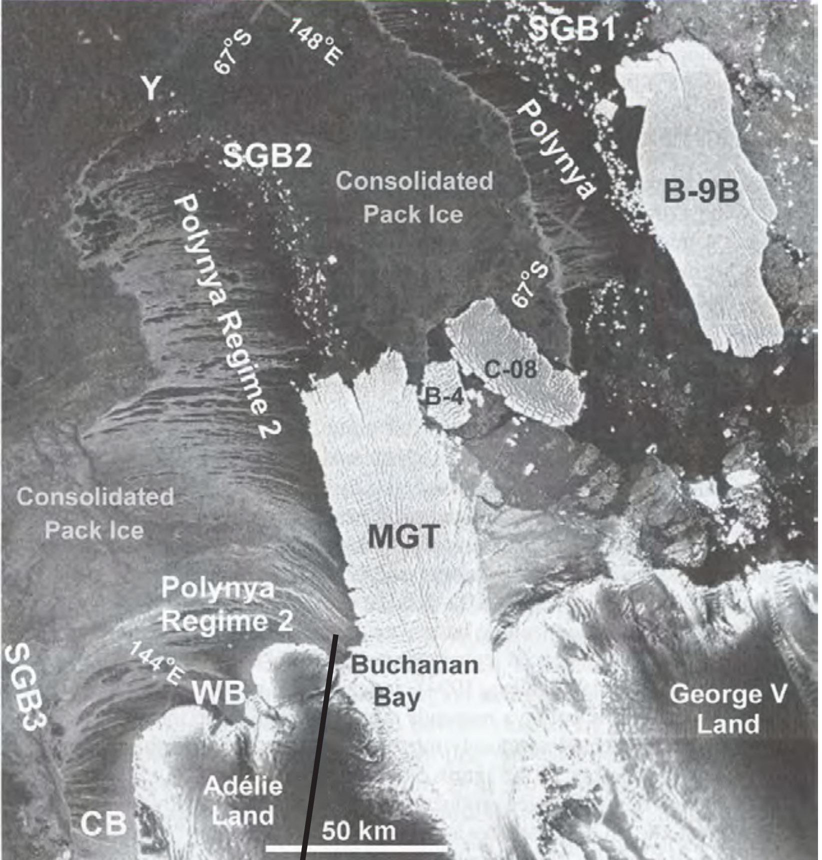
Morales Maqueda, et al., 2004

polynyas ice factories

Mertz Glacier Polynya, located in East Antarctica, covers only 0.001% of the overall Antarctic sea ice zone at its maximum winter extent, but is responsible for 1% of the total sea ice production in the Southern Ocean.

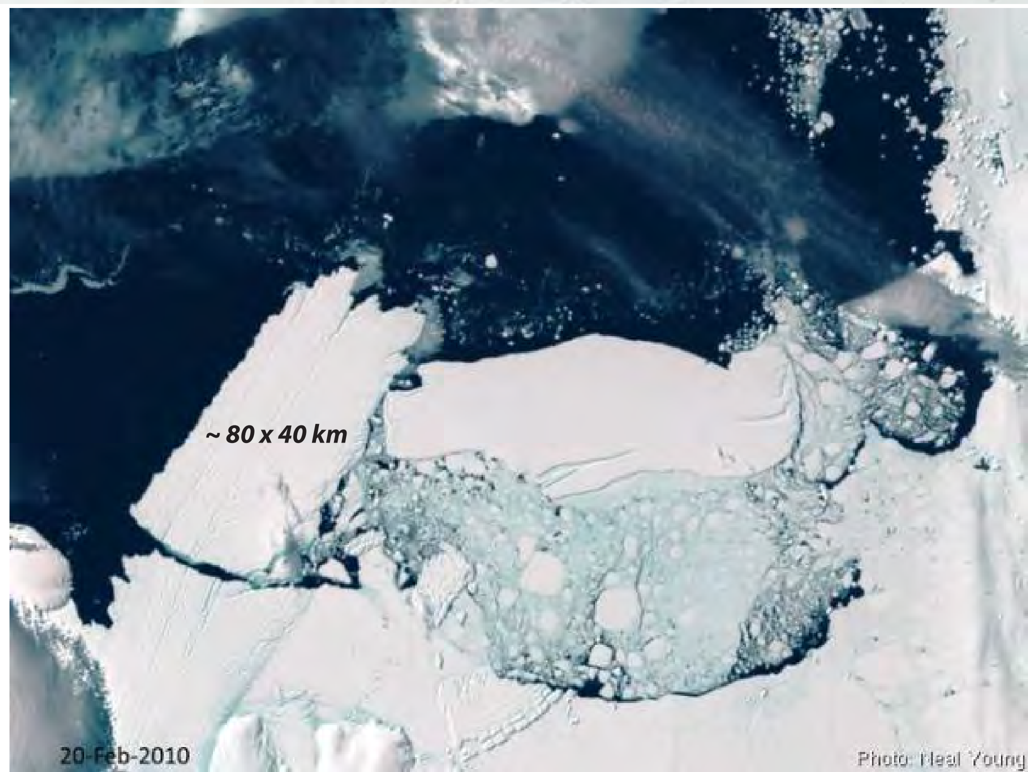


Buchanan Bay

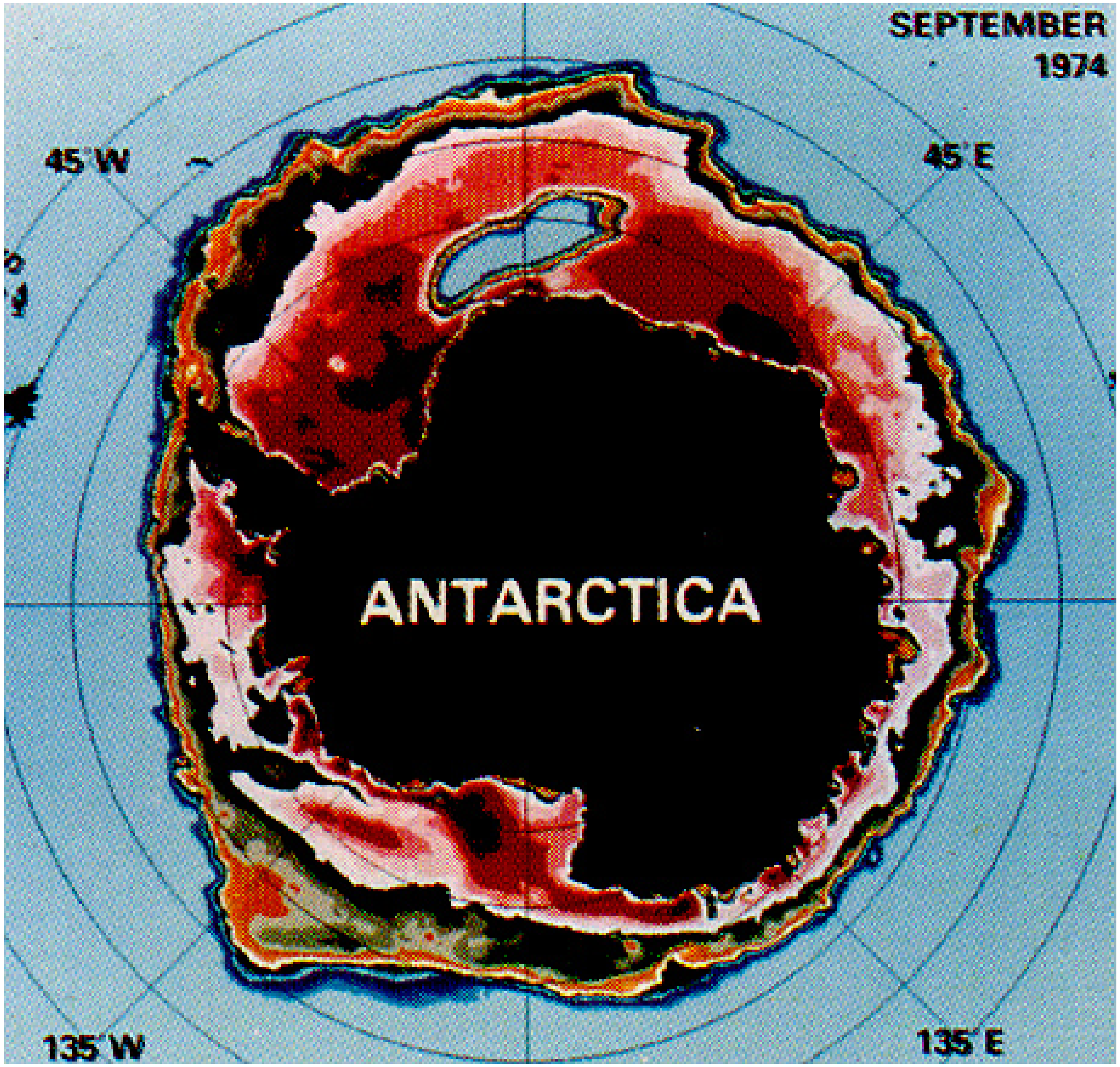

SGB1
67°S
148°E
Y
SGB2
Consolidated
Pack Ice
Polynya
B-9B
Polynya Regime 2
67°S
C-08
B-4
Consolidated
Pack Ice
MGT
Polynya
Regime 2
144°E
WB
SGB3
Buchanan
Bay
George V
Land
Adélie
Land
CB
50 km


iceberg collision!
breaking the Mertz Glacier Tongue, February 2010


Buchanan Bay, July 1999
Mertz Glacier Polynya Experiment


~ 80 x 40 km
20-Feb-2010
Photo: Neal Young

Weddell Polynya



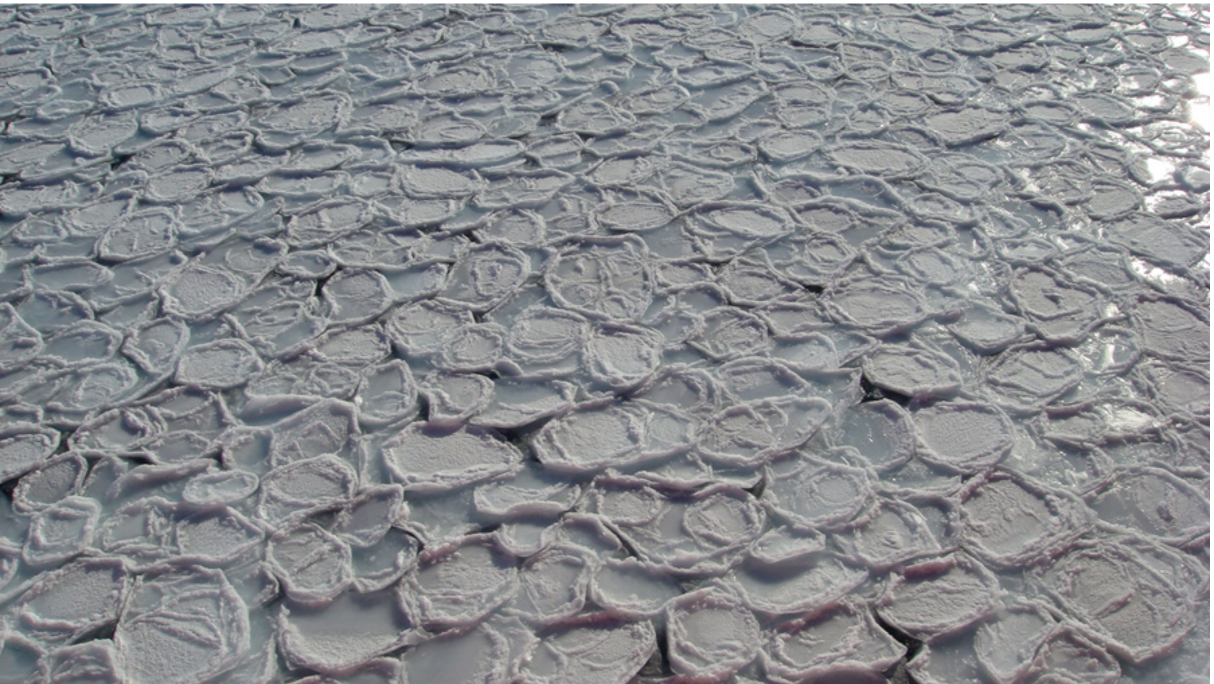
Antarctic Zone Flux Experiment (ANZFLUX) 1994

ocean swells propagating through a vast field of pancake ice

HOMOGENIZATION: long wave sees an effective medium, not individual floes, like long EM wave interacting with brine inclusion microstructure

pancake ice forming in a wave field in the Southern Ocean

pancake ice



“Dynamic” duo

Dynamics

Thermodynamics

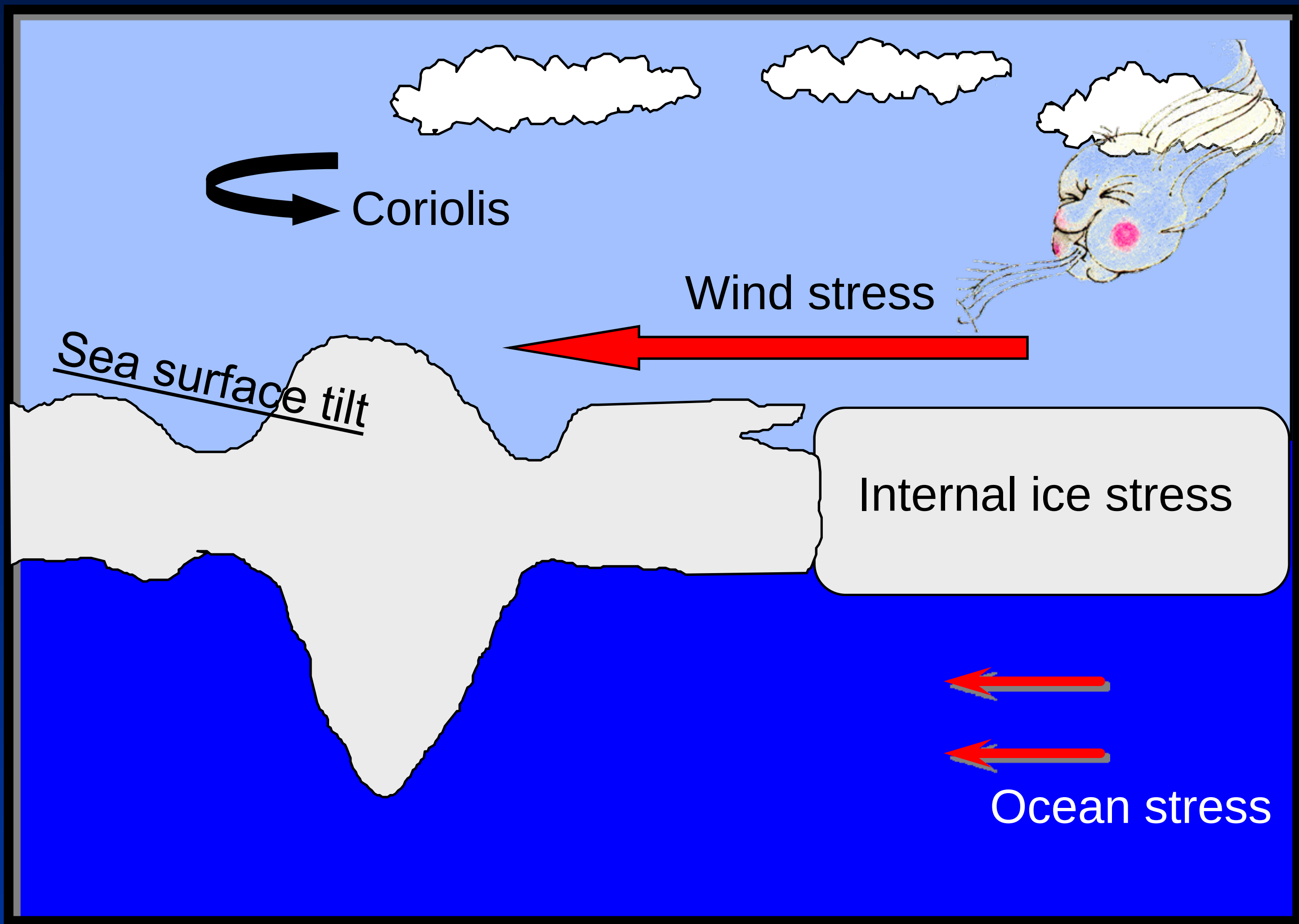
sea ice dynamics

plate tectonics on a fast time scale

measuring ice depth in ridges off Barrow, AK

dynamic sea ice

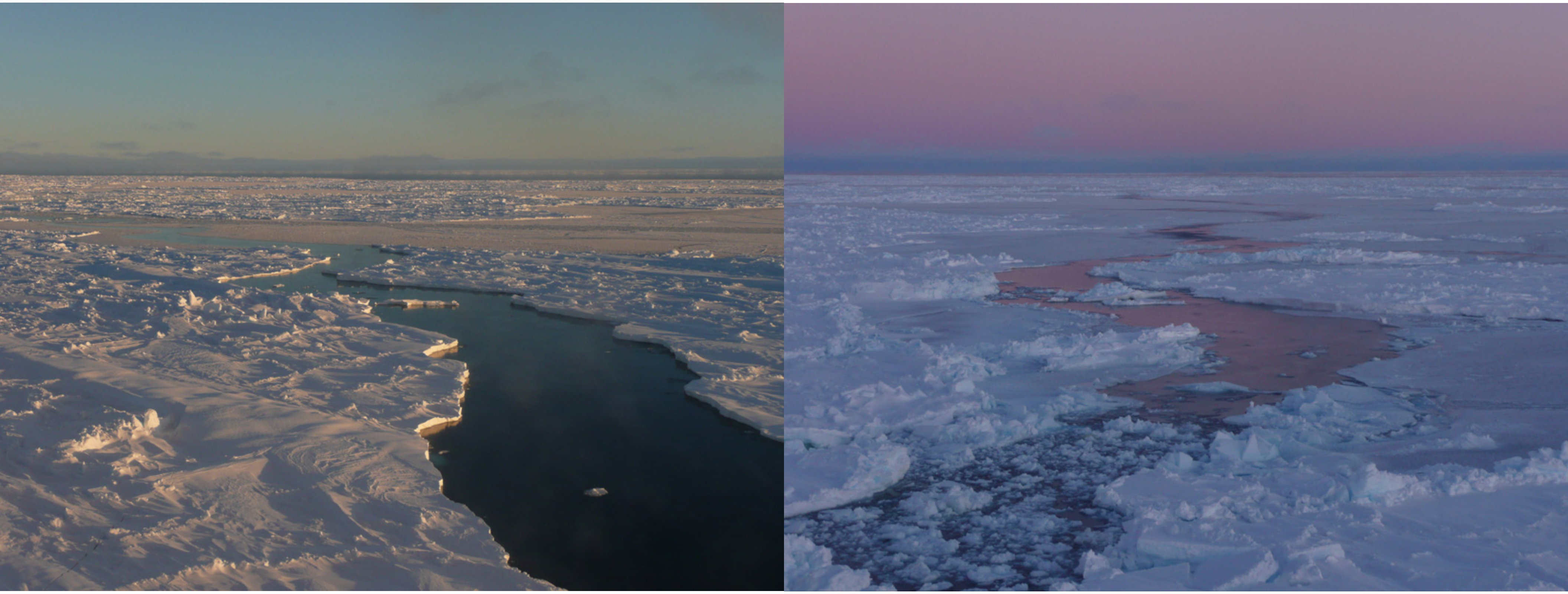
Dynamics



Momentum equation:

Ice acceleration = **wind stress** + ocean stress - Coriolis force
- sea surface tilt + internal ice stress

leads

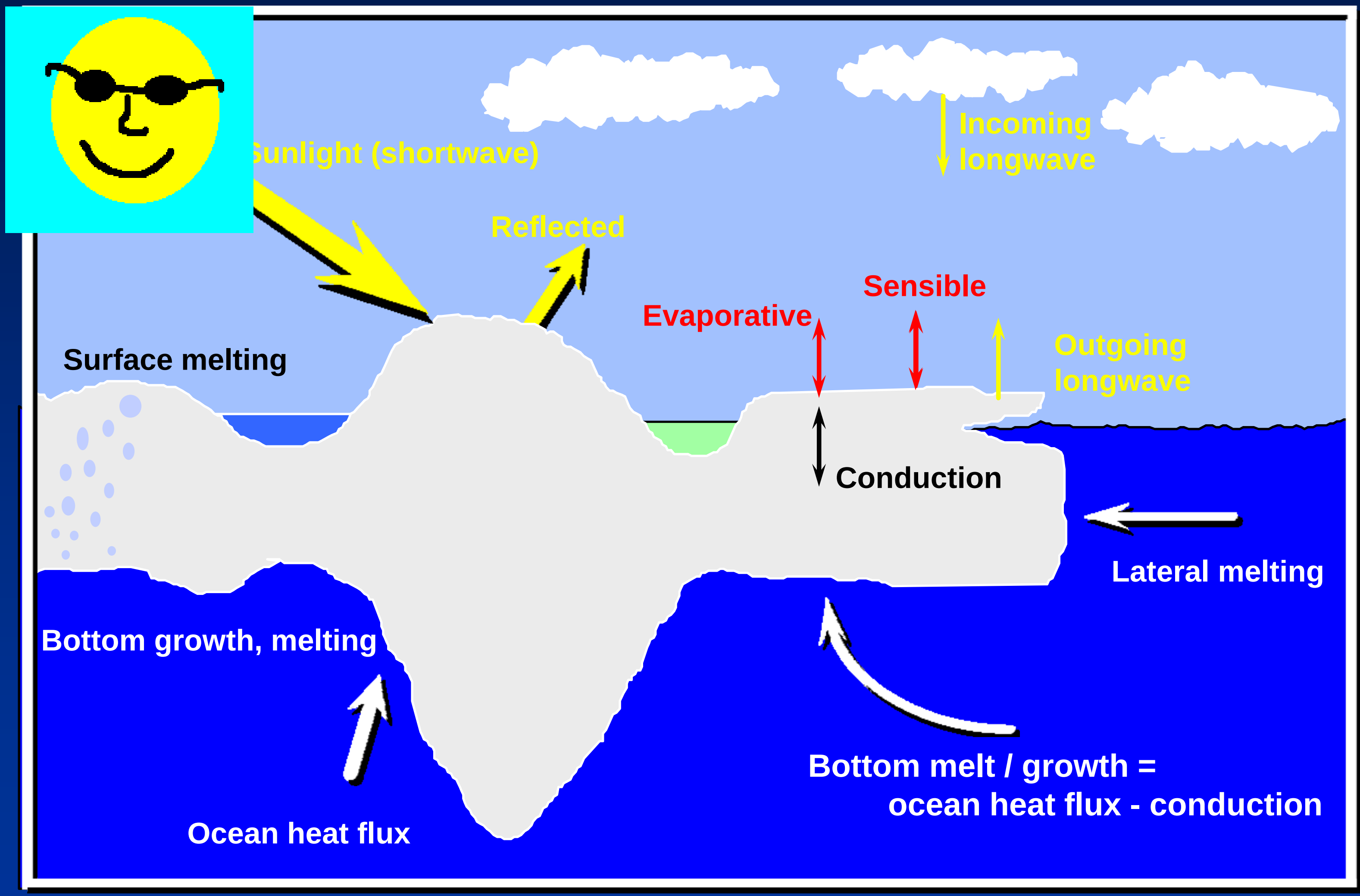


heat flows directly from ocean to atmosphere

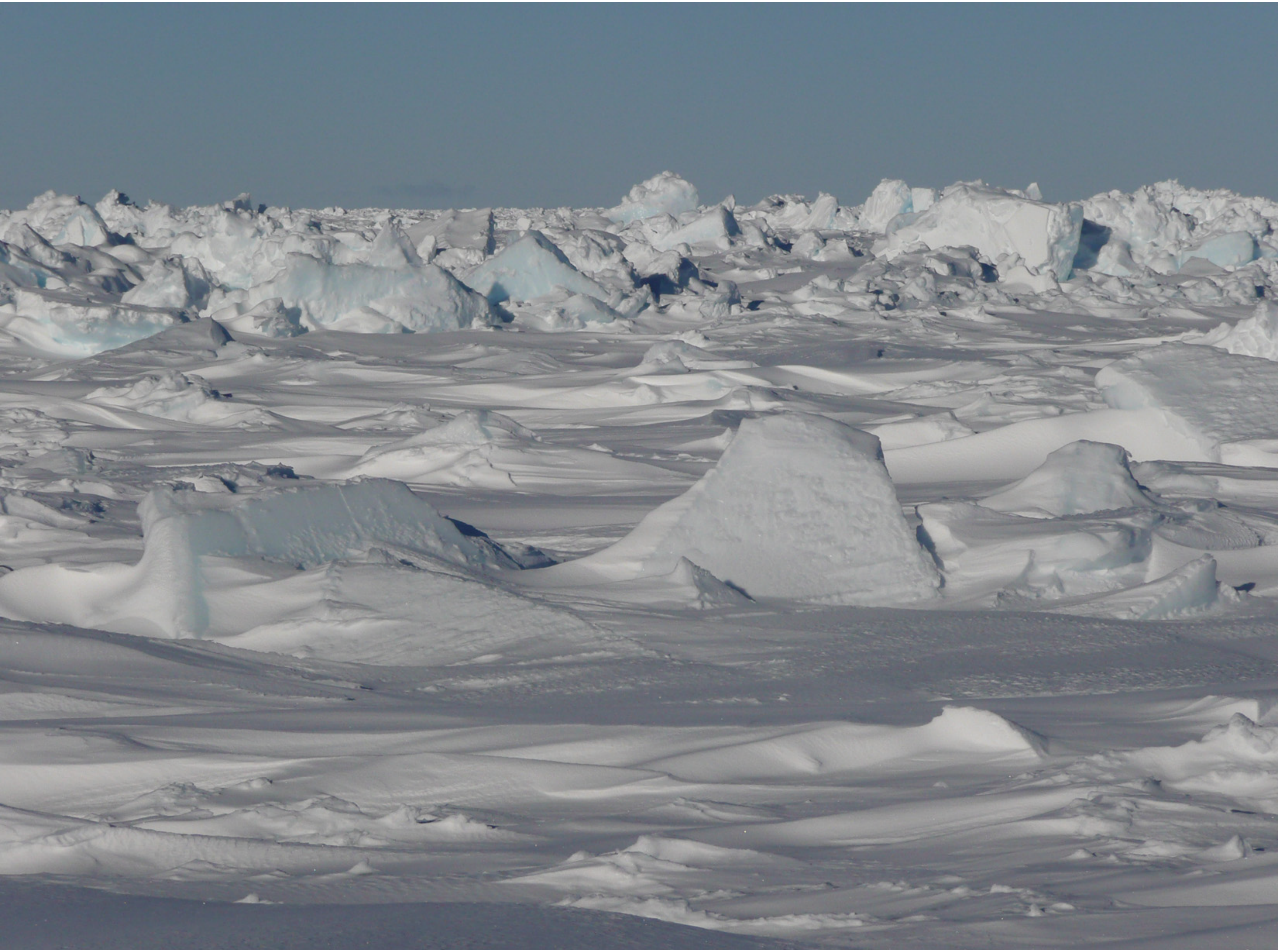
Thermodynamics: 4 ways to melt

Top, bottom, lateral, internal

Heat budgets



Net shortwave + incoming longwave + outgoing longwave + sensible + evaporative + conduction = melt / freeze



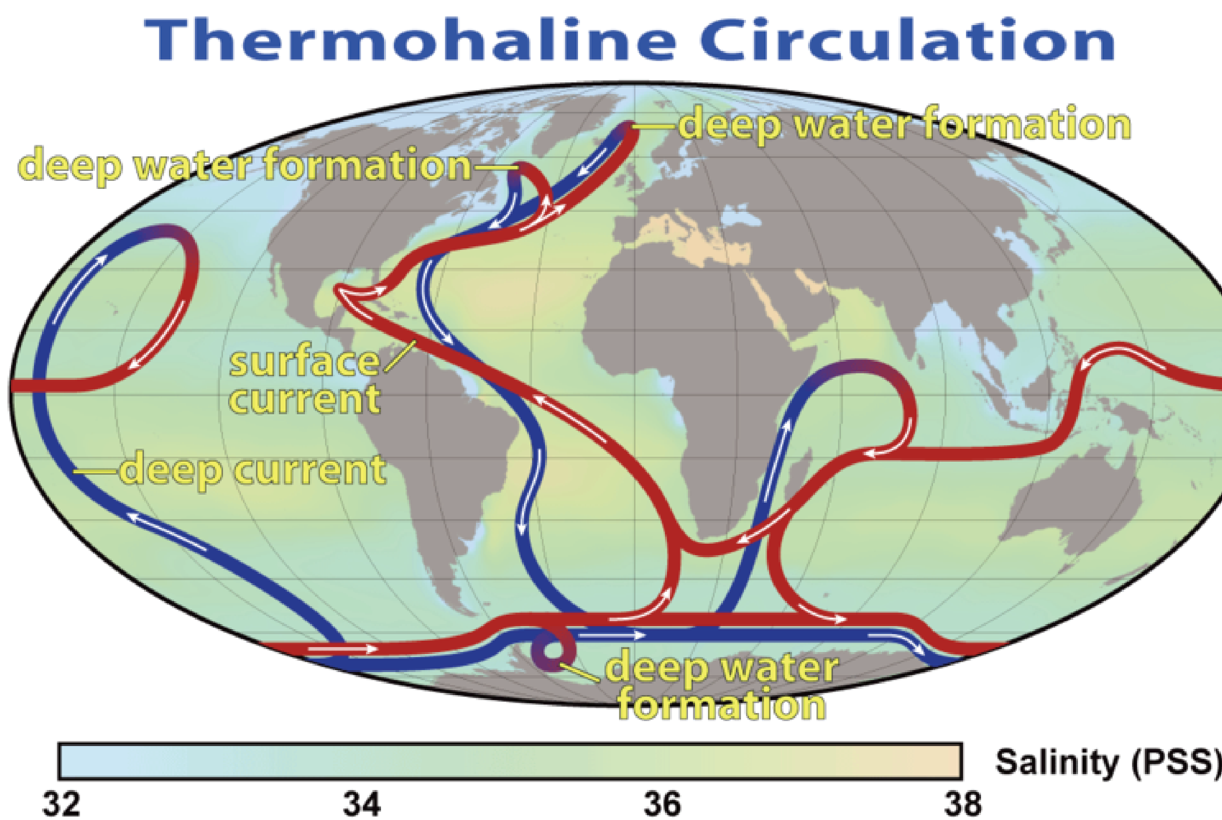
sea ice and global ocean circulation

brine expulsion from freezing sea water results in sinking of dense, saltier water to great depths

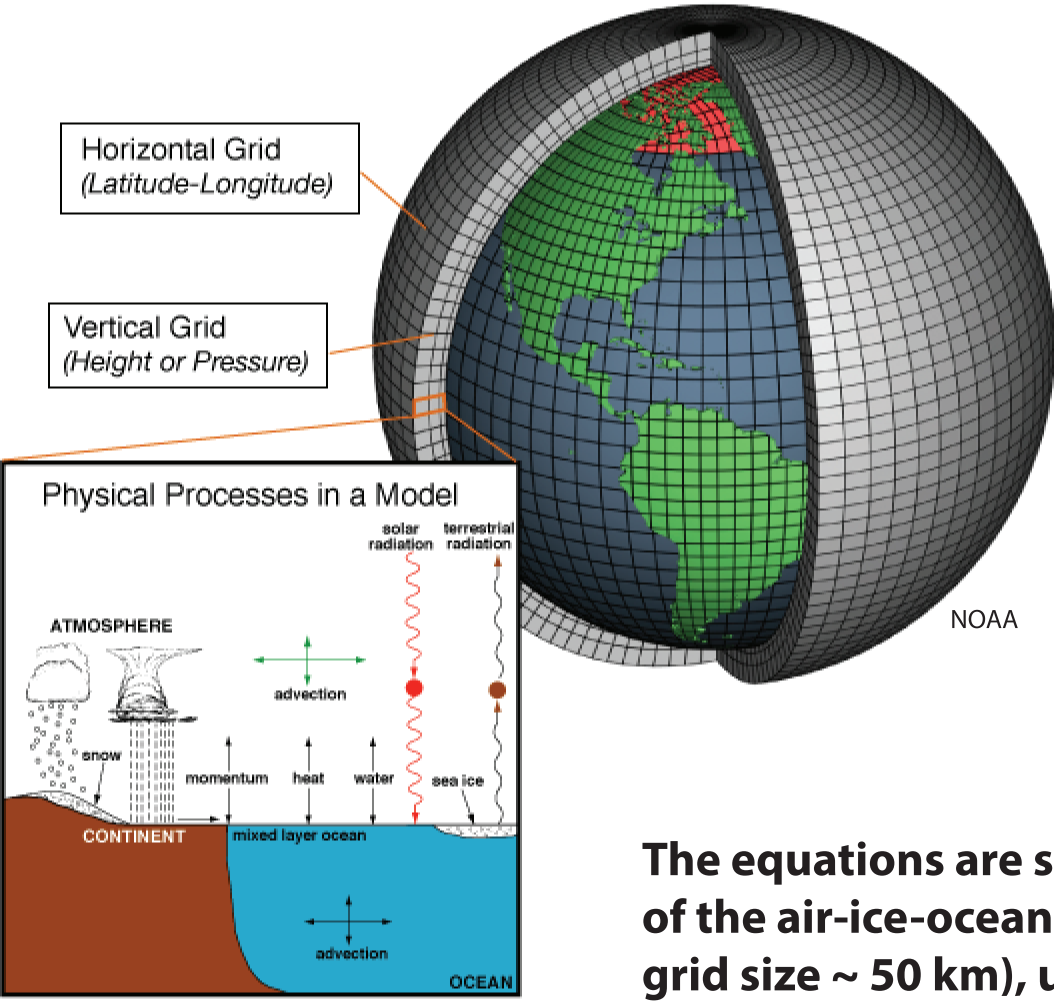
melting sea ice inputs fresher water into the upper ocean

↓

deep-water formation drives circulation in the world's oceans



GLOBAL THERMOHALINE CONVEYOR BELT



NOAA

Global Climate Models

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

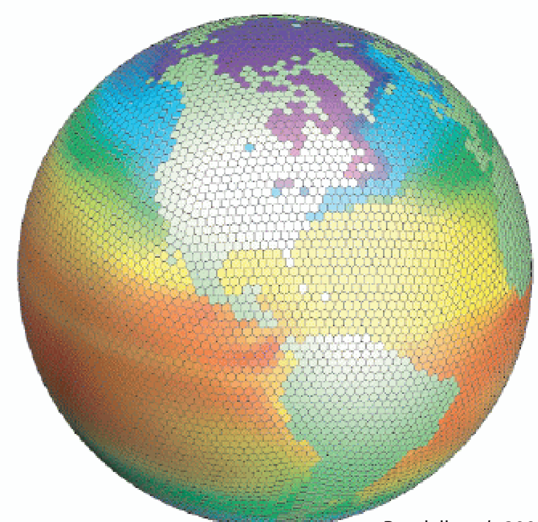
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 50 km), using very powerful computers.

key challenge :

incorporating sub - grid scale processes

linkage of scales


Randall *et al.*, 2002

sea ice components of GCM's

What are the key ingredients -- or ***governing equations***
that need to be solved on grids using powerful computers?

1. Ice thickness distribution $g(x, y, h, t)$ evolution equation

dynamics
+
thermodynamics

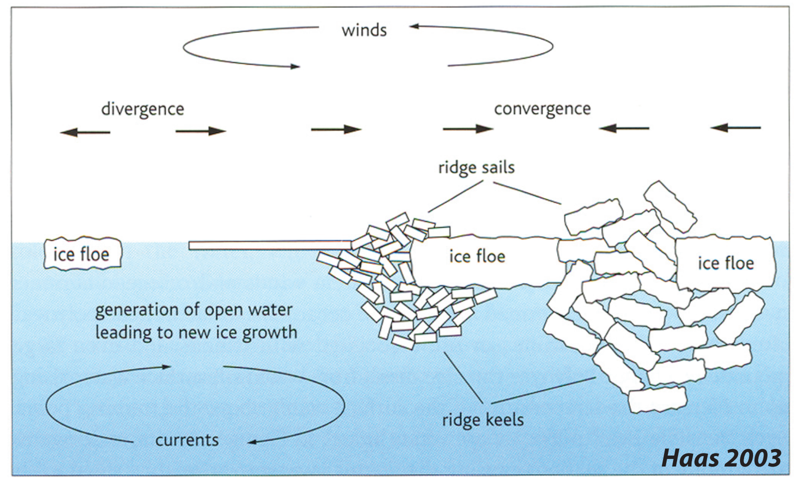
(Thorndike *et al.* 1975)

$$\frac{Dg}{Dt} = -g\nabla \cdot \mathbf{u} + \Psi(g) - \frac{\partial}{\partial h}(\tau g) + \mathcal{L}$$

nonlinear PDE with ice velocity field

ice growth ice melting

mechanical redistribution - ridging and opening



2. Conservation of momentum, stress *vs.* strain relation (Hibler 1979)

$$m\frac{D\mathbf{u}}{Dt} = -mf\mathbf{k} \times \mathbf{u} + \boldsymbol{\tau}_a + \boldsymbol{\tau}_o - mg\nabla H + \mathbf{F}_{int}$$

F = ma **for sea ice** ***dynamics***

3. Heat equation of sea ice and snow

(Maykut and Untersteiner 1971)

$$\frac{\partial T}{\partial t} + \mathbf{u}_{br} \cdot \nabla T = \nabla \cdot k(T)\, \nabla T$$

thermodynamics

\+ ***balance of radiative and thermal fluxes on interfaces***