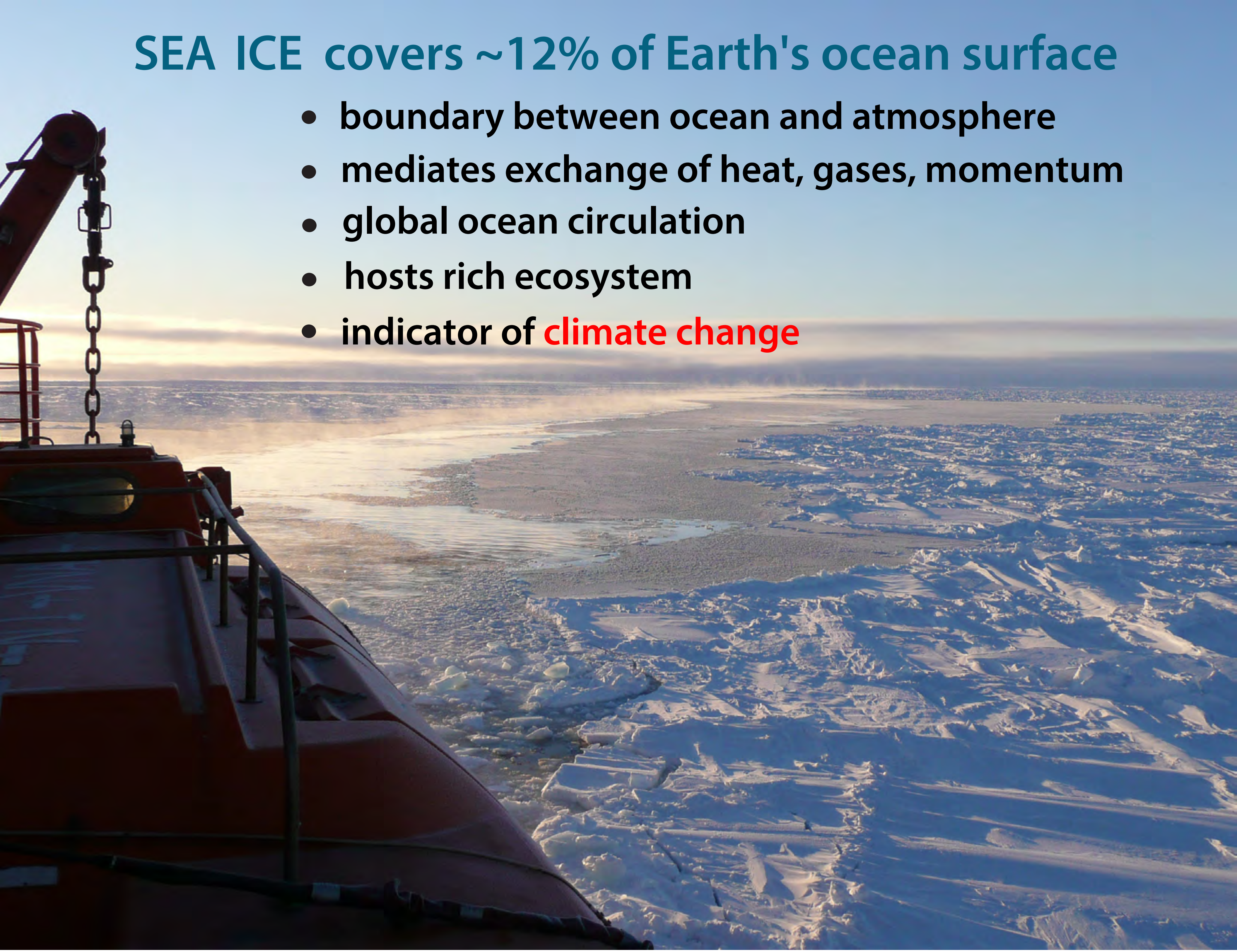
Mathematics of Frozen Seas

Kenneth Golden
University of Utah

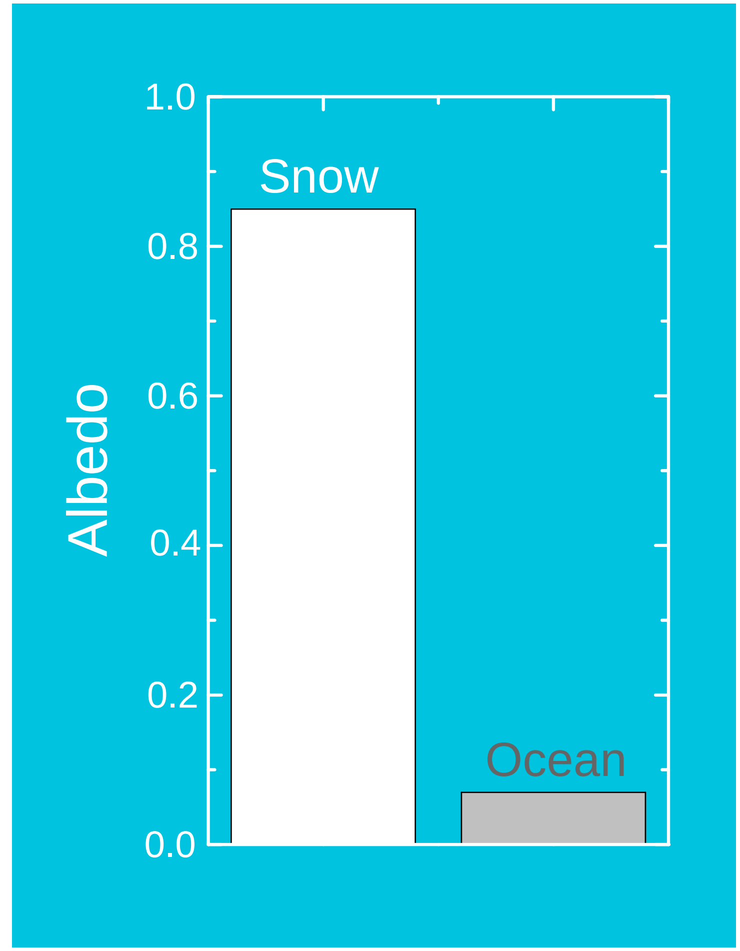
Frasier Colloquium
U.S. Naval Academy

January 29, 2025


SEA ICE covers ~12% of Earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• hosts rich ecosystem
• indicator of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

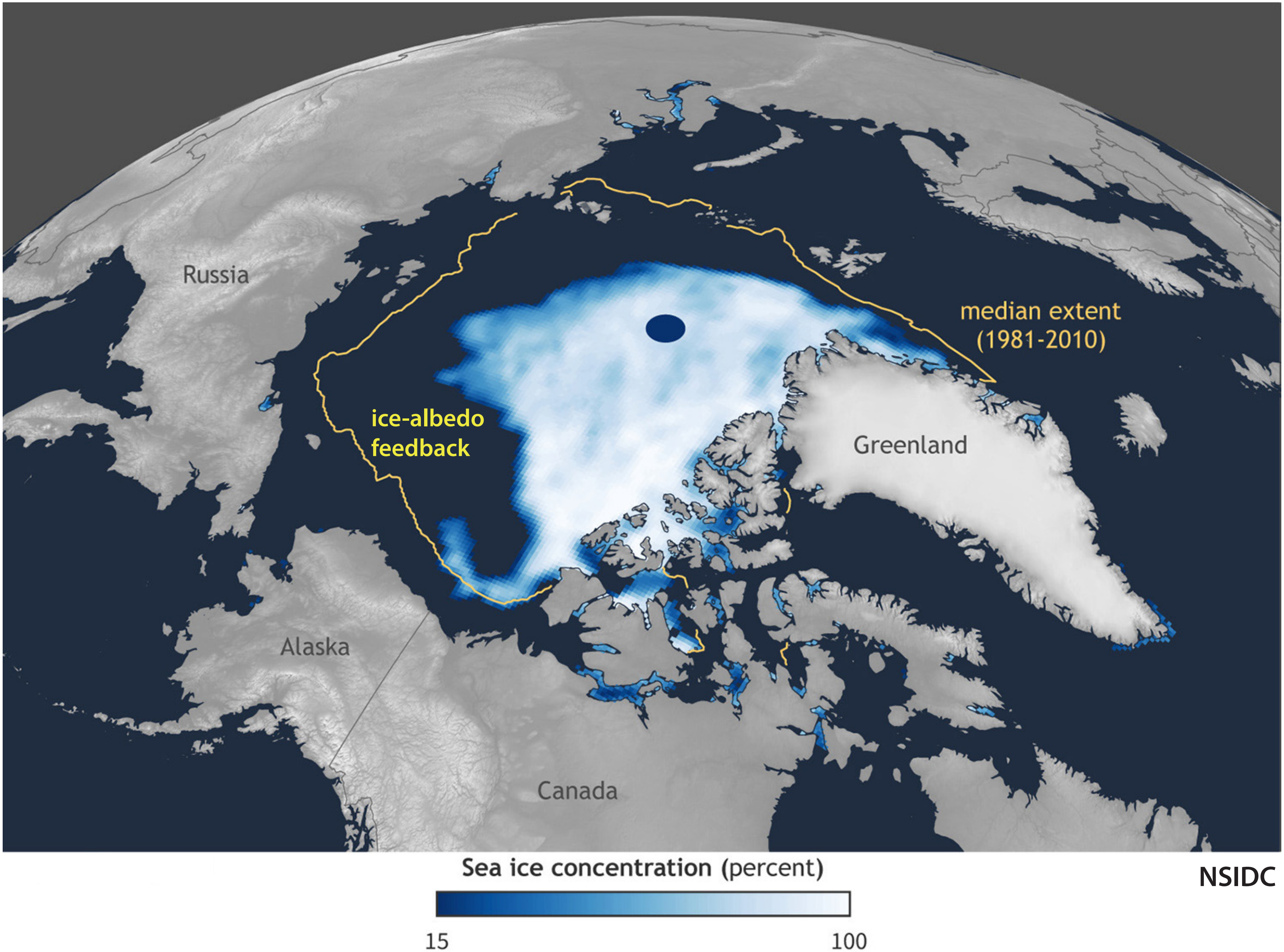
white snow and ice reflect



dark water and land absorb

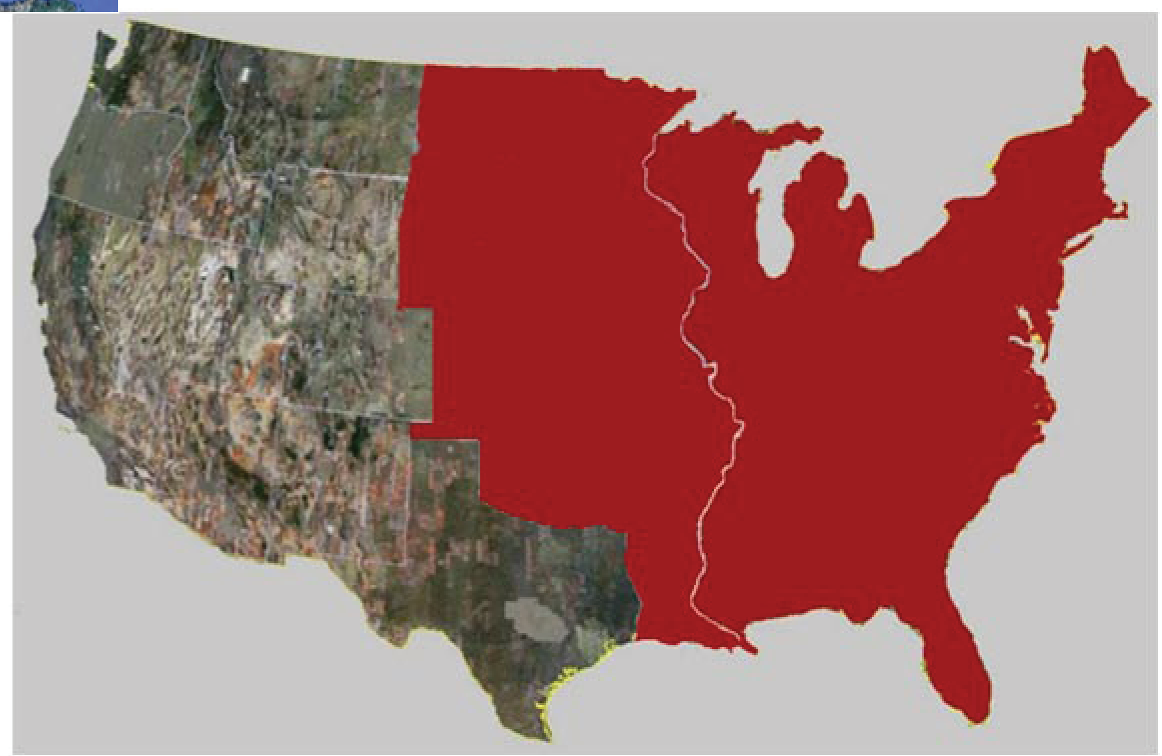
$$\textit{albedo}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Arctic sea ice extent September 15, 2020



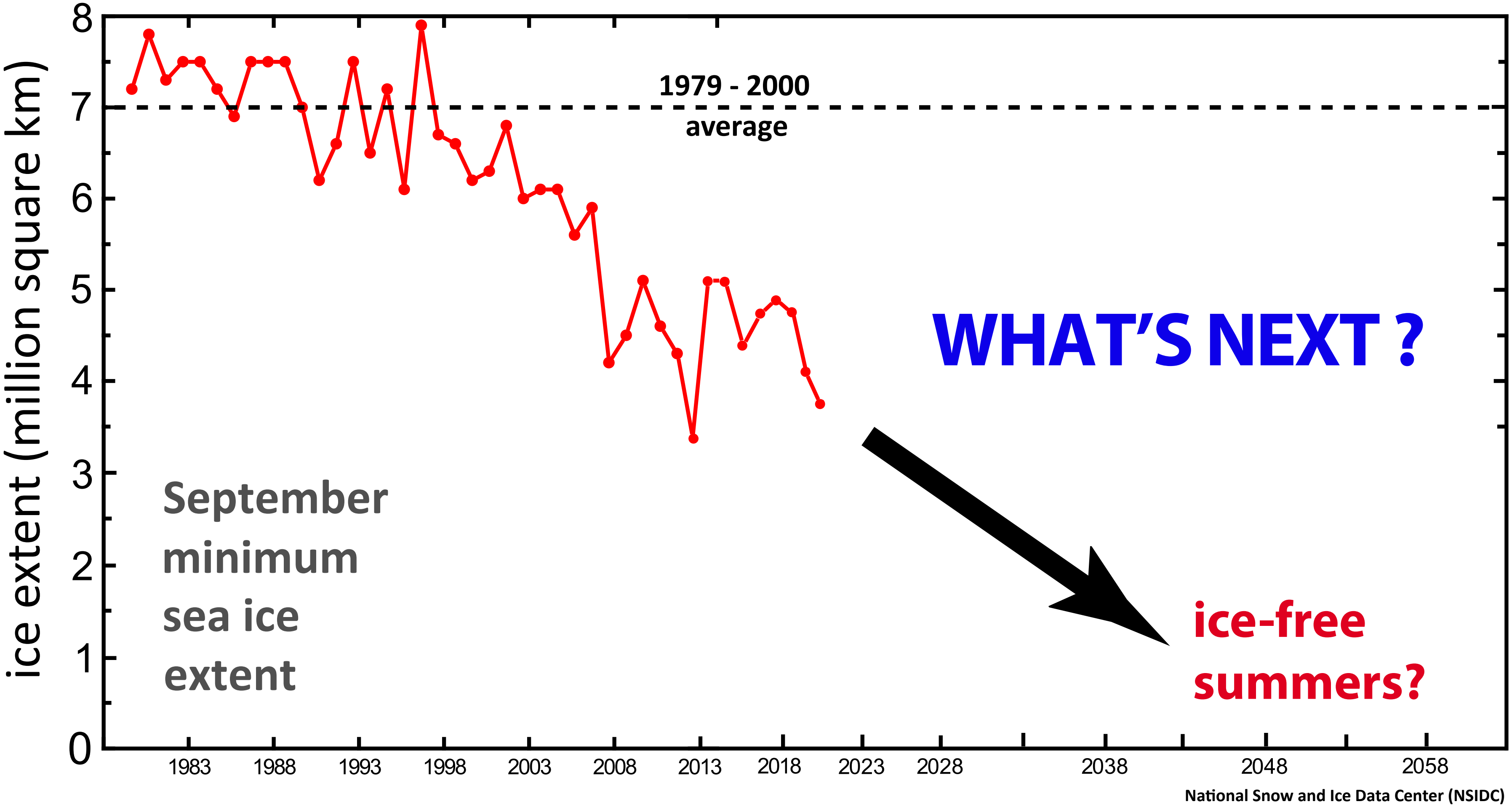
NSIDC

recent losses in comparison to the United States



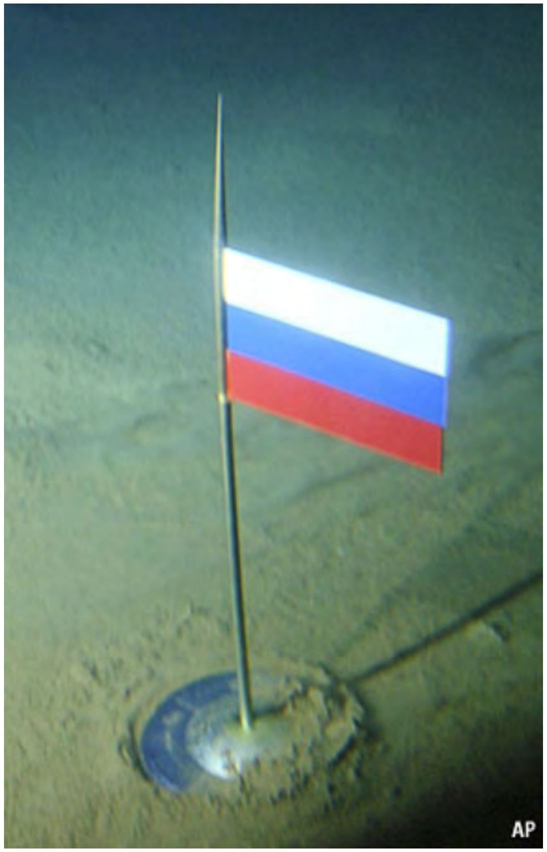
Perovich

ARCTIC summer sea ice loss



predictions require lots of math modeling

As the Arctic Ocean opens up, the United States faces a changing competitive landscape and new threats to national security.



titanium Russian flag on the sea floor at the North Pole, August 2007

record low sea ice extent

Two Russian TU-95 Bears and two Chinese H-6 strategic bombers entered Alaska Air Defense Identification Zone, Bering Sea, July 2024.

Chinese - Russian Joint Naval Operations with 11 warships off the coast of Alaska, August 2023

Russia claims almost 70% of the Arctic Ocean and its natural resources.

Newsweek, December 2023

Natural Resources

USGS: Arctic holds 13% of world's undiscovered oil
30% undiscovered natural gas

Offshore Engineer Magazine

Shipping

Piponwa

Russia & China working together to expand shipping along the Northern Sea Route and build new icebreakers for the line.

atomedia

Military Review

Navigation

The Arctic is warming, but there's still a lot of ice! And it's highly variable.

U.S. Icebreaker Inventory:

1 heavy - the USCGC Polar Star (1976)
1 medium - the USCGC Healy (1999)
1 heavy - the USCGC Polar Sea (decommisioned in 2010)

July 2024 both U.S. icebreakers were sidelined for the near future.

Russian Icebreaker Inventory:

41 including 7 nuclear powered (but many are smaller)

Chinese Icebreaker Inventory: 5

ICE Pact: U.S., Canada, Finland to build and buy lots of icebreakers, announced at July's NATO summit.


Carter, USCG

DOD 2024 Arctic Strategy

The Arctic is "critical to the defense of our homeland."

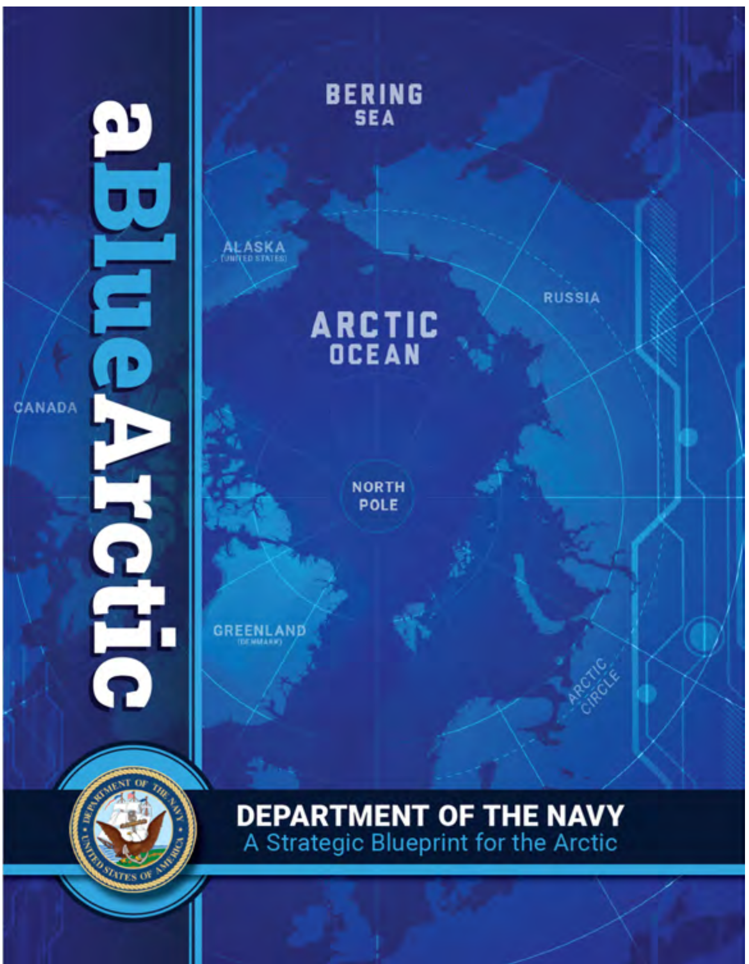
Growing Russian, Chinese military & commercial presence in Arctic with increased cooperation.

These challenges have been amplified because rapidly warming temperatures and thinning ice coverage are enabling this activity.

defense.gov/news


Lt. Pelissero

How to apply naval power as we continue to prepare for a more navigable Arctic Region.



"monitor and respond" - three lines of effort:

1. enhancing capabilities of the joint force
2. greater engagement with allies and partners
3. exercising U.S. presence in the Arctic


U.S. Navy

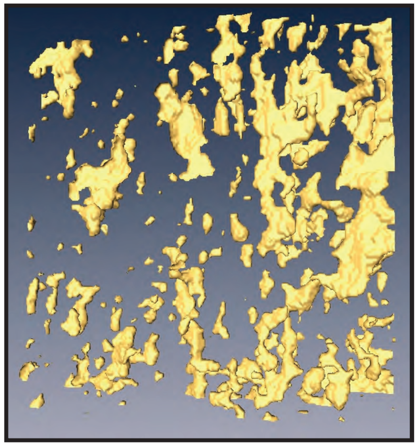
Most operations in the Arctic marine environment involve sea ice and its properties!

Sea Ice is a Multiscale Composite Material

microscale

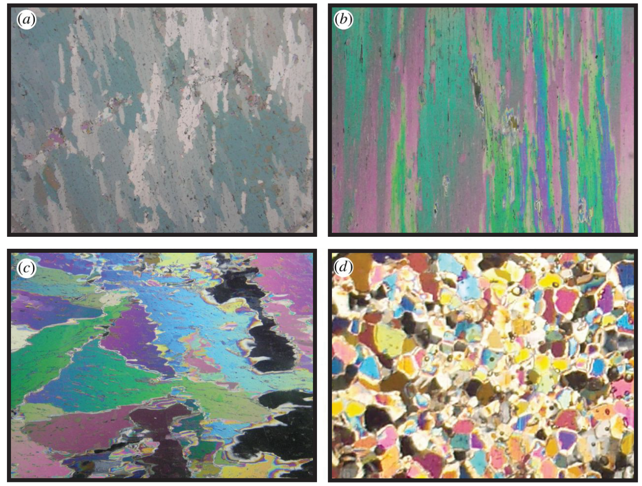
brine inclusions

Weeks & Assur 1969



H. Eicken
Golden *et al. GRL* 2007

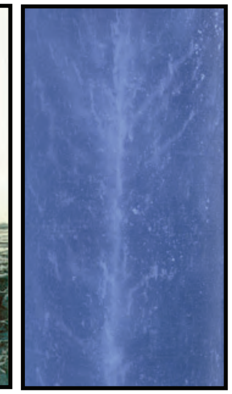
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

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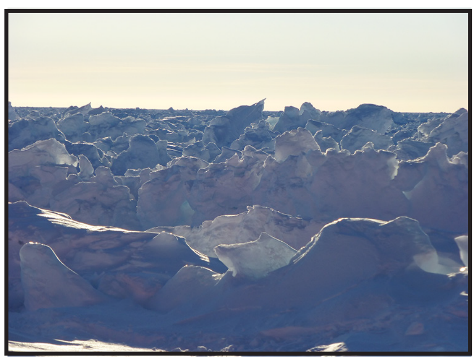
mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges



K. Golden

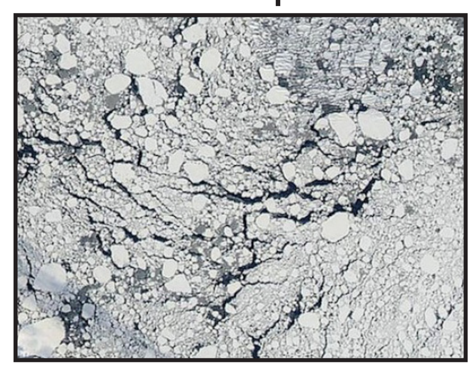
meters

macroscale

sea ice floes

J. Weller

sea ice pack



NASA

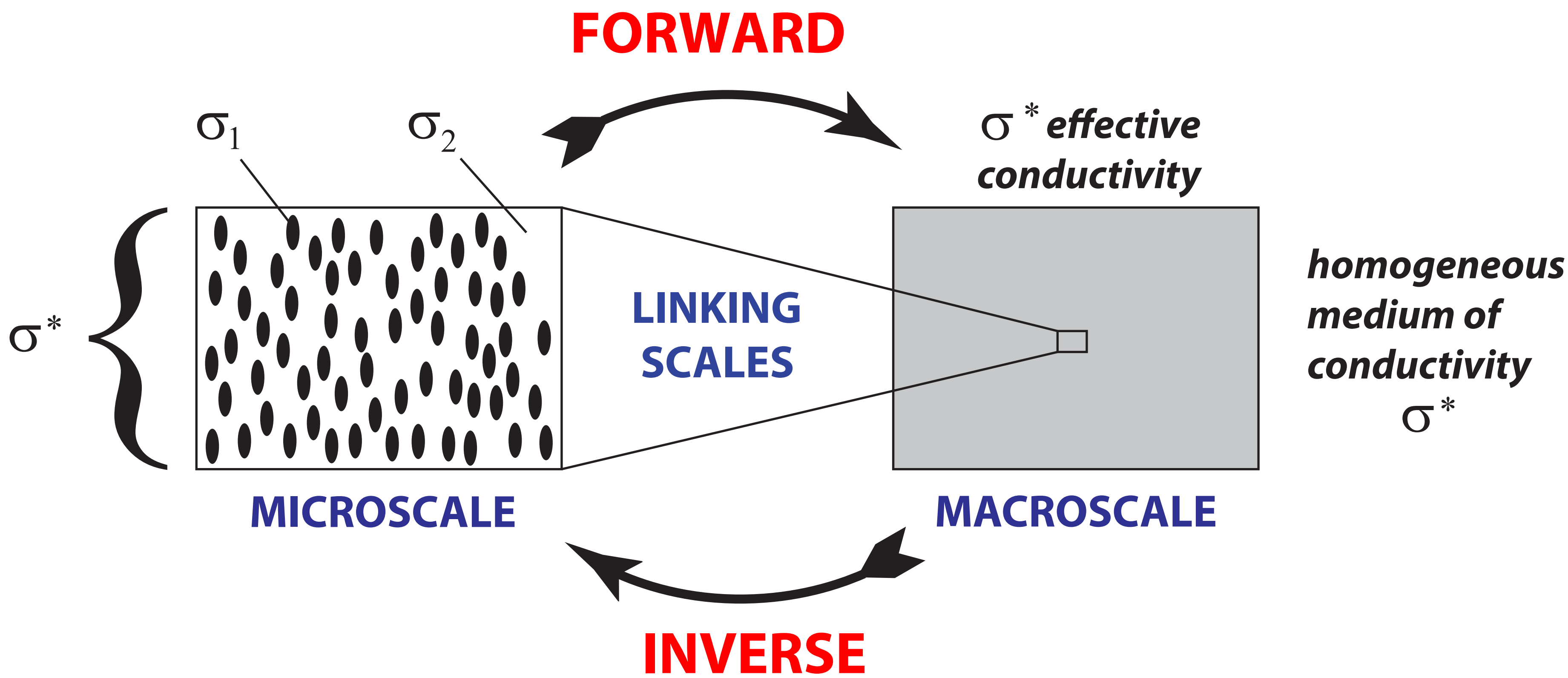
kilometers

Central theme:

How do we use "small scale" information to find effective behavior on larger scales relevant to climate and ecological models?

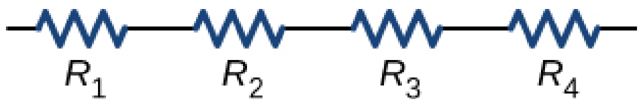
OBJECTIVE: **advance how sea ice is represented in climate models**
improve projections of fate of SEA ICE and its ECOSYSTEMS

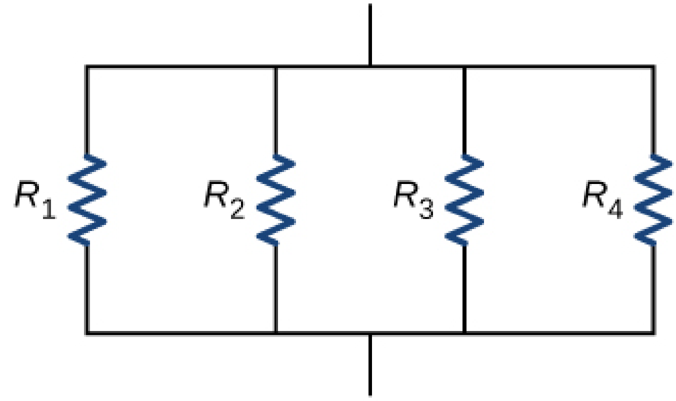
HOMOGENIZATION for Composite Materials



Maxwell 1873, Einstein 1906
Wiener 1912, Hashin and Shtrikman 1962

resistors in series | **resistors in parallel**





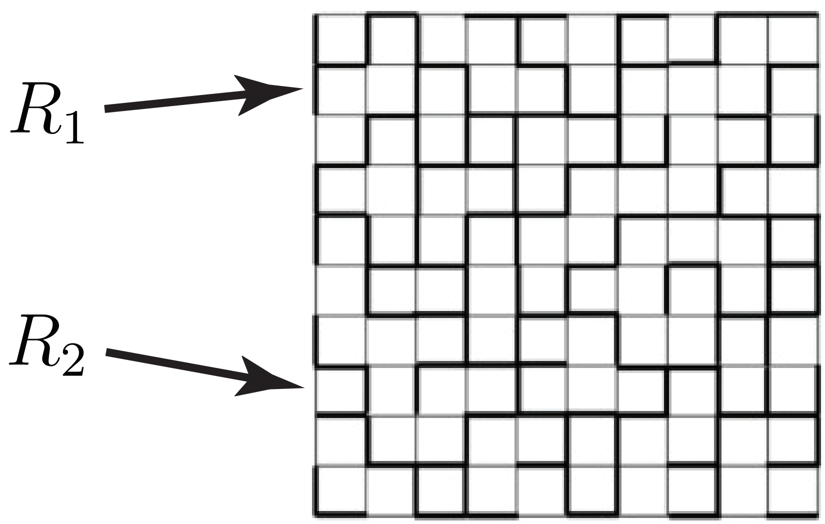
equivalent resistance

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

random resistor network

percolation model



$$R_{eq} = ??$$

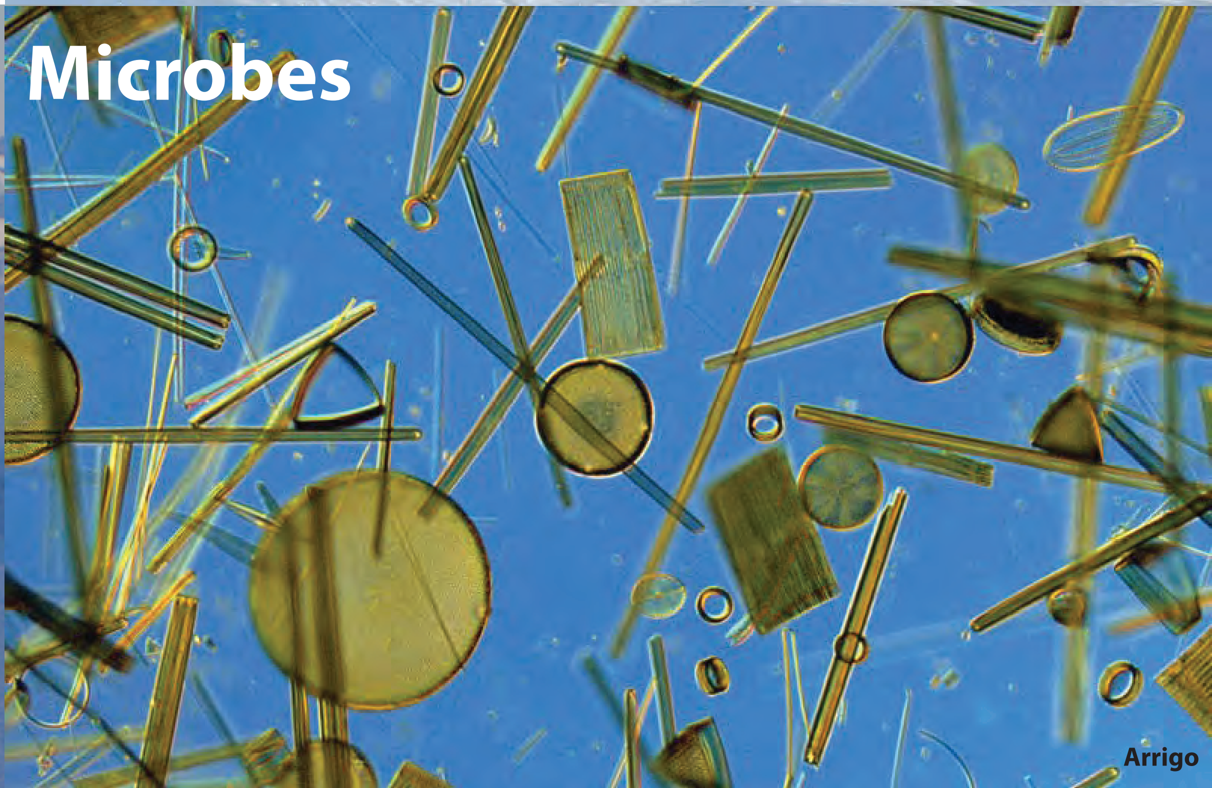
infinite lattice ?

discrete homogenization

Polar Ecology and the Physics of Sea Ice

How do sea ice properties affect the life it hosts?

How does life in and on sea ice affect its physical properties?



Tour a few examples of multiscale modeling of physical and biological processes in the sea ice system.

microscale

mesoscale

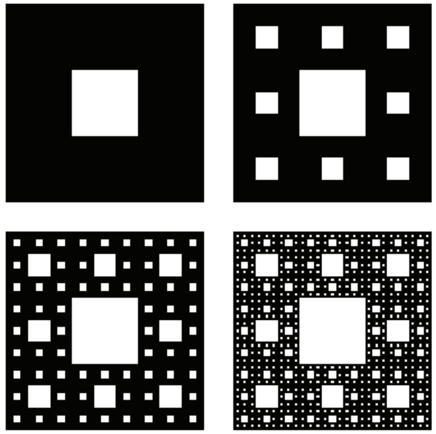
macroscale

Take-away for mathematicians and physicists - our sea ice studies lead us into:

spectral analyis, random matrix theory, topological data analysis, UQ, anomalous diffusion, dynamical systems

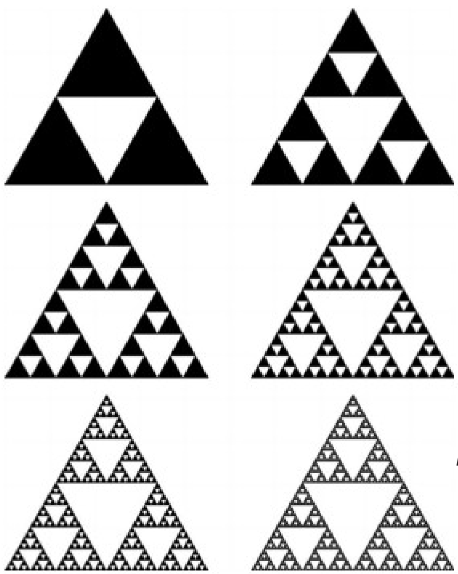
percolation, Anderson localization, mushy layers, phase transitions semiconductors, Ising models, quasicrystals

+ fractal geometry

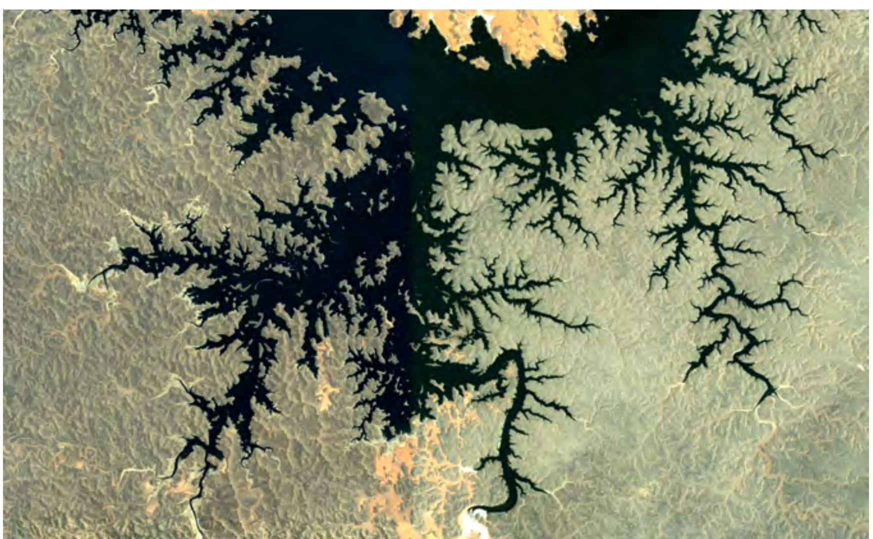


fractals

self-similar structure
non-integer dimension

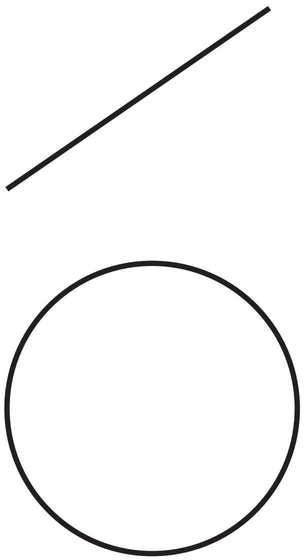


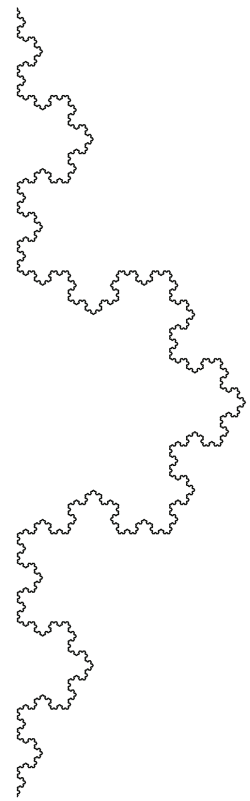
$$D = \frac{\log 3}{\log 2} = 1.585...$$



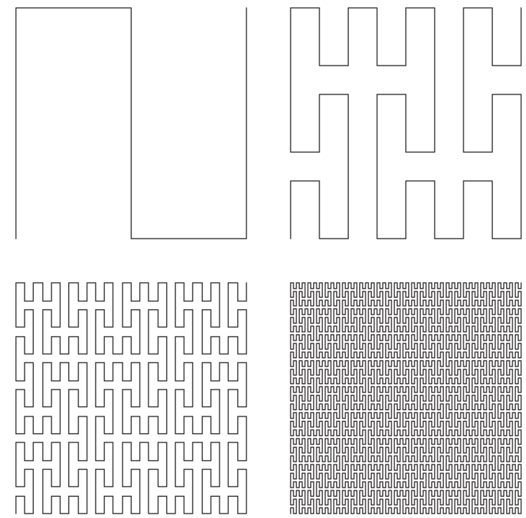
fractal curves in the plane

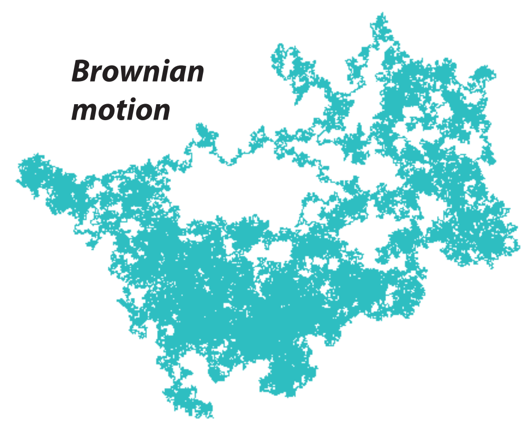
they wiggle so much that their dimension is >1





Peano curve





simple curves

***D* = 1**

Koch snowflake

***D* = 1.26**

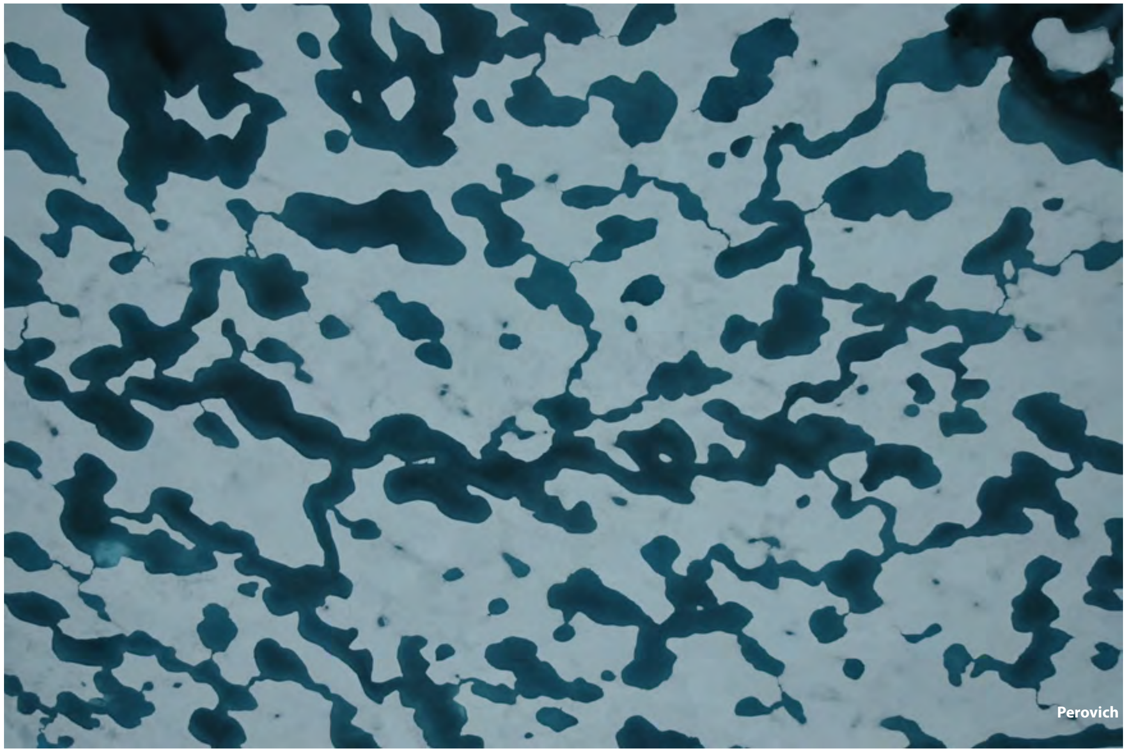
space filling curves

***D* = 2**

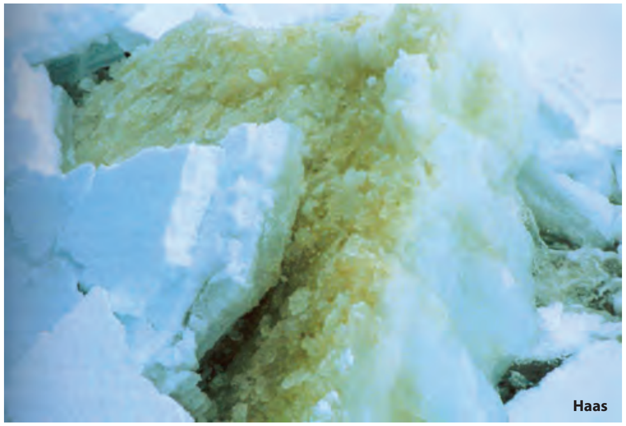
microscale

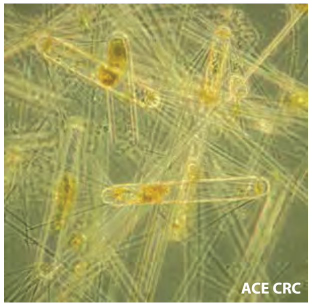
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

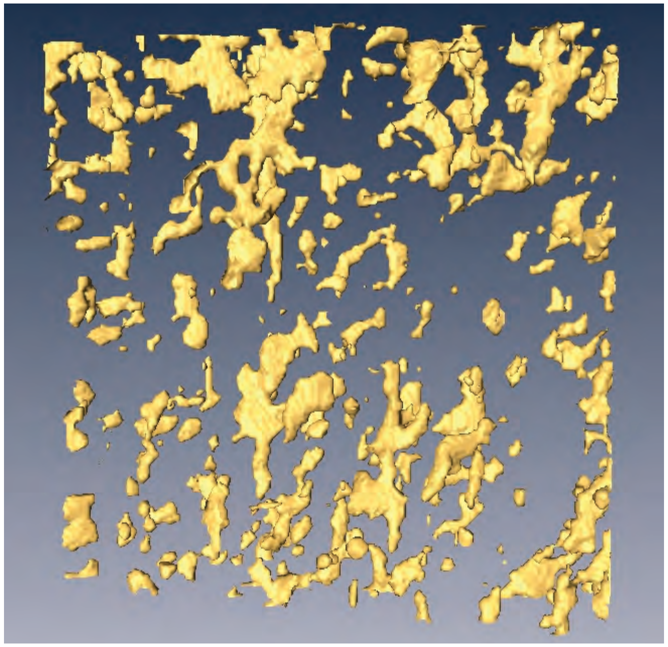


nutrient flux for algal bloom

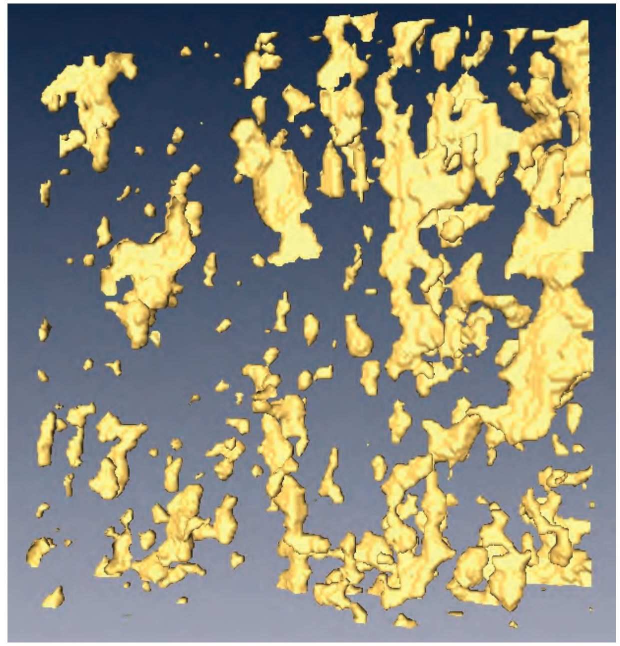




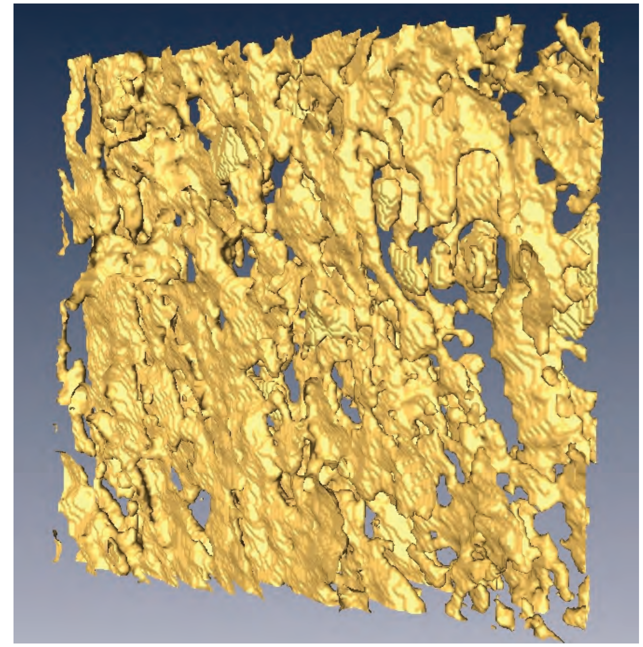
brine volume fraction and ***connectivity*** increase with temperature



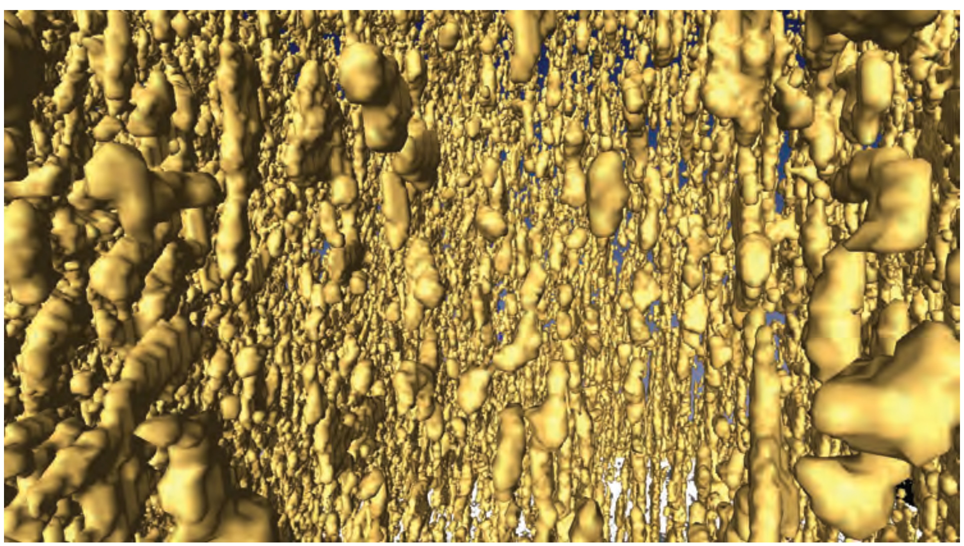
T = -15 °C, $\phi = 0.033$



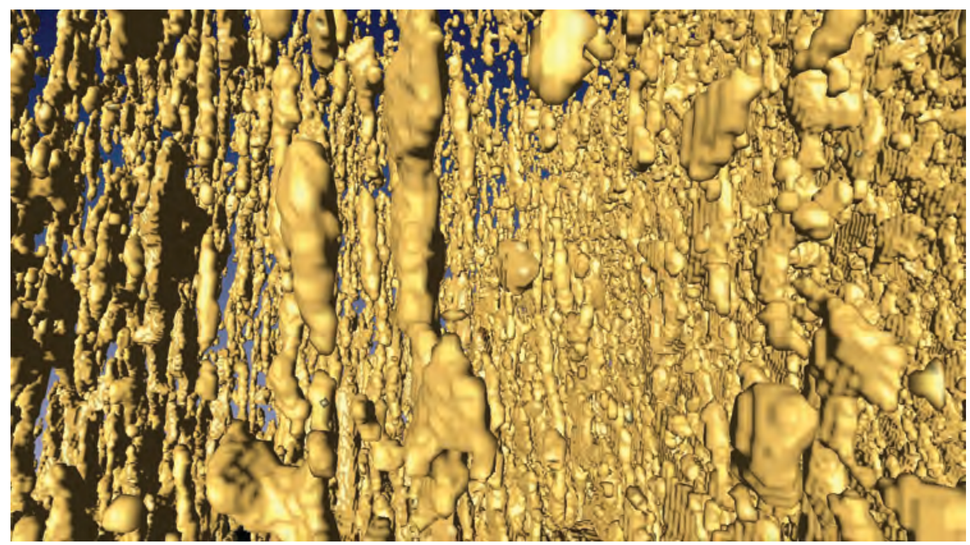
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8 °C, $\phi = 0.057$



T = -4 °C, $\phi = 0.113$

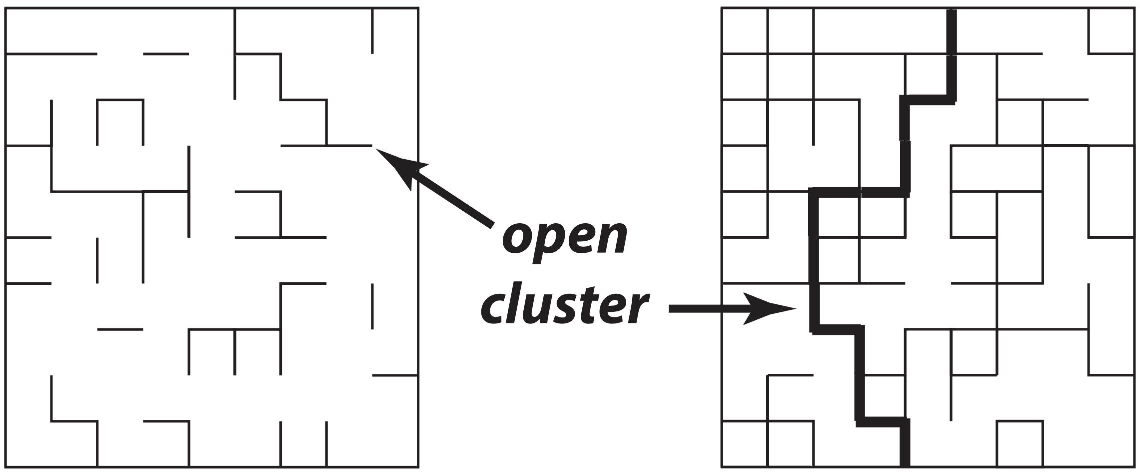
X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

percolation theory

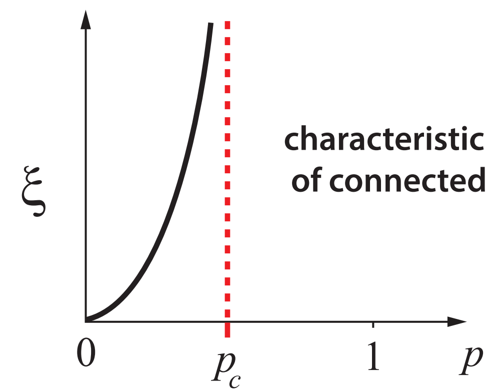
probabilistic theory of connectedness

impermeable | **permeable** | **correlation length**



$p = 1/3$ | $p = 2/3$

development of long range order



percolation threshold

bond ⟶ *open* *with probability* p
closed *with probability* $1\text{-}p$

$$\xi(p) \sim |p - p_c|^{-\nu} \qquad p \to p_c$$

ν **universal: depends only on *d***

p_c depends on type of lattice and *d*

percolation threshold

$$p_c = 1/2 \quad \textbf{for} \;\; \mathbf{d = 2}$$

smallest *p* for which there is an infinite open cluster

Critical behavior of fluid transport in sea ice

Arctic field data

vertical fluid permeability k (m^2)

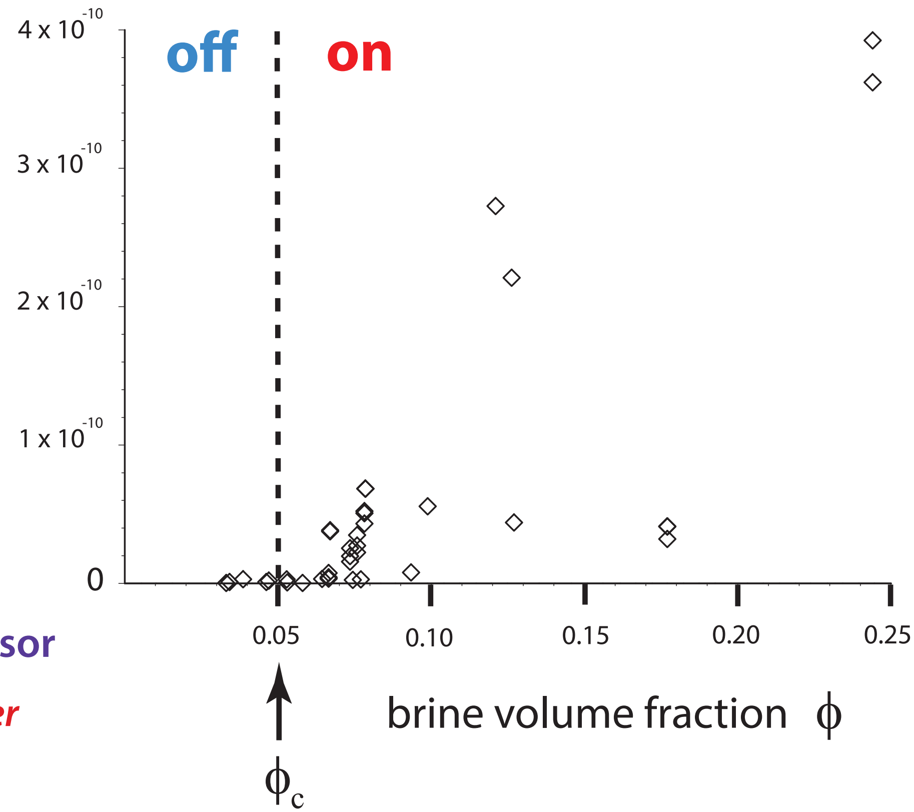
Darcy's Law

$$\mathrm{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

$\mathbf{k}$ = fluid permeability tensor

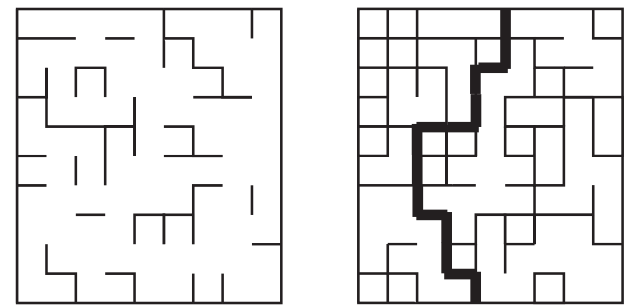
homogenized parameter

impermeable permeable



"on - off" switch for bulk fluid flow

impermeable permeable



$p = 1/3$ $p = 2/3$

lattice percolation

PERCOLATION THRESHOLD $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ \mathrm{C}$, $S \approx 5$ ppt

RULE OF FIVES

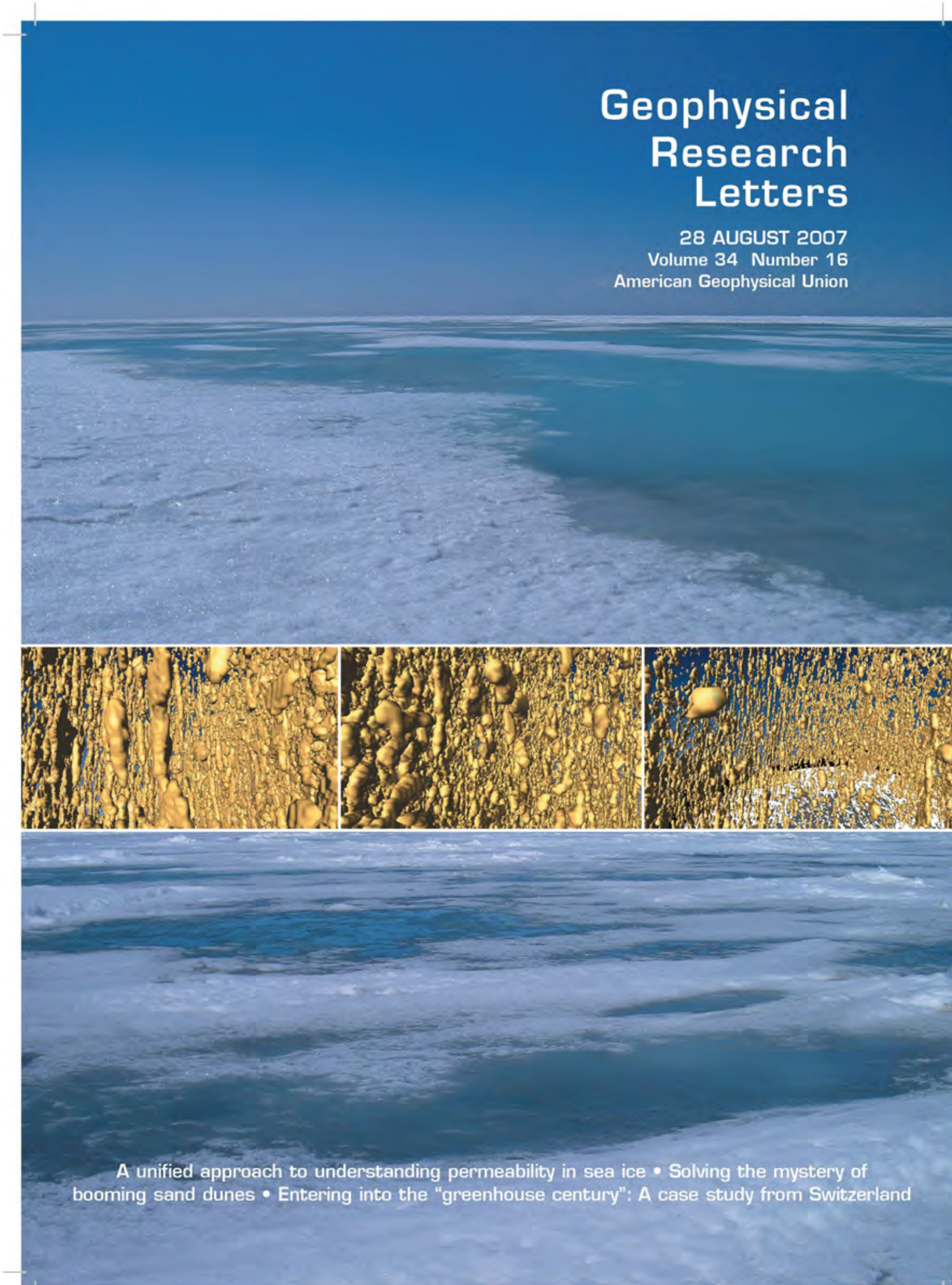
Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, GRL 2007

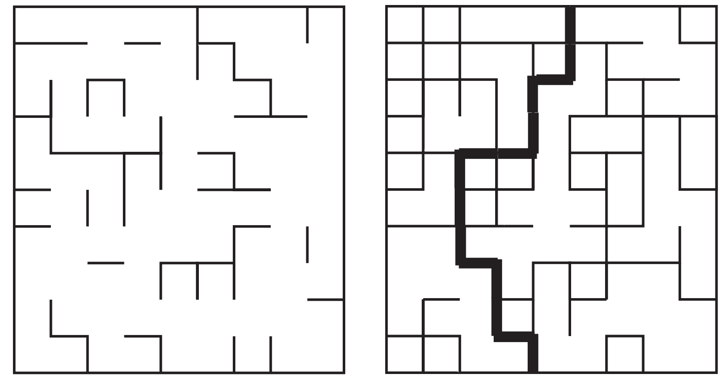
macro

micro



X-ray CT

percolation theory for fluid permeability



percolation threshold, critical exponents

confirms 'rule of fives'

Golden et al., Science 1998

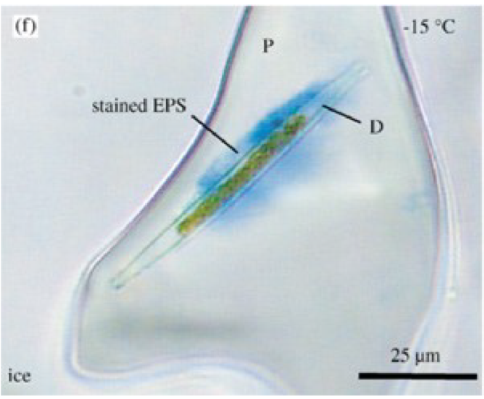
Pringle, Eicken, Miner, Golden
J. Geophys. Res. 2009

theory agrees closely with field data

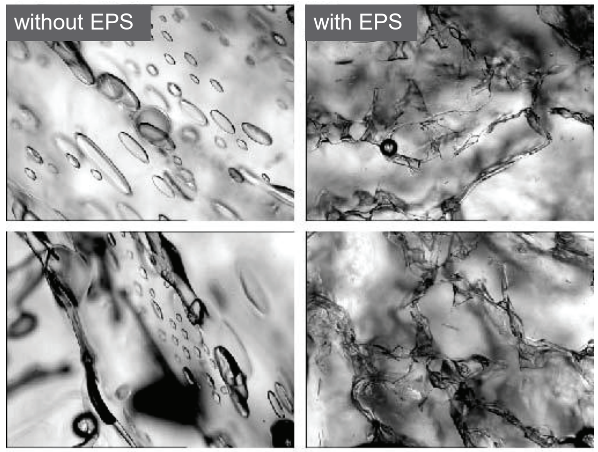
Sea ice algae secrete exopolymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport? **How does the biology affect the physics?**

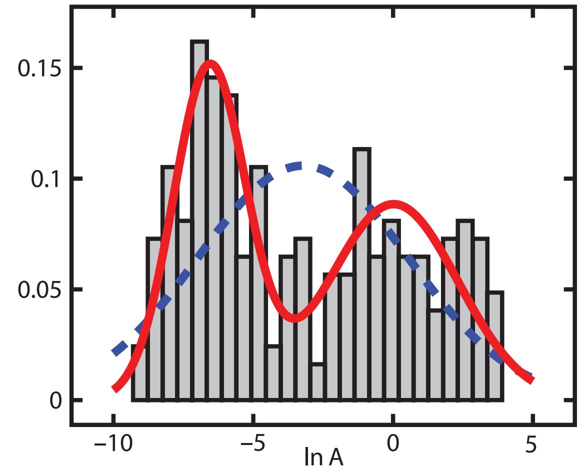
FRACTAL



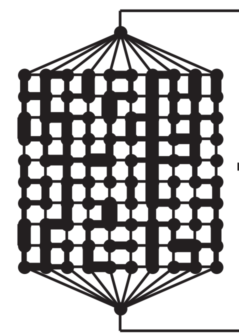
Krembs

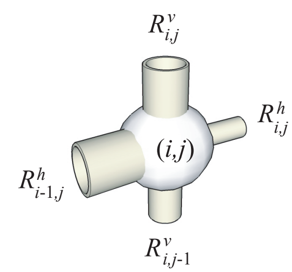


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability *k;* results predict observed drop in *k*

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

***SIAM News* June 2024**

EPS - Algae Model Jajeh, Reimer, Golden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Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and the Mathematics of Transport in Sea Ice

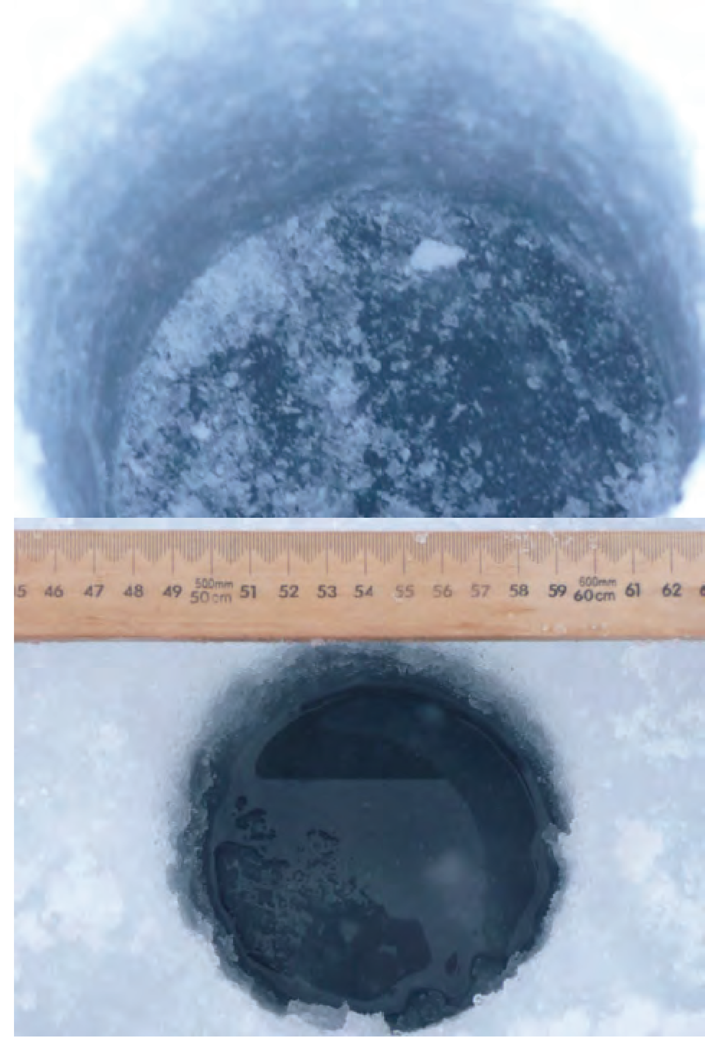
page 562

Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

page 586

Real analysis in polar coordinates (see page 613)

photo by Jan Lieser

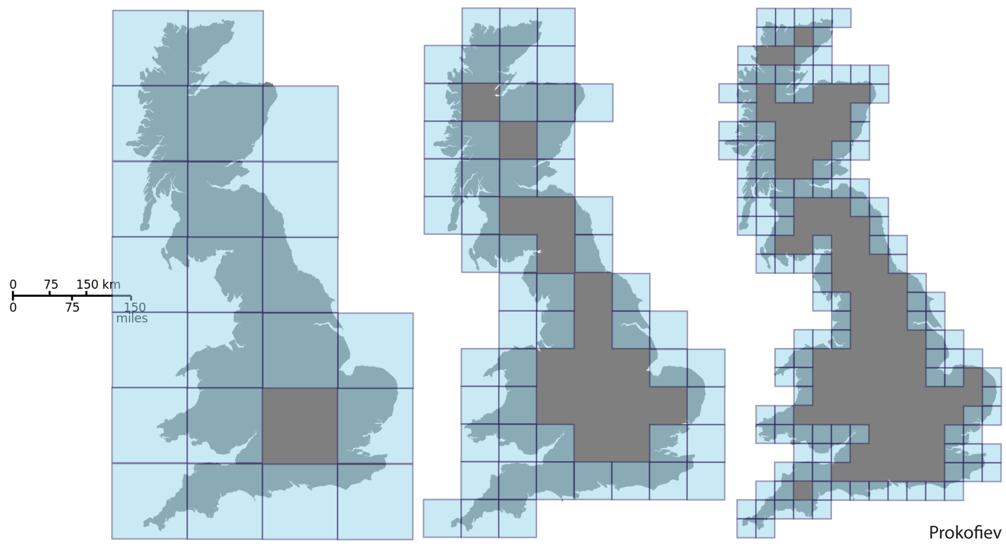


measuring fluid permeability of Antarctic sea ice

SIPEX 2007

Thermal Evolution of Brine Fractal Geometry in Sea Ice

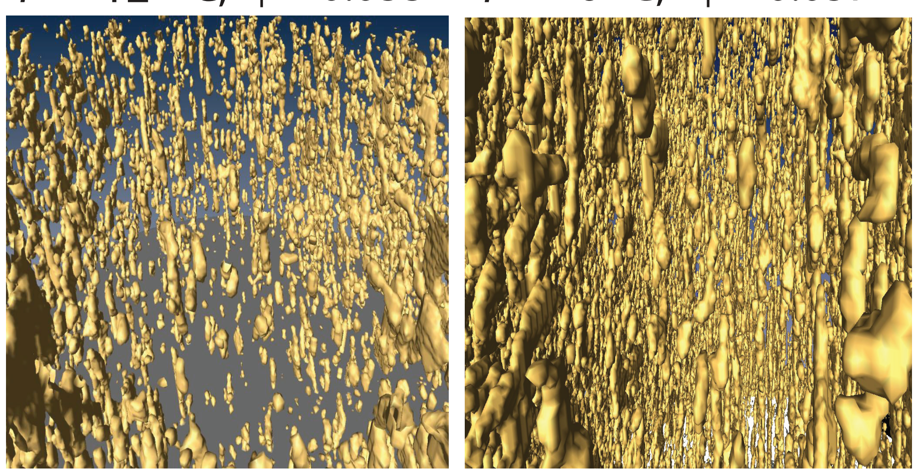
Nash Ward, Daniel Hallman, Benjamin Murphy, Jody Reimer,
Marc Oggier, Megan O'Sadnick, Elena Cherkaev and Kenneth Golden, 2025



fractal dimension of the coastline of Great Britain by box counting

$$N(\epsilon) \sim \epsilon^{-D}$$

$T = -12°$ C, $\phi = 0.033$ $T = -8°$ C, $\phi = 0.057$



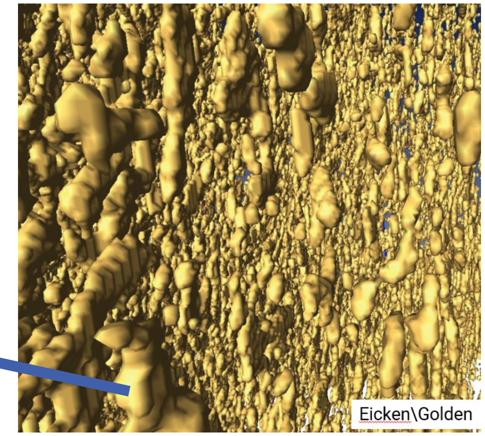
Golden, Eicken, et al. *GRL*, 2007

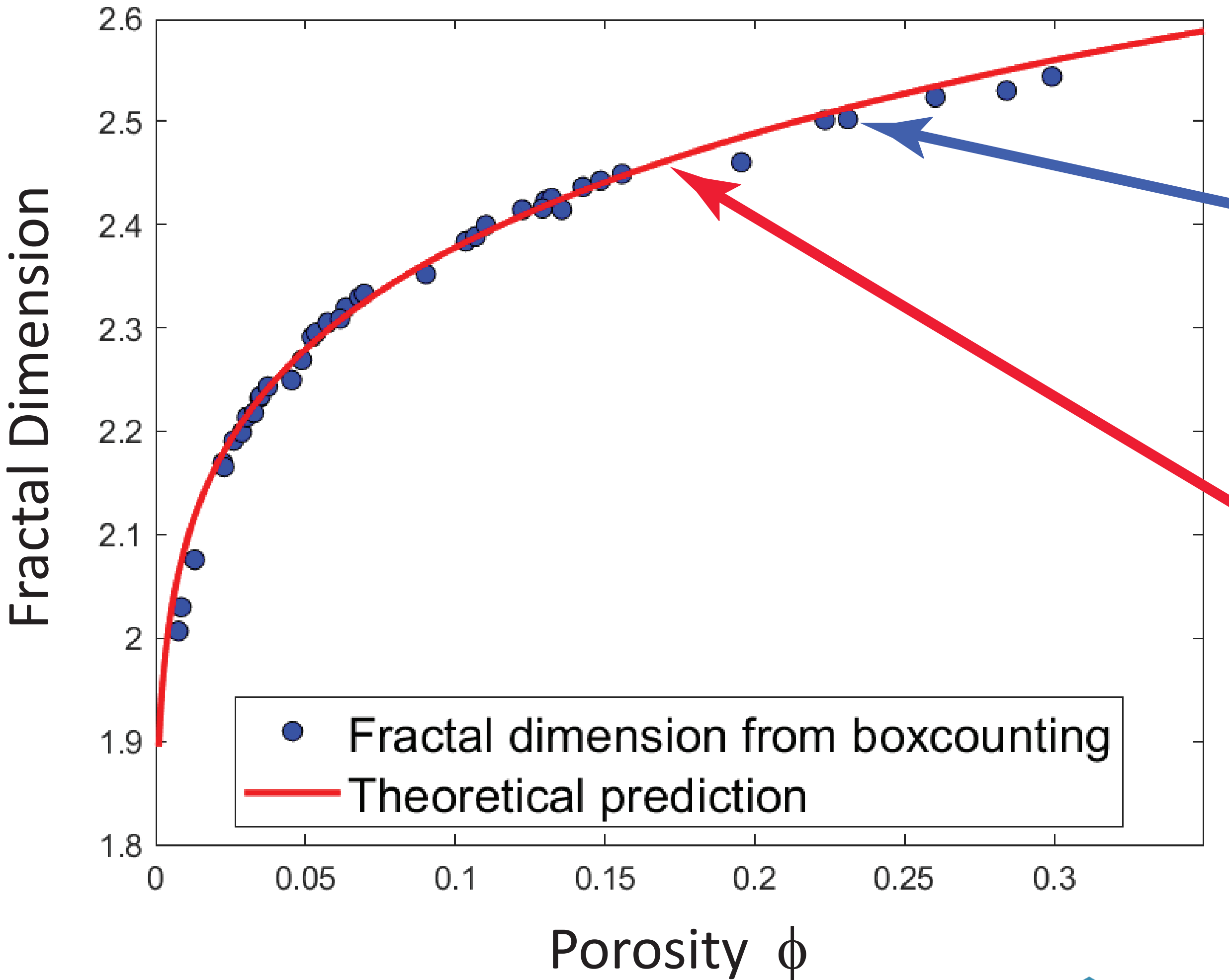
brine channels and inclusions "look" like fractals (from 30 yrs ago)

X-ray computed tomography of brine in sea ice

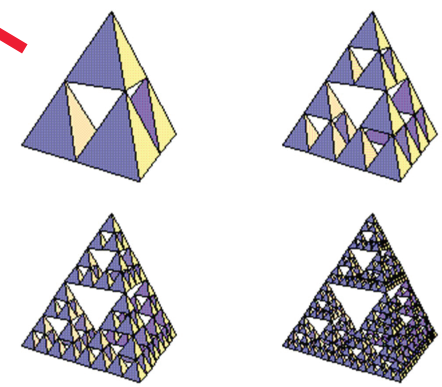
columnar and granular

The first quantitative study of the fractal dimension of brine in sea ice and its strong dependence on temperature and porosity.





Follows same curve as exactly self-similar Sierpinski tetrahedron

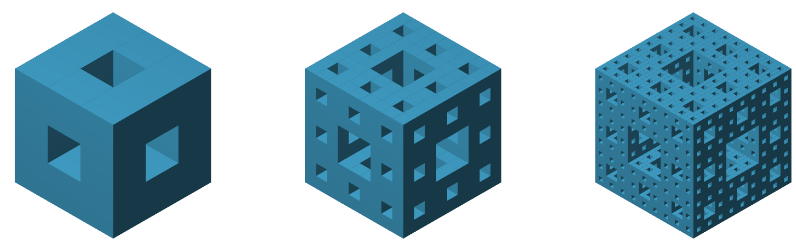


D. Eppstein

red curve $$F_d = d_E - \frac{\ln \phi}{\ln(\lambda_{min}/\lambda_{max})}$$

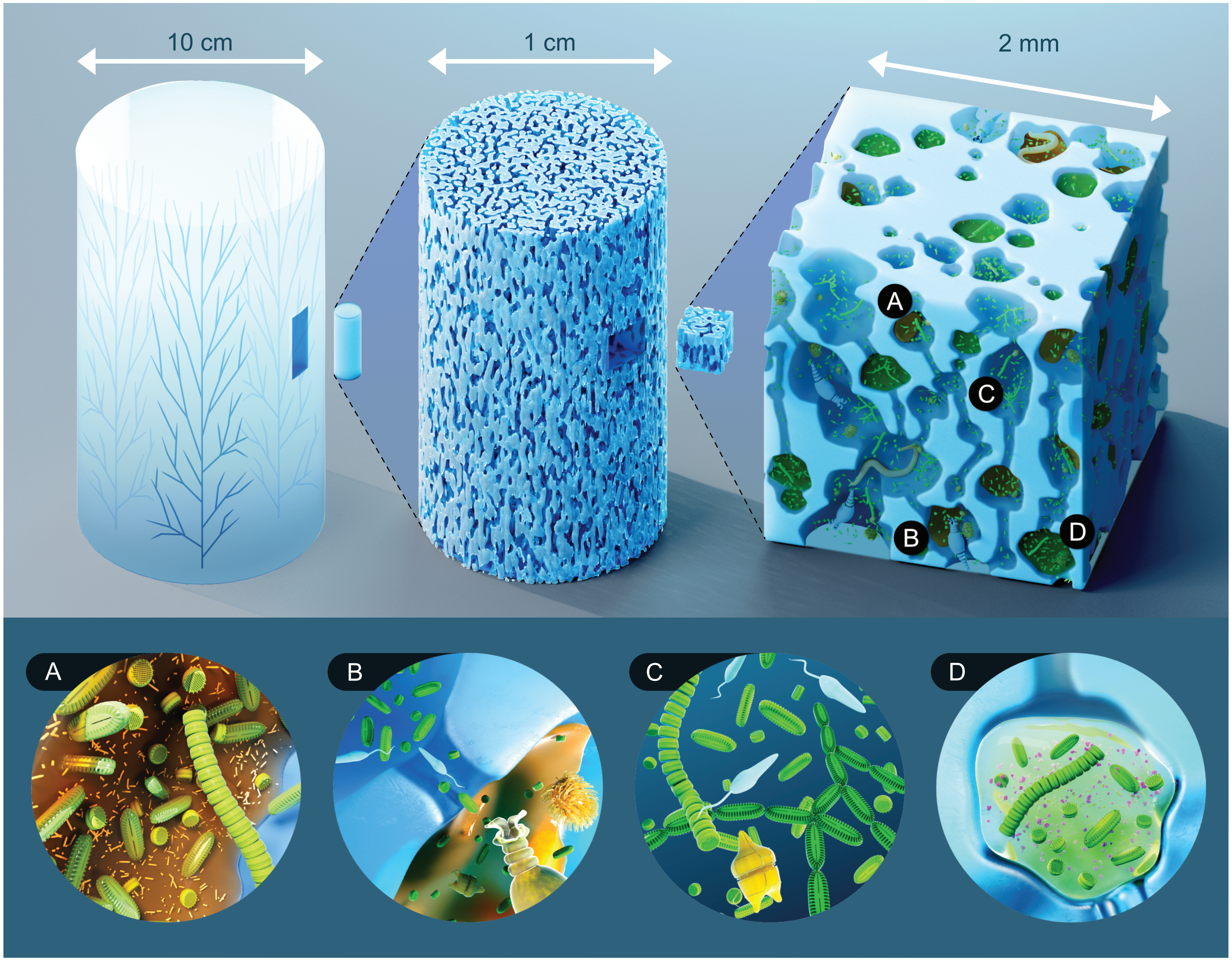
Katz and Thompson,1985; Yu and Li, 2001

discovered for sandstones
statistically self-similar porous media



Fractal geometry of brine in sea ice, Ward, *et al.* 2025

Implications of brine fractal geometry on sea ice ecology and biogeochemistry



Brine inclusions are home to ice endemic organisms, e.g., bacteria, diatoms, flagellates, rotifers, nematodes.

The habitability of sea ice for these organisms is inextricably linked to its complex brine geometry.

(A) Many sea ice organisms attach themselves to inclusion walls; inclusions with a higher fractal dimension have greater surface area for colonization.

(B) Narrow channels prevent the passage of larger organisms, leading to refuges where smaller organisms can multiply without being grazed, as in (C).

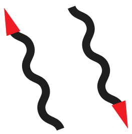
(D) Ice algae secrete extracellular polymeric substances (EPS) which alter incusion geometry and may further increase the fractal dimension.

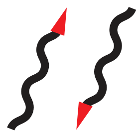
Golden

Remote Sensing of Sea Ice

with radar, microwaves, ...

interaction of EM waves with brine and polycrystalline microstructures, rough surfaces

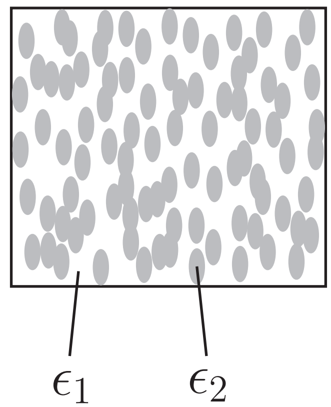




INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data ε^*

Effective complex permittivity of a composite in the quasistatic (long wavelength) limit



$$\Big\} \; \epsilon^*$$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

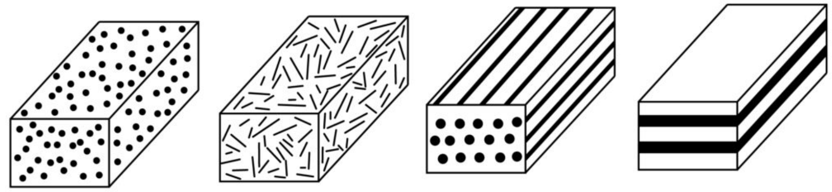
p_1 , p_2 = volume fractions of the components

electrical conductivity
thermal conductivty
magnetic permeability
diffusivity

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \;,\; \text{composite geometry} \right)$$

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

complexities of mixture geometry



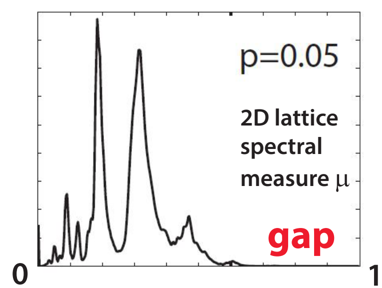
distilled

distilled

Analytic Continuation Method

Stieltjes Integral Representations
for Homogenized Parameters

Bergman 1978, Milton 1979
Golden & Papanicolaou 1983
Milton 2002



spectral properties of operator (matrix)
Hamiltonian, quantum states, energy levels for atoms

eigenvectors **eigenvalues**

EXTEND to: polycrystals, advection diffusion, waves through ice pack

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Proc. Roy. Soc. A   8 Feb 2015

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

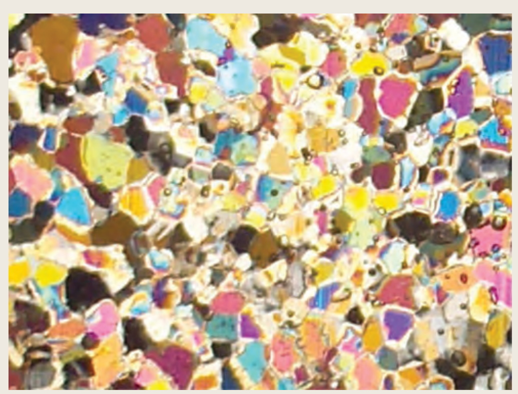
  Milton (1981, 2002), Barabash and Stroud (1999), ...

- **Forward and inverse bounds**

  orientation statistics

- **Applied to sea ice using two-scale homogenization**

- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



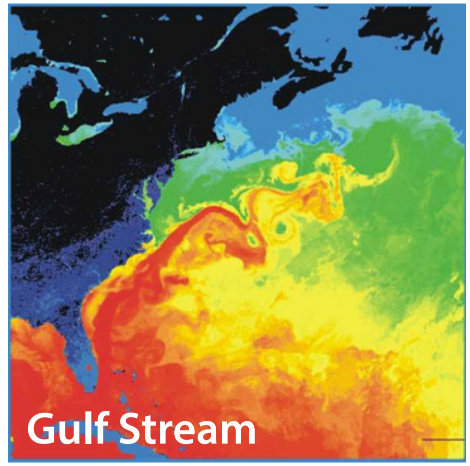
21/01/15 1:47 PM

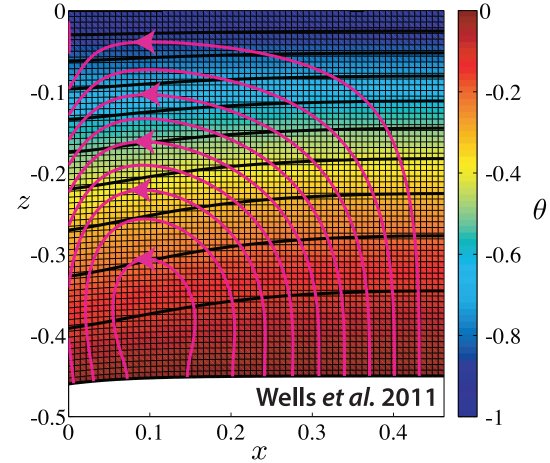
mesoscale

advection enhanced diffusion

HOMOGENIZE

heat transport in sea ice with convection
nutrient and salt transport in sea ice
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies





advection diffusion equation with velocity field $\vec{u}$

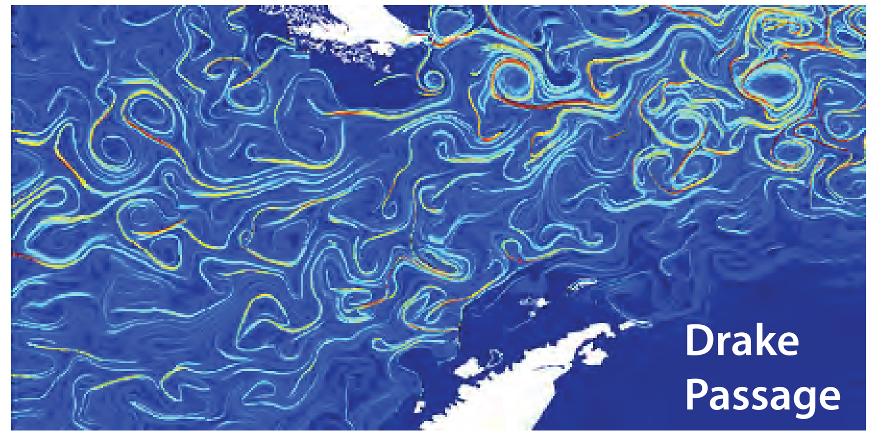
$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize

heat equation

$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$



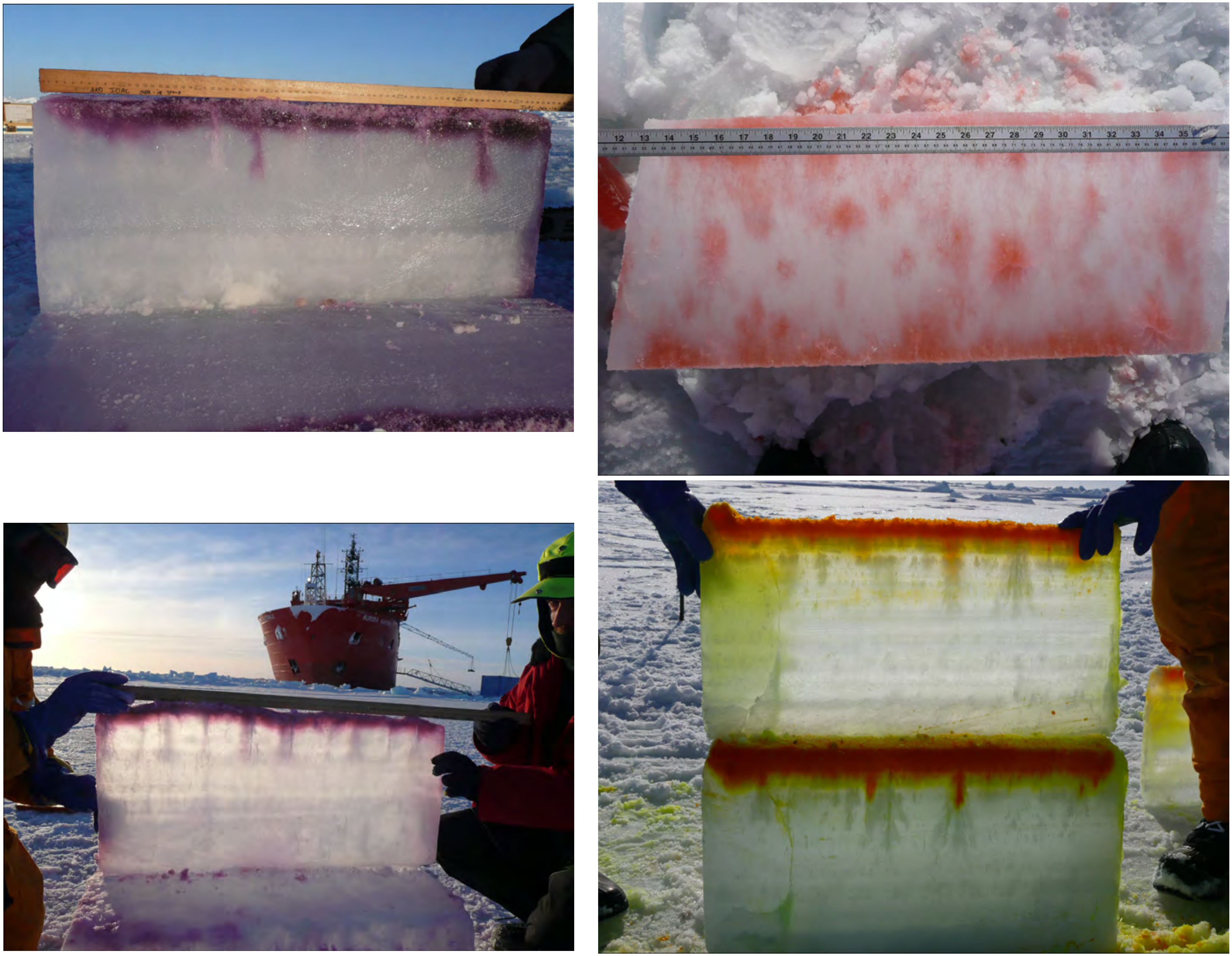
κ^* ***effective diffusivity***

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017
Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020

tracers flowing through inverted sea ice blocks



PROCEEDINGS
OF THE ROYAL SOCIETY A

MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

Homogenization for convection-enhanced thermal transport in sea ice

N. Kraitzman, R. Hardenbrook,
H. Dinh, N. B. Murphy, E. Cherkaev,
J. Zhu and K. M. Golden

August 2024

First rigorous mathematical theory of thermal conductivity of sea ice with convective fluid flow; captures data.

missing in climate models

ocean wave propagation through the sea ice pack

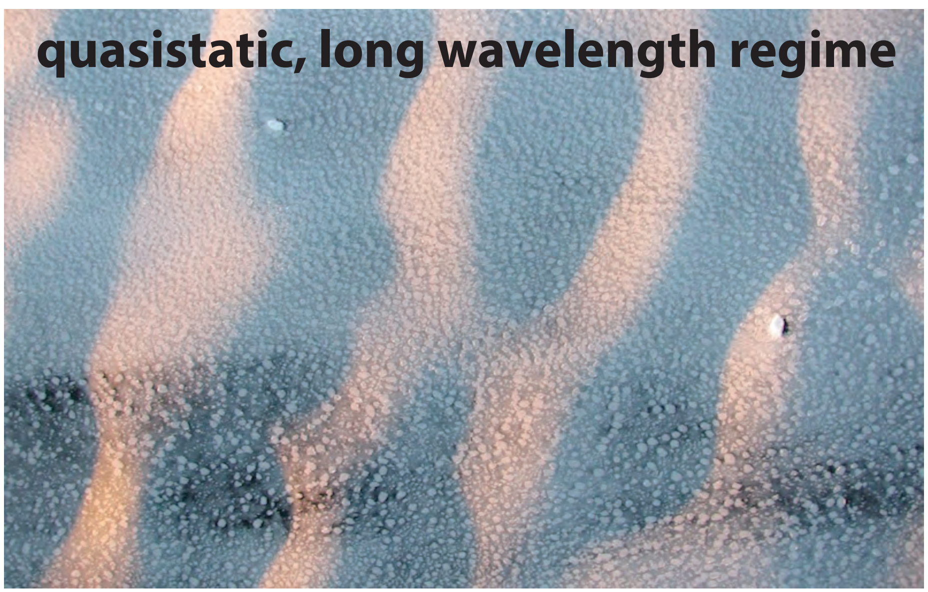
- wave-ice interactions critical to growth and melting processes
- break-up; pancake promotion floe size distribution

HOMOGENIZE
ice-ocean composite

find effective complex viscoelasticity

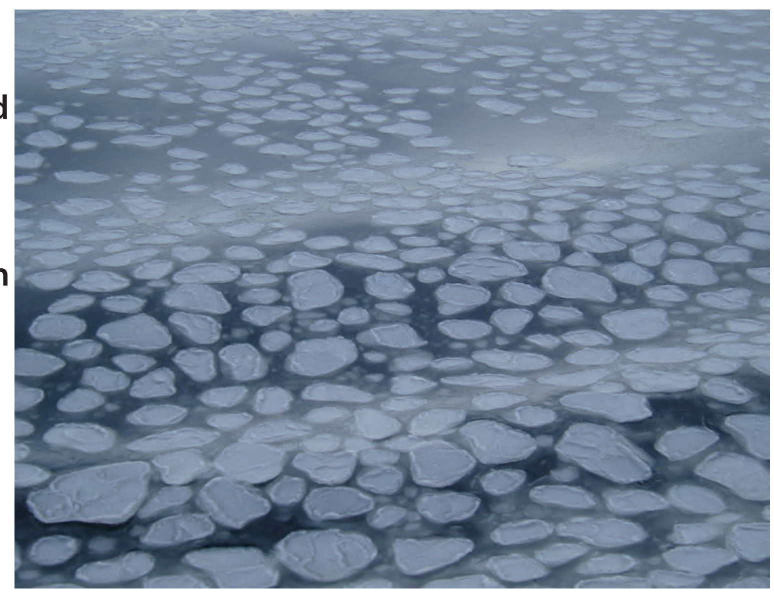
[Anderson localization]

Keller 1998
Mosig, Montiel, Squire 2015
Wang, Shen 2012



homogenized parameter depends on sea ice concentration and ice floe geometry

like EM waves

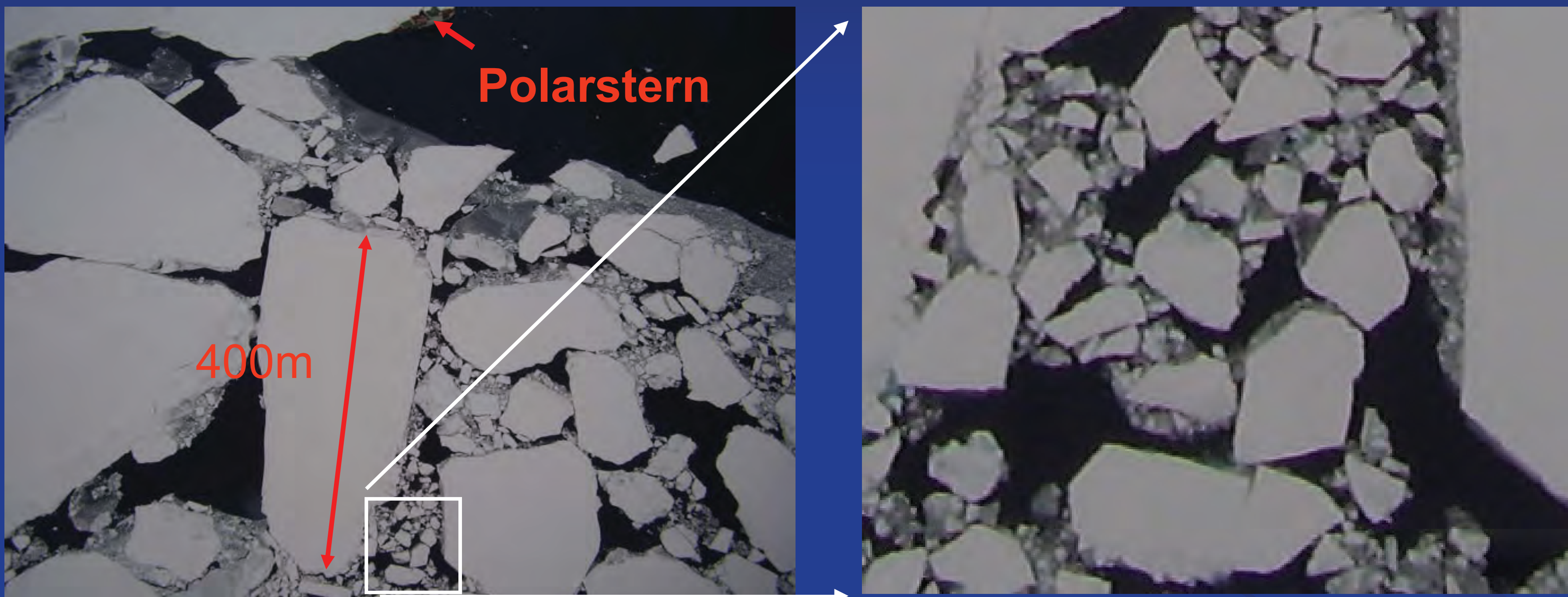


Sampson, Murphy, Hallman, Cherkaev, Golden 2025

The sea ice pack has fractal structure.

Self-similarity of sea ice floes

Weddell Sea, Antarctica



fractal dimensions of Okhotsk Sea ice pack
smaller scales D~1.2, larger scales D~1.9

fractal dim. *vs.* floe size exponent
Adam Dorsky, Nash Ward, Ken Golden 2025

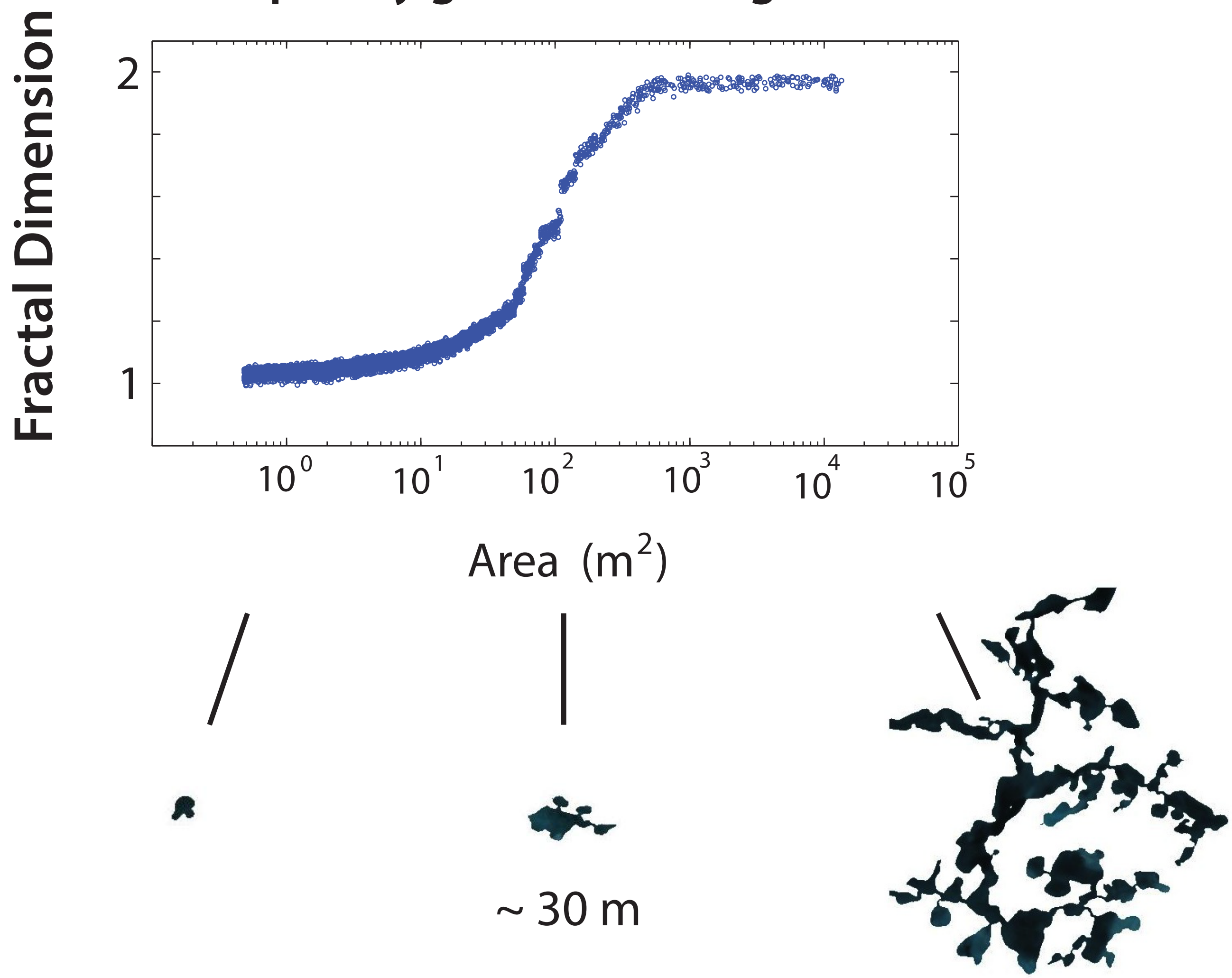
Toyota, *et al. Geophys. Res. Lett.* 2006
Rothrock and Thorndike, *J. Geophys. Res. 1984*

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

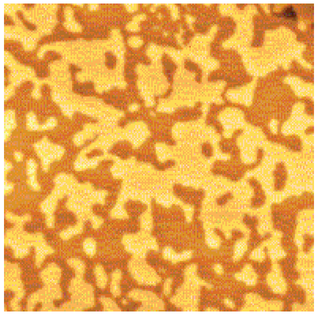
complexity grows with length scale



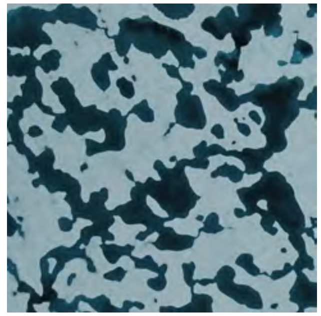
simple pond ***transitional pond*** ***complex pond***

From magnets to melt ponds

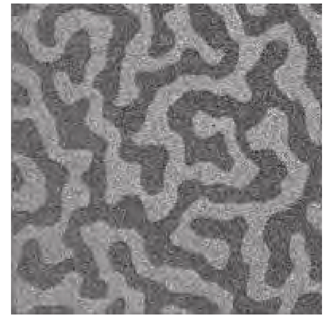
100 year old model for magnetic materials used to explain melt pond geometry



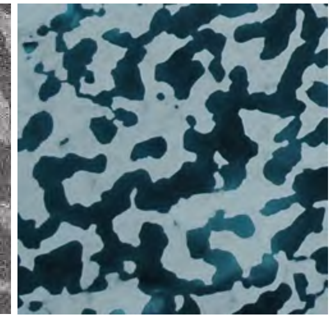
magnetic domains in cobalt



Arctic melt ponds

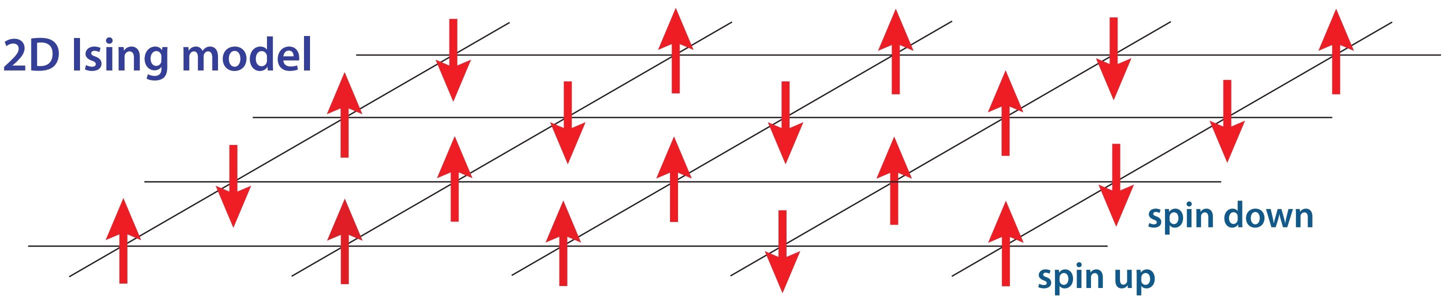


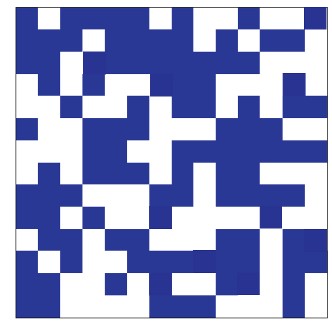
magnetic domains in cobalt-iron-boron



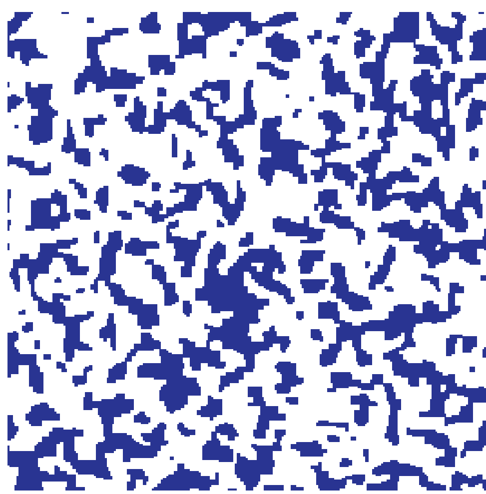
Arctic melt ponds

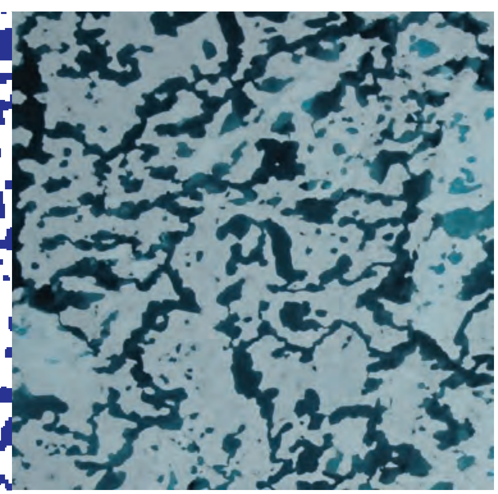
2D Ising model





model



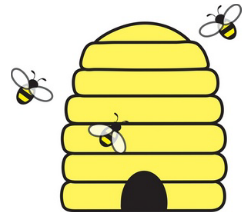


real ponds (Perovich)

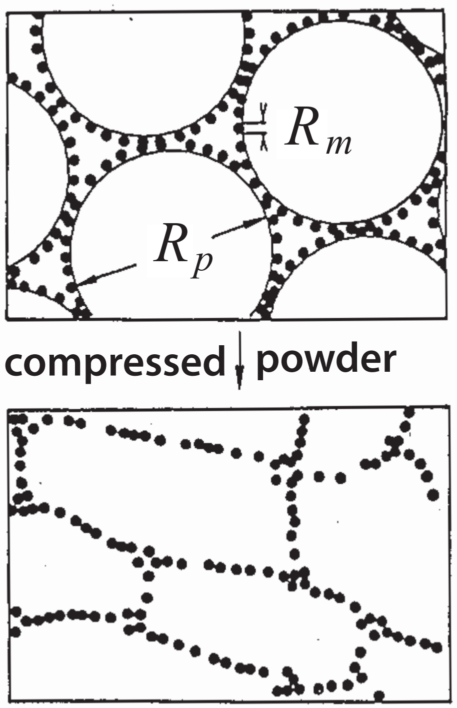
Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Scientific American, EOS, PhysicsWorld, ...

cross pollination

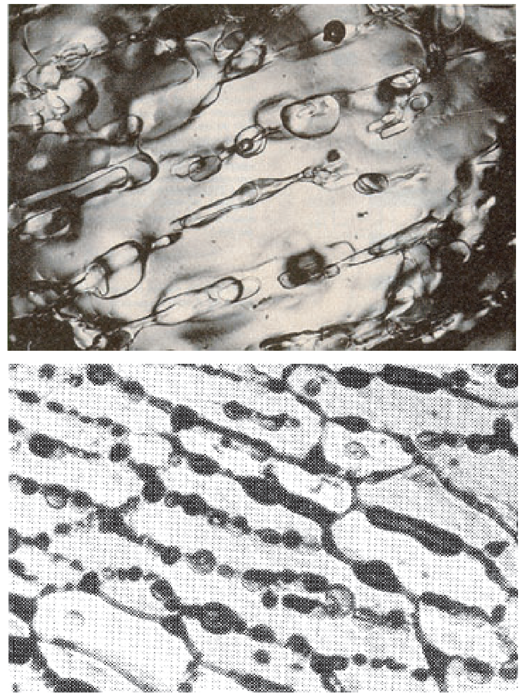


The math doesn't care if it's sea ice or bone!



radar absorbing coating

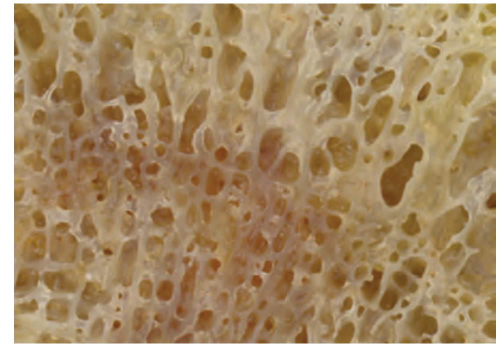
Kusy & Turner
Nature 1971



sea ice

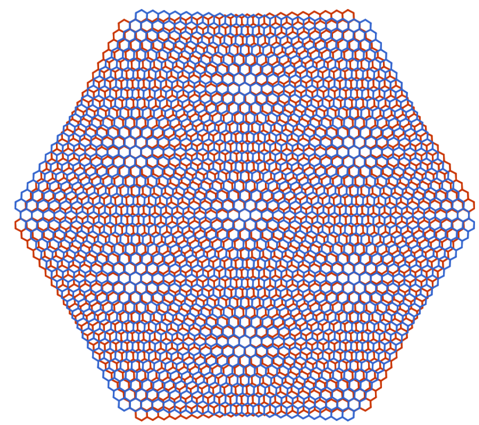
Golden, Ackley, Lytle
Science 1998

Rule of Fives
fluid flow



human bone

Golden, Murphy, Cherkaev
J. Biomechanics 2011



twisted bilayer composites

Morison, Murphy, Cherkaev, Golden
Communications Physics 2022

stealth technology, climate science, medical imaging, twistronics

macroscale

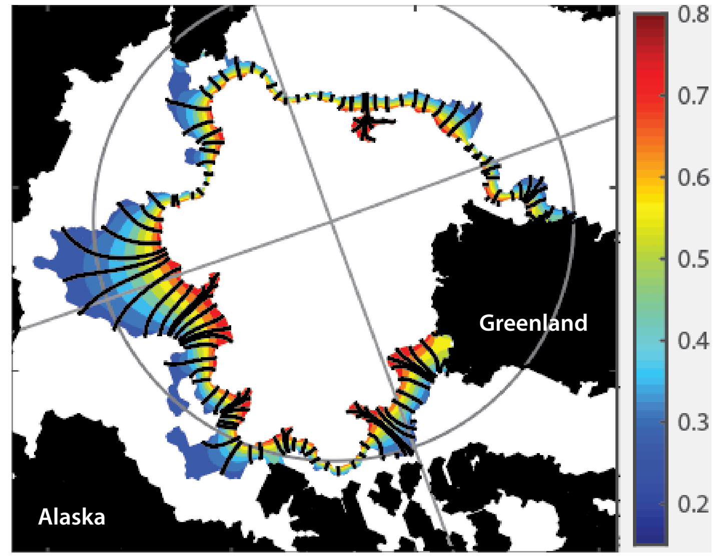
Marginal Ice Zone (MIZ)

transitional region between

dense interior pack $\Psi > 0.8$

sparse outer fringes $\Psi < 0.15$

- biologically active region
- intense wave-ice interactions
- strong air-ice-ocean exchanges



MIZ WIDTH

fundamental length scale of ecological and climate dynamics

streamlines of harmonic Ψ

MIZ width definition from medical imaging

Strong, *Climate Dynamics* 2012

Strong & Rigor, *GRL* 2013

39% widening
1979 - 2012

Strong, Foster, Cherkaev, Eisenman, Golden, *J. Atmos. Oceanic Tech.* 2017

Strong & Golden, *SIAM News* 2017

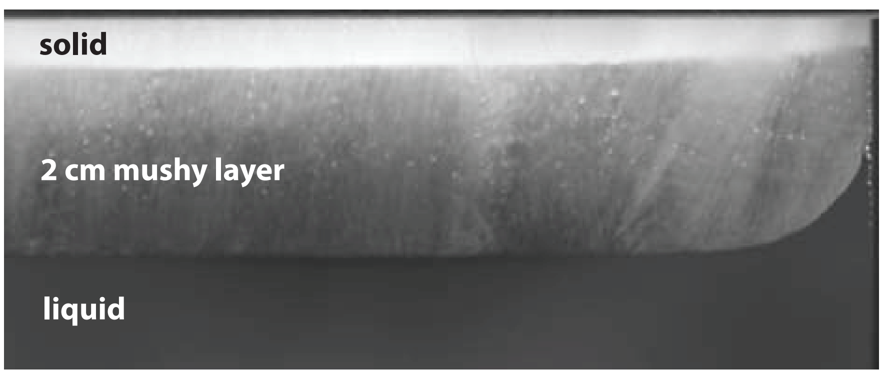
Multiscale mushy layer model for marginal ice zone dynamics

Strong, Cherkaev, Golden *Scientific Reports* 2024

MIZ - transitional region between dense pack ice and open ocean

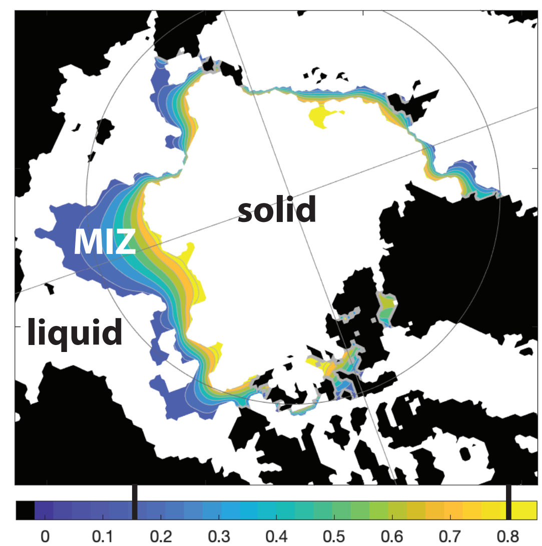
OBJECTIVE: model & predict dramatic annual cycle
impacts climate dynamics, polar ecology, human activities

mushy layer physics in the lab



NaCl-H2O in lab
(Peppin et al., 2007, J. Fluid Mech.)

Arctic MIZ as a mushy layer

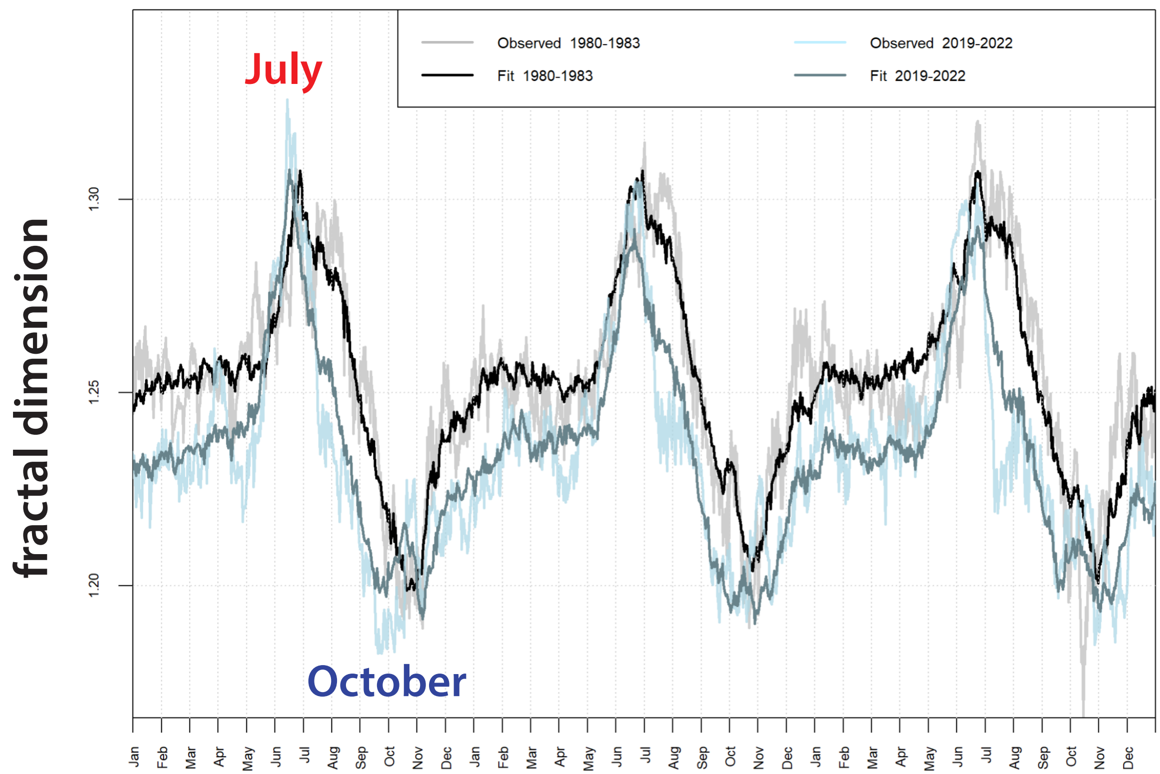


sea ice concentration ψ

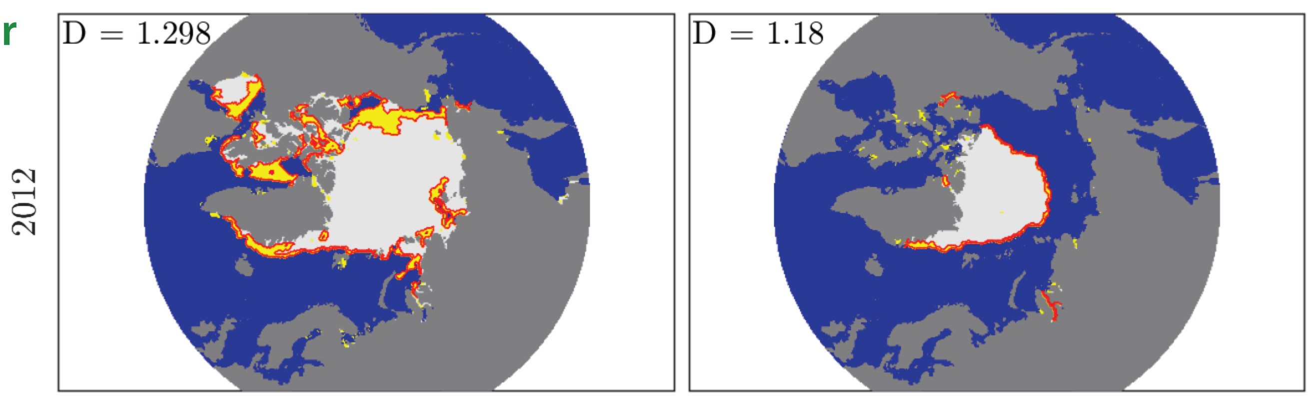
Identifying Fractal Geometry in Arctic Marginal Ice Zone Dynamics

Julie Sherman, Court Strong, Ken Golden 2025

Compute the fractal dimension of the boundary of the Arctic MIZ by boxcounting methods; analyze seasonal cycle and long term trends.



early summer



early autumn

wave and thermal interactions with fractal boundary

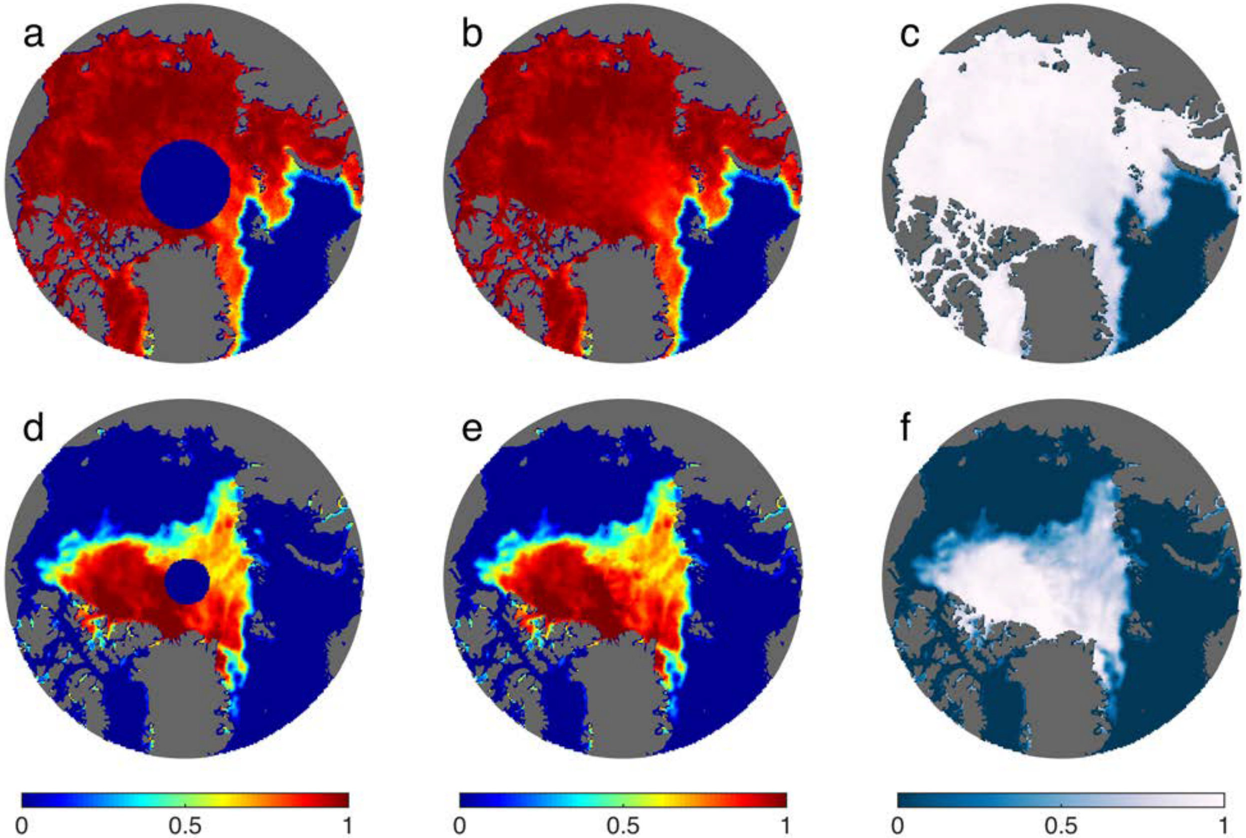
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



$$\Delta\psi = 0$$

fill = harmonic function satisfying satellite BC's plus learned stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

Global Sea Ice Concentration Climate Data Records, 2022
Lavergne, Sorensen, et al., Norwegian Met. Inst., ... OSI SAF

ISSN 0002-9920 (print)
ISSN 1088-9477 (online)

Notices

of the American Mathematical Society

November 2020 | Volume 67, Number 10

AMS AMERICAN MATHEMATICAL SOCIETY
Advancing research. Creating connections.

The cover is based on "Modeling Sea Ice," page 1535.

Conclusions

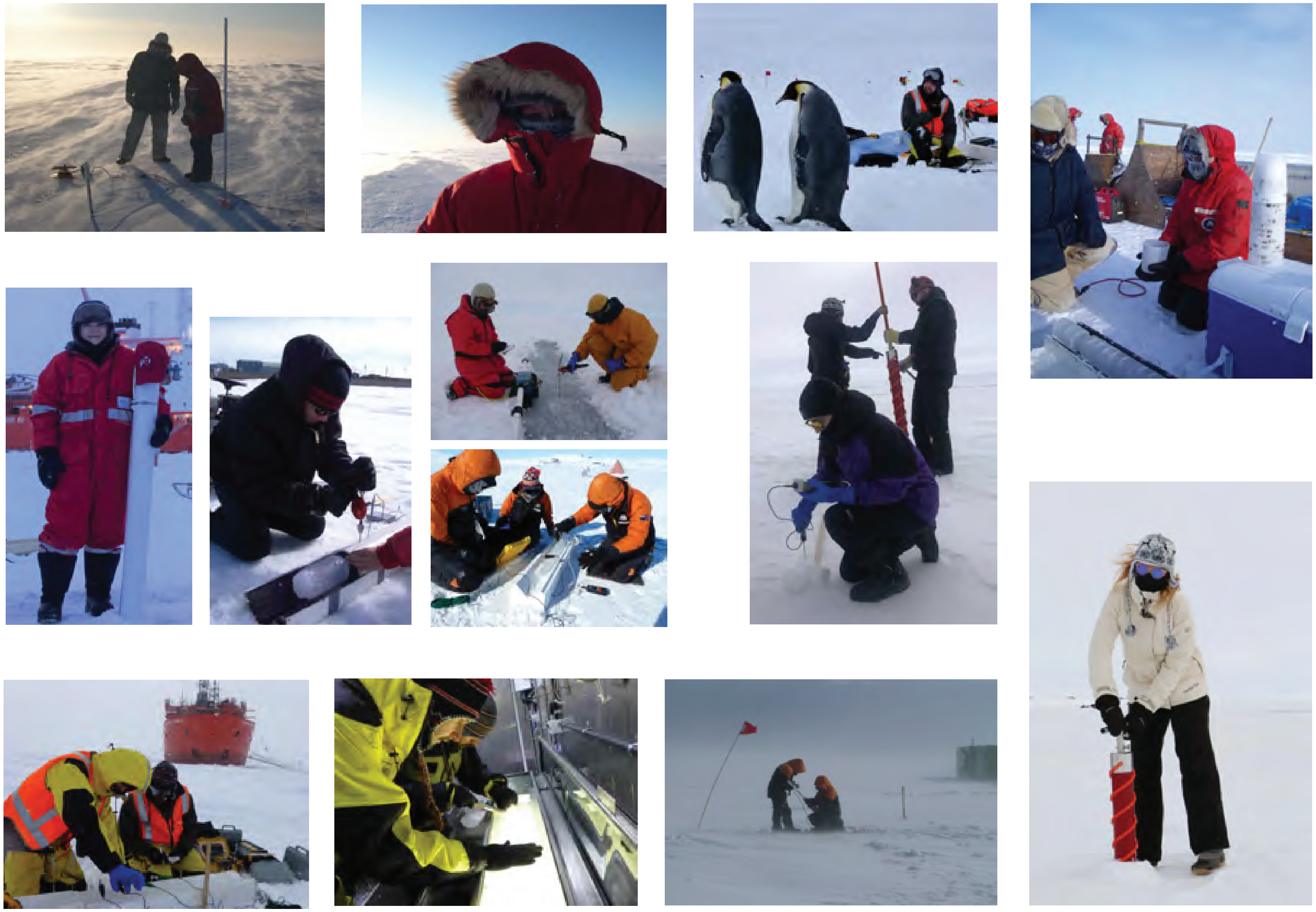
Our research is helping to improve projections of Earth's climate, its sea ice packs, and the fate of the ecosystems they support.

Cross pollination helps us study sea ice, while our sea ice math advances the theory of composites, inverse problems, etc.

Sea ice is really cool --- modeling it leads to unexpected areas of math and physics.

Mathematics is the operating system of science and engineering!

Thank you to so many postdocs, graduate students, undergraduates, high school students and colleagues who contributed to this work!



U. of Utah students in the Arctic and Antarctic (2003-2022): closing the gap between theory and observation - making math models come alive and experiencing climate change firsthand.

THANK YOU

Office of Naval Research

Applied and Computational Analysis Program

Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences

Division of Polar Programs

UNITED STATES COAST GUARD
SEMPER PARATUS
1790

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Department of the Environment and Water Resources
Australian Antarctic Division

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ANTARCTIC CLIMATE & ECOSYSTEMS
COOPERATIVE RESEARCH CENTRE

NSF

Antarctica New Zealand

UAF
UNIVERSITY OF ALASKA FAIRBANKS

THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

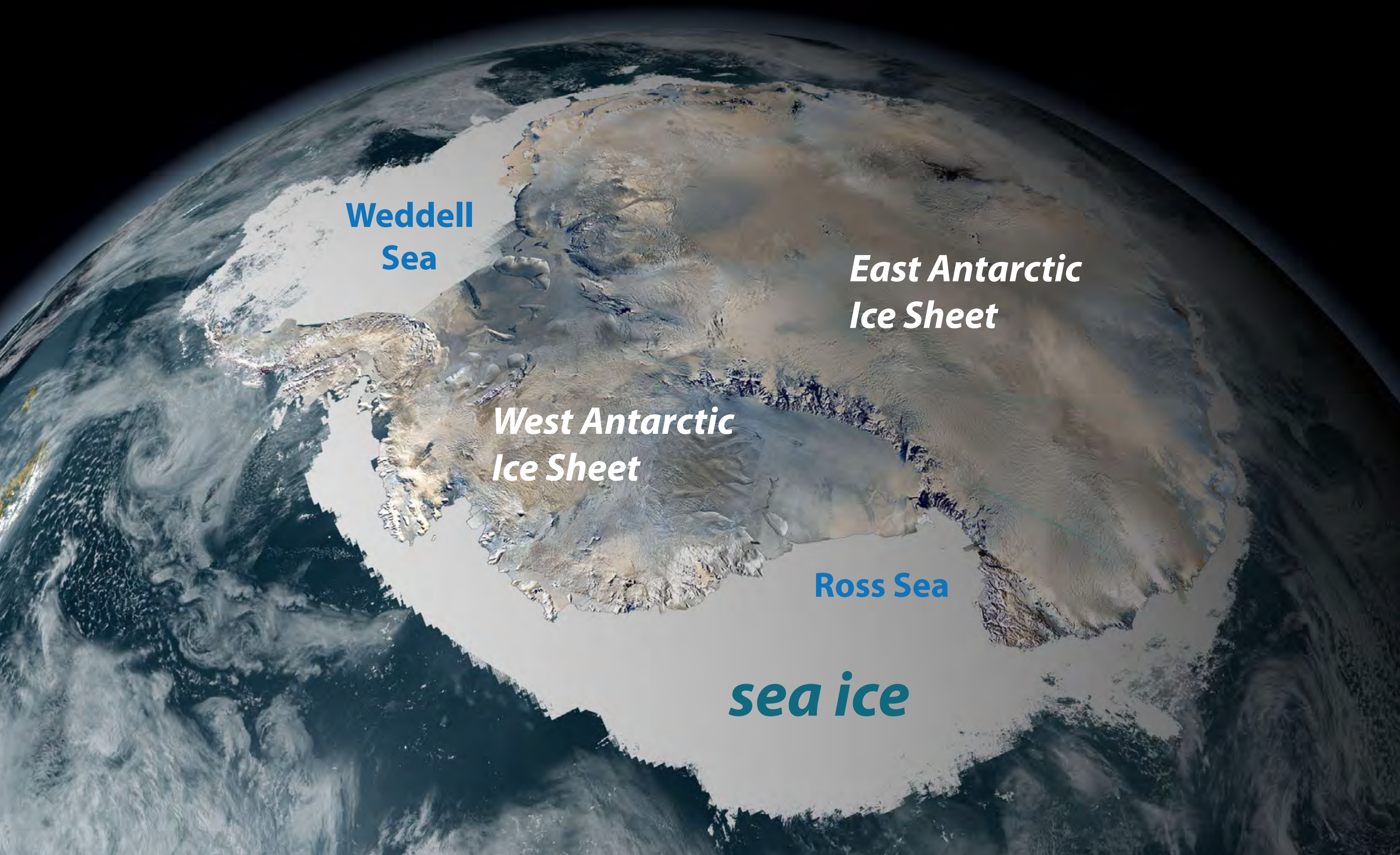
A Journey to the Arctic: Mathematical Modelers Meet Real World Ice

Ken Golden & Jody Reimer

David Gluckman, Daniel Hallman, Anthony Jajeh, Anthony Lee, Kathy Lin, *Marco Lozzi*, Delaney Mosier, Nash Ward

ANTARCTICA

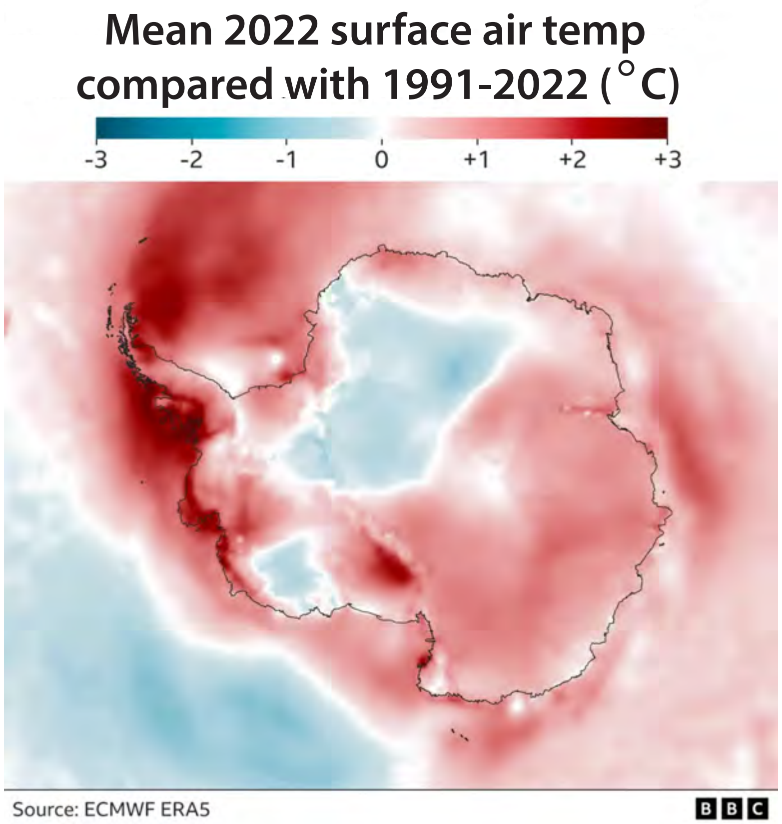
southern cryosphere



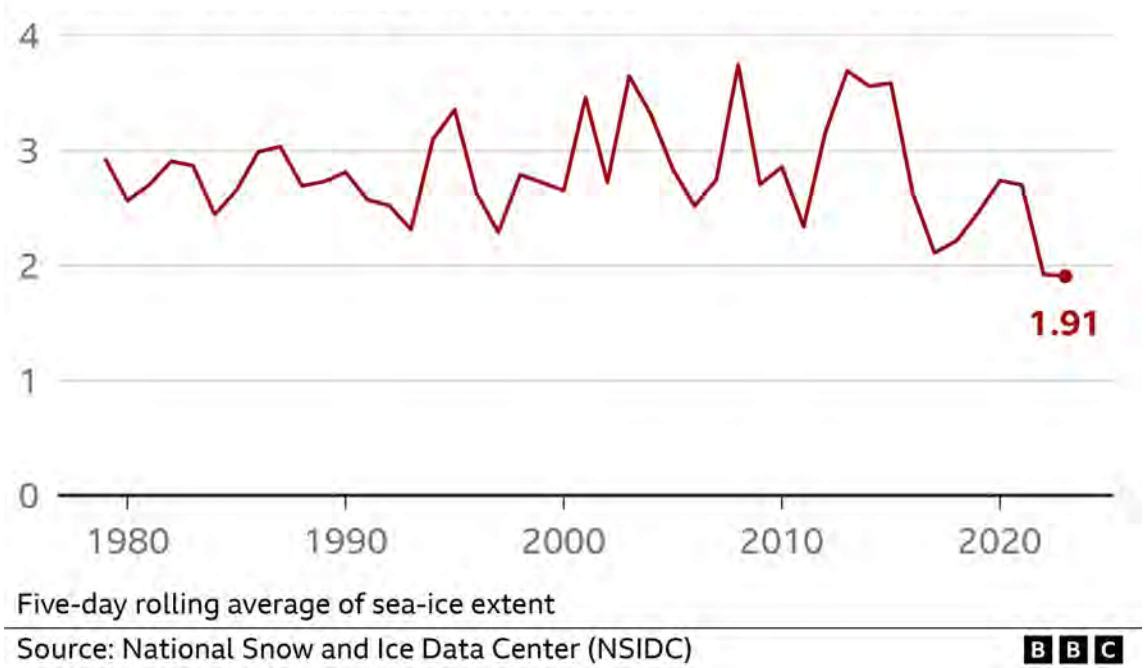
New Record Low for Antarctic Sea Ice

February 13, 2023

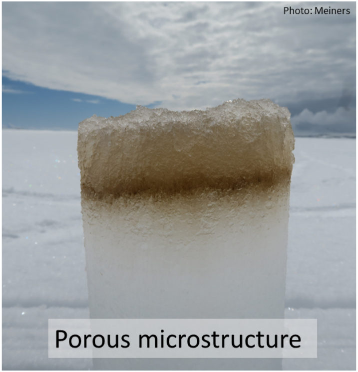
Much of Antarctica warmer than average

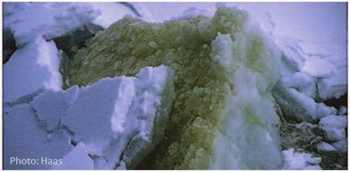


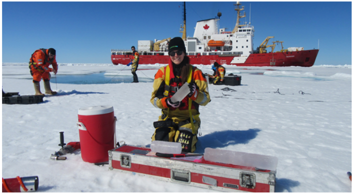
Minimum extent 1979-2023 (million sq km)



SEA ICE ALGAE high level of local heterogeneity







Can we improve agreement between algae models and data?

80% of polar bear diet can be traced to ice algae*.

* Brown TA, et al. (2018). *PloS one*, 13(1), e0191631

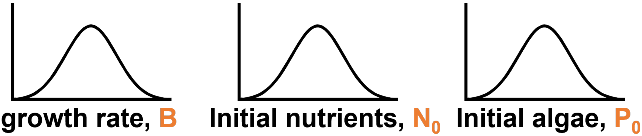
HETEROGENEITY in PARAMETERS & CONDITIONS

At each location within a larger region, consider

$$\frac{dN}{dt} = \alpha - BNP - \eta N$$
$$\frac{dP}{dt} = \gamma BNP - \delta P$$

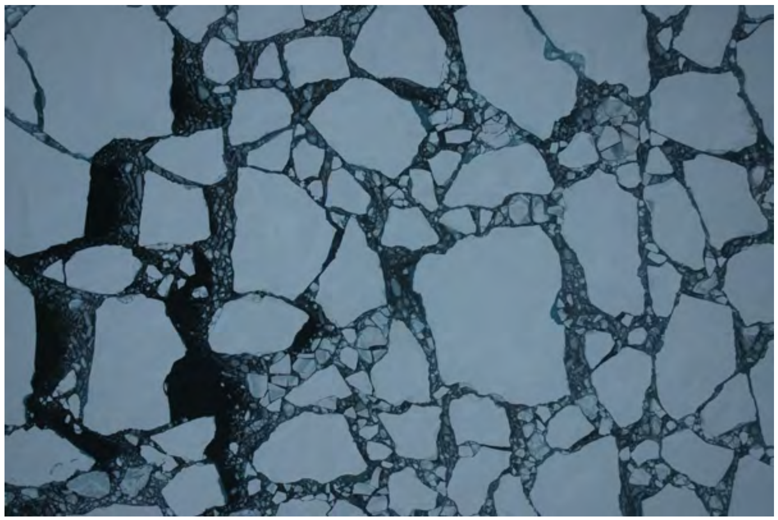
treating parameters as random variables

$$N(0) = N_0, \qquad P(0) = P_0$$

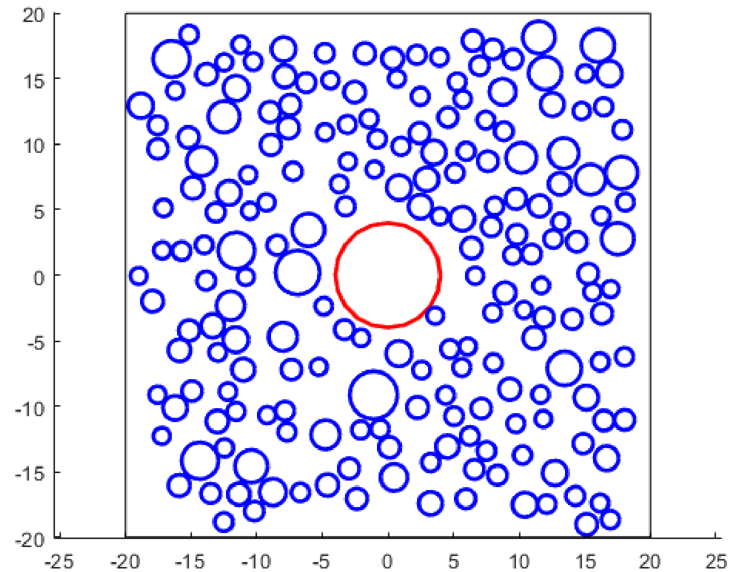


But, Monte Carlo for Full Algae Model: 8 hours X 10,000

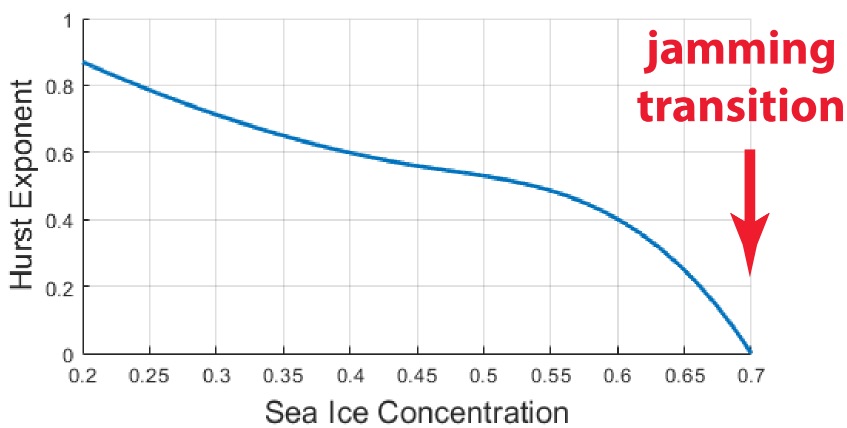
Arctic sea ice pack with tagged particle



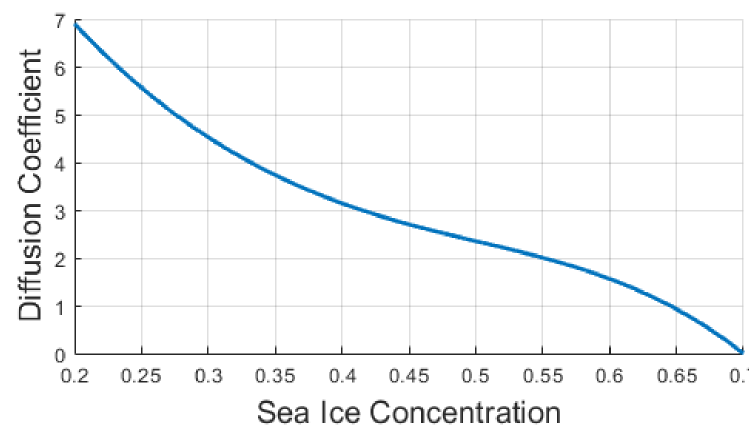
Einstein's pollen grain



Hurst exponent



diffusion coefficient



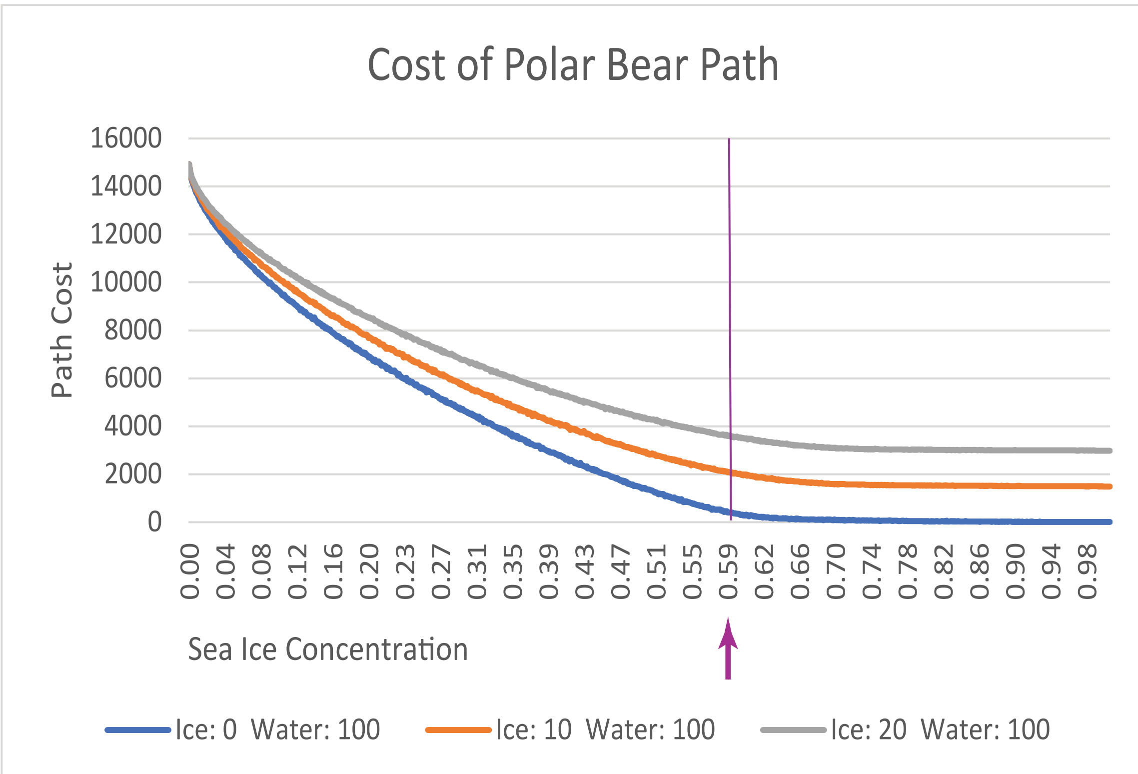
sea ice ecosystem

sea ice algae
support life in the polar oceans

Polar Bear Percolation

Optimal Movement of a Polar Bear in a Heterogenous Icescape

$C(p)$



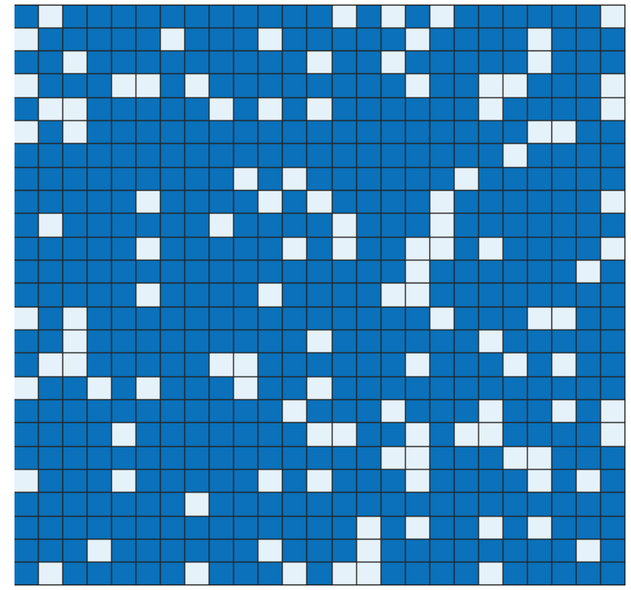
$h = 0.2$

$h = 0.1$

$h = 0$

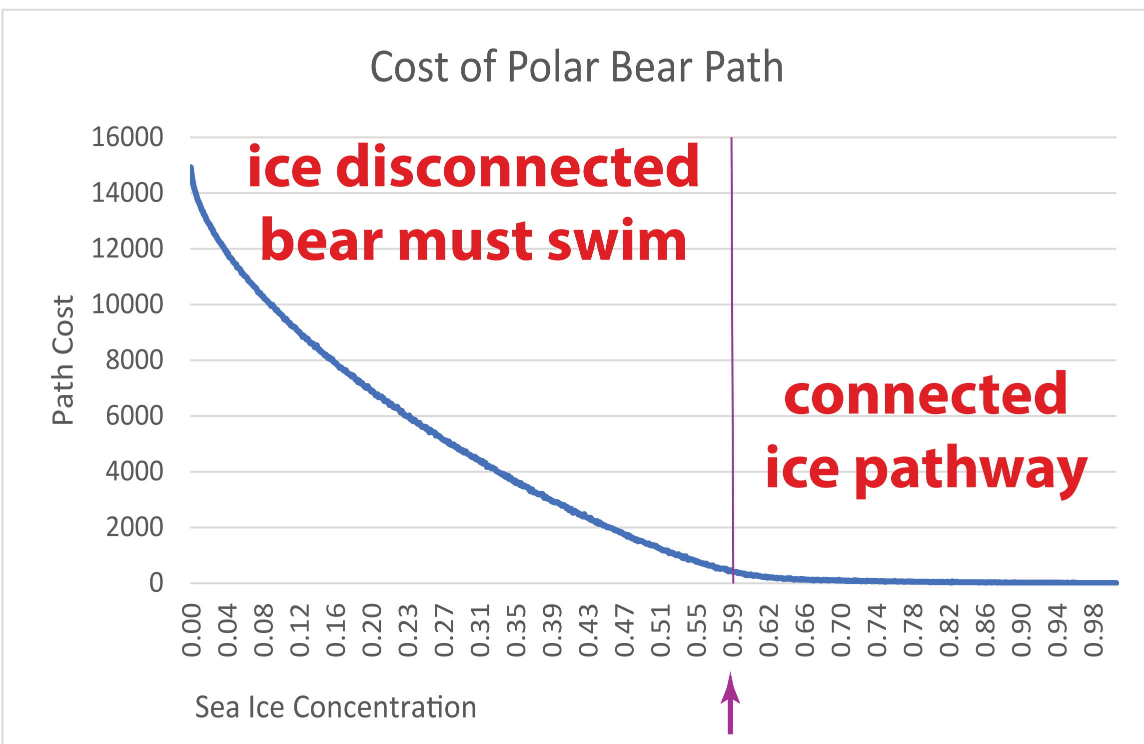
$$h = \frac{c_i}{c_w}$$

ratio of local "conductivities"



20% Ice

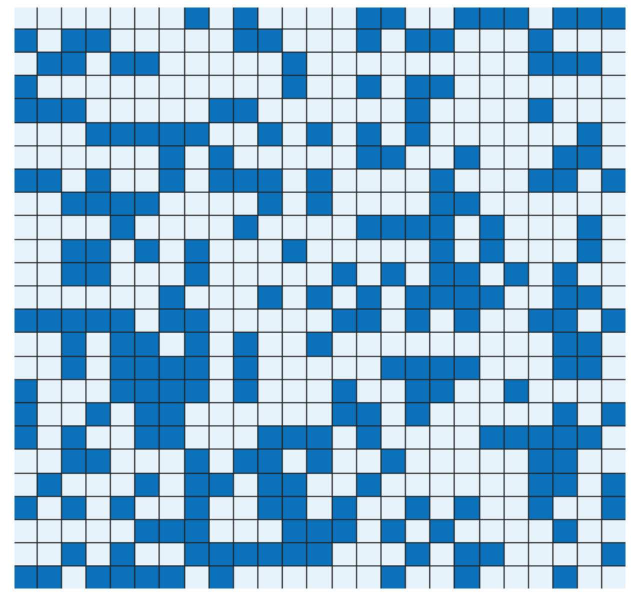
$C(p)$



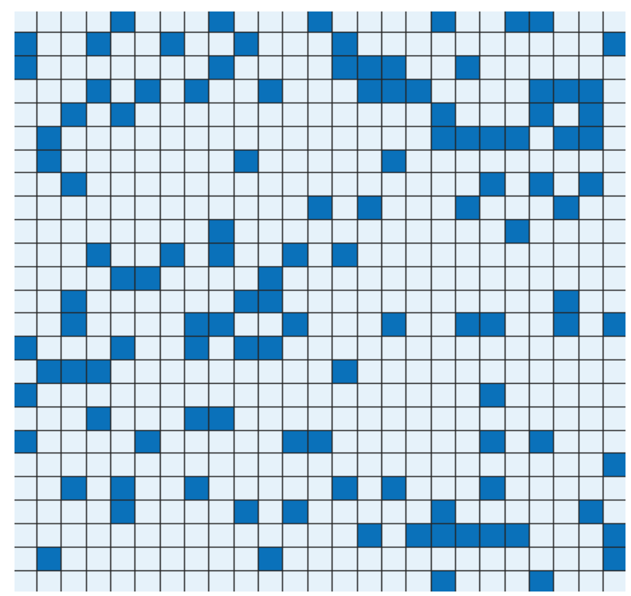
$h = 0$

site percolation threshold

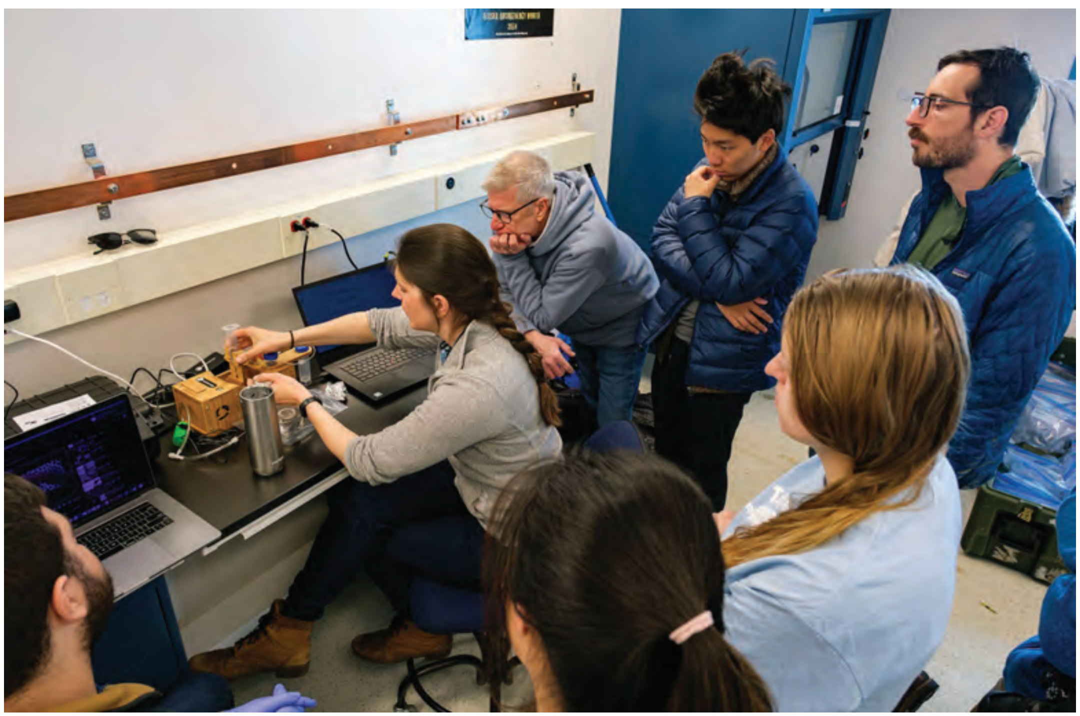
$p_c = 0.59 \;\; \text{for} \;\; d = 2$



60% Ice

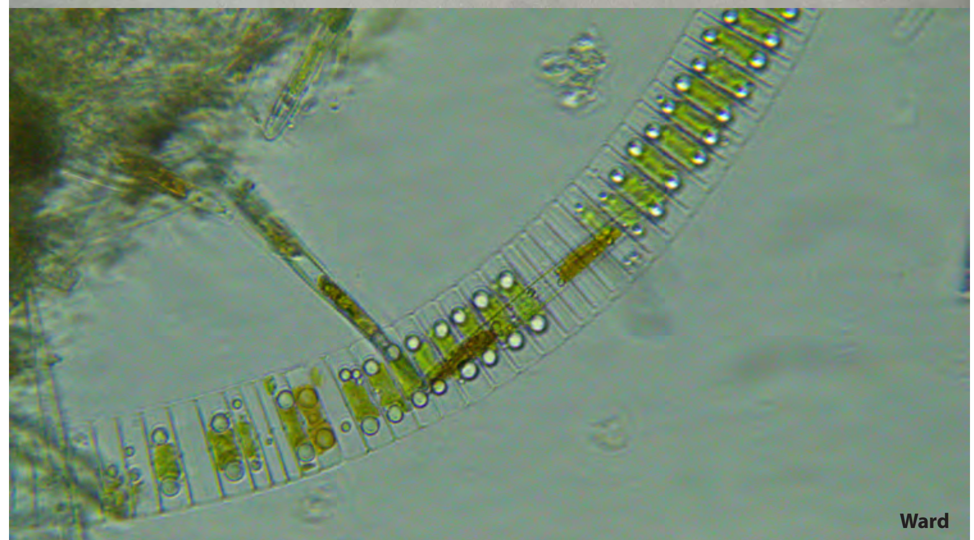


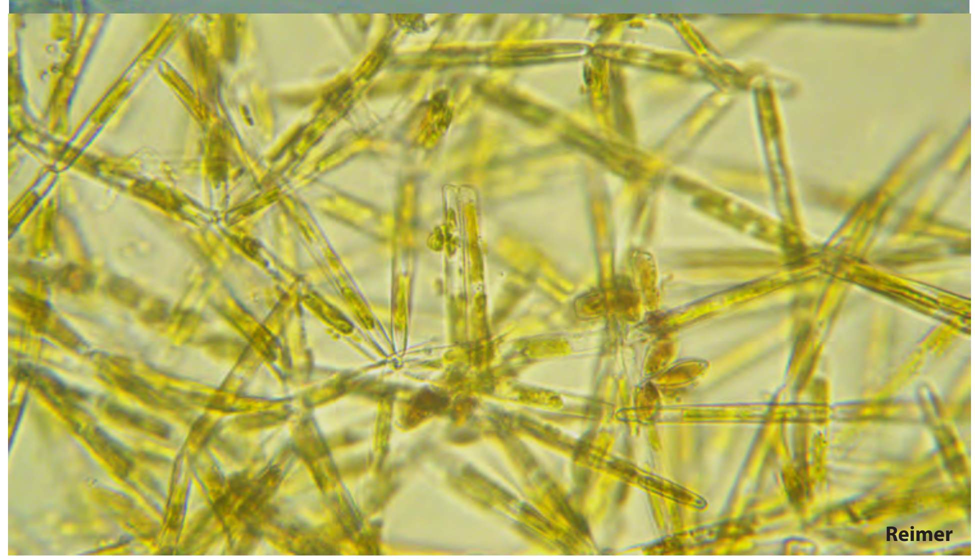
80% Ice




DEWALT
DEWALT

bottom of a sea ice core



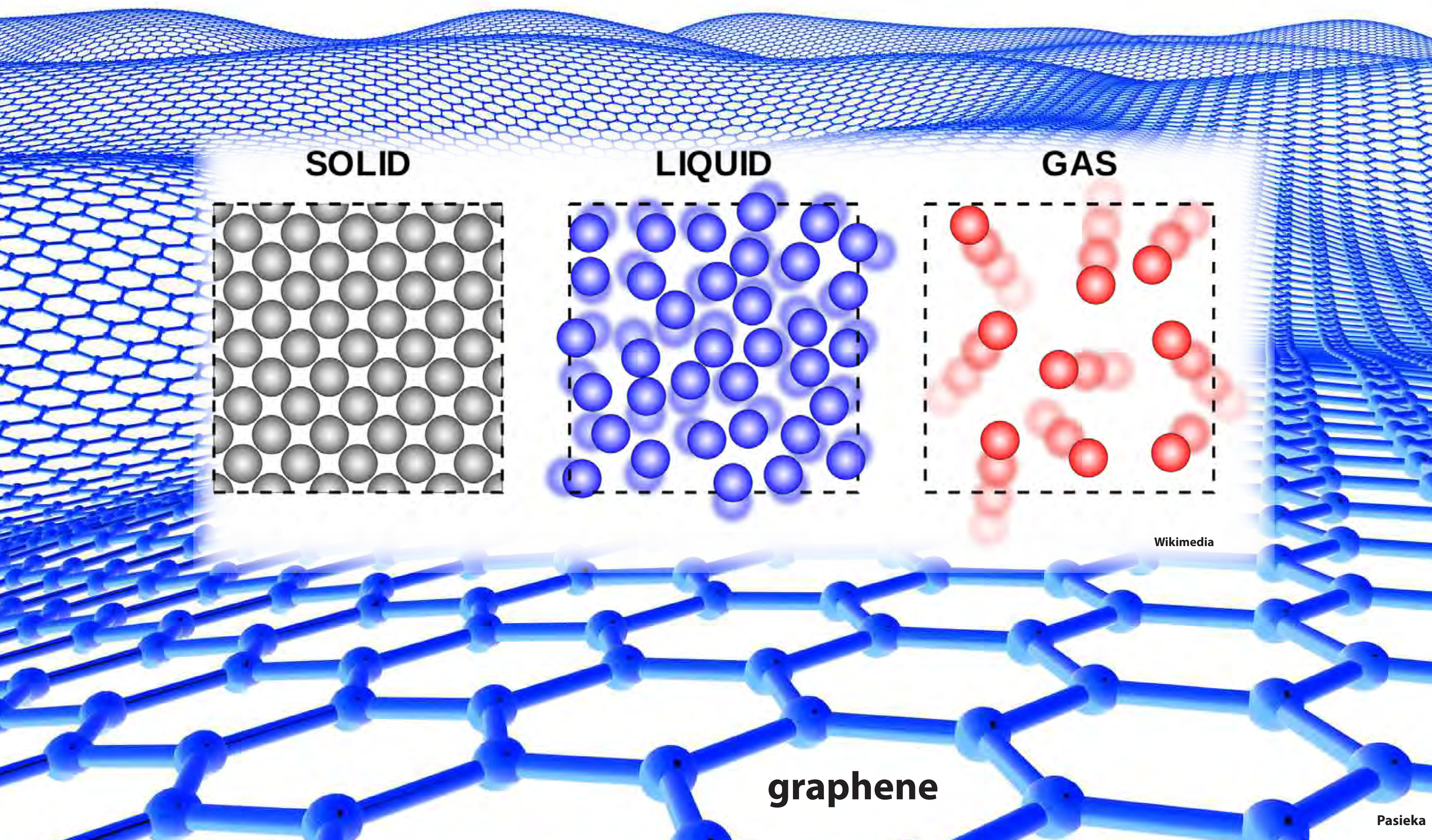


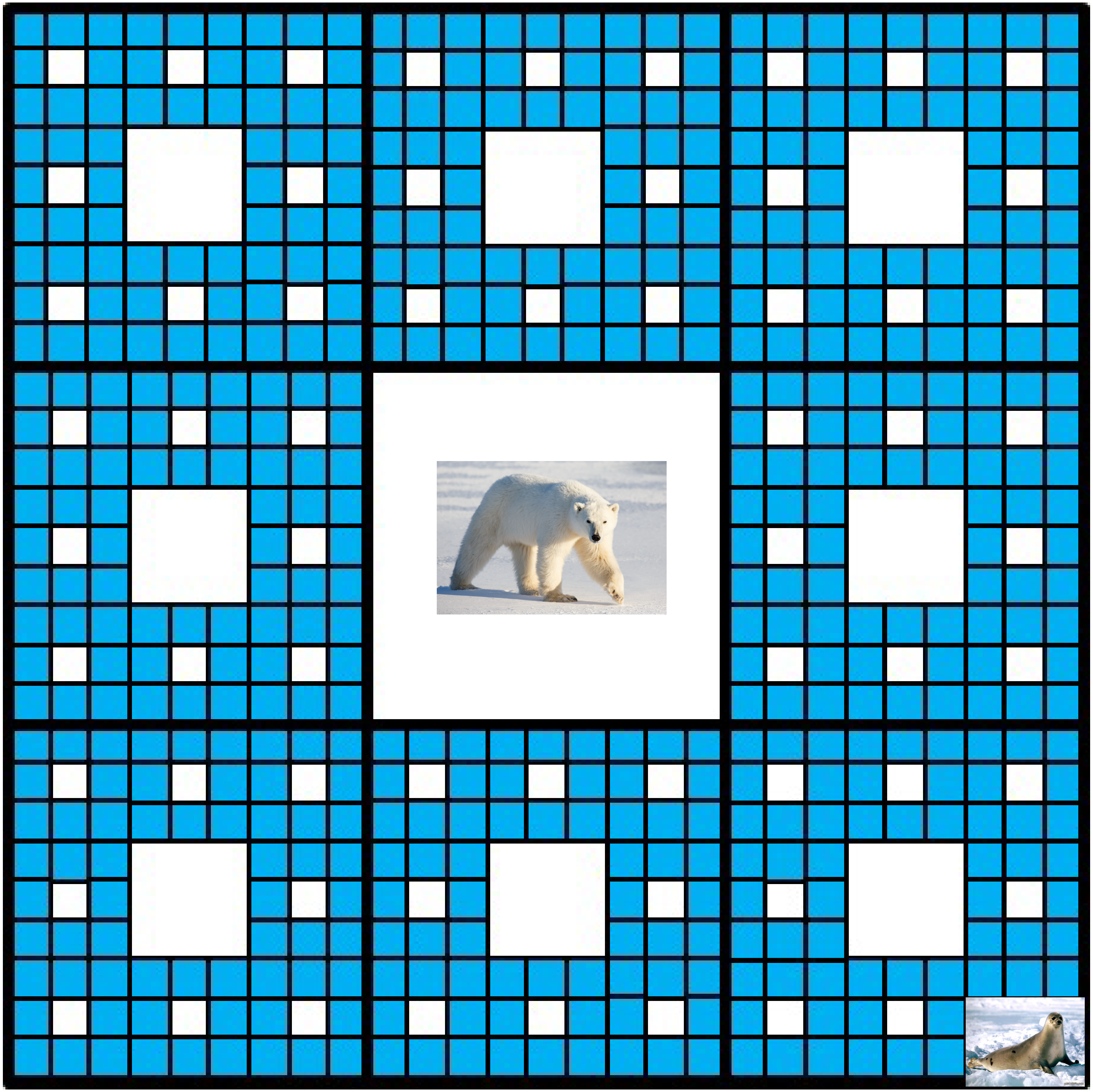
STATISTICAL PHYSICS

percolation, phase transitions
solid state, semiconductors

How do microscopic laws determine macroscopic behavior?

Banwell, Burton, Cenedese, Golden, Astrom, Physics of the Cryosphere, *Nature Reviews Physics* 2023





polar bear foraging in a fractal icescape

Nicole Forrester
Jody Reimer
Ken Golden

It costs the polar bear 5 times the energy to swim through water than to walk on sea ice.

What pathway to a seal minimizes energy spent?