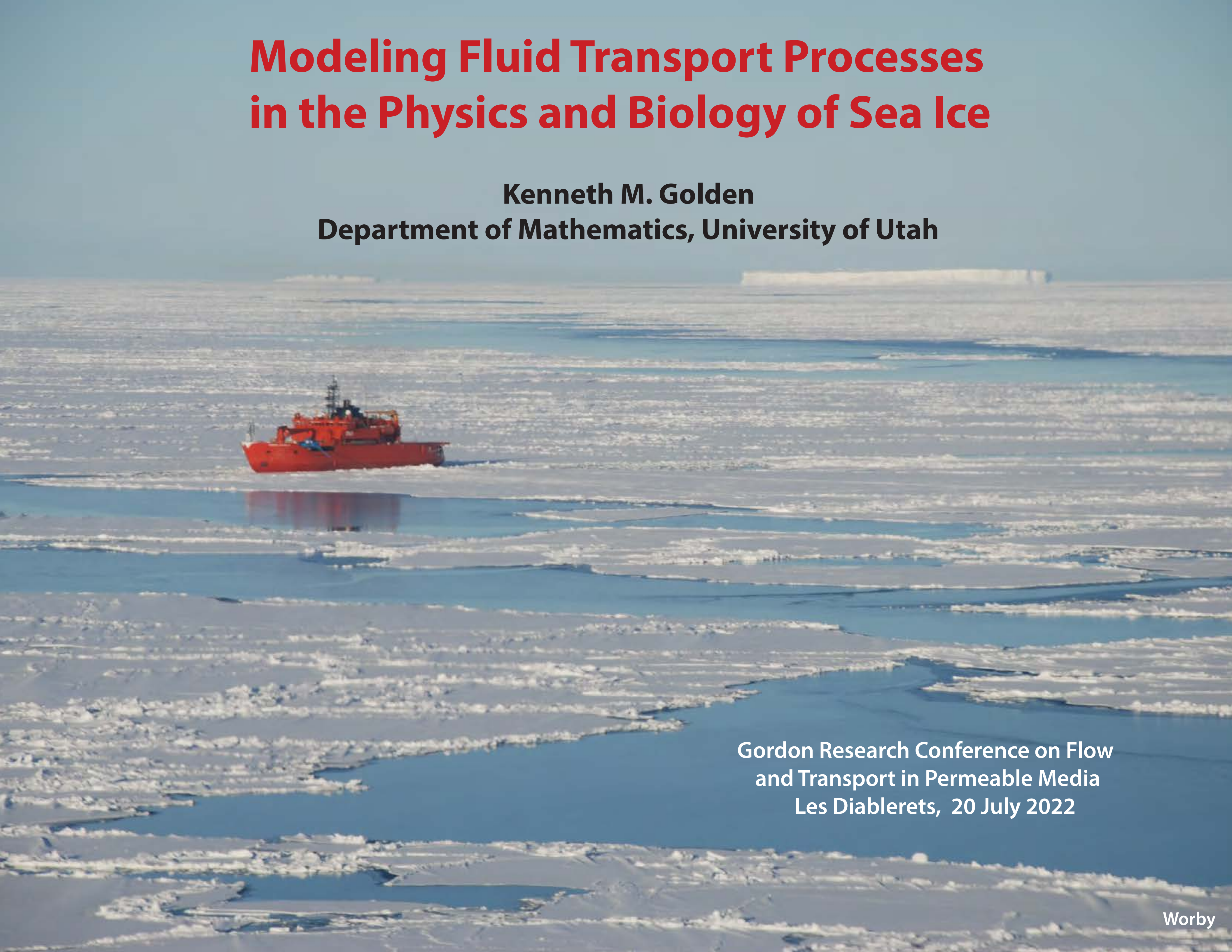
Modeling Fluid Transport Processes in the Physics and Biology of Sea Ice

Kenneth M. Golden
Department of Mathematics, University of Utah

Gordon Research Conference on Flow
and Transport in Permeable Media
Les Diablerets, 20 July 2022



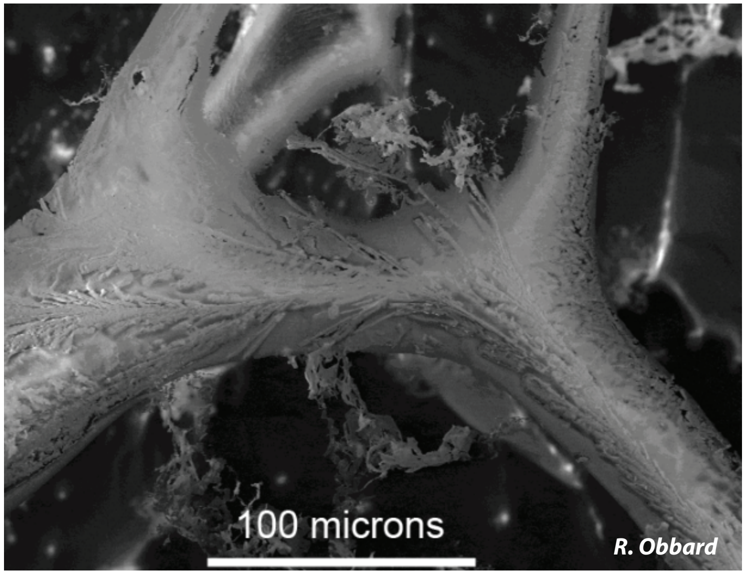
Worby

sea ice may appear to be a barren, impermeable cap …

Golden


Weeks & Assur

brine inclusions in sea ice (mm)



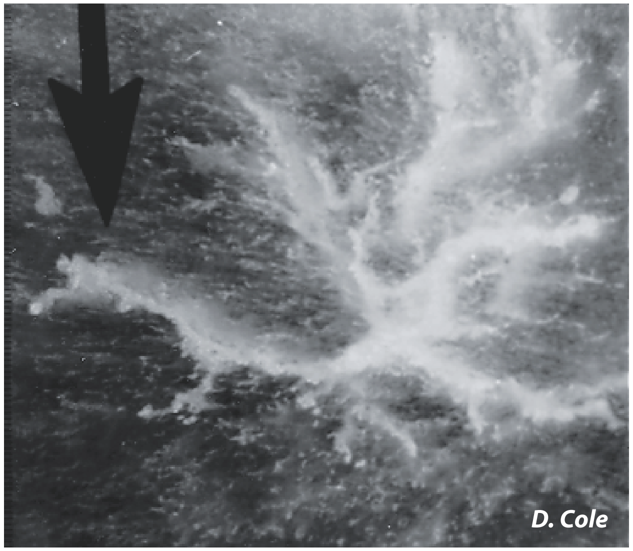
R. Obbard

micro - brine channel (SEM)

sea ice is a porous composite

pure ice with brine, air, and salt inclusions

brine channels (cm)


D. Cole

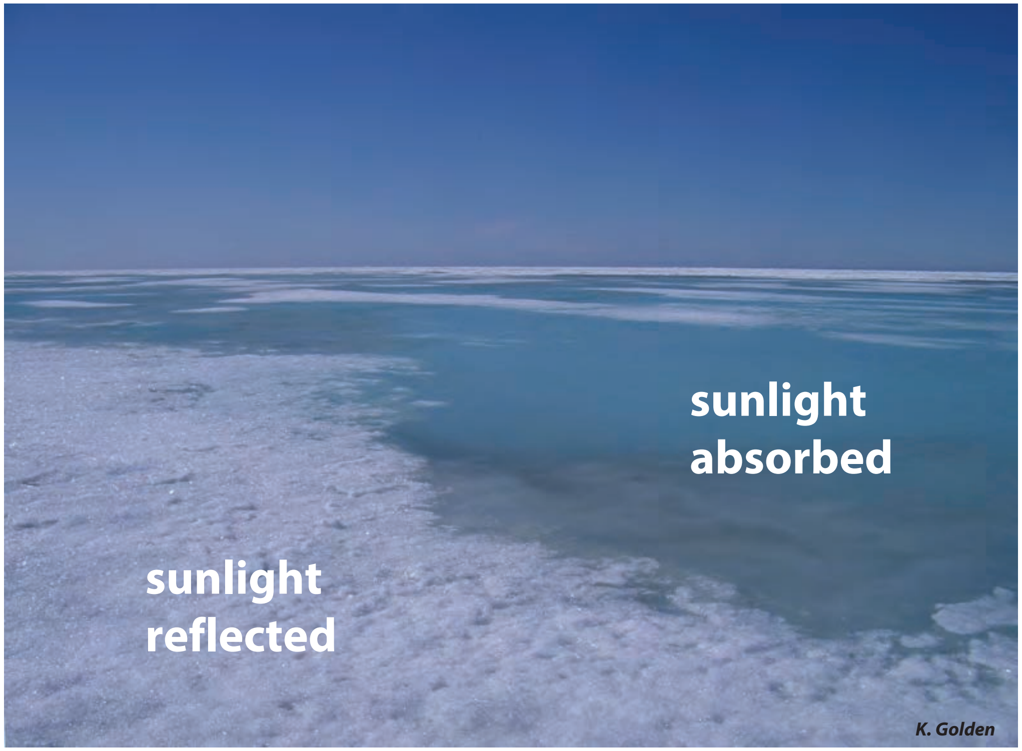
horizontal section


Golden

vertical section

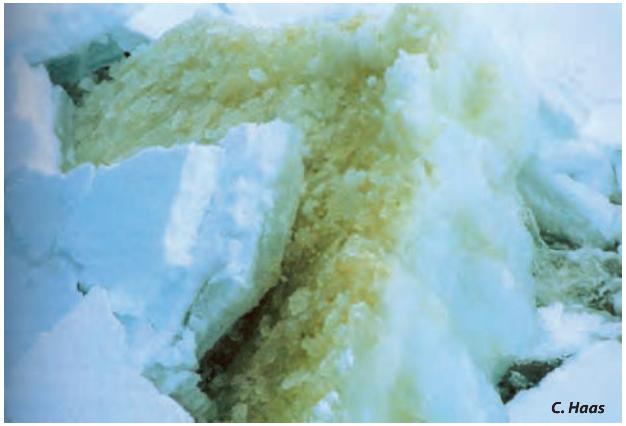
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

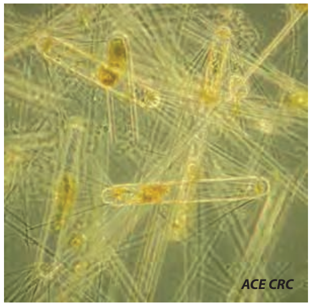


K. Golden

nutrient flux for algal communities

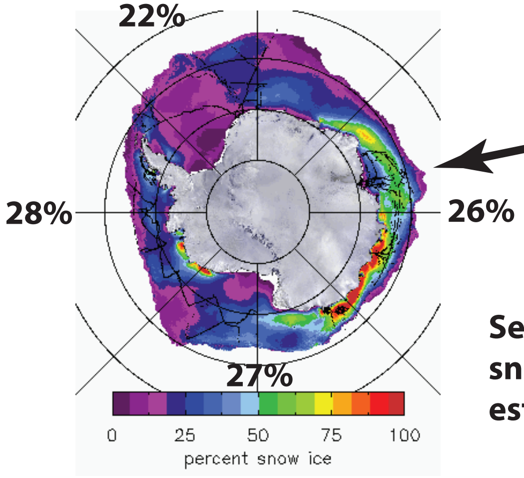


C. Haas



ACE CRC

C. Krembs



T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

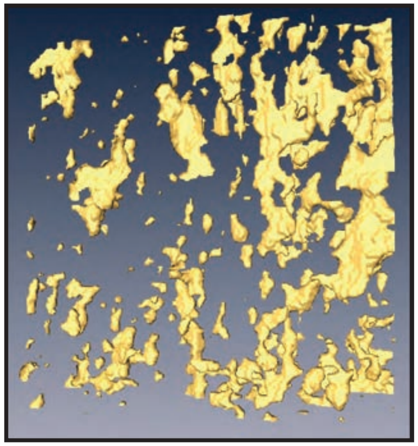
- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO_2*

Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

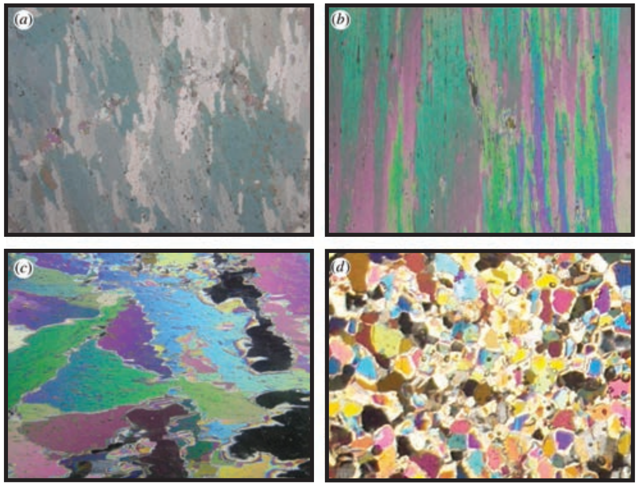
Weeks & Assur 1969



H. Eicken

Golden *et al. GRL* 2007

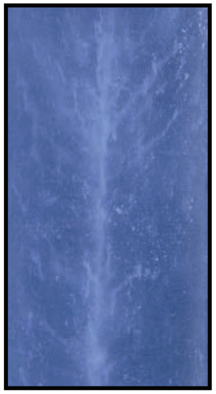
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

---

mesoscale

macroscale

Arctic melt ponds

K. Frey

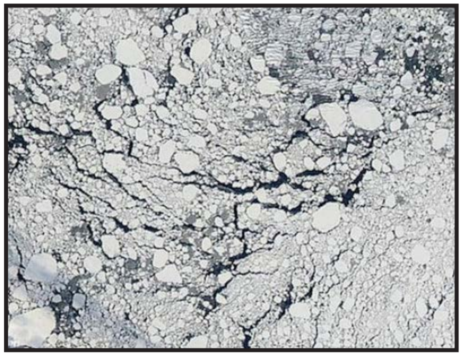
Antarctic pressure ridges

K. Golden

sea ice floes

J. Weller

sea ice pack

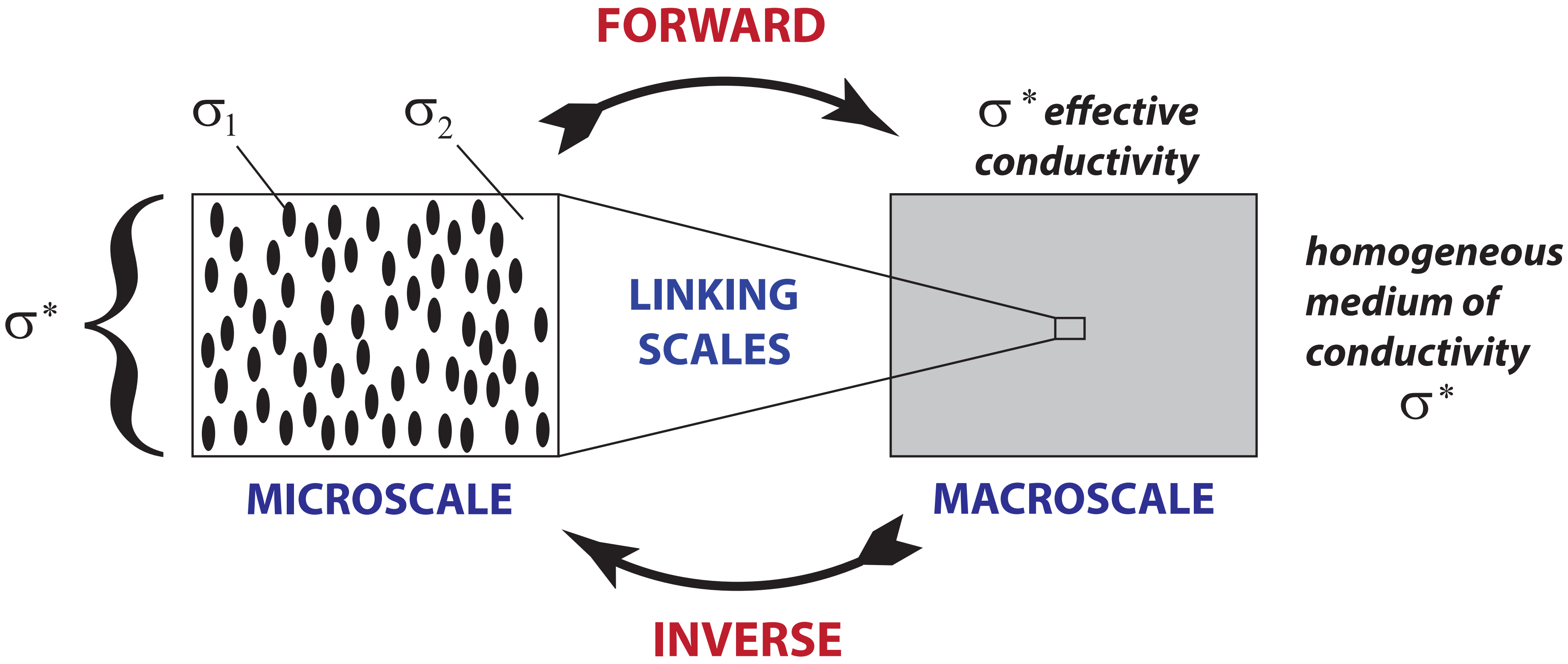


NASA

meters

kilometers

HOMOGENIZATION for Composite Materials



FORWARD

σ_1 σ_2

σ^*

σ^* effective conductivity

LINKING SCALES

MICROSCALE

MACROSCALE

homogeneous medium of conductivity σ^*

INVERSE

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

A tour of fluid transport processes in sea ice modeling

Inputs, Ingredients

COMPOSITE MATERIALS

- electrical engineering, stealth technology
- porous media, oil extraction
- statistical mechanics of ferromagnets
- Anderson localization, semiconductor physics
- random matrix theory
- differential equations

MODELING SEA ICE

microscale

mesoscale

macroscale

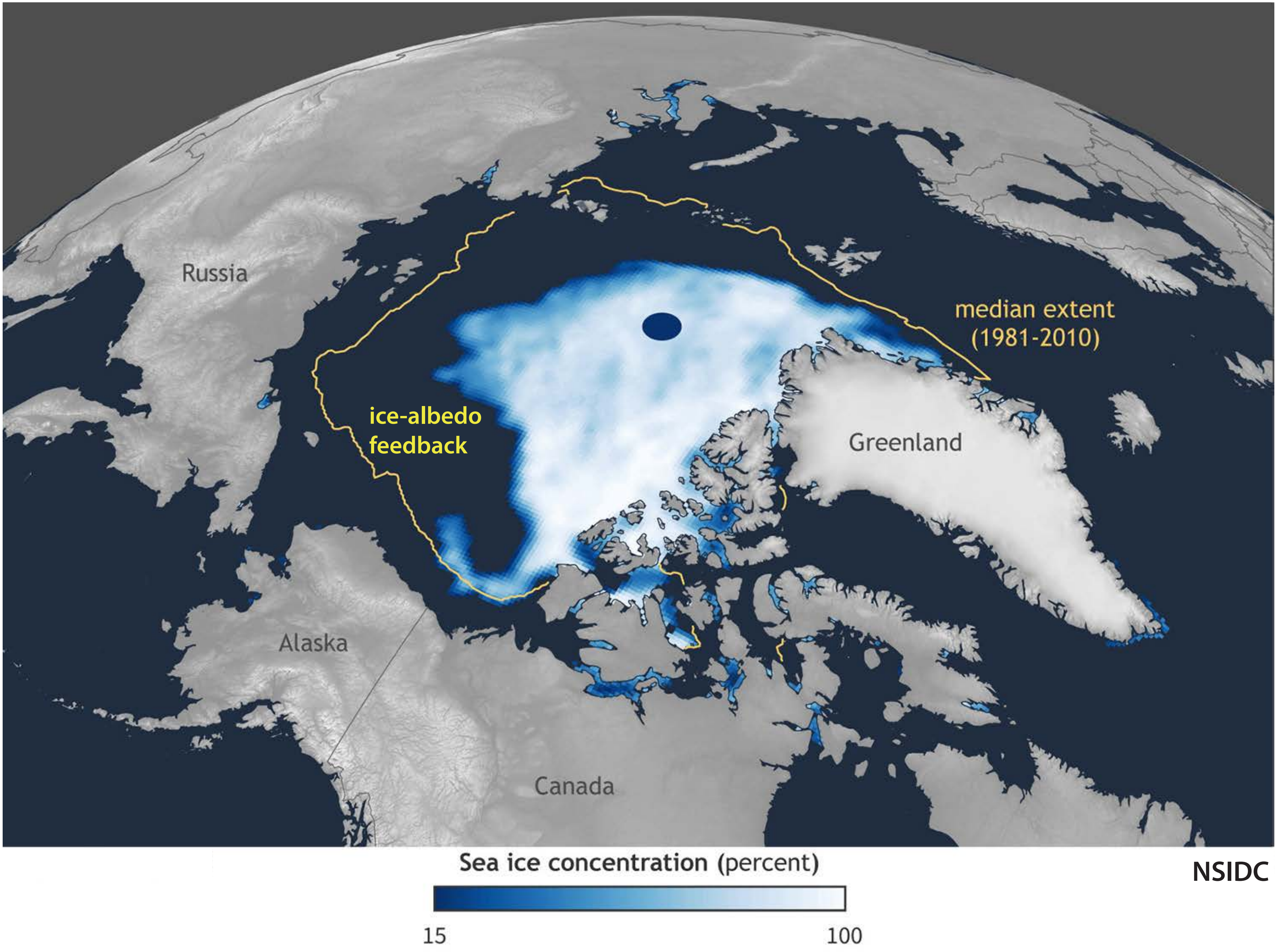
Outputs, Impacts

CLIMATE MODELING

- sea ice physics & biology
- composites, polycrystals
- remote sensing
- advection diffusion
- biomedical imaging, biomaterials, EPS
- polar microbial ecology

Physics of sea ice drives advances in many areas of science and engineering.

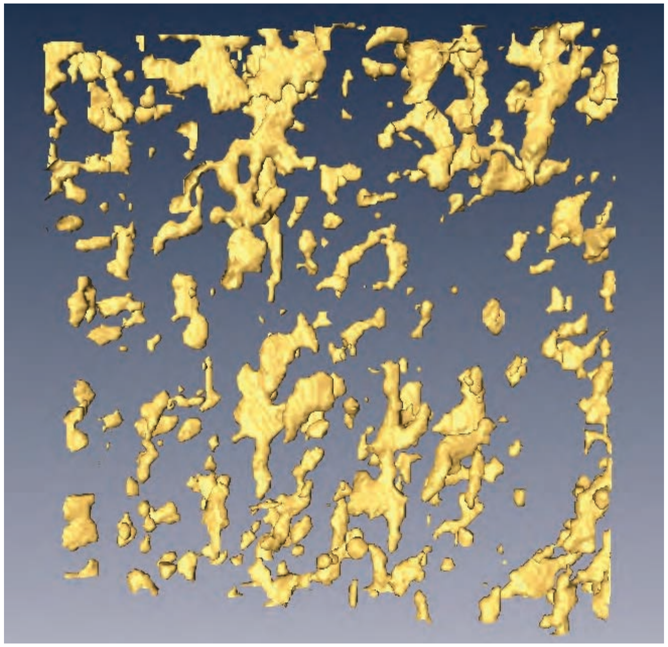
Arctic sea ice extent September 15, 2020



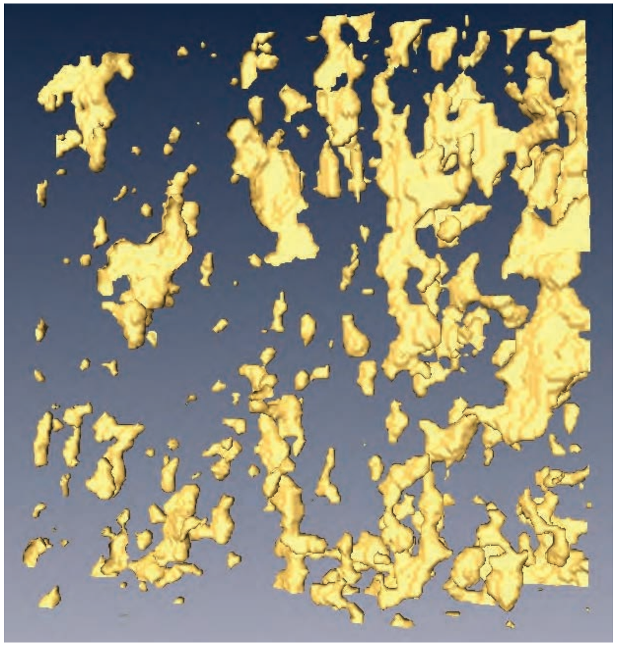
NSIDC

microscale

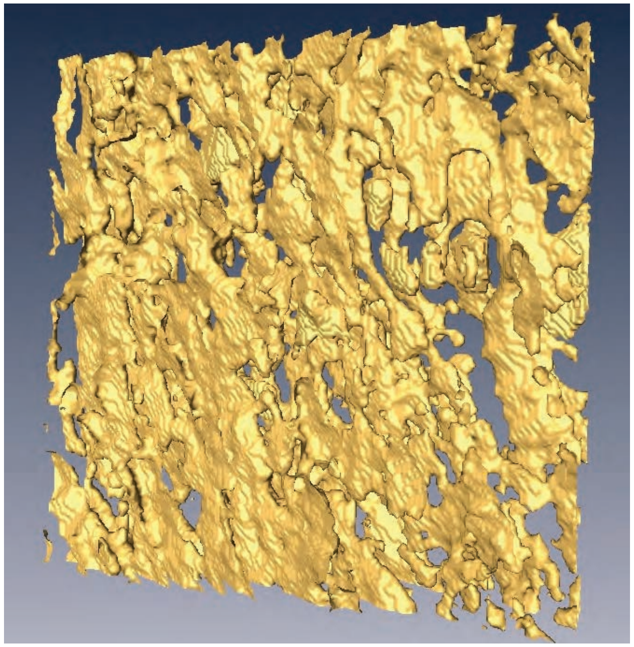
brine volume fraction and ***connectivity*** increase with temperature



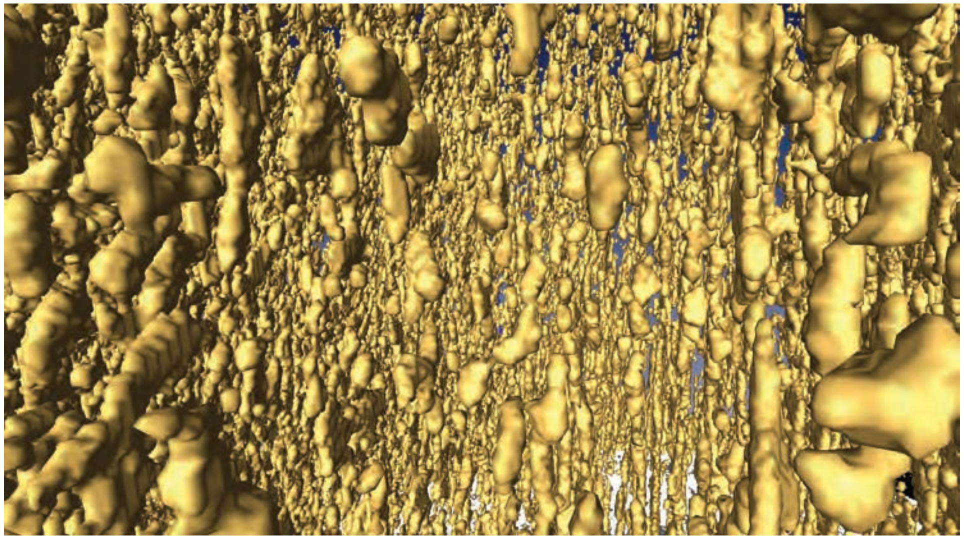
$T = -15\,^\circ\mathrm{C}, \ \phi = 0.033$



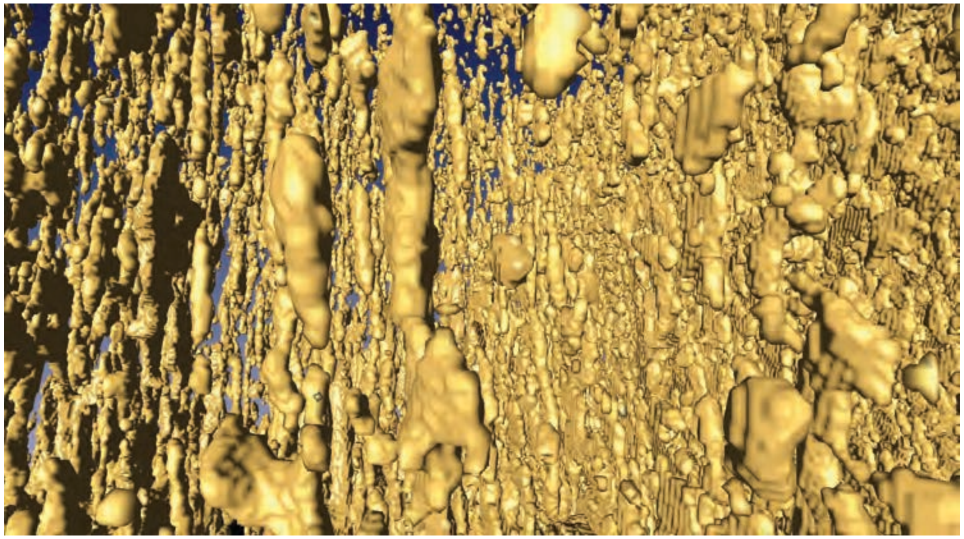
$T = -6\,^\circ\mathrm{C}, \ \phi = 0.075$



$T = -3\,^\circ\mathrm{C}, \ \phi = 0.143$



$T = -8\,^\circ\mathrm{C}, \ \phi = 0.057$



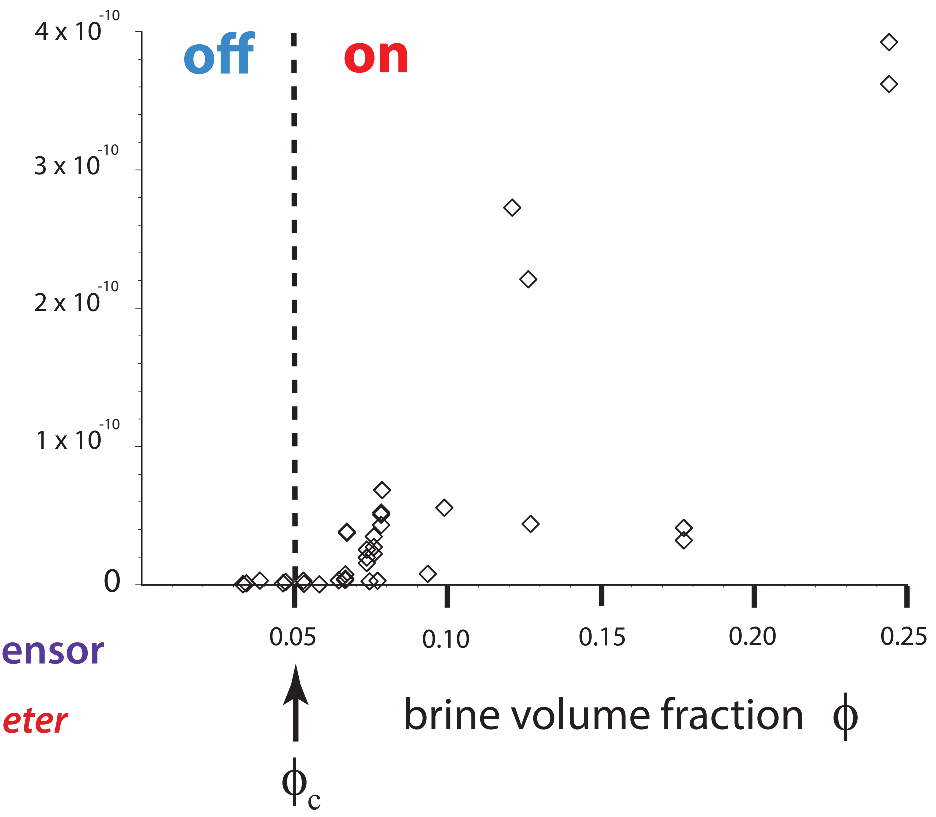
$T = -4\,^\circ\mathrm{C}, \ \phi = 0.113$

X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

Critical behavior of fluid transport in sea ice

impermeable   permeable



Arctic field data

vertical fluid permeability k (m^2)

"on - off" switch for fluid flow

Darcy's Law

$$v = -\frac{\mathbf{k}}{\eta} \nabla p$$

$\mathbf{k}$ = fluid permeability tensor

homogenized parameter

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C, \; S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

sea ice algal communities

D. Thomas 2004

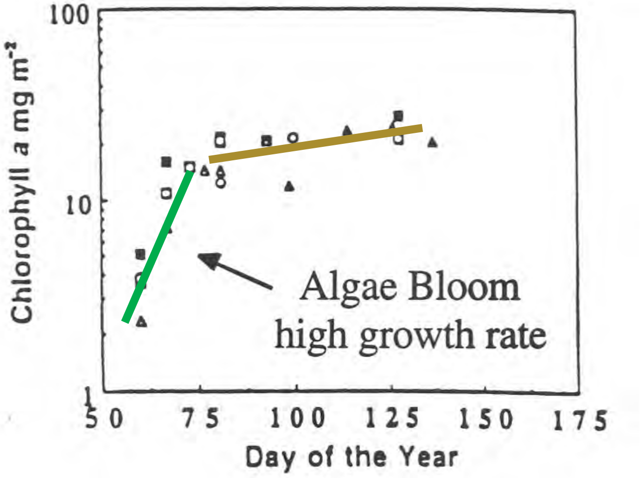
nutrient replenishment controlled by ice permeability

biological activity turns on or off according to *rule of fives*

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

critical behavior of microbial activity

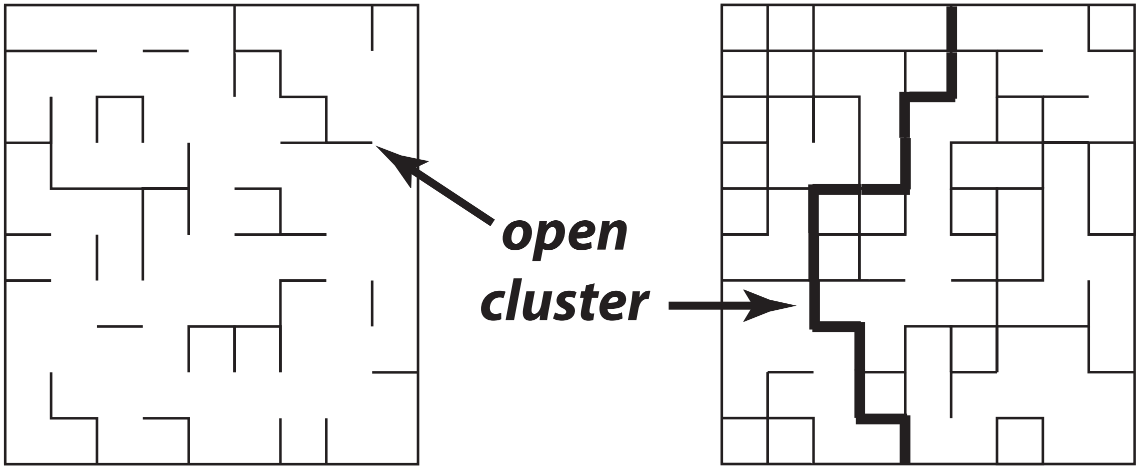


Convection-fueled algae bloom
Ice Station Weddell

percolation theory

probabilistic theory of connectedness

impermeable permeable



$p = 1/3$ $p = 2/3$

bond → **open** *with probability* p
closed *with probability* $1-p$

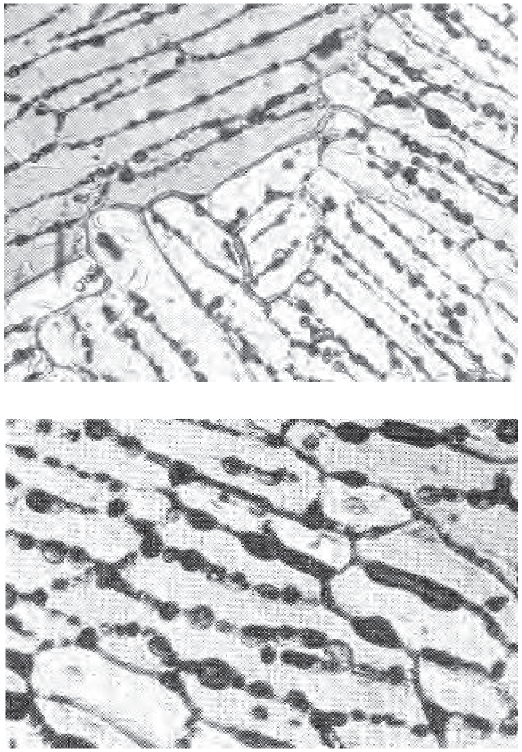
percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

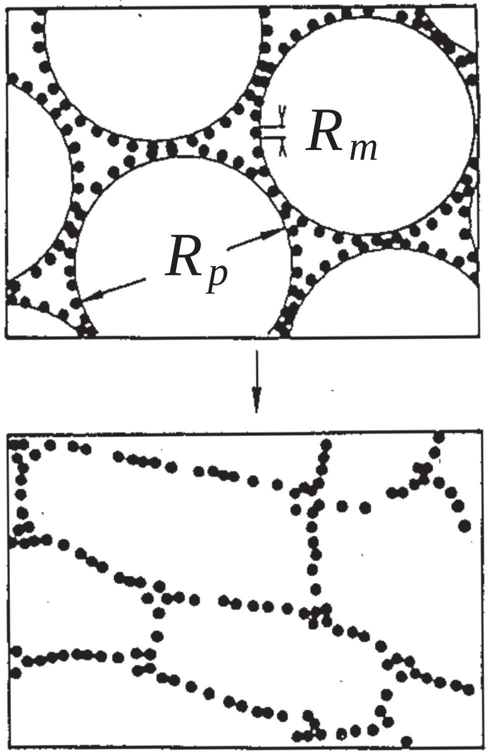
smallest *p* for which there is an infinite open cluster

Continuum percolation model for ***stealthy*** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science*, 1998

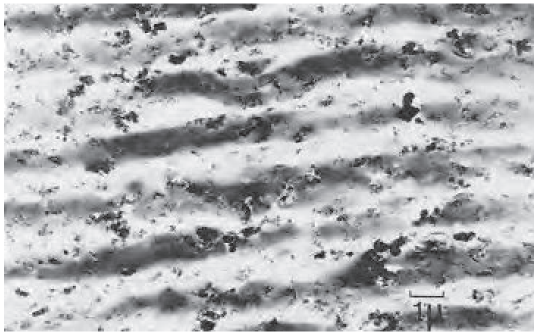


sea ice



compressed powder

B-2 Stealth Bomber
F-117 Nighthawk
Stealthy Helicopters

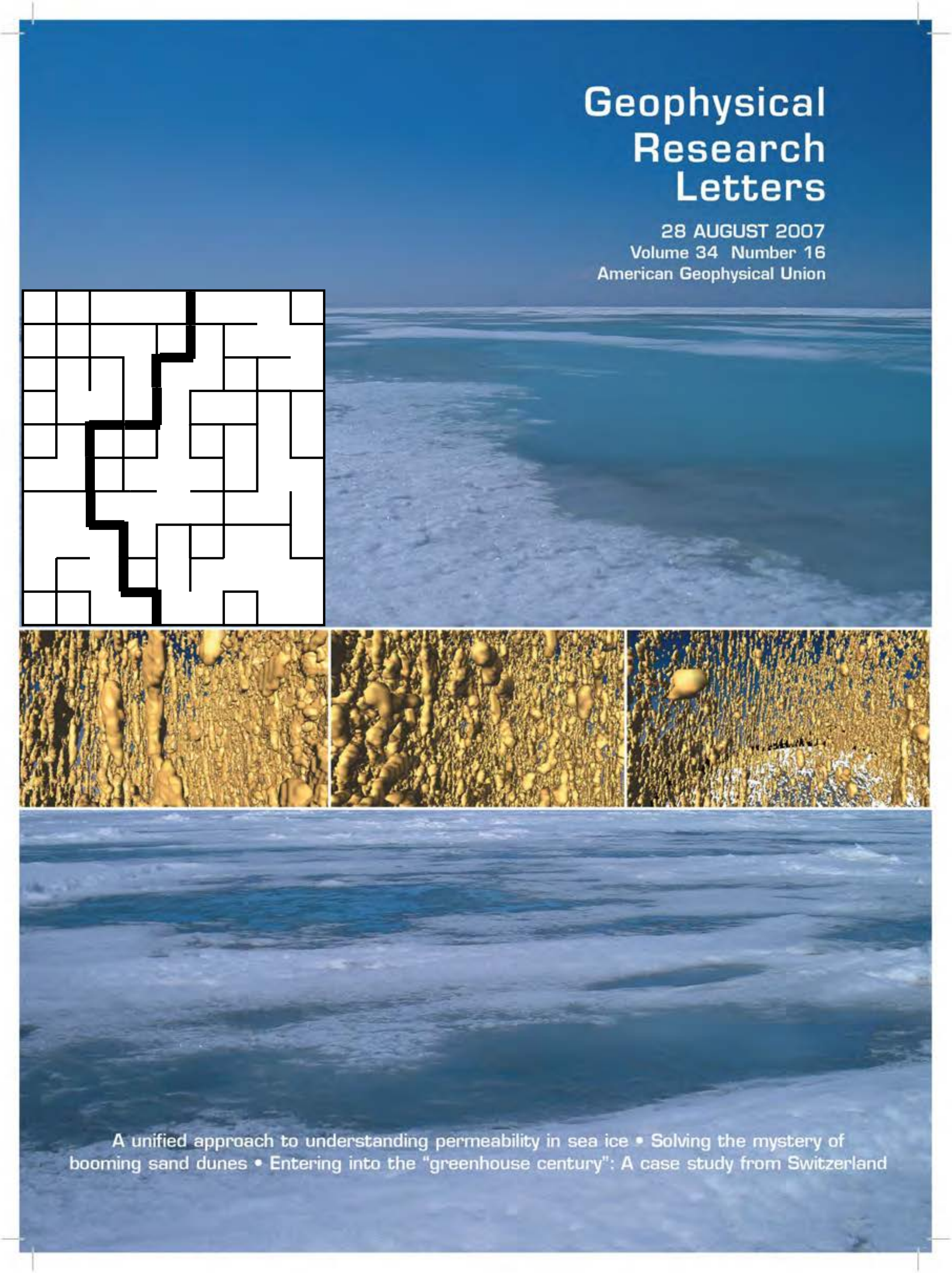


radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton*, Miner, Pringle, Zhu, Geophysical Research Letters 2007



Geophysical Research Letters
28 AUGUST 2007
Volume 34 Number 16
American Geophysical Union

A unified approach to understanding permeability in sea ice • Solving the mystery of booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland

microscale

governs

mesoscale processes

melt pond evolution

percolation theory for fluid permeability

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

from critical path analysis in **hopping conduction**

hierarchical model

rock physics

network model

rigorous bounds

X-ray tomography for brine inclusions

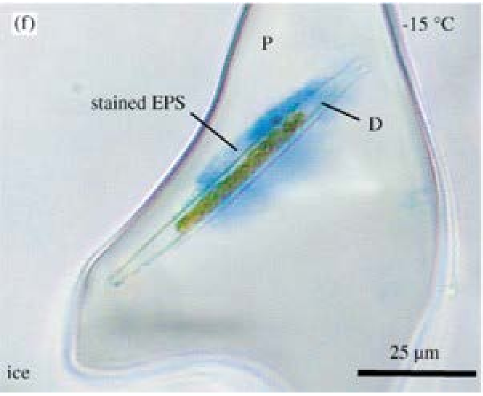
confirms rule of fives

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

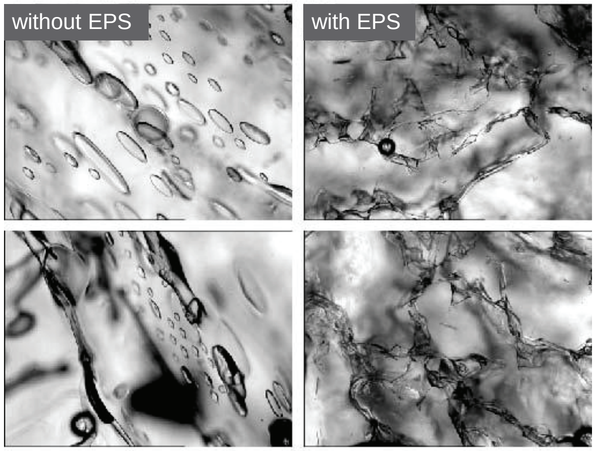
theories agree closely with field data

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

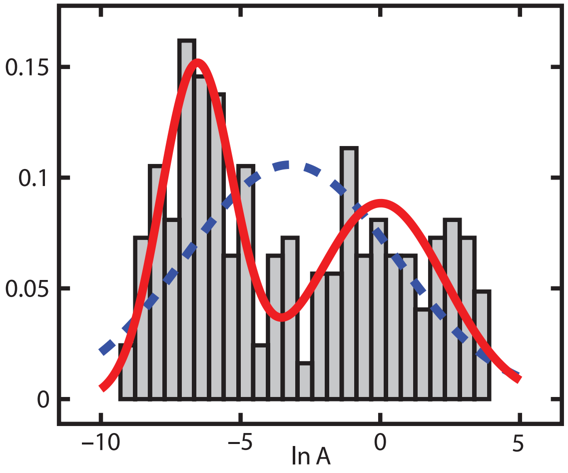
How does EPS affect fluid transport? **How does the biology affect the physics?**



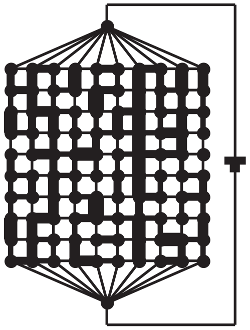
Krembs

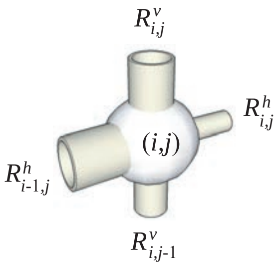


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





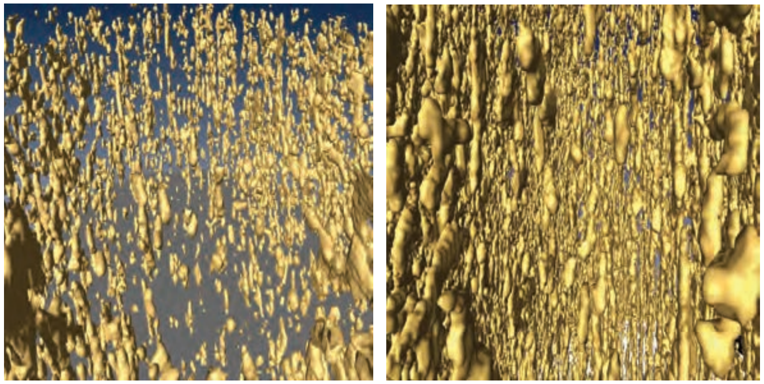
- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability *k;* results predict observed drop in *k*

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
***Multiscale Modeling and Simulation*, 2018**

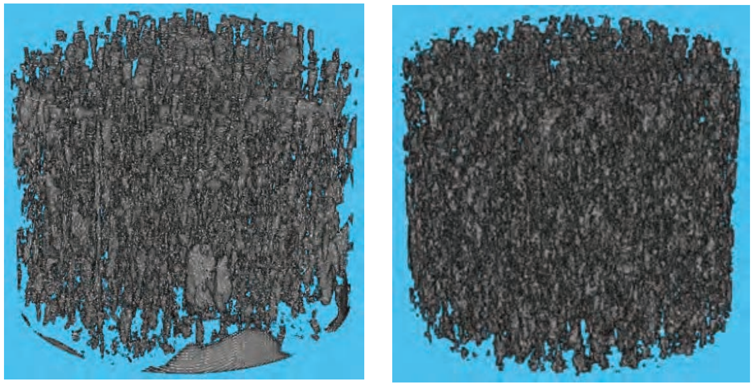
Zhu, Jabini, Golden,
Eicken, Morris
***Ann. Glac.* 2006**

Thermal evolution of the fractal geometry of the brine microstructure in sea ice

N. Ward, D. Hallman, J. Reimer, H. Eicken, M. Oggier and K. M. Golden, 2022



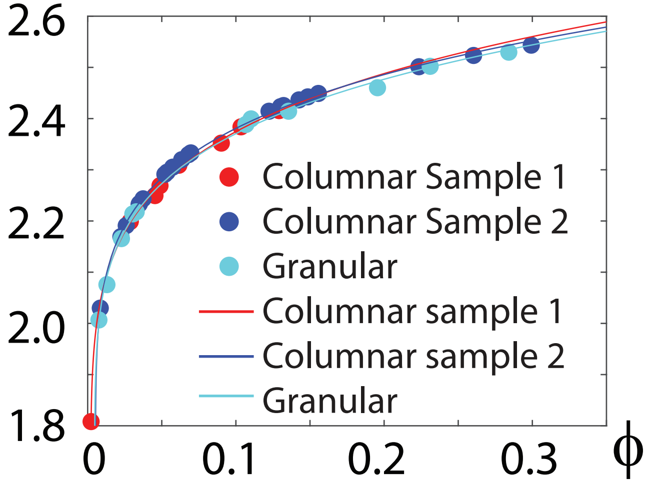
$T = -12\ ^\circ\mathrm{C},\ \phi = 0.033$ $T = -8^\circ\mathrm{C},\ \phi = 0.057$



X-ray tomography DLA model

+ implications for brine phase as a habitat

Fractal dimension



brine volume fraction (porosity)

theory of porosity as a function of fractal dimension

invert

excellent correspondence with data

Katz and Thompson, *PRL*, 1985

Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*
2014	*Arctic*	*Chukchi Sea*

ISSN 0002-9920

Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and the Mathematics of Transport in Sea Ice

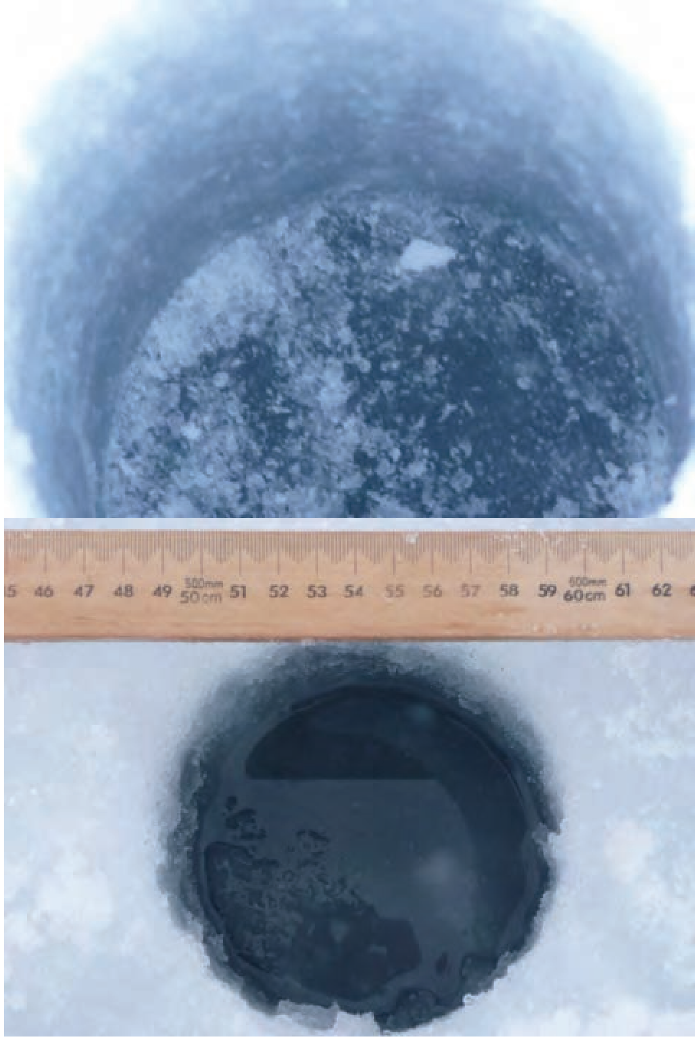
page 562

Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

page 586

Real analysis in polar coordinates (see page 613)

photo by Jan Lieser



measuring fluid permeability of Antarctic sea ice

SIPEX 2007

electrical measurements

Wenner array

vertical conductivity

Zhu, Golden, Gully, Sampson ***Physica B*** 2010

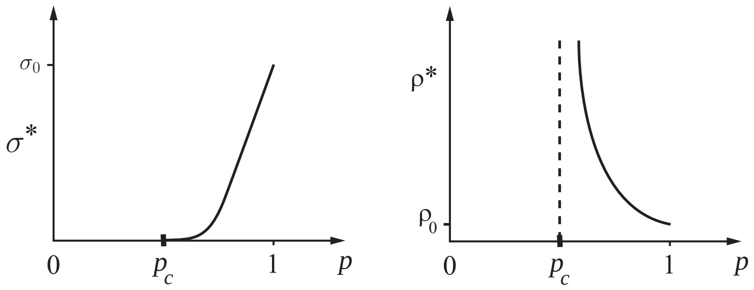
Sampson, Golden, Gully, Worby ***Deep Sea Research*** 2011

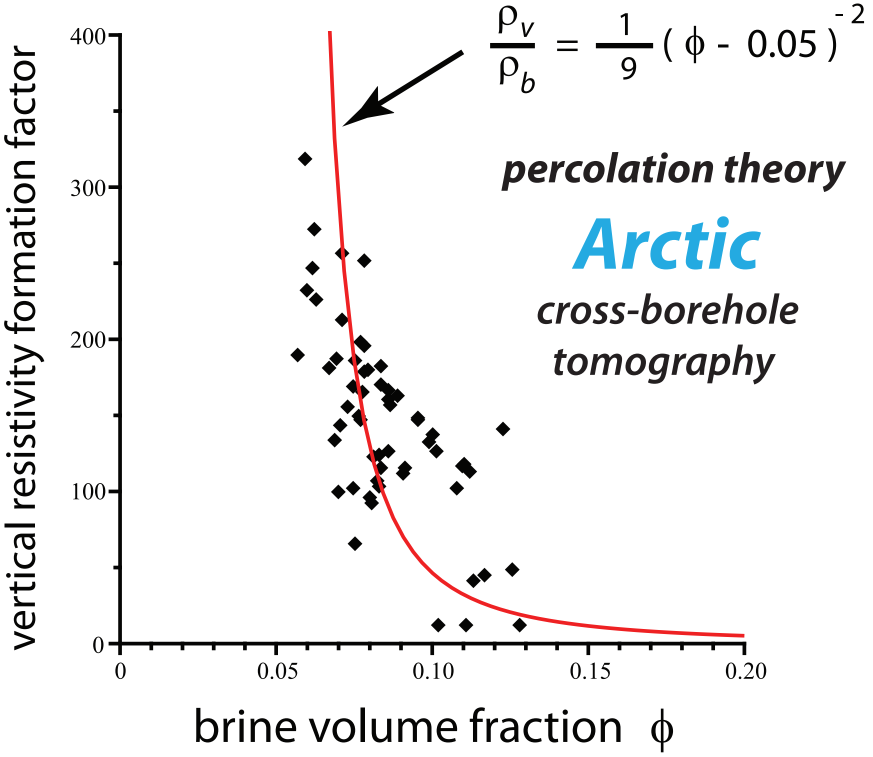
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow

same universal critical exponent as for fluid permeability

studied for over 50 years but no previous observations or theory of critical behavior



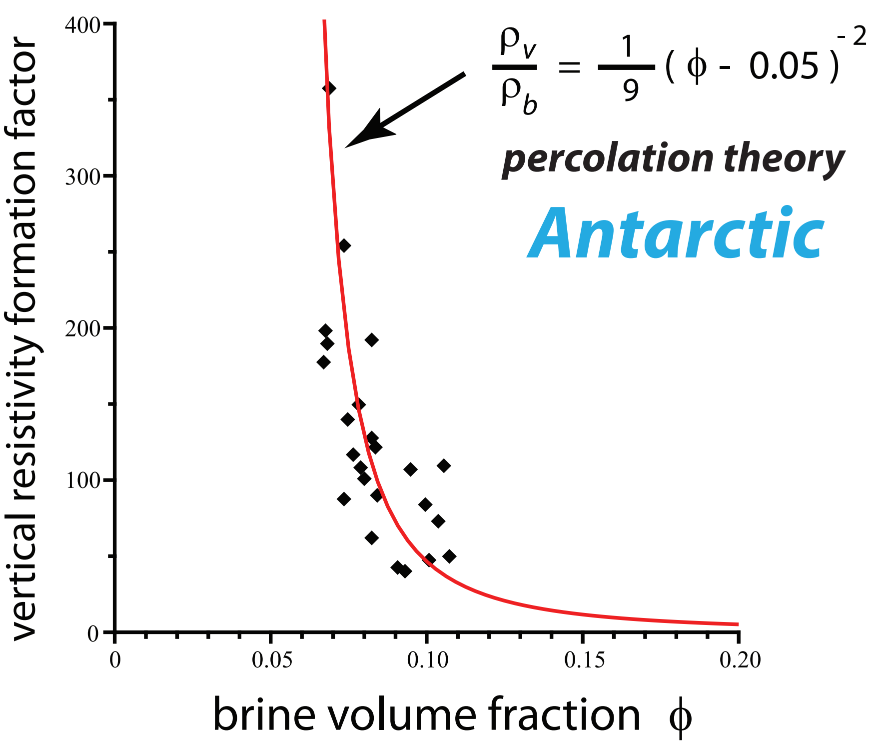


$$\frac{\rho_v}{\rho_b} = \frac{1}{9}\,(\,\phi - 0.05\,)^{-2}$$

percolation theory

Arctic

cross-borehole tomography



$$\frac{\rho_v}{\rho_b} = \frac{1}{9}\,(\,\phi - 0.05\,)^{-2}$$

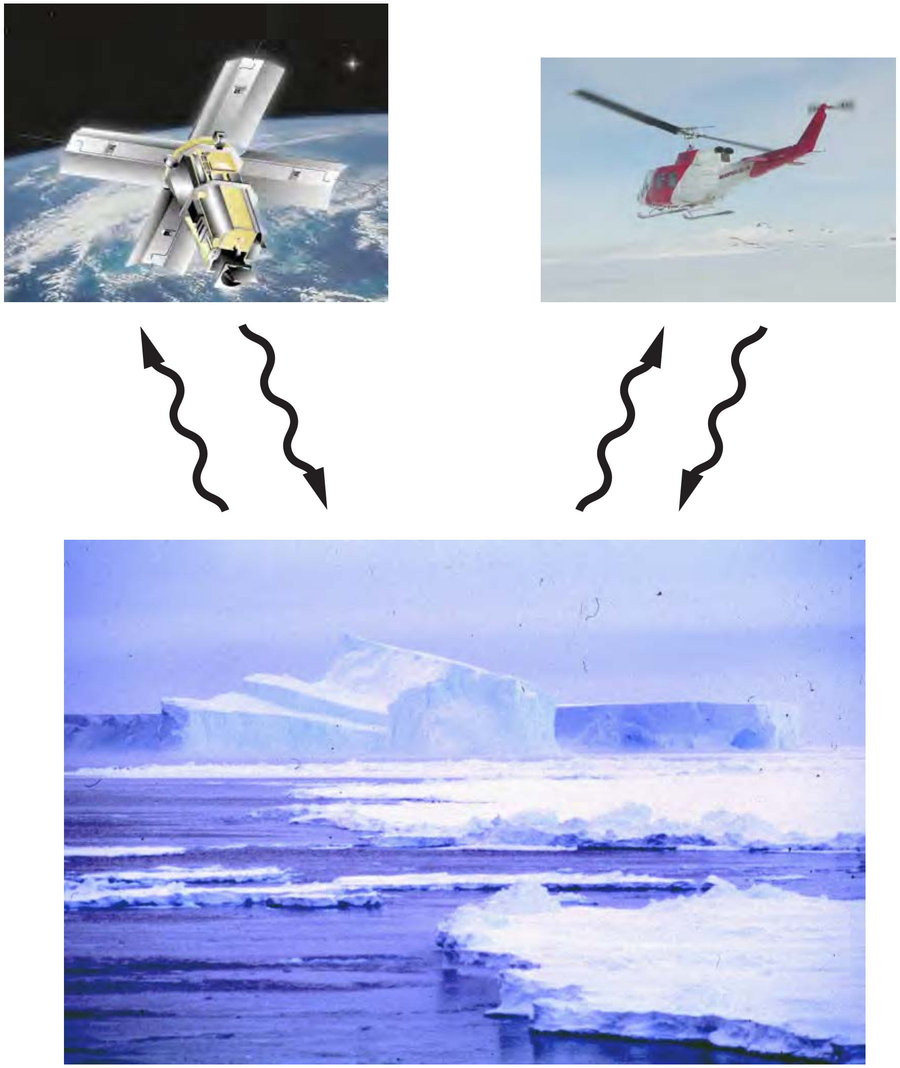
percolation theory

Antarctic

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2022

Golden

Remote sensing of sea ice



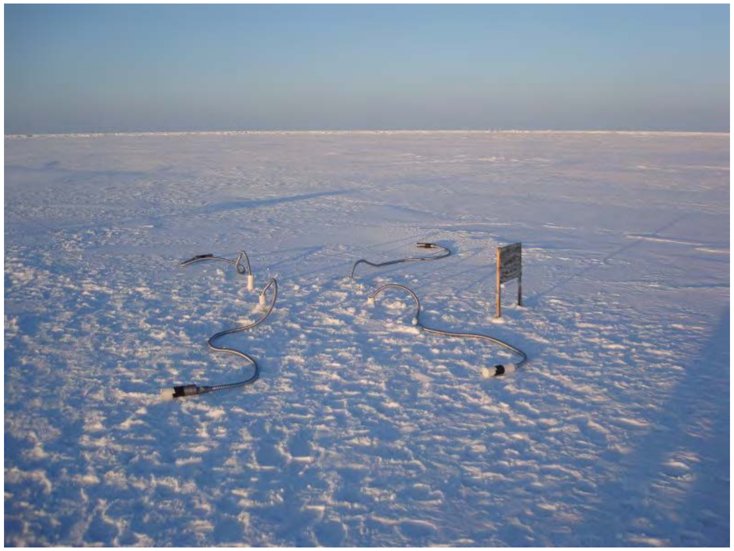
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice
properties from
electromagnetic
(EM) data

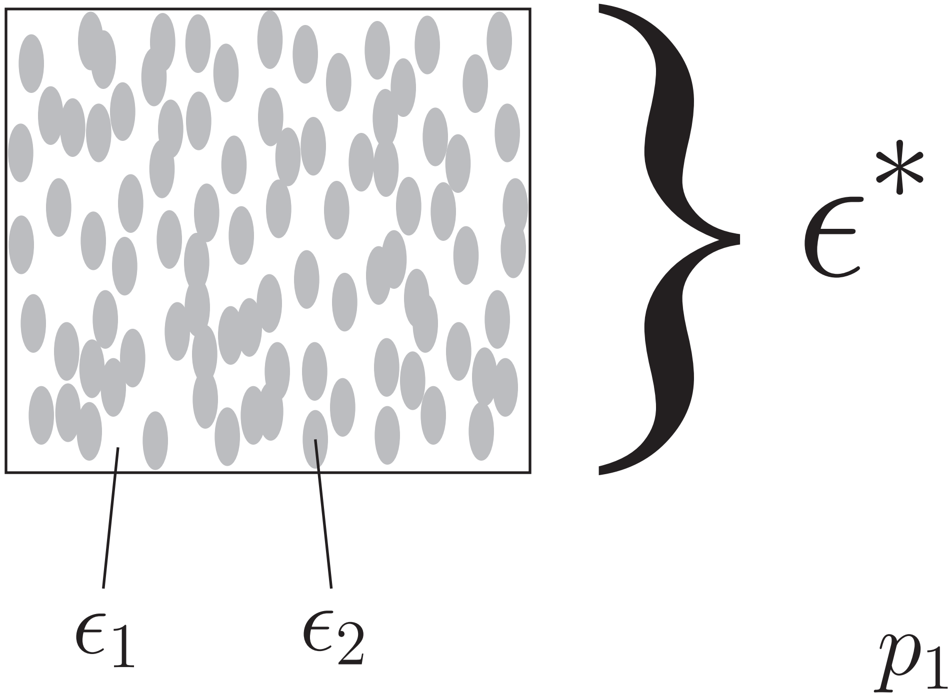
$$\varepsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \text{ , composite geometry } \right)$$

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method for Homogenization

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

separates geometry from parameters

geometry

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

material parameters

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

μ

- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on *n*-point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$$

$\chi =$ characteristic function of the brine phase

$$E = s\,(s + \Gamma\chi)^{-1} e_k$$

$\Gamma\chi$: microscale → macroscale

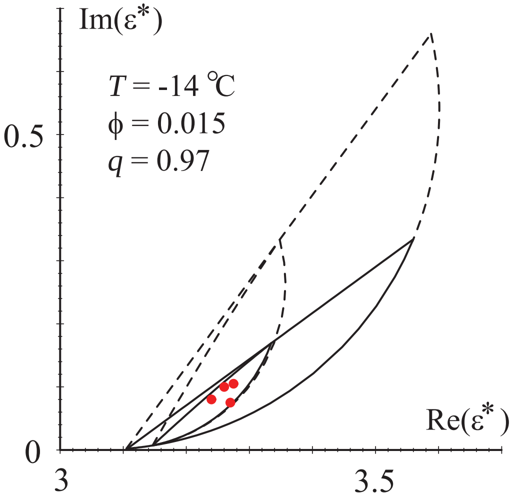
$\Gamma\chi$ *links scales*

Golden and Papanicolaou, Comm. Math. Phys. 1983

This representation distills the complexities of mixture geometry into the spectral properties of an operator like the Hamiltonian in physics.

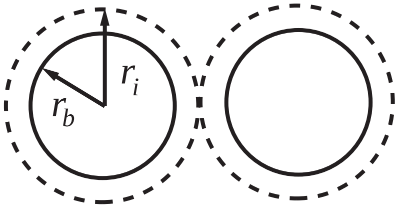
forward and inverse bounds on the complex permittivity of sea ice

forward bounds



4.75 GHz data

matrix particle

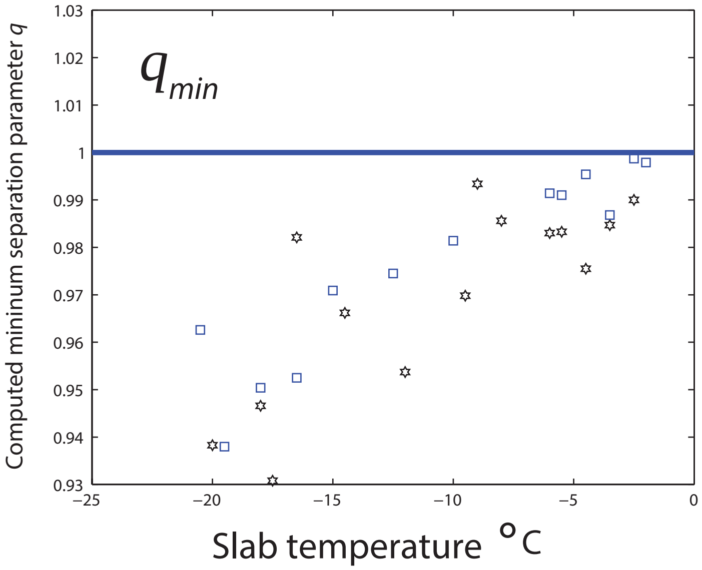


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



Inverse Homogenization

Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001), McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)

ε^* → ***composite geometry*** **(spectral measure μ)**

inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

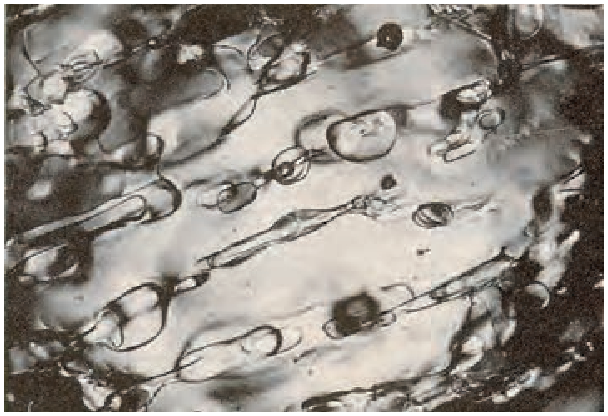
inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

rigorous inverse bound
on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

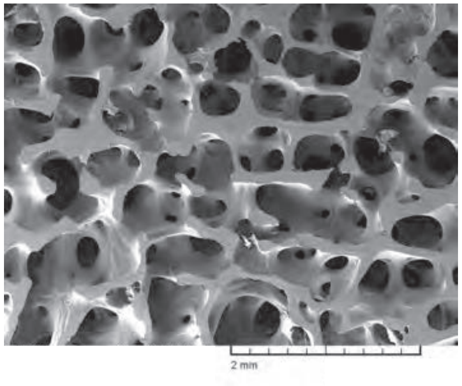
SEA ICE



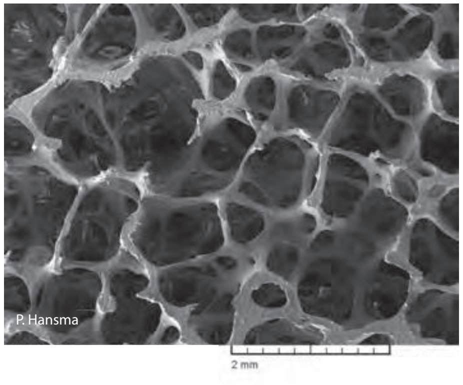
HUMAN BONE

spectral characterization of porous microstructures in human bone

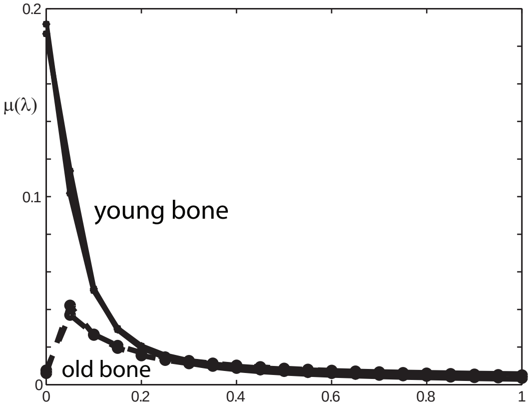
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures from complex permittivity data



use regularized inversion scheme

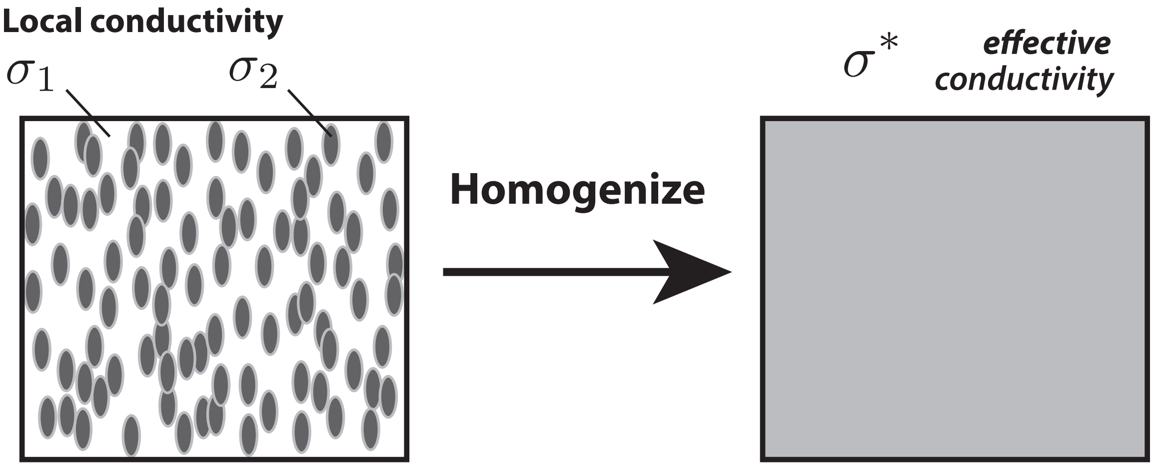
apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

Homogenization for polycrystalline materials

Two-component composites



Local conductivity

σ_1 σ_2

Homogenize

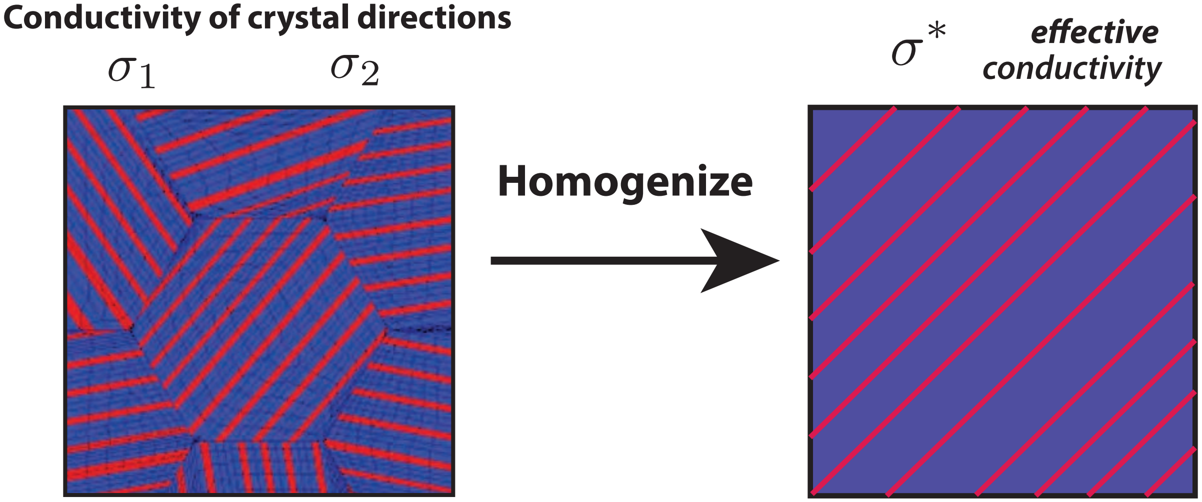
σ^* ***effective conductivity***

inhomogeneous medium

homogeneous medium

Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Polycrystalline media



Conductivity of crystal directions

σ_1 σ_2

Homogenize

σ^* ***effective conductivity***

inhomogeneous medium

homogeneous medium

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

  Milton (1981, 2002), Barabash and Stroud (1999), ...

- **Forward and inverse bounds**

  orientation statistics

- **Applied to sea ice using two-scale homogenization**

- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**

Proc. Roy. Soc. A 8 Feb 2015

Proc. R. Soc. A | Volume 471 | Issue 2174 | 8 February 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A

350 YEARS
OF SCIENTIFIC
PUBLISHING

An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques

A computer model to determine how a human should walk so as to expend the least energy

THE
ROYAL
SOCIETY
PUBLISHING

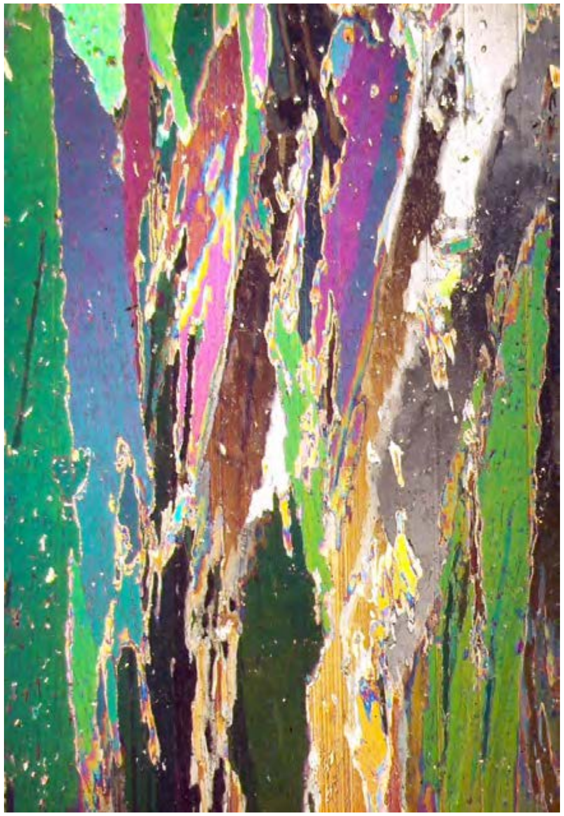
21/01/15 1:47 PM

higher threshold for fluid flow in granular sea ice

microscale details impact "mesoscale" processes

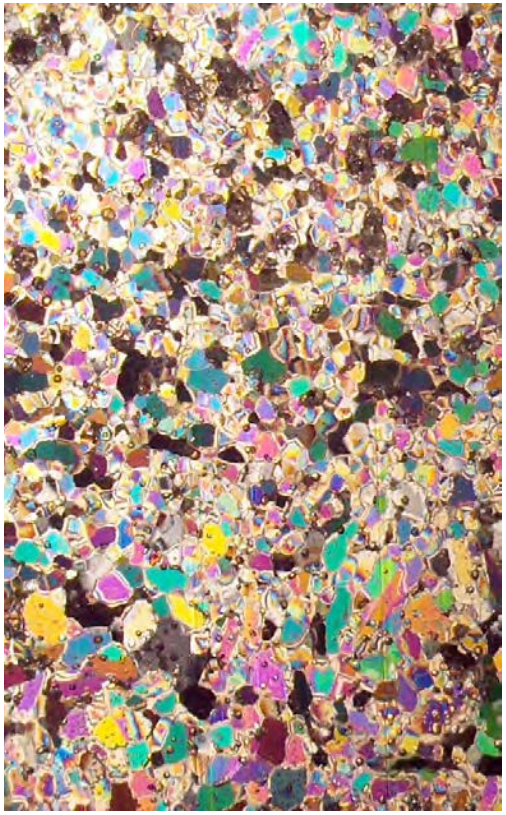
nutrient fluxes for microbes
melt pond drainage
snow-ice formation

columnar



5%

granular



10%

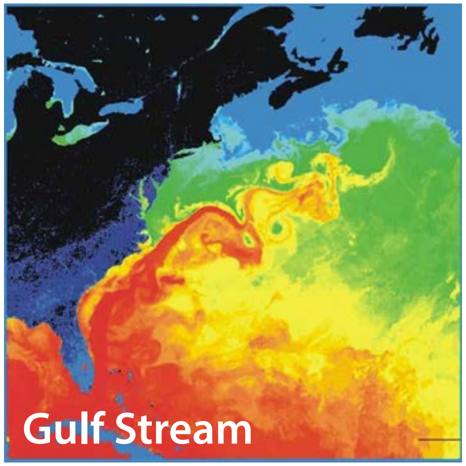
Golden, Sampson, Gully, Lubbers, Tison 2022

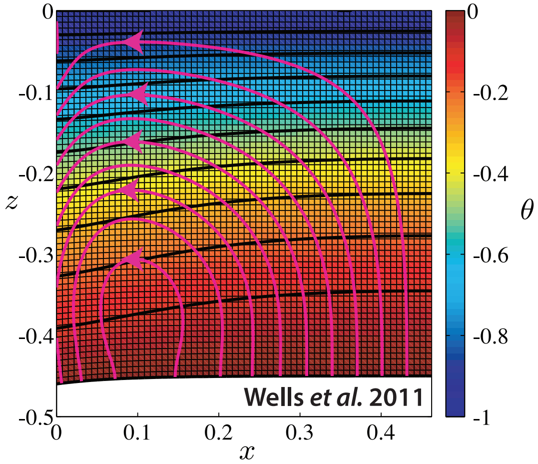
electromagnetically distinguishing ice types
Kitsel Lusted, Elena Cherkaev, Ken Golden

advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice
heat transport in sea ice with convection
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

$$\Big\downarrow \quad \textit{homogenize}$$

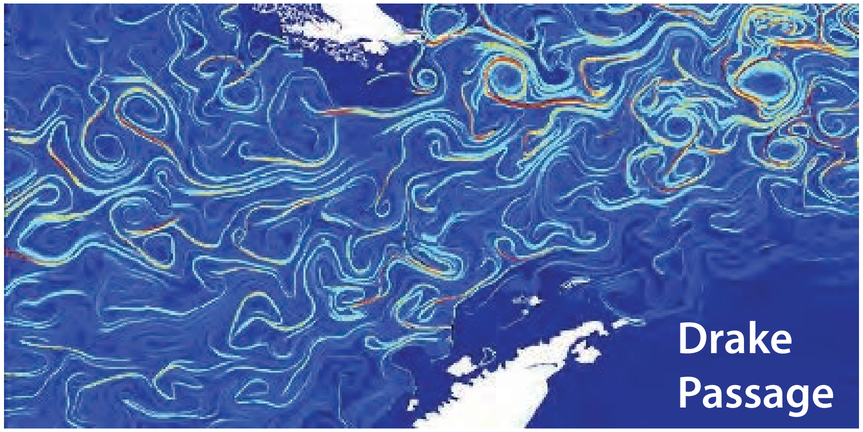
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity***

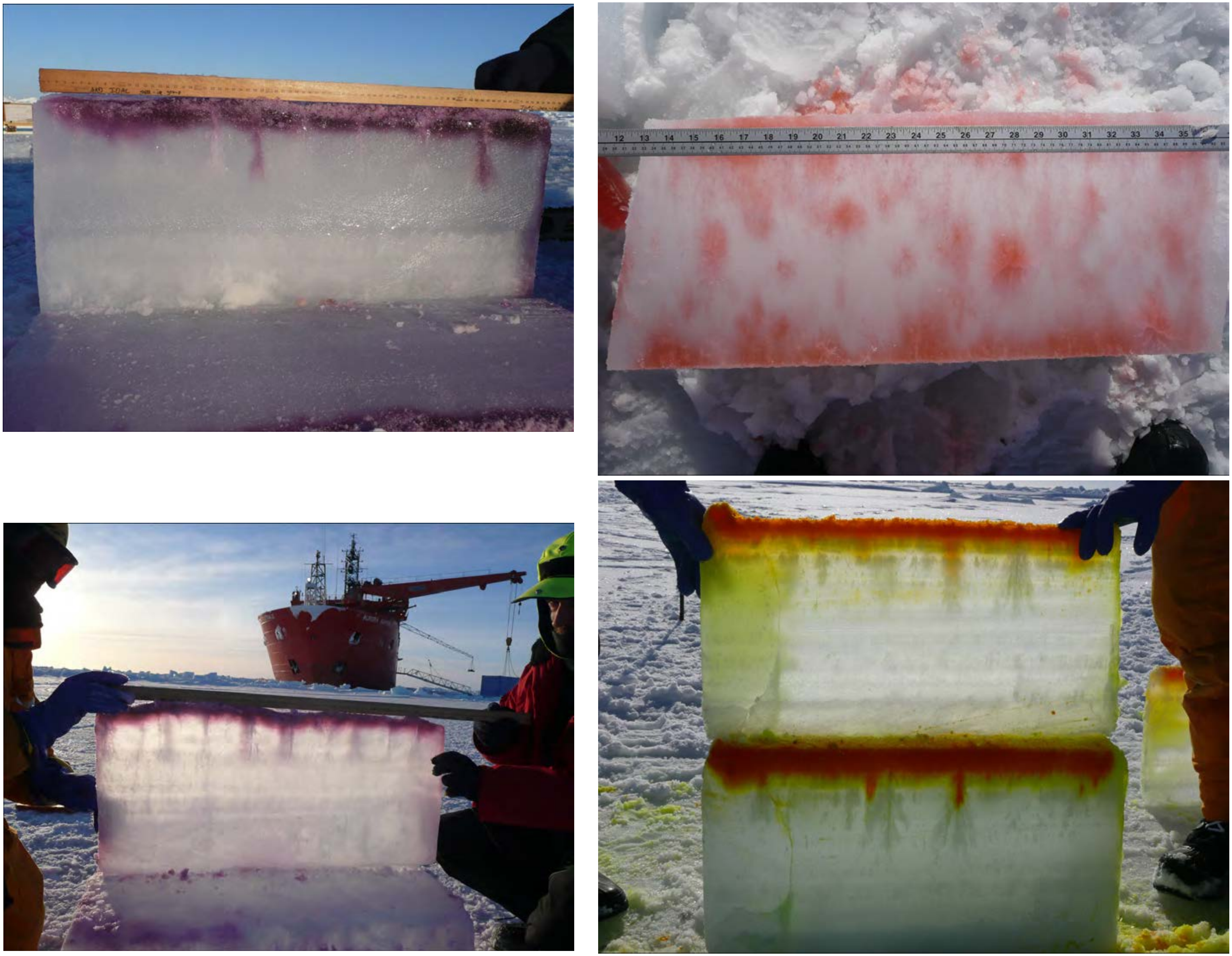
Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017
Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020



tracers flowing through inverted sea ice blocks



Stieltjes Integral Representation for Advection Diffusion

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020

$$\kappa^* = \kappa\left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2+\tau^2}\right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2+\tau^2}$$

- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H\Gamma$
- $H =$ stream matrix , $\kappa =$ local diffusivity
- $\Gamma := -\nabla(-\Delta)^{-1}\nabla\cdot$, Δ is the Laplace operator
- $i\Gamma H\Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ-plane

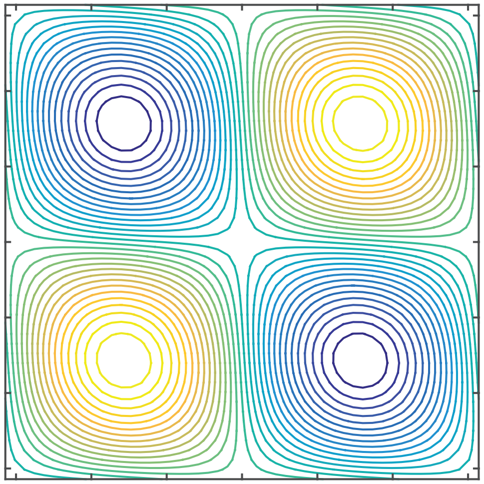
rigorous framework for numerical computations of spectral measures and effective diffusivity for model flows

new integral representations, theory of moment calculations

separation of material properties and flow field

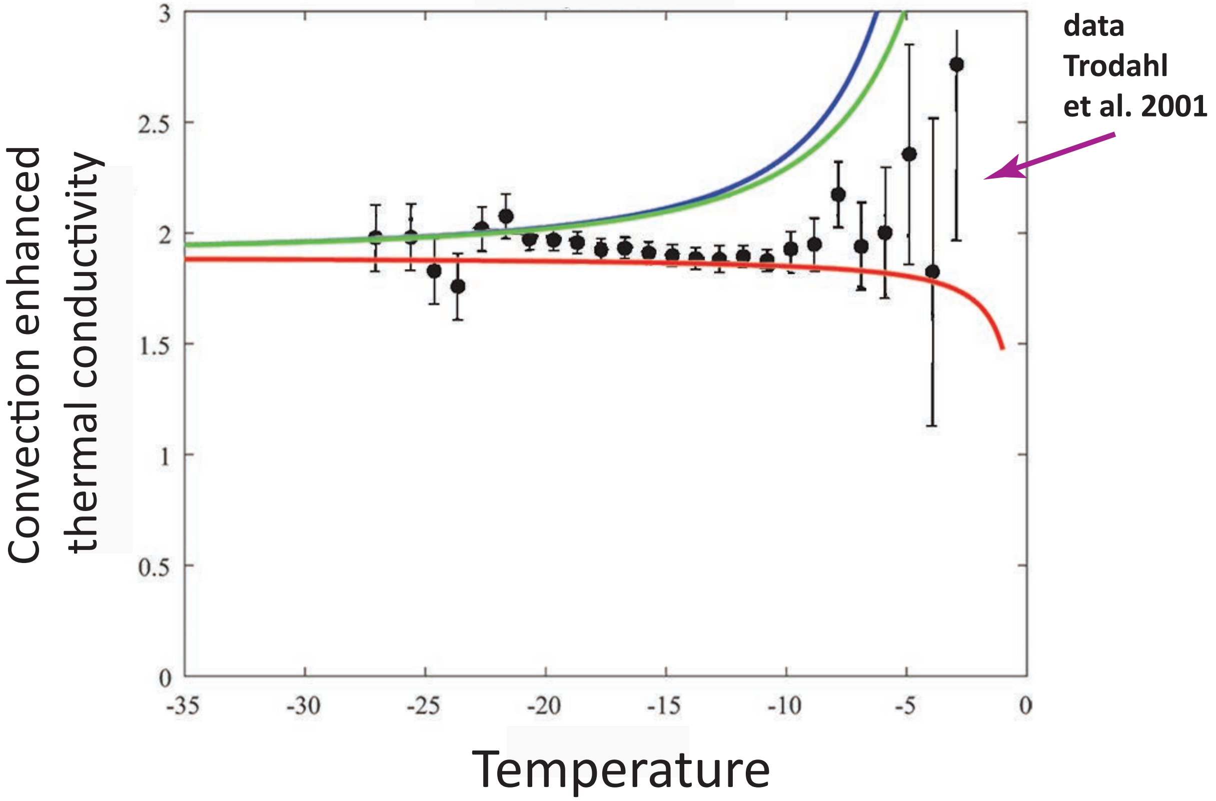
Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Dinh, Murphy, Zhu, Cherkaev, Golden 2022



cat’s eye flow model for
brine convection cells

similar bounds
for shear flows



rigorous Padé bounds from Stieltjes integral +
analytical calculations of moments of measure

wave propagation in the marginal ice zone (MIZ)

Stieltjes integral representation and bounds for the complex viscoelasticity of the ice - ocean layer

Sampson, Murphy, Cherkaev, Golden 2022

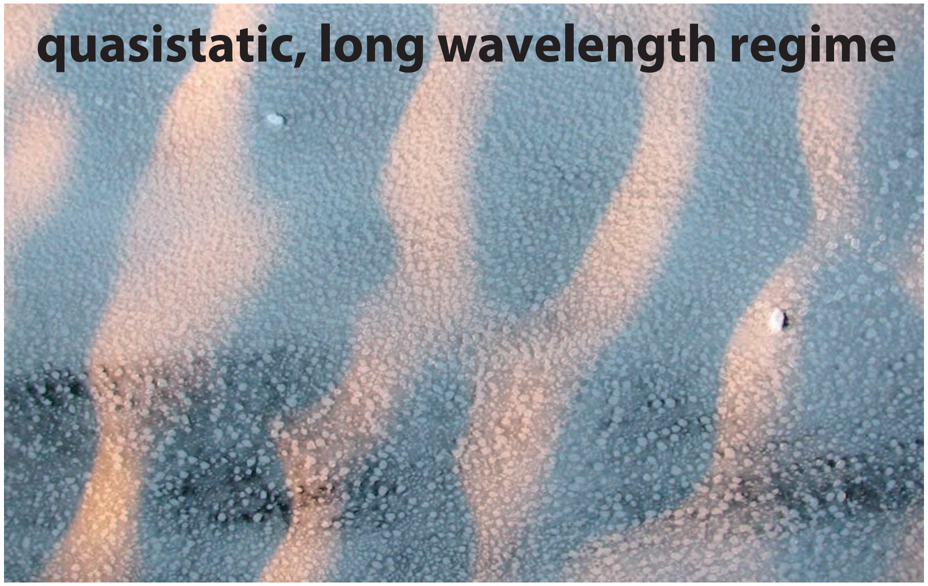
first theory of key parameter in wave-ice interactions only fitted to wave data before

Keller, 1998
Mosig, Montiel, Squire, 2015
Wang, Shen, 2012

Analytic Continuation Method

Bergman (78) - Milton (79)
integral representation for ε^*
Golden and Papanicolaou (83)

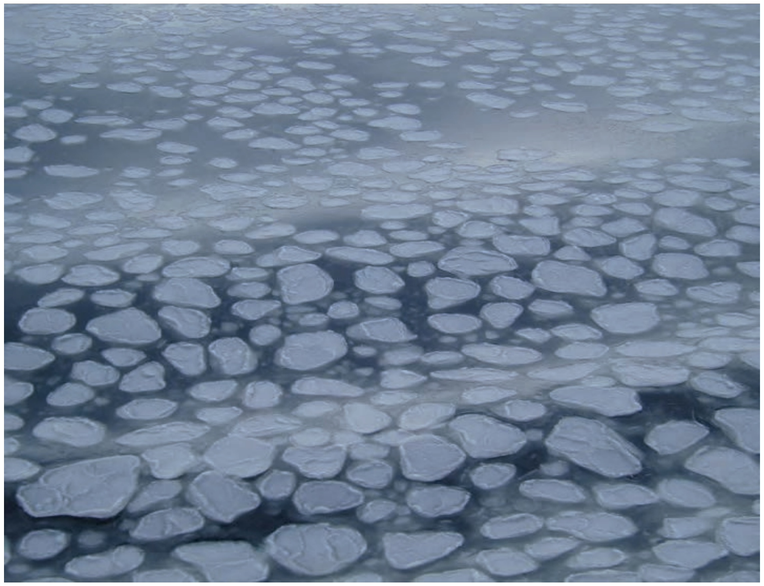
Milton, *Theory of Composites* (02)



quasistatic, long wavelength regime

homogenized
parameter
depends on
sea ice
concentration
and ice floe
geometry

like EM waves



melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

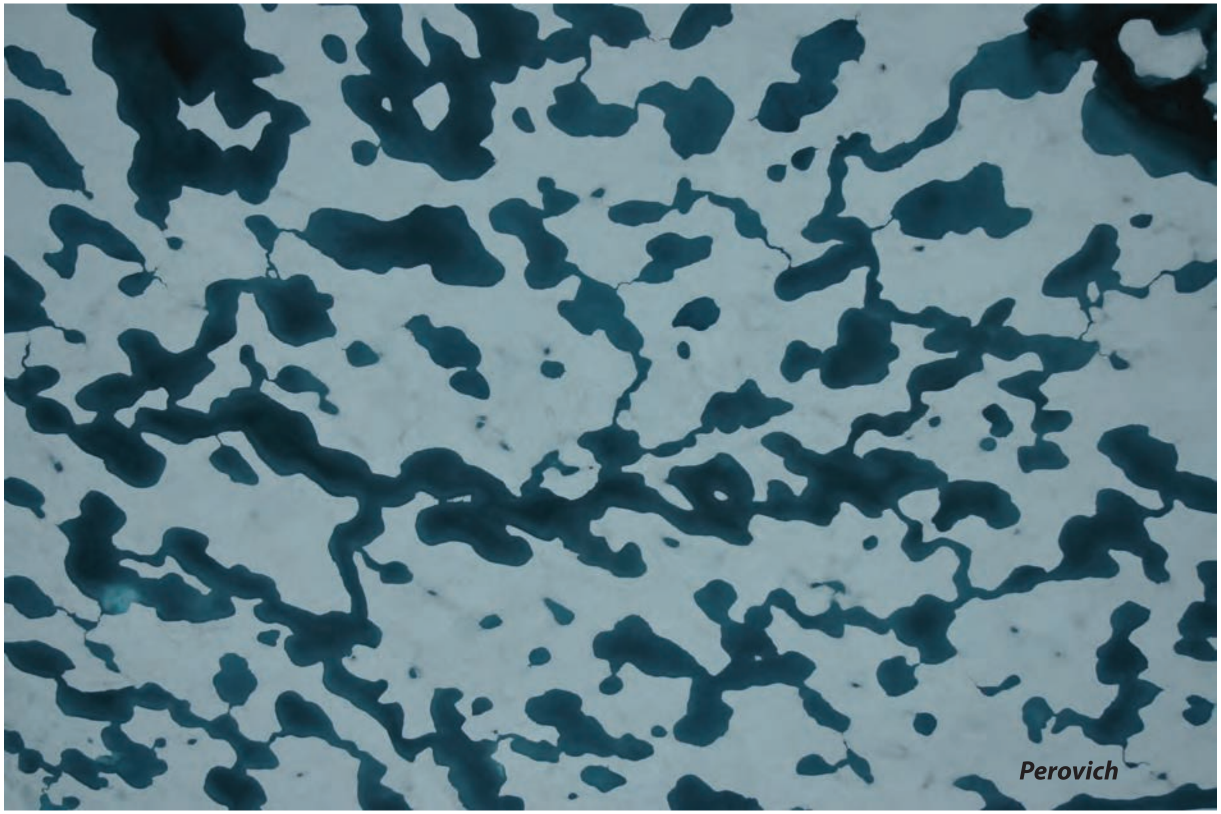
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

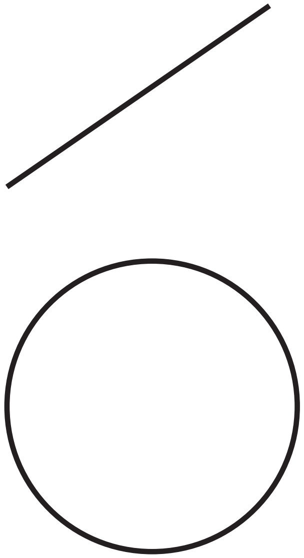
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

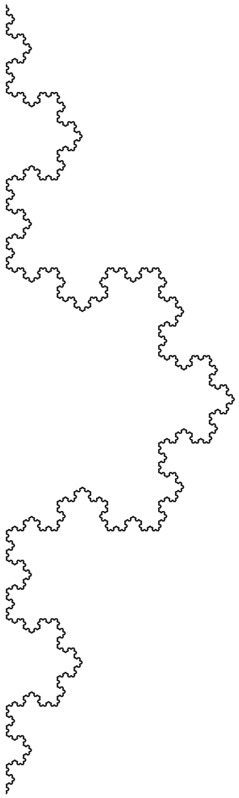
fractal curves in the plane

they wiggle so much that their dimension is >1



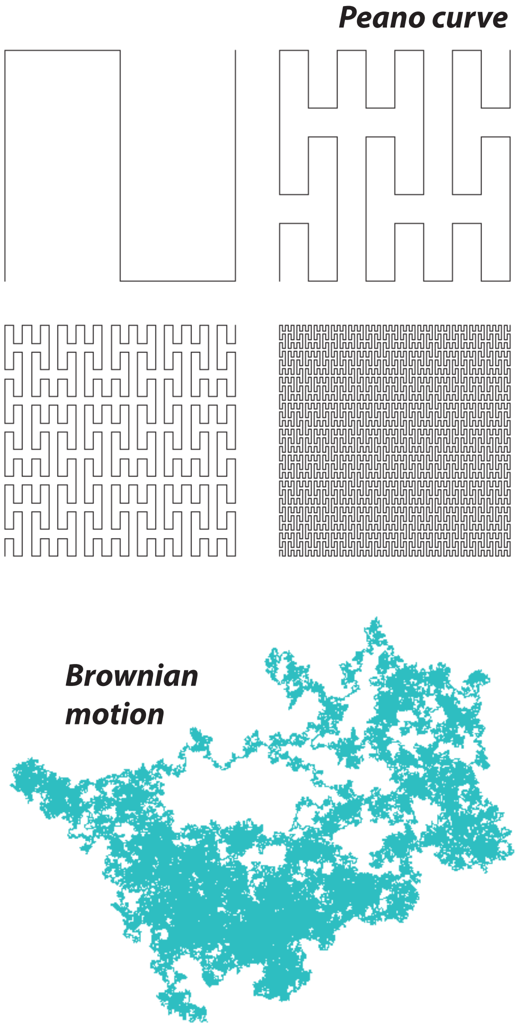
simple curves

$D = 1$



Koch snowflake

$D = 1.26$



space filling curves

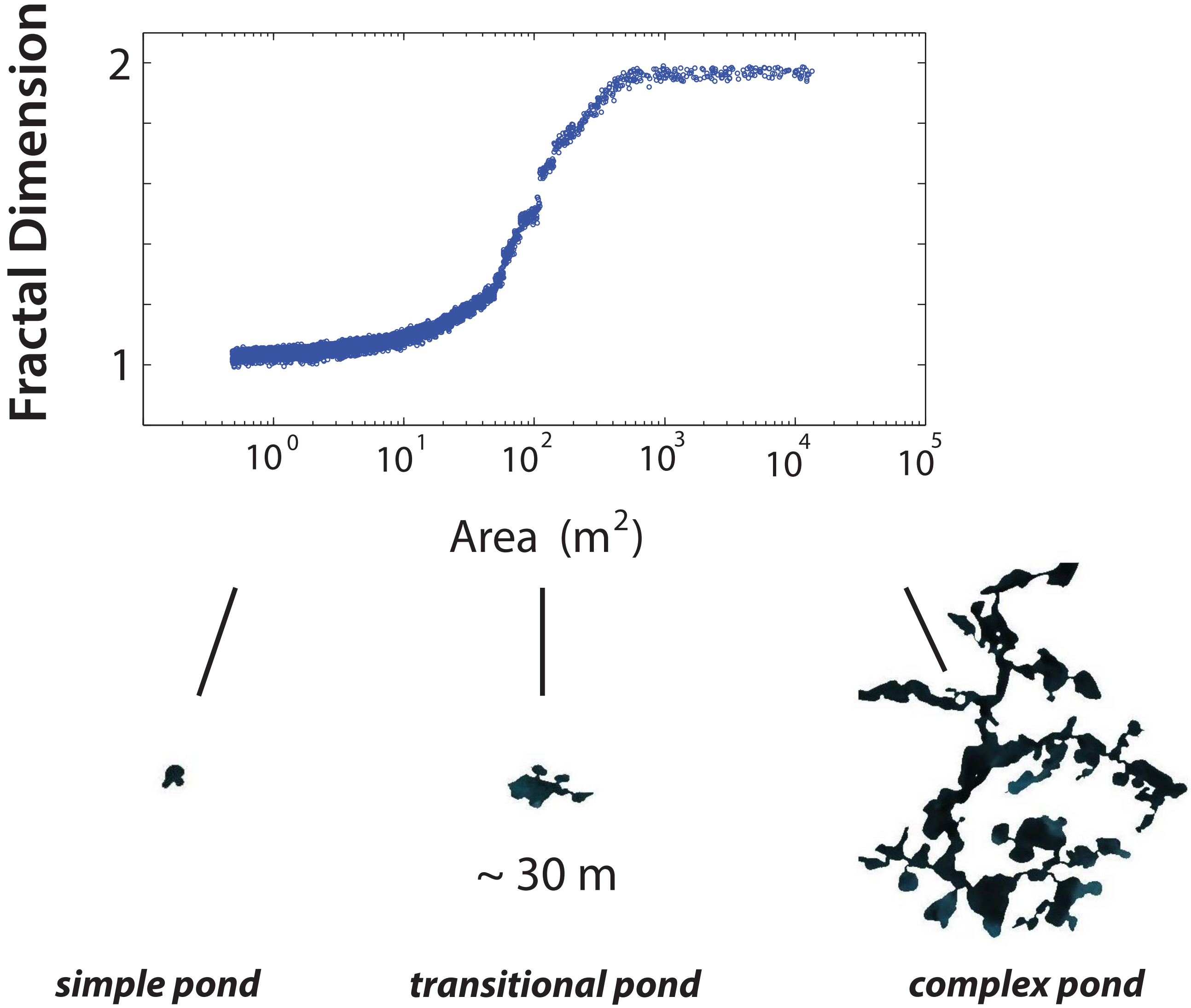
$D = 2$

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

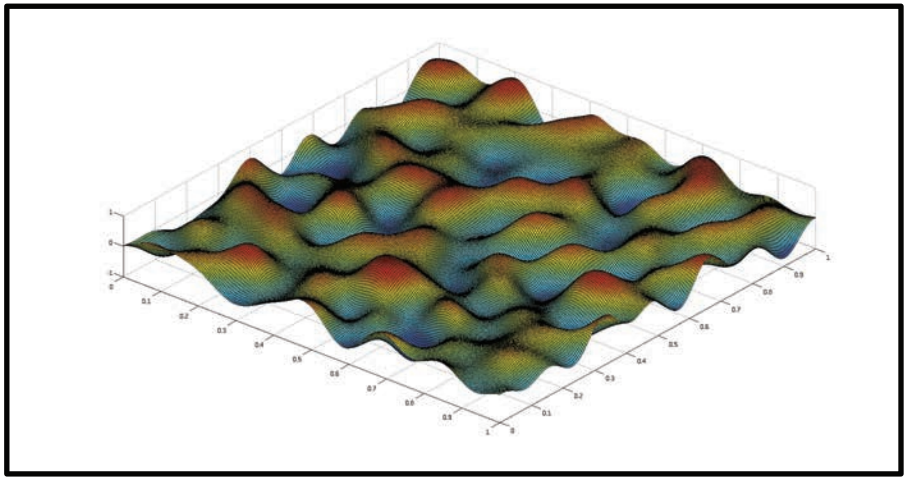
complexity grows with length scale



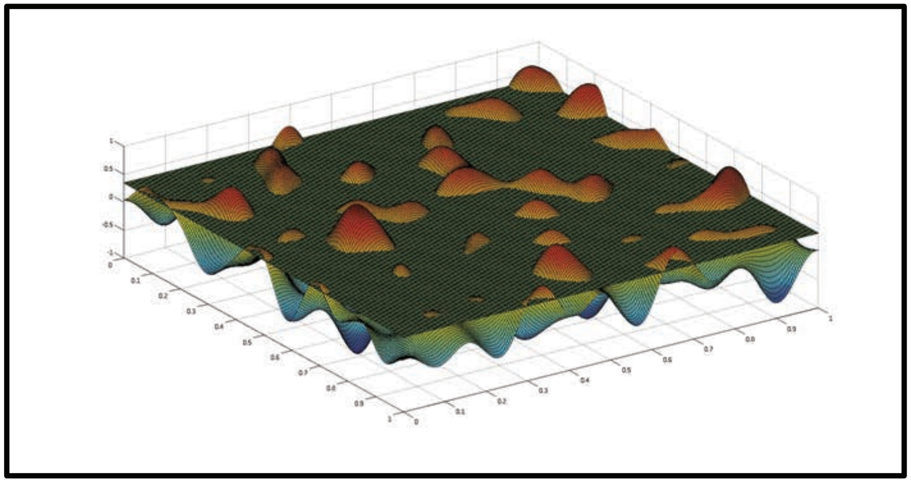
Continuum percolation model for melt pond evolution

level sets of random surfaces

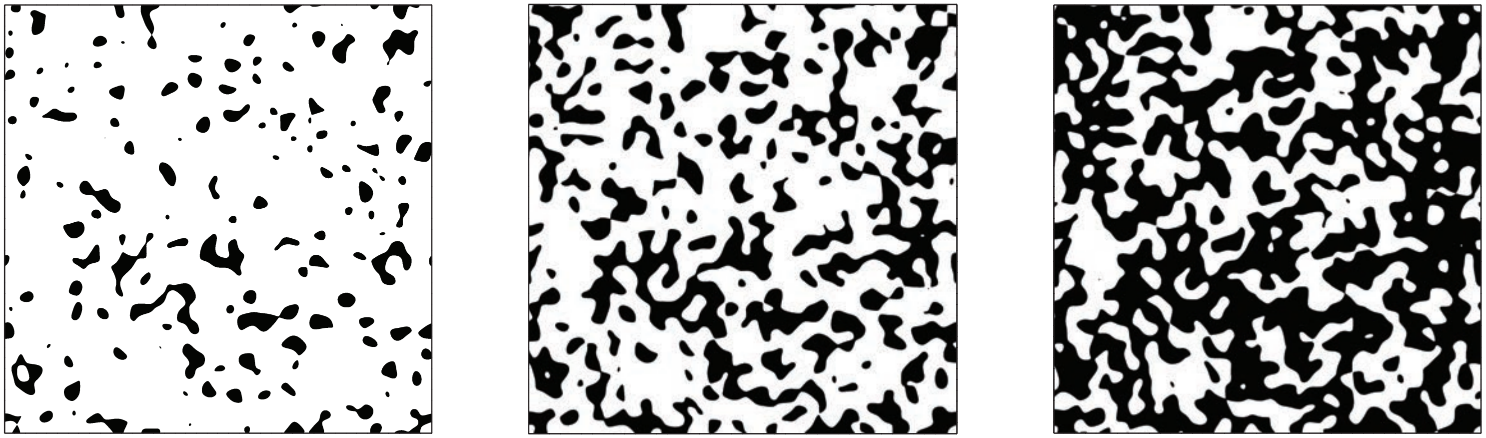
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

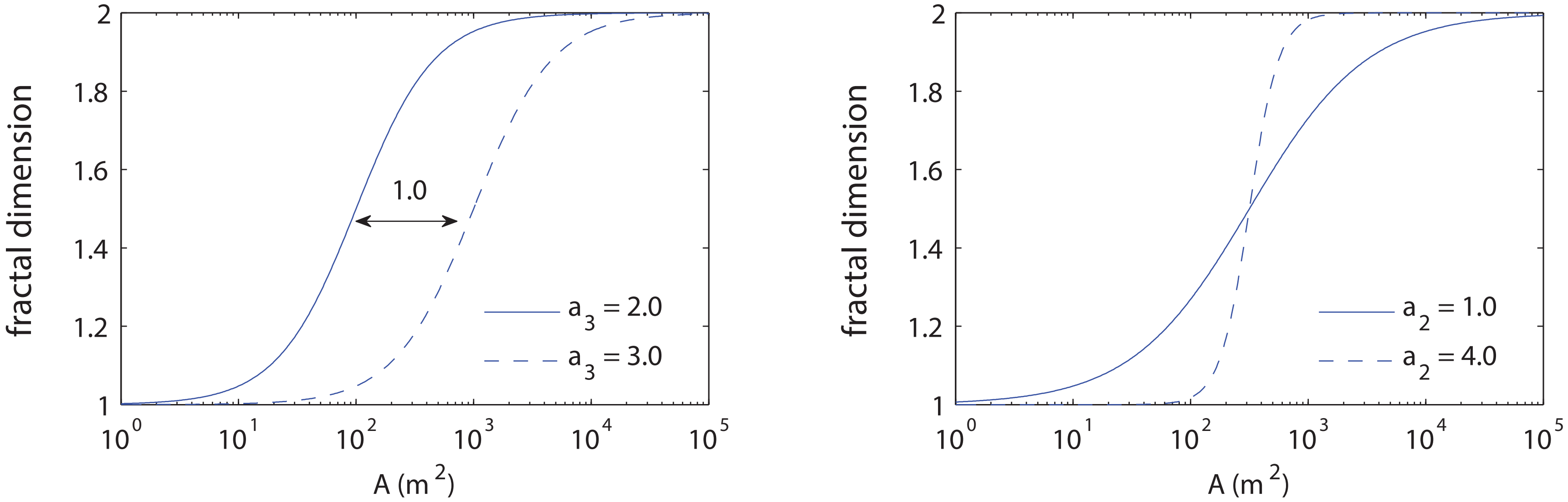


intersections of a plane with the surface define melt ponds



electronic transport in disordered media   *diffusion in turbulent plasmas*   *Isichenko, Rev. Mod. Phys., 1992*

fractal dimension curves depend on statistical parameters defining random surface



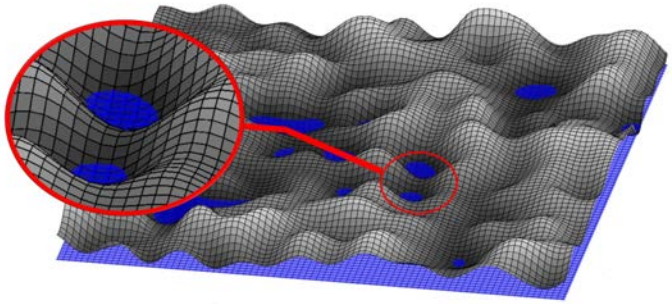
Topology of the sea ice surface and the fractal geometry of Arctic melt ponds

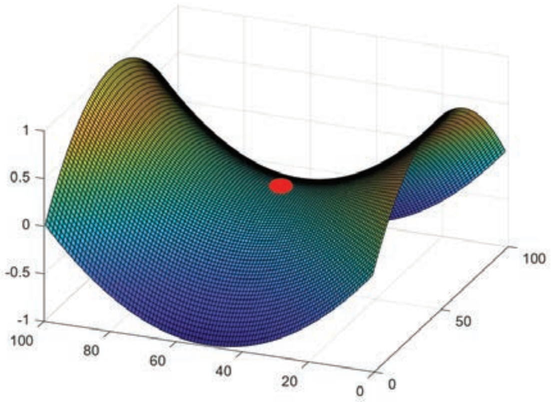
Physical Review Research (invited, under revision)

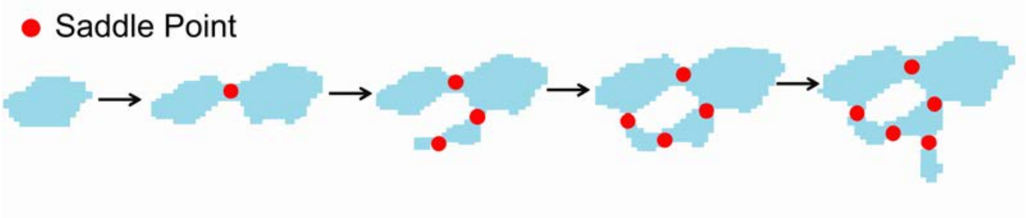
**Ryleigh Moore, Jacob Jones, Dane Gollero,
Court Strong, Ken Golden**

Several models replicate the transition in fractal dimension, but none explain how it arises.

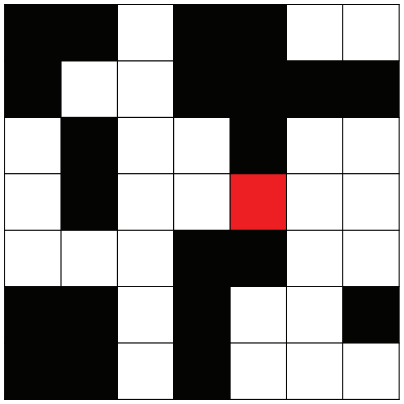
We use Morse theory applied to the random surface model to show that saddle points play the critical role in the fractal transition.



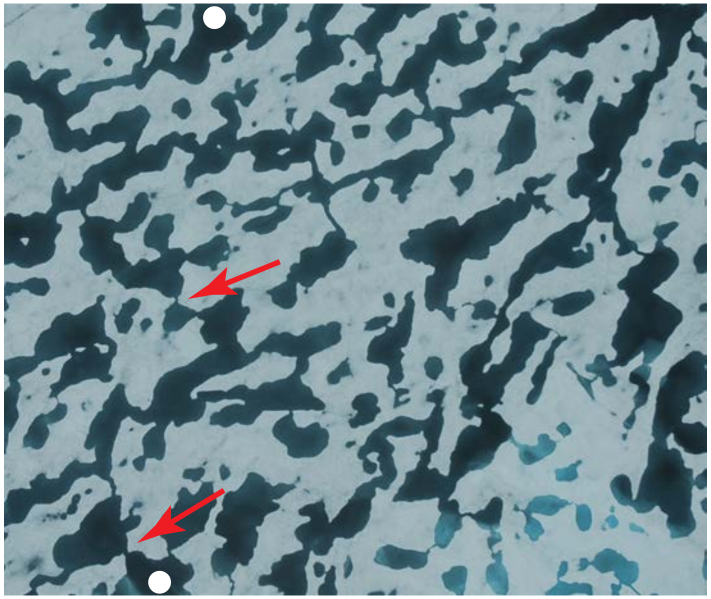




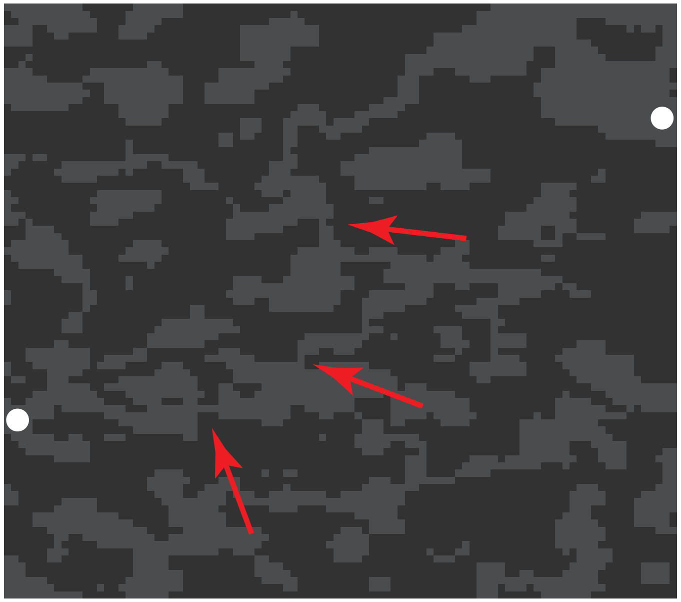
ponds coalesce
(change topology) and
complexify at saddle points



- Ponds connect through saddle points (Morse Theory).
- Red bonds in lattice percolation theory ~ saddle points.



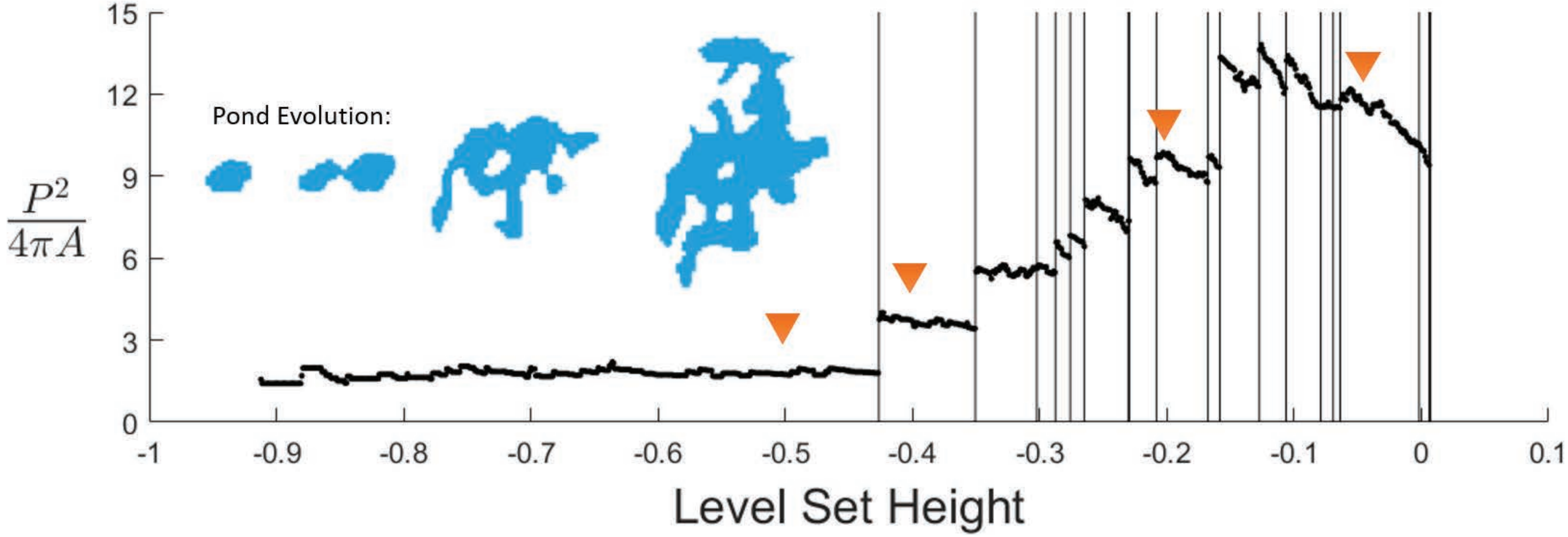
saddles



“red squares”

Main results

Isoperimetric quotient - as a proxy for fractal dimension - increases in discrete jumps when ponds coalesce at saddle points.

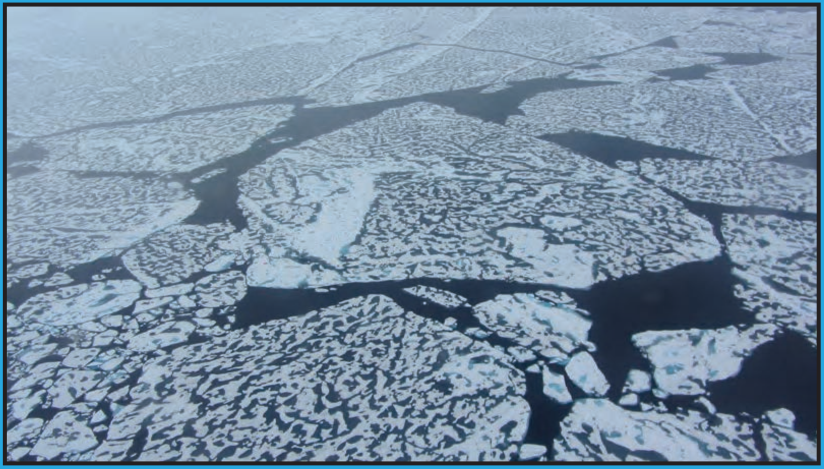


Horizontal fluid permeability "controlled" by saddles ~ electronic transport in 2D random potential.

drainage processes, seal holes

High connectivity of melt pond networks allows vast expanses of melt water to drain down seal holes, thaw holes, and into leads.

P. Nicklen



NPEO

C. Lydersen

P. McGowan

meted.ucar.edu

ice floe break-up

melt pond evolution depends also on large-scale “pores” in ice cover

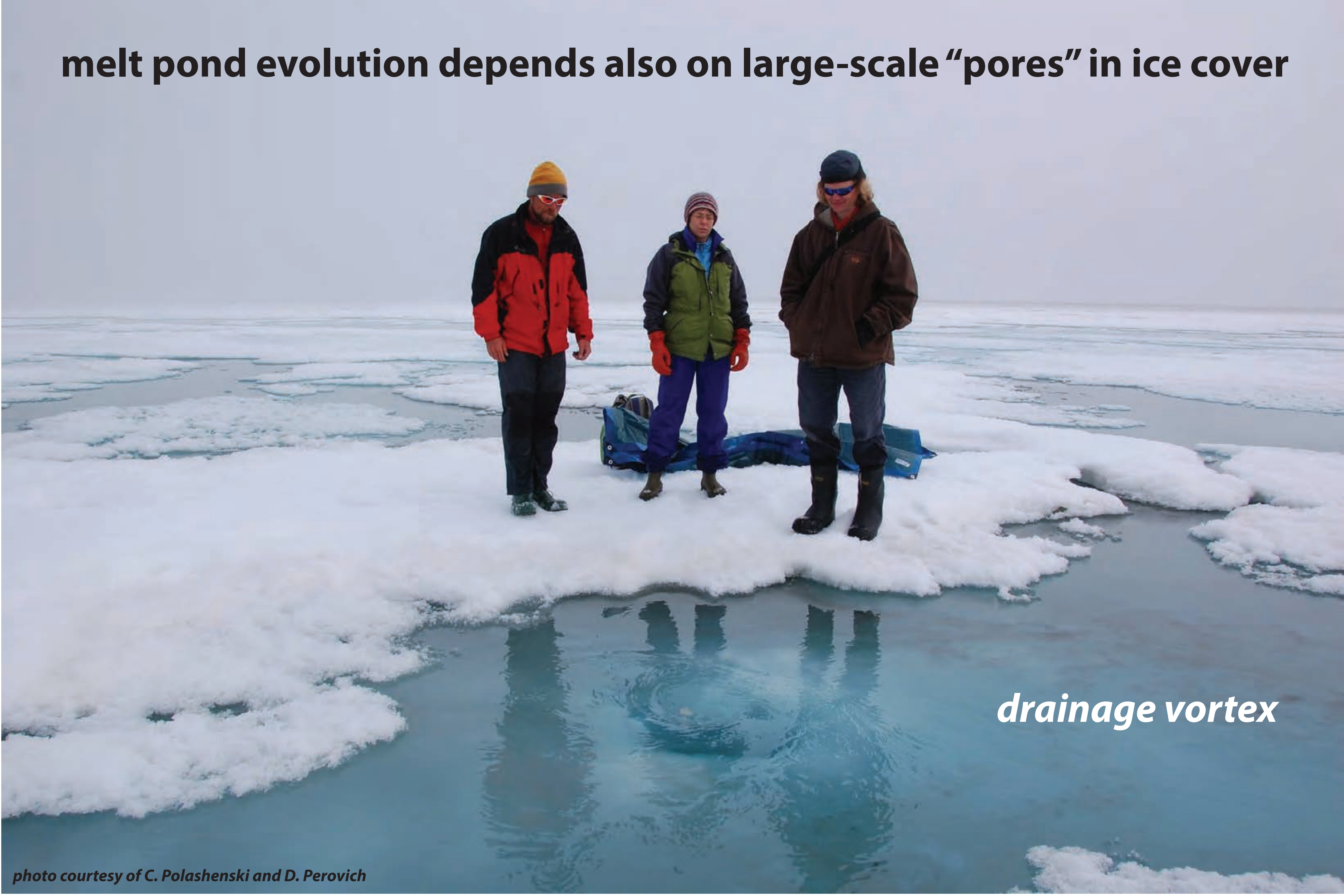


photo courtesy of C. Polashenski and D. Perovich

Melt pond connectivity enables vast expanses of melt water to drain down seal holes, thaw holes, and leads in the ice.

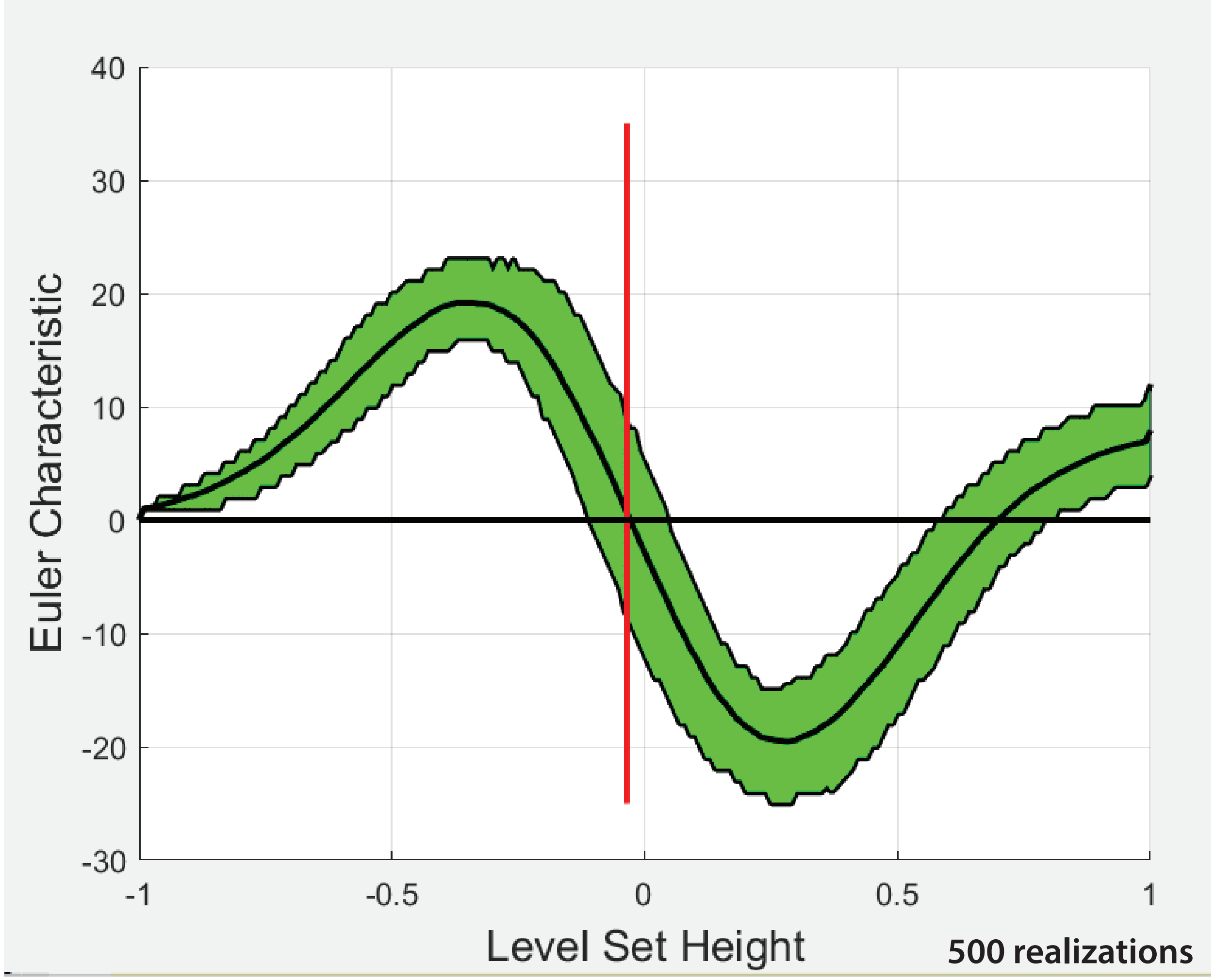
Topological Data Analysis

Euler characteristic = # maxima + # minima - # saddles

topological invariant

persistent homology

filtration - sequence of nested topological spaces, indexed by water level



500 realizations

Expected
Euler Characteristic Curve (ECC)

tracks the evolution of the EC of the flooded surface as water rises

zero of ECC ~ percolation

percolation on a torus
creates a giant cycle

Bobrowski &
Skraba, 2020

Carlsson, 2009

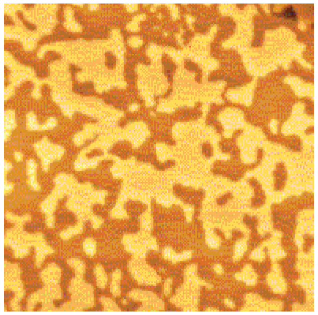
Vogel, 2002 GRF

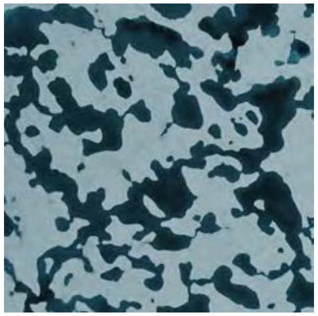
porous media
cosmology
brain activity

melt pond donuts

From magnets to melt ponds

100 year old model for magnetic materials used to explain melt pond geometry

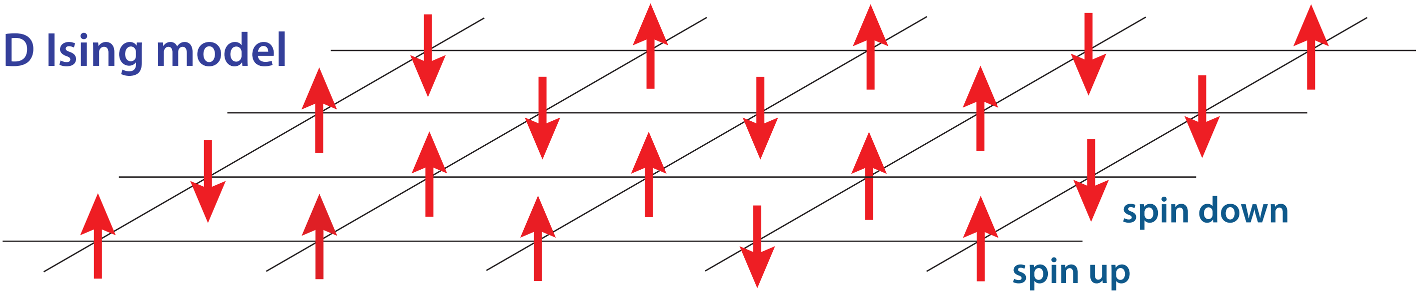

magnetic domains in cobalt

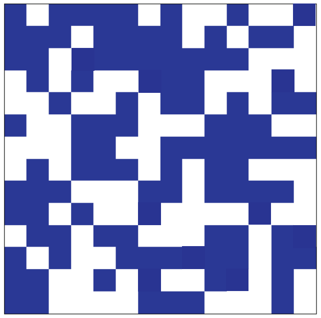

Arctic melt ponds


magnetic domains in cobalt-iron-boron

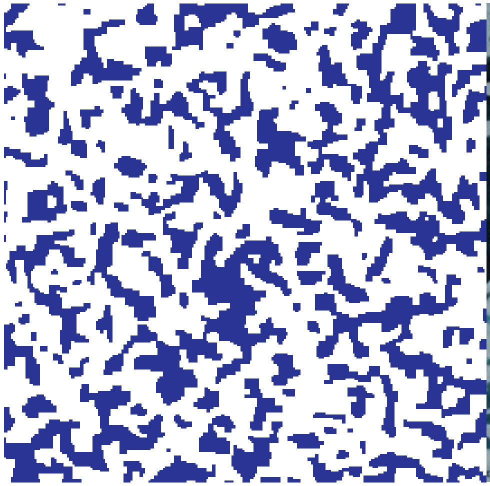

Arctic melt ponds

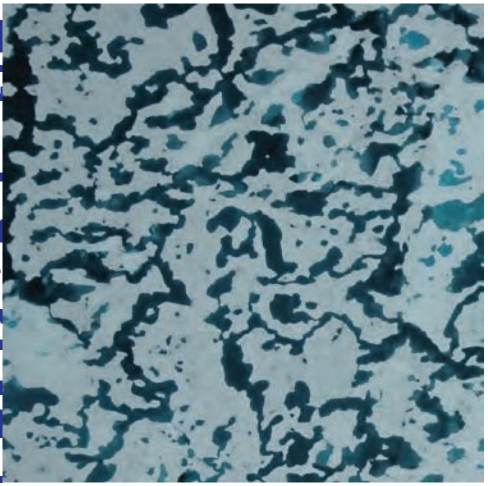
2D Ising model





model





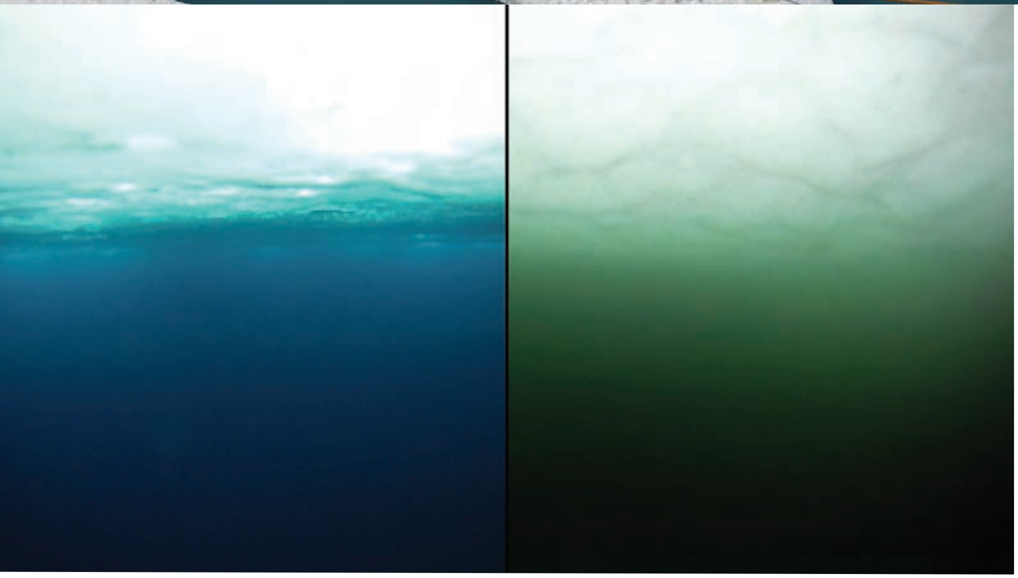
real ponds (Perovich)

Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom

bloom

massive under-ice algal bloom

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder,
Flocco, Feltham, ***Science Advances*** 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden
Geophys. Res. Lett. 2019

(2015 AMS MRC)

Conclusions

1. Sea ice is a fascinating multiscale porous composite with structure similar to many other natural and man-made materials.

2. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.

3. Homogenization and statistical physics provide rigorous methods to find effective behavior of sea ice; and advance the theory of composites.

4. Field experiments are essential to developing relevant mathematics.

5. Our research is advancing how sea ice is represented in climate models, and improving projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

ISSN 0002-9920 (print)
ISSN 1088-9477 (online)

Notices

of the American Mathematical Society

November 2020

Volume 67, Number 10

AMS
AMERICAN
MATHEMATICAL
SOCIETY
Advancing research. Creating connections.

The cover is based on "Modeling Sea Ice,"
page 1535.

THANK YOU

Office of Naval Research

Applied and Computational Analysis Program

Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences

Division of Polar Programs

United States Coast Guard
Semper Paratus
1790

Department of the Navy
ONR
Science & Technology

MCRN
Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

ace
CRC
ANTARCTIC CLIMATE & ECOSYSTEMS
COOPERATIVE RESEARCH CENTRE

NSF

Antarctica New Zealand

UAF
UNIVERSITY OF ALASKA FAIRBANKS

THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999

University of Utah Sea Ice Modeling Group (2017-2021)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Adjunct Assistant Professor of Mathematics

Postdoctoral Researchers: Noa Kraitzman (now at ANU), Jody Reimer

Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)
Christian Sampson (now at UNC Chapel Hill with Chris Jones)
Huy Dinh (now a sea ice MURI Postdoc at NYU/Courant)
Rebecca Hardenbrook
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier
Daniel Hallman

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich,
Dane Gollero, Samir Suthar, Anna Hyde,
Kitsel Lusted, Ruby Bowers, Kimball Johnston,
Jerry Zhang, Nash Ward, David Gluckman

High School Students: Jeremiah Chapman, Titus Quah, Dylan Webb

---

Sea Ice Ecology Group Postdoc Jody Reimer, Grad Student Julie Sherman,
Undergraduates Kayla Stewart, Nicole Forrester

Measuring sea ice thickness

direct calculation of spectral measures

Murphy, Hohenegger, Cherkaev, Golden, *Comm. Math. Sci.* 2015

- depends only on the composite geometry
- discretization of microstructural image gives binary network
- fundamental operator becomes a random matrix
- spectral measure computed from eigenvalues and eigenvectors

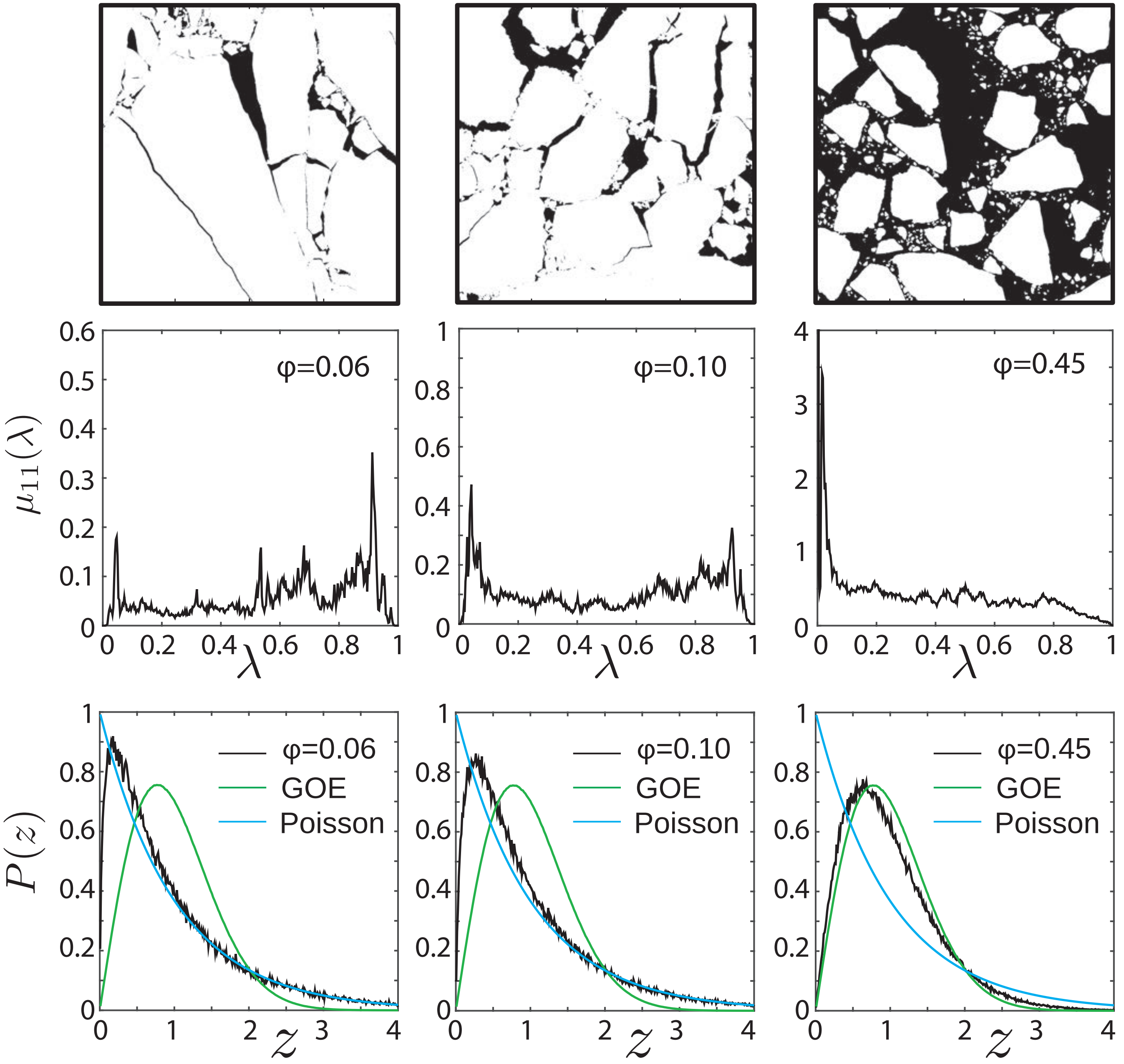
once we have the spectral measure μ it can be used in Stieltjes integrals for other transport coefficients:

electrical and thermal conductivity, complex permittivity, magnetic permeability, diffusion, fluid flow properties

earlier studies of spectral measures

Day and Thorpe 1996
Helsing, McPhedran, Milton 2011

Spectral computations for sea ice floe configurations

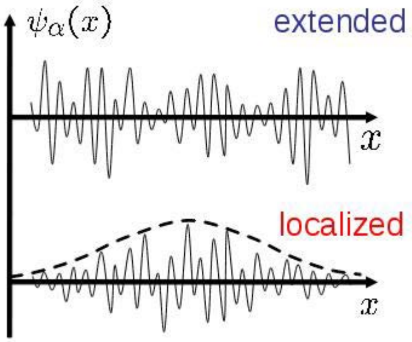


spectral measures

eigenvalue spacing distributions

UNIVERSAL Wigner-Dyson distribution

uncorrelated → **level repulsion**



electronic transport in semiconductors

metal / insulator transition

localization

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

Anderson transition in wave physics:
quantum, optics, acoustics, water waves, ...

from analysis of spectral measures for brine, melt ponds, ice floes

we find percolation-driven

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

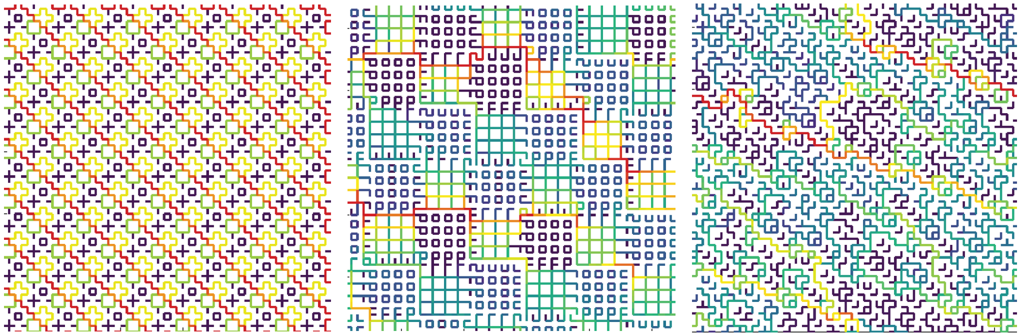
PERCOLATION TRANSITION → **universal eigenvalue statistics (GOE)**
extended states, mobility edges

-- but with NO wave interference or scattering effects ! --

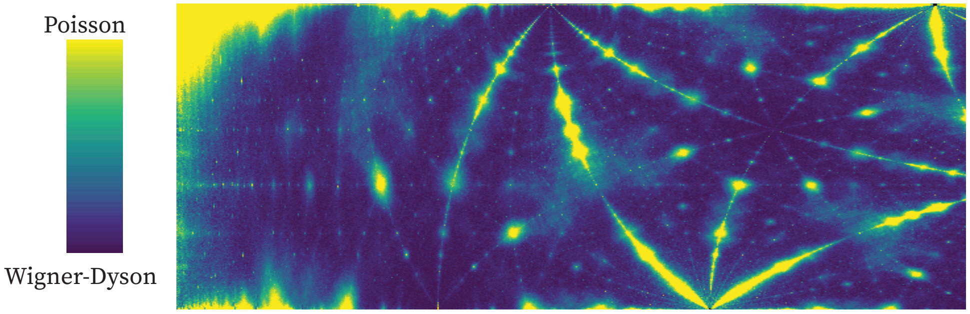
Order to disorder in quasiperiodic composites

Morison, Murphy, Cherkaev, Golden, *Commun. Phys.* 2022

Parameterized Moiré Pattern Creates Tunable Microgeometry

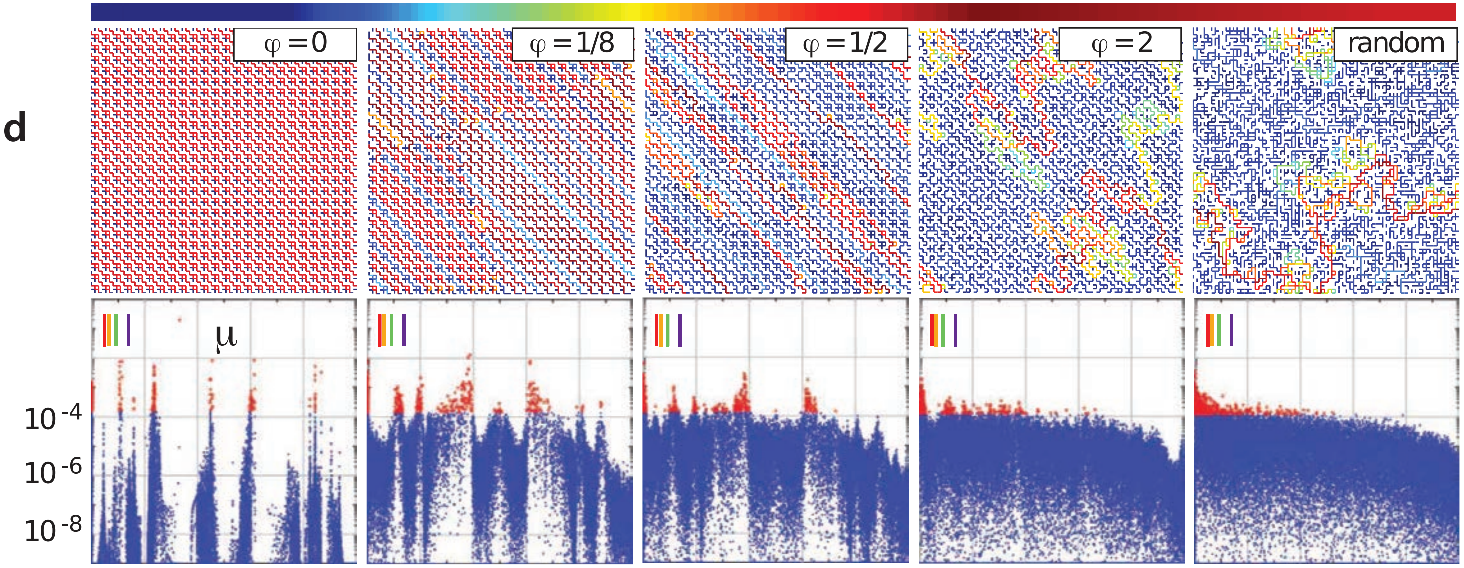


constellation of periodic systems in a sea of randomness



parameter space

periodic ⟶ **quasiperiodic**



electric field strength

spectral measure

RRN at percolation threshold

we bring the framework of solid state physics of electronic transport and band gaps in semiconductors to classical transport in periodic and quasiperiodic composites

photonic crystals and quasicrystals