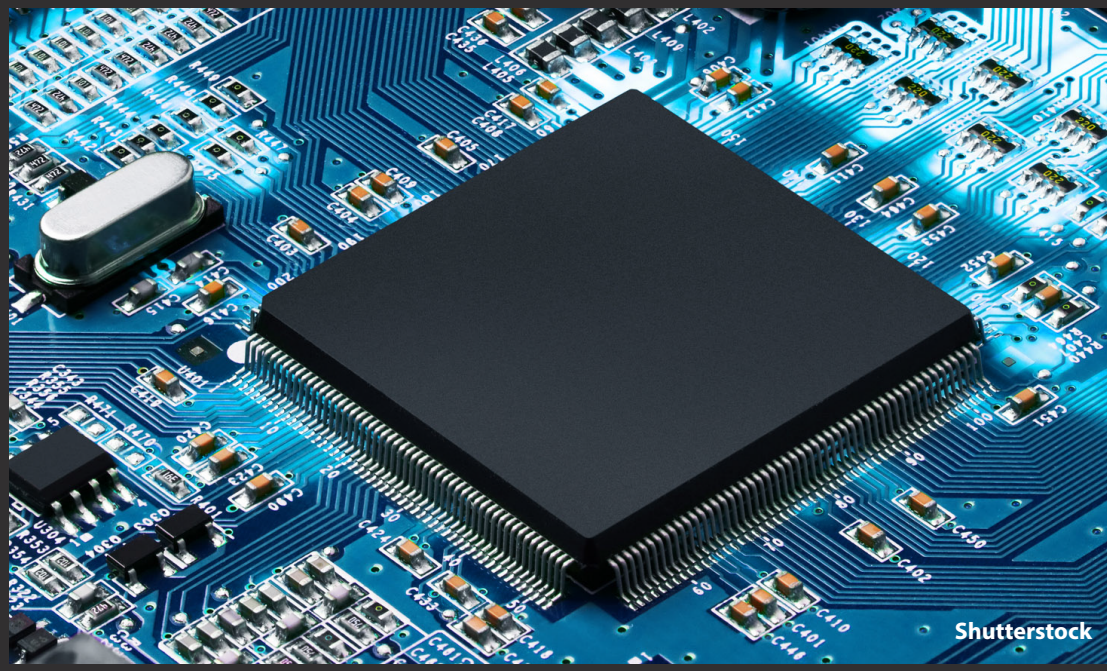
sea ice

semiconductor



invisibility cloak

some final thoughts...

Arctic Marginal Ice Zone

sea ice concentration

$$0.15 < \psi < 0.80$$

transitional region between dense pack ice and open ocean

- biologically active region
- intense ocean-ice-atmosphere interactions
- significant wave activity

MIZ "WIDTH" ??

fundamental length scale of ecological and climate dynamics

Where the action is!

Objective method for measuring MIZ width motivated by medical imaging and diagnostics

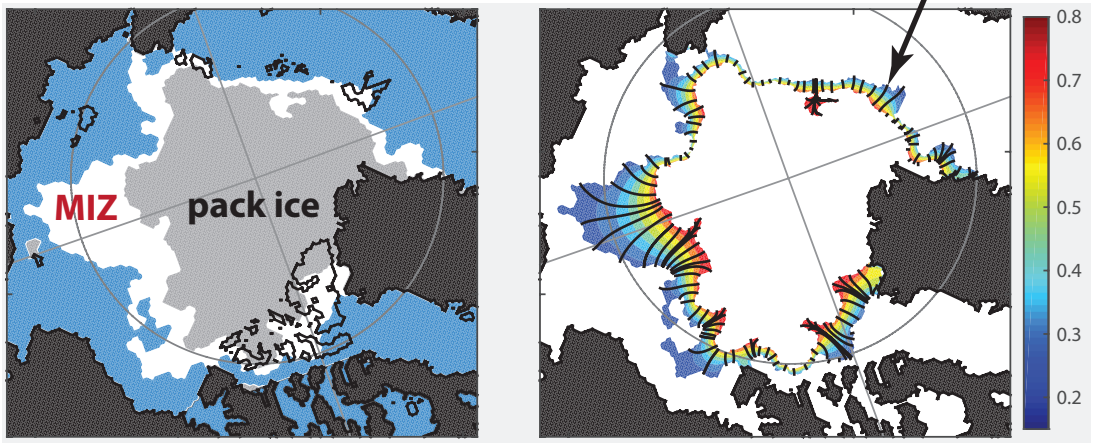
Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

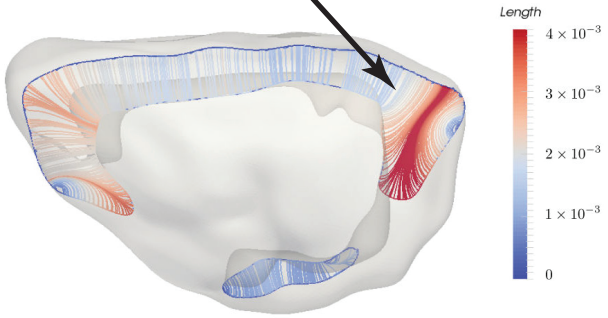
39% widening
1979 - 2012

streamlines of a solution to Laplace's equation

$$\nabla^2 \psi = 0$$

"average" lengths of streamlines





crossection of the cerebral cortex of a rodent brain

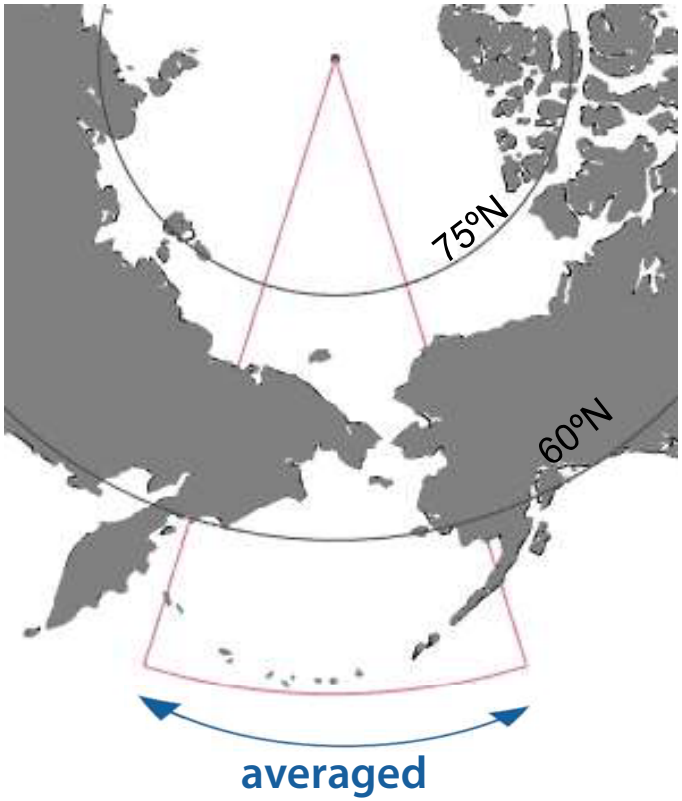
analysis of different MIZ WIDTH definitions

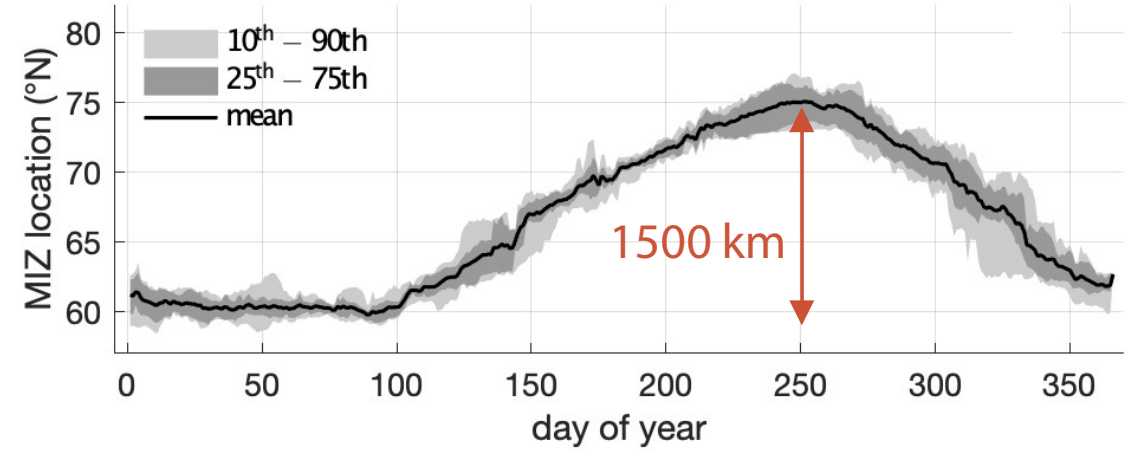
Strong, Foster, Cherkaev, Eisenman, Golden
***J. Atmos. Oceanic Tech.* 2017**

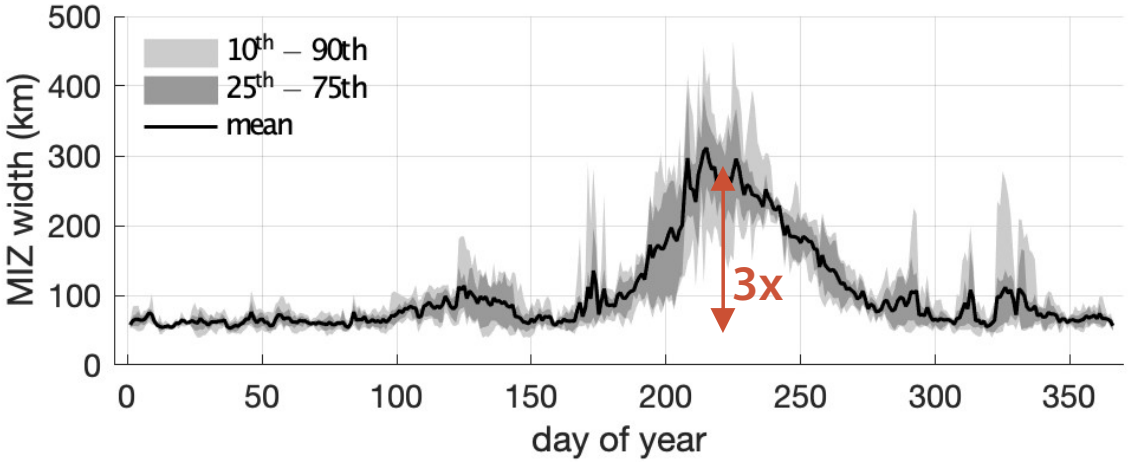
Strong and Golden
***SIAM News*, April 2017**

Observed MIZ location and width

Observational analysis of annual cycle in Bering-Chukchi Sea sector 2000-2004





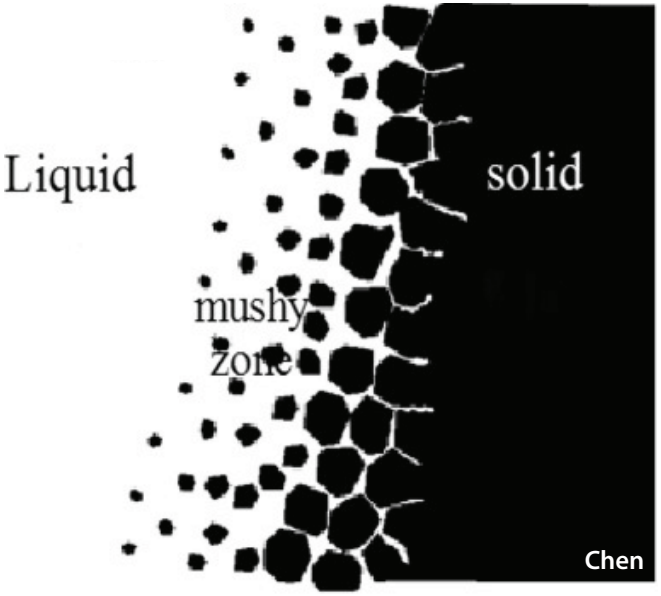


widens by a factor of 3-4 and moves poleward by 1500 km

In processes in geophysics and materials science, a region where solid and liquid phases co-exist is known as a mushy layer.

e.g. in solidification of binary liquids and alloys

solid porous matrix
with liquid inclusions

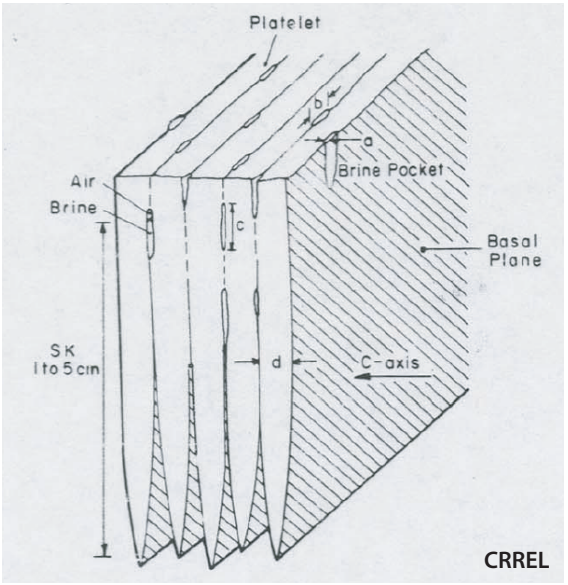


sea ice

cooling metal

Earth's mantle

dendritic interface between
sea ice and sea water



Sea Ice is a mushy layer, Feltham, et al., *GRL* 2006
Phase evolution of young sea ice, Wettlaufer, et al., *GRL* 1997

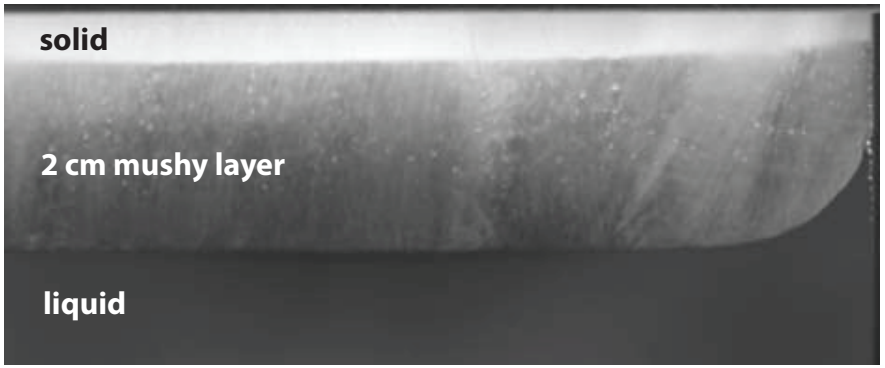
Multiscale mushy layer model for marginal ice zone dynamics

Strong, Cherkaev, Golden *Scientific Reports* 2024

MIZ - transitional region between dense pack ice and open ocean

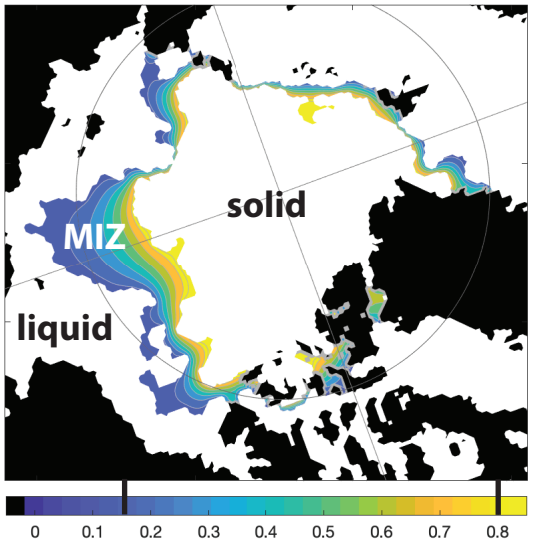
OBJECTIVE: model & predict dramatic annual cycle
impacts climate dynamics, polar ecology, human activities

mushy layer physics in the lab



NaCl-H2O in lab
(Peppin et al., 2007, J. Fluid Mech.)

Arctic MIZ as a mushy layer



sea ice concentration ψ

Sea Ice is a mushy layer, Feltham, et al., *GRL* 2006

Phase evolution of young sea ice, Wettlaufer, et al., *GRL* 1997

MIZ as a moving phase transition region

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + S$$

$$S = [\rho(c_l - c_s)T + \rho L] \frac{\partial \psi}{\partial t}$$

$$\psi = 1 - \left(\frac{T - T_s}{T_l - T_s} \right)^{\alpha}$$

$$k_x = \left(\frac{\psi}{k_s} + \frac{1 - \psi}{k_l} \right)^{-1}$$

$$k_z = \psi k_s + (1 - \psi) k_l$$

homogenization

ρ effective density

T temperature

c specific heat

L latent heat of fusion

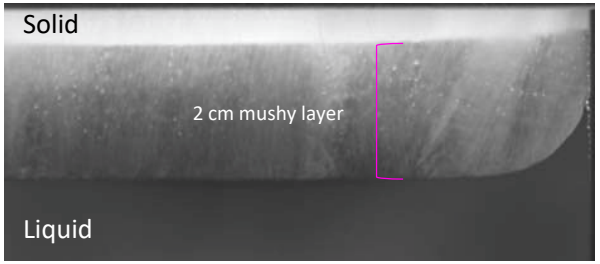
S models nonlinear phase change

ψ sea ice concentration

k effective diffusivity

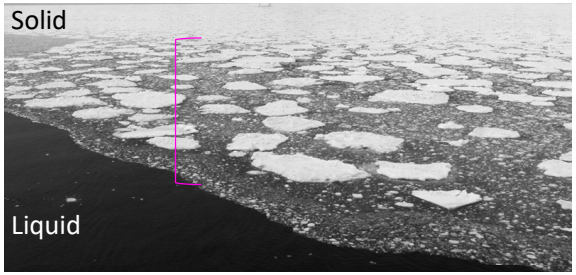
l liquid, s solid

Classical small-scale application



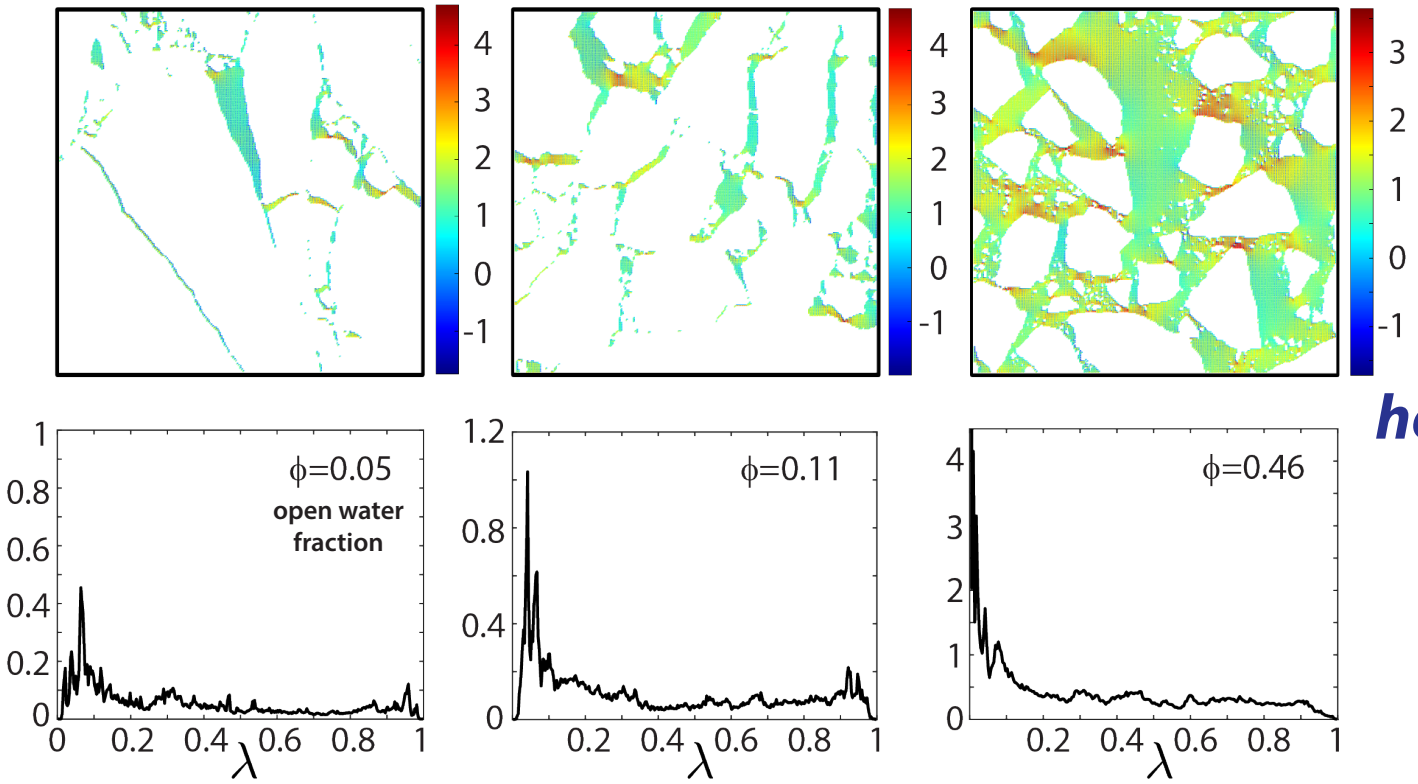
NaCl-H_2O in lab
(Peppin et al., 2007;, J. Fluid Mech.)

Macroscale application



- Develop multiscale PDE model for simulating phase transition fronts to predict MIZ seasonal cycles and decadal trends
- Model simulates MIZ as a large-scale mushy layer with effective thermal conductivity derived from physics of composite materials

thermal flow field through the ice cover: multiscale granular composite



homogenize

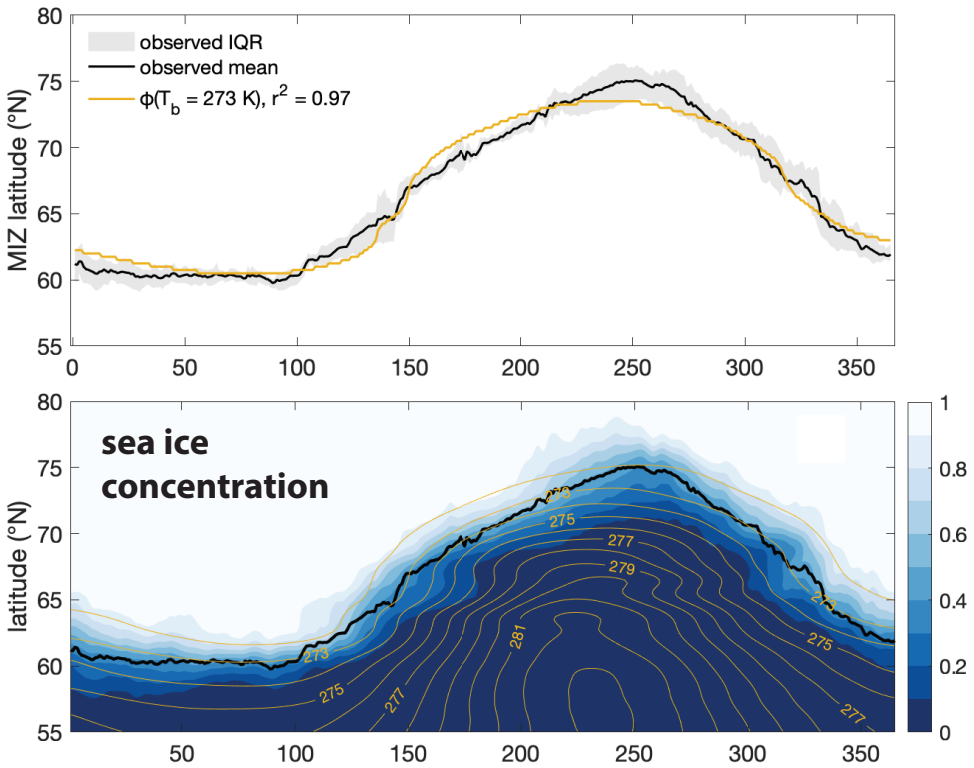
spectral measures for 2D horizontal thermal conductivity

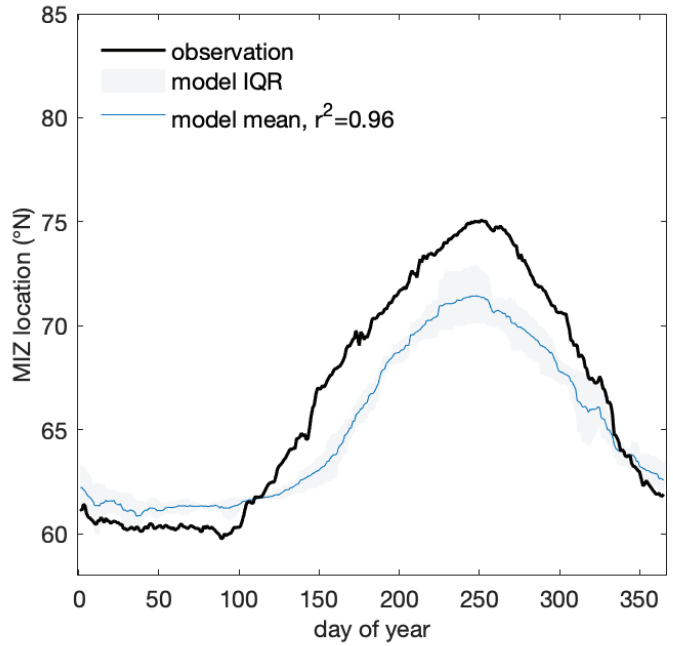
homogenized thermal conductivity is a key parameter in MIZ mushy layer model

MIZ observations

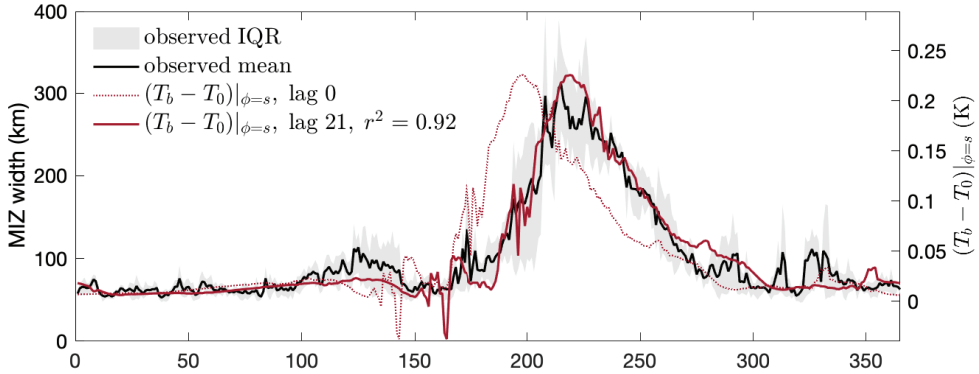
MIZ model vs. observations

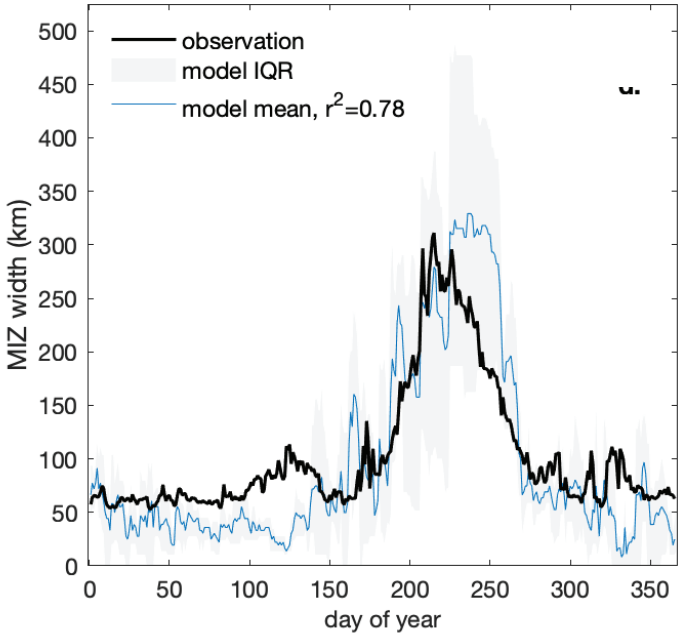
location





width





Model captures basic physics of MIZ dynamics.

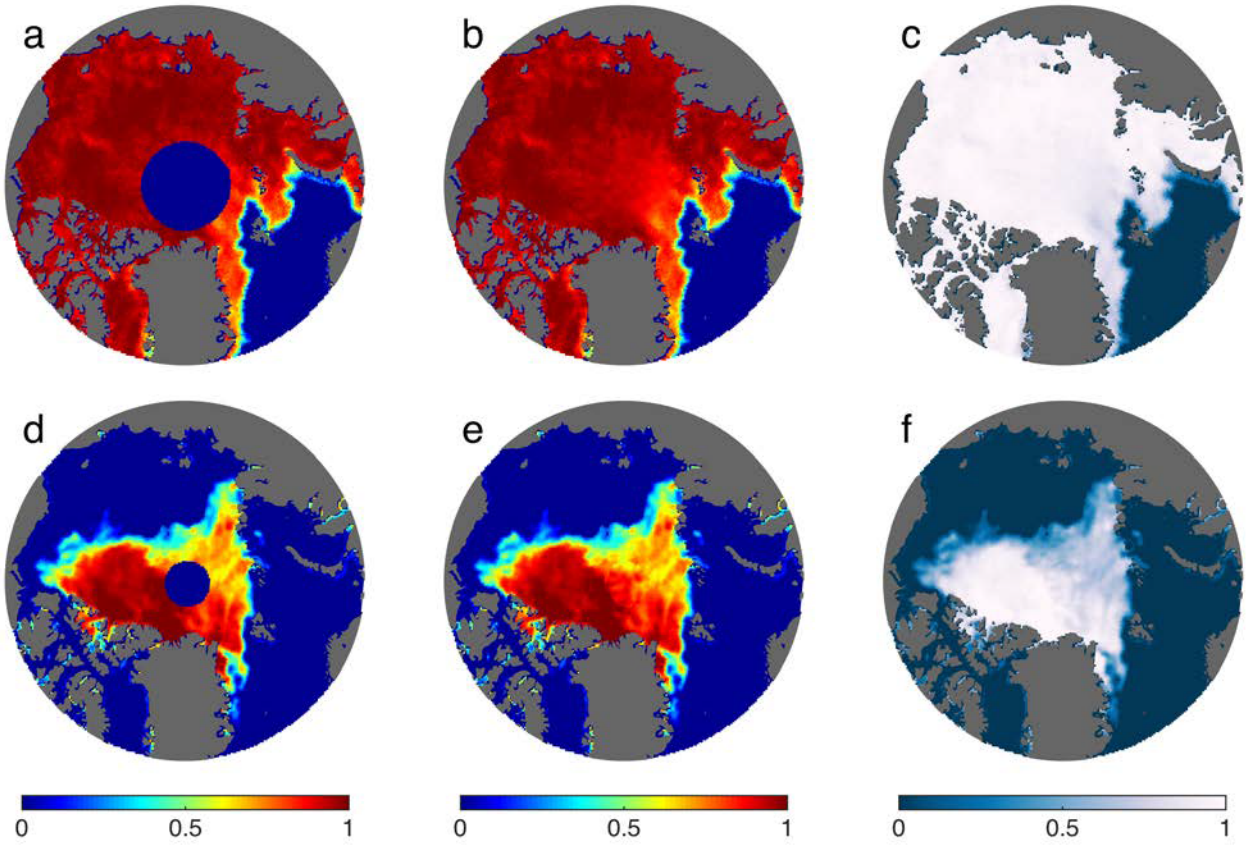
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



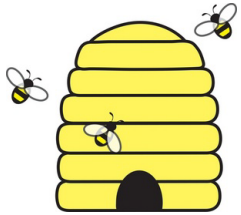
$$\Delta\psi = 0$$

fill = harmonic function satisfying satellite BC's plus learned stochastic term

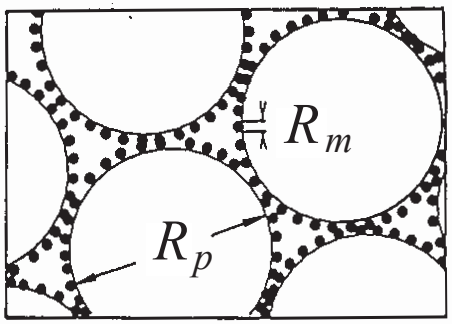
Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

Global Sea Ice Concentration Climate Data Records, 2022
Lavergne, Sorensen, et al., Norwegian Met. Inst., ... OSI SAF

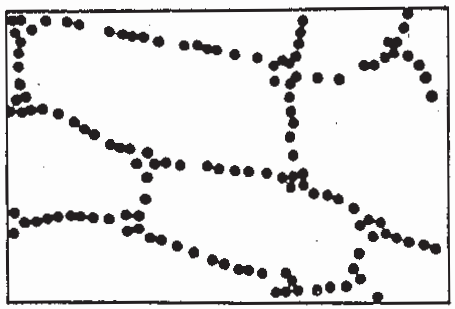
cross pollination



The math doesn't care if it's sea ice or bone!

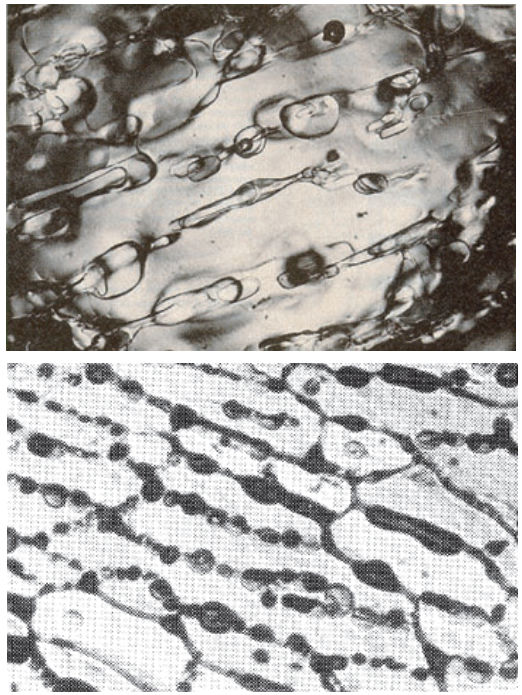


compressed powder



radar absorbing coating

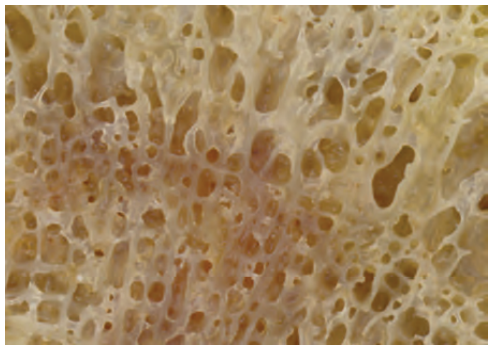
Kusy & Turner
Nature 1971



sea ice

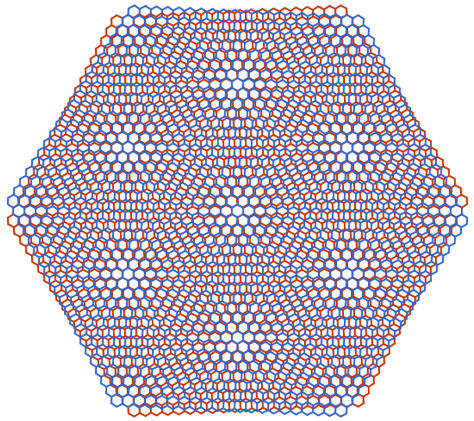
Golden, Ackley, Lytle
Science 1998

Rule of Fives
fluid flow



human bone

Golden, Murphy, Cherkaev
J. Biomechanics 2011

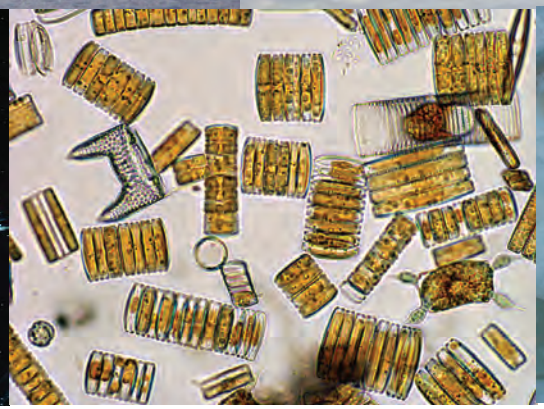


twisted bilayer composites

Morison, Murphy, Cherkaev, Golden
Communications Physics 2022

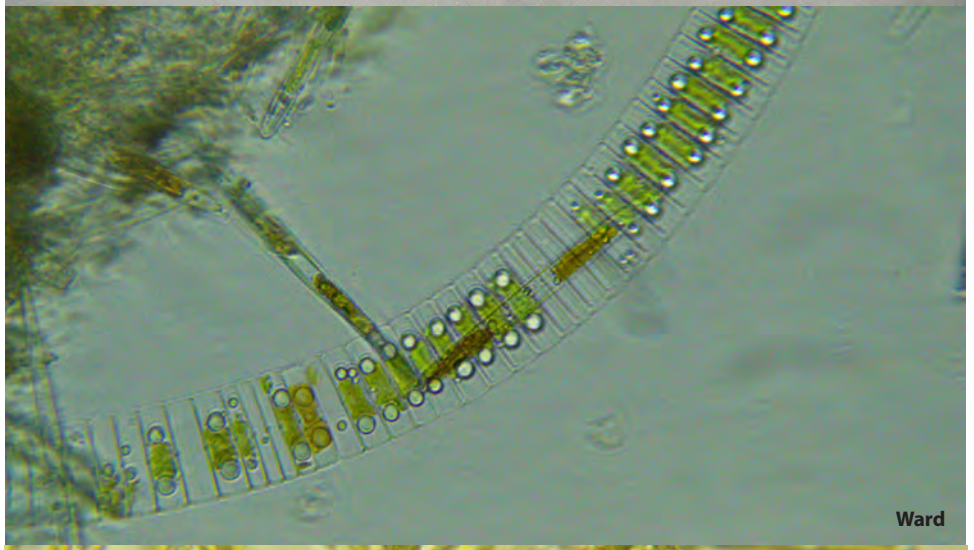
stealth technology, climate science, medical imaging, twistronics

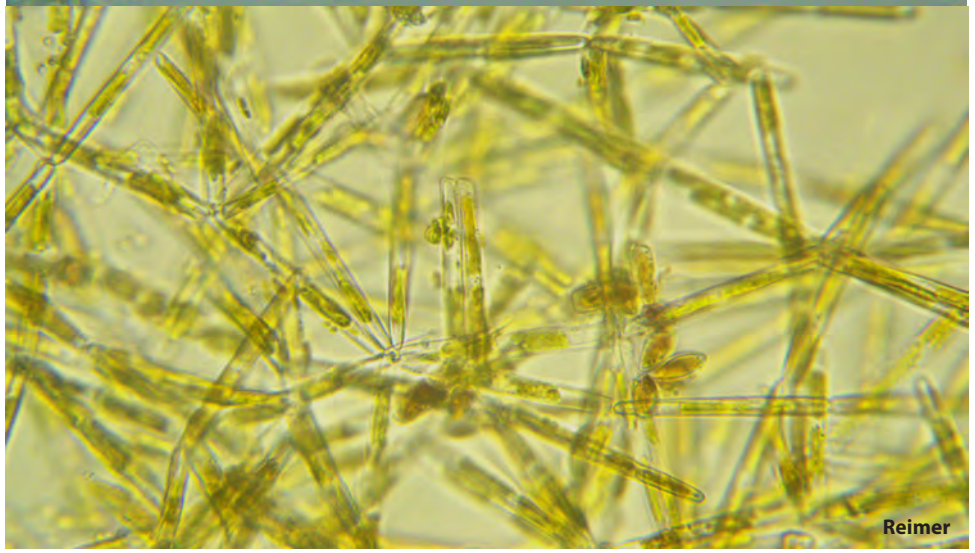
sea ice ecosystem



sea ice algae
support life in the polar oceans

bottom of a sea ice core





OPEN DIRECTIONS

- macroscale constitutive laws; fracture mechanical homogenization for sea ice pack
- wave-ice interactions; propagation of energy
- machine learning for sea ice parameters
- low order models
- UQ for sea ice predictions & climate modeling
- critical transitions
- cryosphere tipping points role of feedback mechanisms

OPEN DIRECTIONS

- downstream implications of ice loss and changes in ice pack composition - climate, ecology
- what does Earth's sea ice ecosystem tell us about possibilities for life on icy moons and planets
- EPS and strategies for life in extreme environments
- sea ice community dynamics - across scales
- tipping points in polar ecology; species competition

Conclusions

Our research is helping to improve projections of Earth's climate, its sea ice packs, and the fate of the ecosystems they support.

Cross pollination helps us study sea ice, while our sea ice math advances the theory of composites, inverse problems, etc.

Sea ice is really cool --- modeling it leads to unexpected areas of math and physics.

Mathematics is the operating system of science and engineering!

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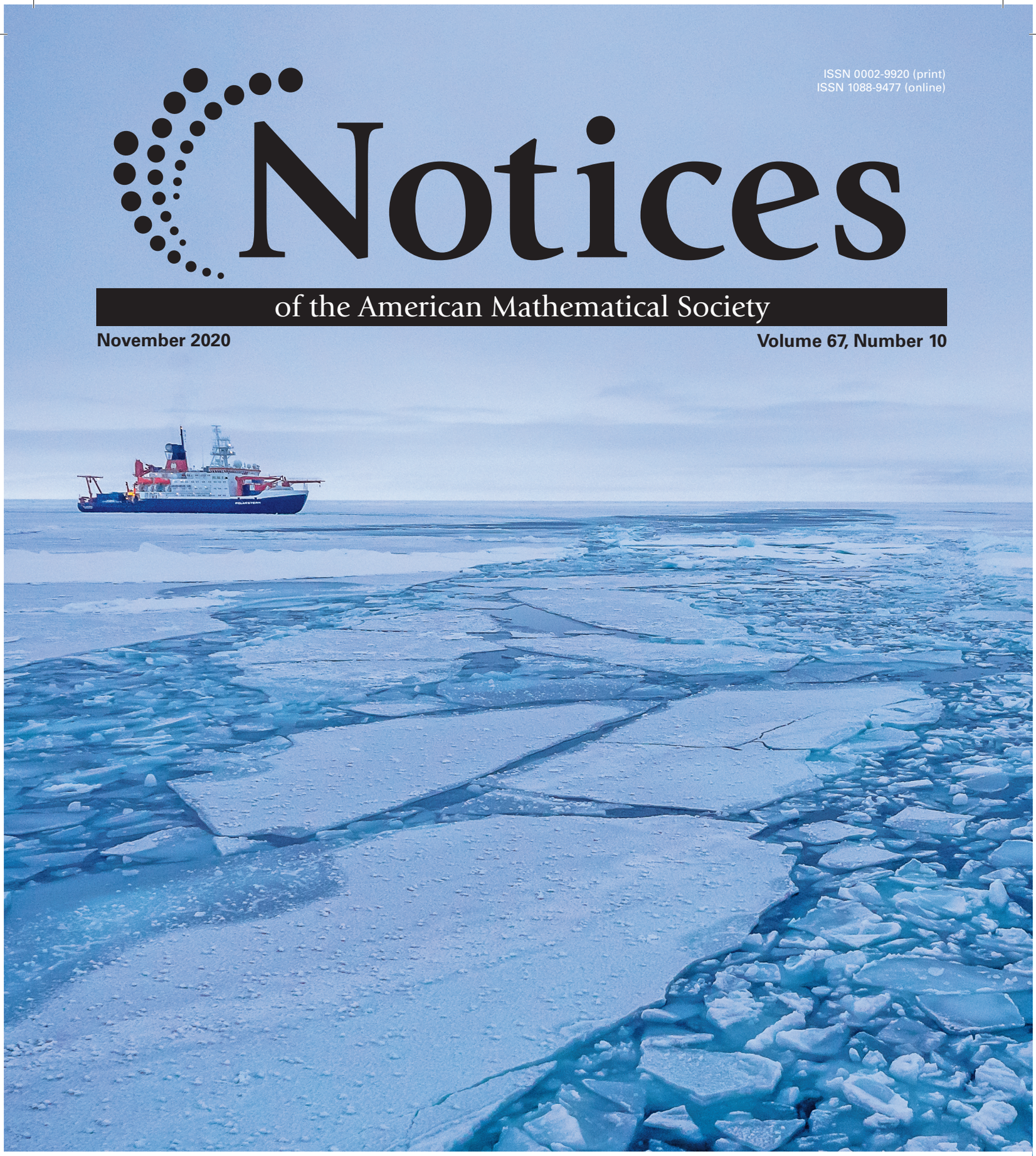
Notices

of the American Mathematical Society

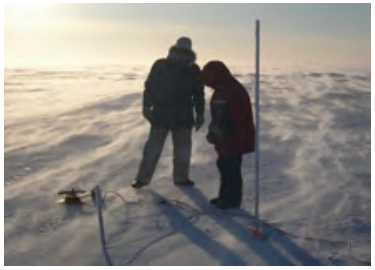
November 2020 Volume 67, Number 10



AMS American Mathematical Society
Advancing research. Creating connections.

The cover is based on "Modeling Sea Ice," page 1535.

Thank you to so many postdocs, graduate students, undergraduates, high school students and colleagues who contributed to this work!

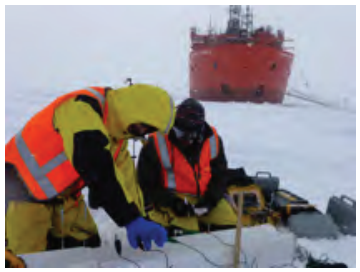
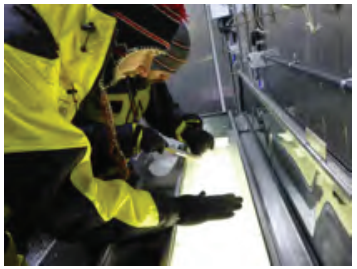











U. of Utah students in the Arctic and Antarctic (2003-2022): closing the gap between theory and observation - making math models come alive and experiencing climate change firsthand.

THANK YOU SLMath & UAF

Office of Naval Research

Applied and Computational Analysis Program

Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences

Division of Polar Programs

MCRN Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

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UAF UNIVERSITY OF ALASKA FAIRBANKS

THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999