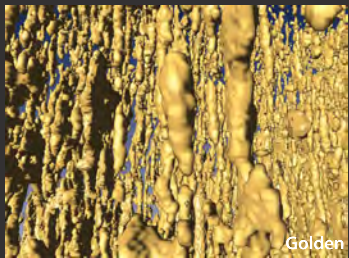
millimeters



centimeters

meters

kilometers

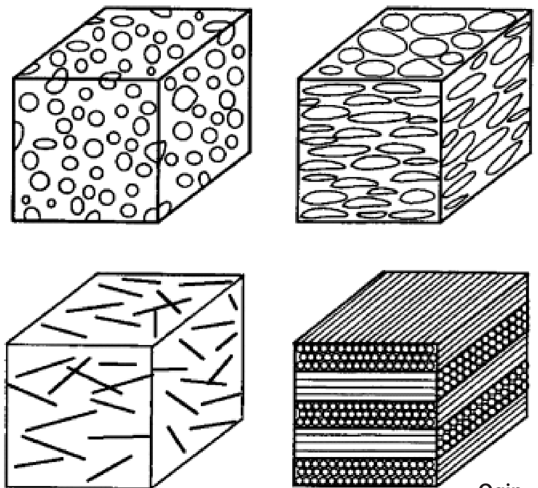
10^3 kilometers

Mathematics of Frozen Seas

Ken Golden, University of Utah

Homogenization for Composite Materials

composites & metamaterials



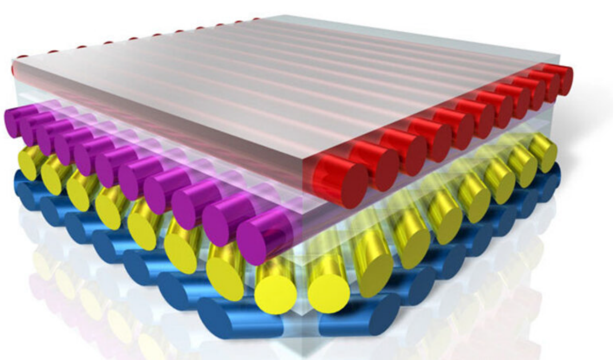
Ogin

porous rock

Gaspari

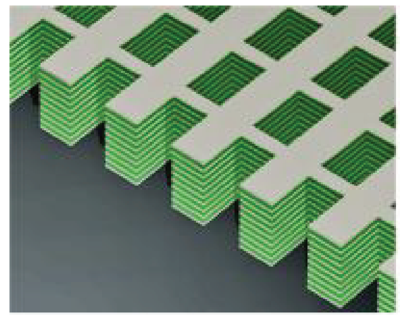
porous sea ice

Weeks

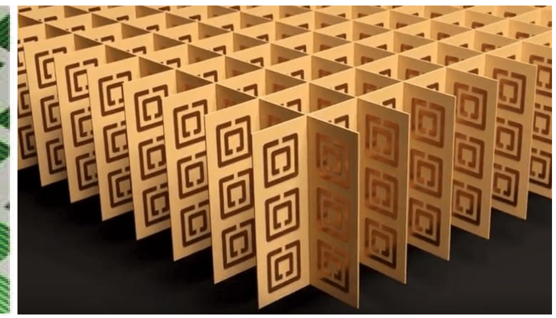


TheTech Co

negative index of refraction metamaterials

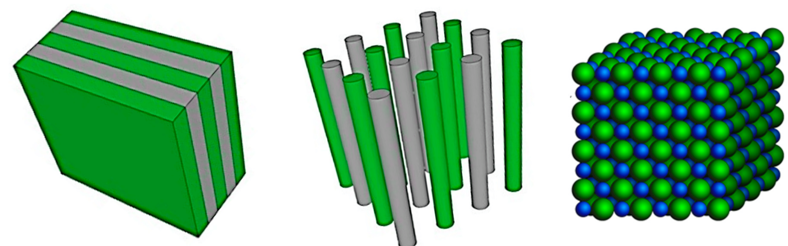


UC Berkeley



AccSci

acoustic and seismic metamaterials

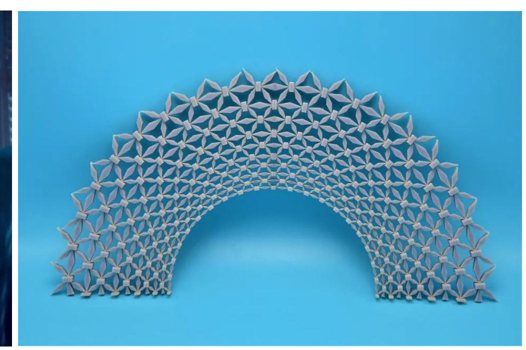


Qahtan

invisibility cloak

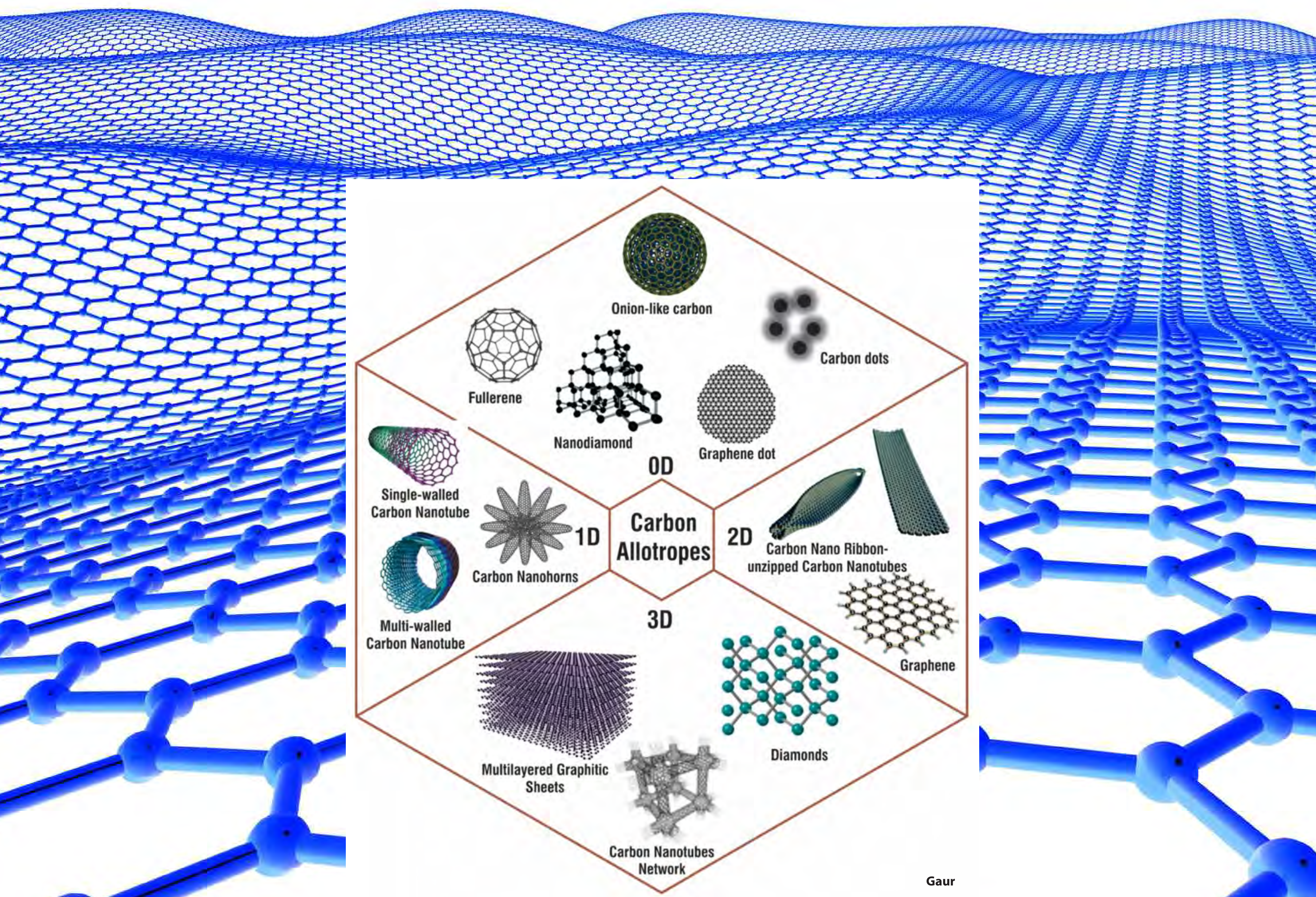
Warner Bros.

structural cloak



U. Missouri

Carbon chemistry and nanomaterials



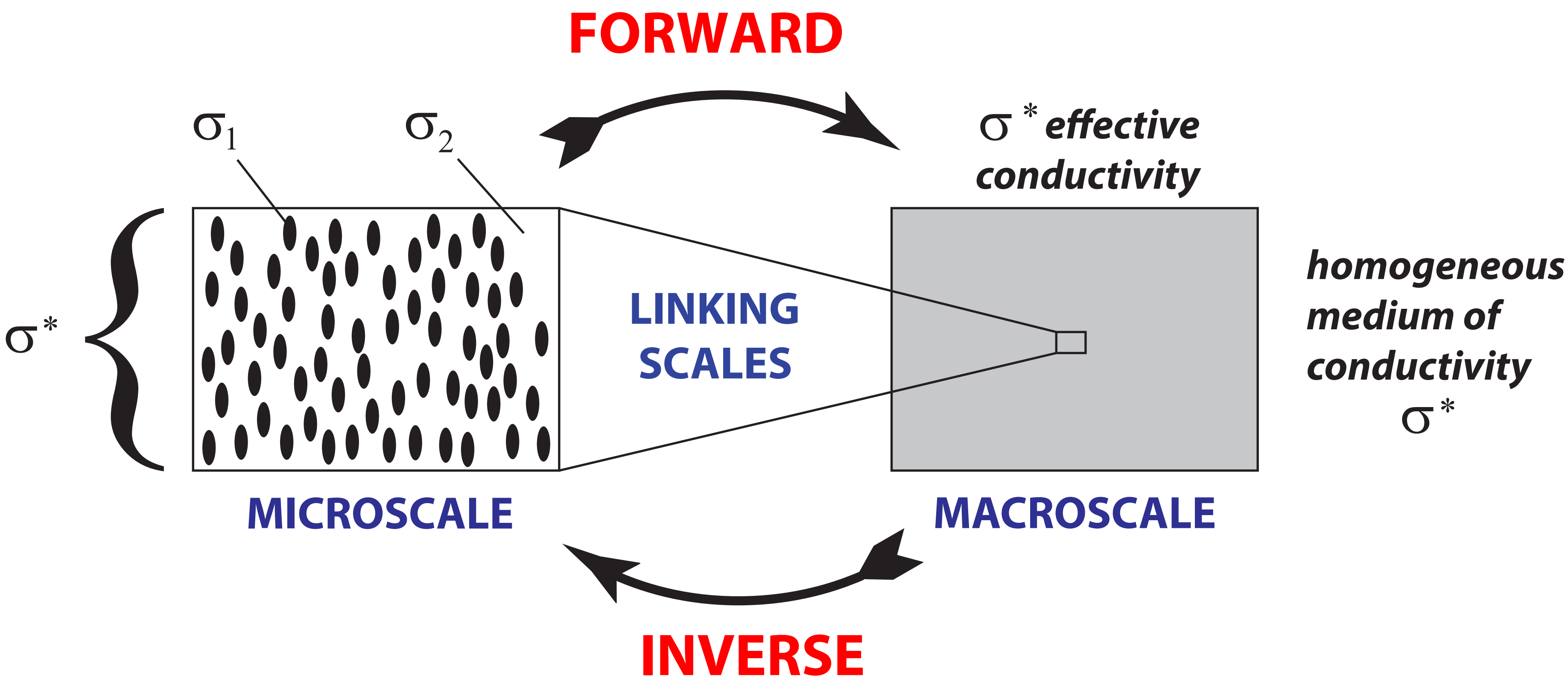
Gaur

Central theme:

How do we use “small scale” information to find effective behavior on larger scales relevant to climate and ecological models?

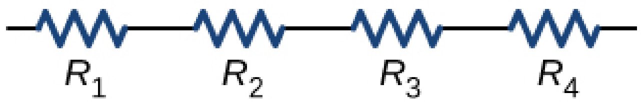
OBJECTIVE: **advance how sea ice is represented in climate models improve projections of fate of SEA ICE and its ECOSYSTEMS**

HOMOGENIZATION for Composite Materials

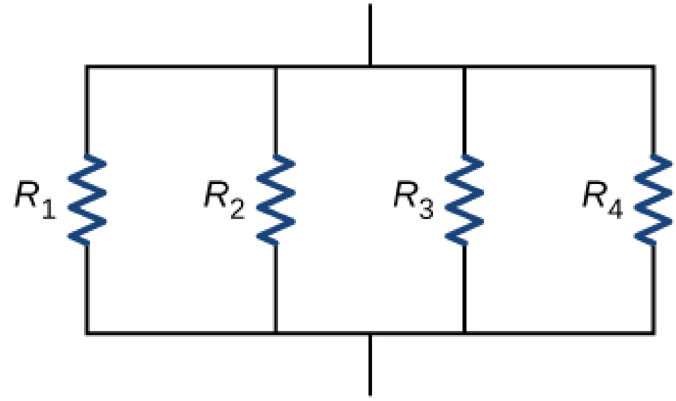


Maxwell 1873, Einstein 1906
Wiener 1912, Hashin and Shtrikman 1962

resistors in series



resistors in parallel



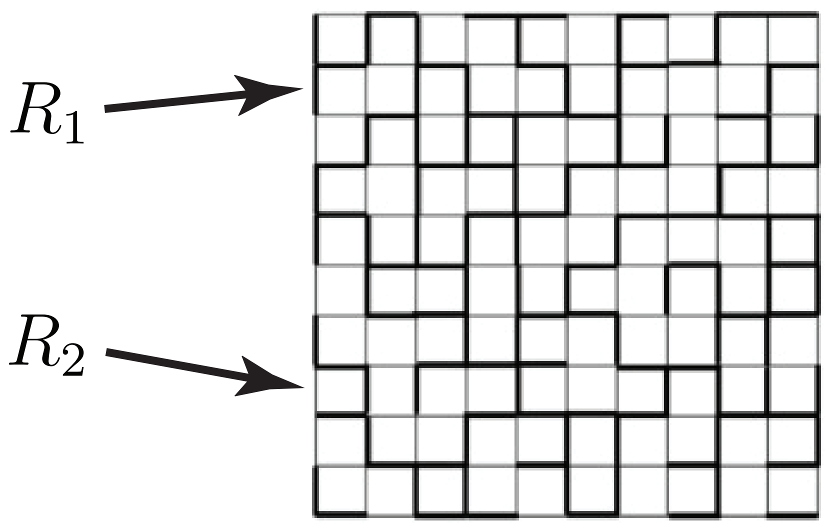
equivalent resistance

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

random resistor network

percolation model

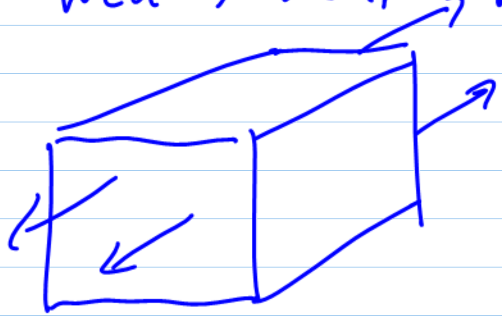


$R_{eq} = ??$

infinite lattice ?

discrete homogenization

weak, cheap, light

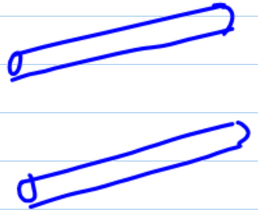


concrete

breaks apart under tension

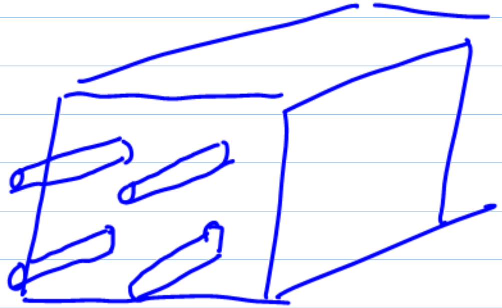
+

strong, expensive, heavy



steel rods

=

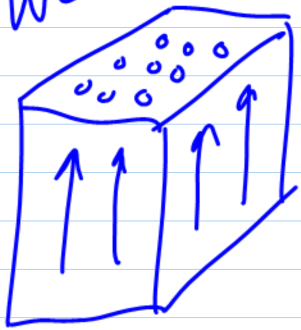


reinforced concrete

strong, cheap, light

---

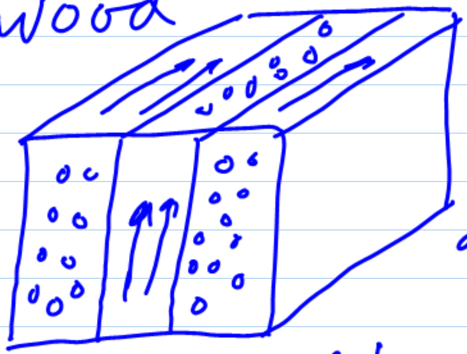
wood



strong in fiber direction

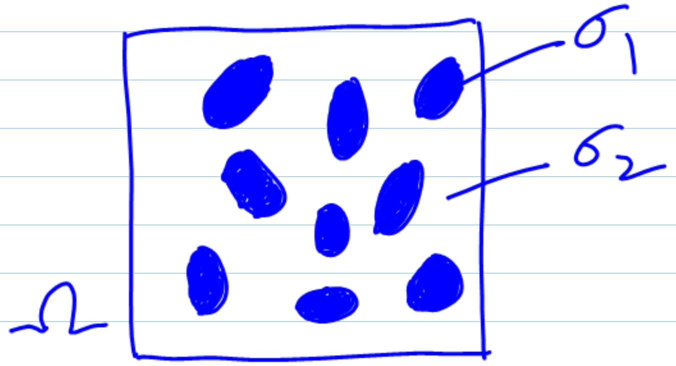
but fibers can be pulled apart

plywood



strong in 2 (or all) directions

like a poly crystal!



$\sigma(x)$ electrical conductivity

$$J(x) = \sigma(x) E(x)$$

↑ current density — ↑ electric field

$$\langle E \rangle = \frac{1}{|\Omega|} \int_\Omega E \, dx$$

$\langle E \rangle = e_k$

B.C. on ϕ

Gauss: $\nabla \cdot J = \rho$

↑ sources $= 0$

$E = -\nabla \phi$

ϕ = electric potential

$\nabla \times E = 0$

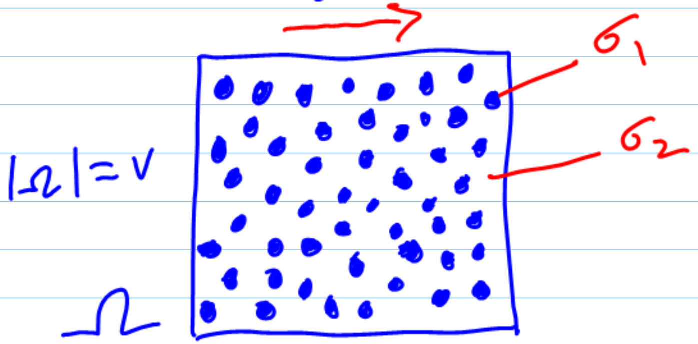
$$\nabla \cdot \vec{J} = 0 \implies \boxed{\nabla \cdot (\sigma \nabla \phi) = 0}$$

Laplace in each phase BUT...

"equivalent" classical transport problems:

dielectrics	displacement D	elec field $\vec{E}$ $D = \varepsilon E = -\varepsilon \nabla \phi$	permittivity ε
magnetism	mag. induction B	mag field H $B = \mu H$	mag permeability μ
thermal cond.	heat current q	temp. gradient ∇T $q = -k \nabla T$	thermal cond k
diffusion	particle flux q	conc. gradient ∇c $q = -D \nabla c$	diffusivity D
Darcy's Law fluid flow	fluid vel. v	pressure gradient ∇P $v = -K \nabla P$	permeability K

Homogenization



$$\sigma(x) = \sigma_1 \chi_1(x) + \sigma_2 \chi_2(x)$$

$$\chi_1(x) = \begin{cases} 1 & x \in \text{med } 1 \\ 0 & x \in \text{med } 2 \end{cases}$$

$$\chi_2 = 1 - \chi_1$$

$$\langle \vec{E} \rangle = \frac{1}{V} \int_\Omega E \, dx = e_k$$

$$\nabla \cdot J = 0$$

$$\nabla \times E = 0$$

locally $\quad J(x) = \sigma(x) E(x)$

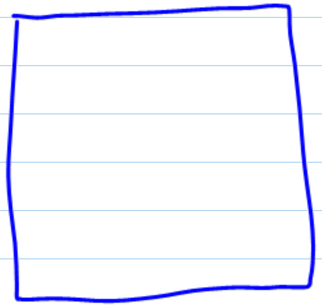
$\downarrow$

homogenized $\quad \langle J \rangle = \sigma^* \langle E \rangle$

↑ effective conductivity

$$\sigma^* = \frac{1}{V} \int_\Omega \sigma E_k \, dx$$

Variational Formulation of Effective Conductivity



Ω

$\langle E \rangle = e_k$

energy dissipated in a conducting medium Ω per unit vol.

$$U = \frac{1}{2V} \int_{\Omega} J \cdot \bar{E} \, dx$$

$$= \frac{1}{2V} \int_{\Omega} \sigma(x) E(x) \cdot \bar{E}(x) \, dx$$

if homogeneous, $\sigma(x) = \sigma^*$, $E(x) = e_k$ then $U = \frac{1}{2V} \sigma^* e_k \cdot \bar{e}_k$

$$\frac{1}{2}\sigma^* e_k \cdot \bar{e}_k = U = \frac{1}{2}\frac{1}{V}\int_\Omega \sigma E \cdot \bar{E}\, dx$$

$$\sigma^* = \frac{1}{V}\int \sigma E \cdot \bar{E}\, dx$$

↑ energy integral

Now, do variational calculation of energy integral

$E = -\nabla\varphi \qquad \varphi \to \varphi + \delta\varphi \ , \ \delta\varphi\big|_{\partial\Omega} = 0$

subject to condition $\nabla \times E = 0$,

obtain minimum that solves $\nabla \cdot J = 0$ **!** (Exercise!)

$$\Longrightarrow \quad U = \min_{\nabla\times F=0} \frac{1}{V}\int \sigma F\cdot\bar{F}\,dx \qquad \text{solution satisfies } \nabla\cdot\sigma F = 0$$

Dual Variational Principle

$$\text{vary } U = \frac{1}{2V}\int_\Omega \frac{1}{\sigma} J\cdot\bar{J}\,dx \qquad \text{subject to } \nabla\cdot J = 0$$

minimum satisfies $\nabla\times\left(\frac{J}{\sigma}\right)=0$

$$\Longrightarrow \quad \sigma^* = \min_{\nabla\times E=0} \frac{1}{V}\int_\Omega \sigma(x)\,E(x)\cdot\bar{E}(x)\,dx \qquad (1)$$

$$\frac{1}{\sigma^*} = \min_{\nabla\cdot J=0} \frac{1}{V}\int_\Omega \frac{1}{\sigma(x)}\,J(x)\cdot\bar{J}(x)\,dx \qquad (2)$$

Obtain bounds by putting in a "trial field" into the variational principle

simplest try: $E = e_k$ in (1)

$J = e_k$ in (2)

(1) $\Rightarrow$ $\sigma^* \le \langle \sigma \rangle$ Arithmetic Mean Bound

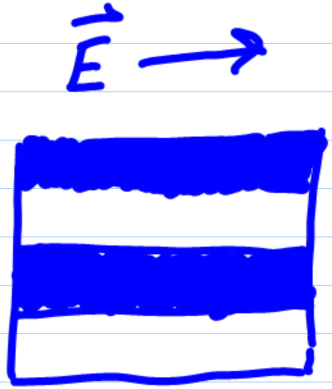
(2) $\Rightarrow$ $\frac{1}{\sigma^*} \le \left\langle \frac{1}{\sigma} \right\rangle$ Harmonic Mean Bound

harmonic mean $\frac{1}{\left\langle \frac{1}{\sigma} \right\rangle} \le \sigma^* \le \langle \sigma \rangle$ arithmetic mean

These bounds are optimal
i.e. ∃ actual composite geometries
that attain the bounds

2 phase materials

$$\frac{1}{\frac{p_1}{\sigma_1} + \frac{p_2}{\sigma_2}} \le \sigma^* \le p_1\sigma_1 + p_2\sigma_2$$



Assume isotropy   Hashin & Shtrikman 1963

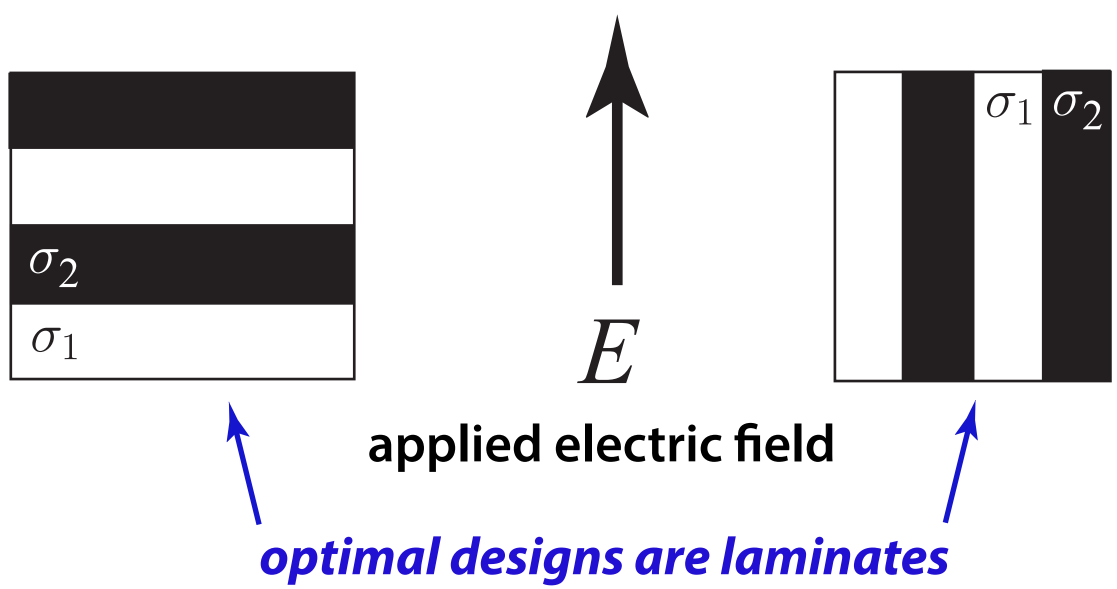
coated spheres
optimal

arithmetic and harmonic mean bounds on transport properties

effective electrical conductivity σ^* for two phase composite of σ_1 and σ_2

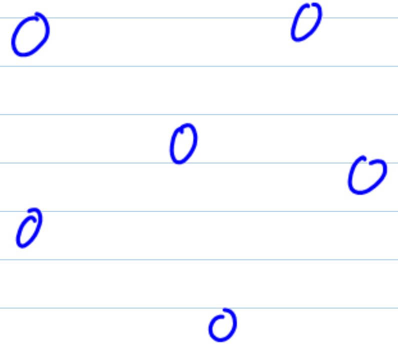
optimal bounds on σ^* for known volume fractions p_1 and p_2:

$$\frac{1}{\frac{p_1}{\sigma_1} + \frac{p_2}{\sigma_2}} \leq \sigma^* \leq p_1\sigma_1 + p_2\sigma_2$$



Wiener 1912, ...

other approaches

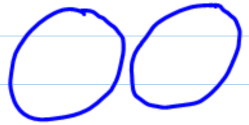


- Effective Medium Theories, CPA
  replace surroundings with
  homogeneous medium TBD

- Volume fraction expansions (small p) $\quad p \ll 1$

- Expansion around homogeneous medium $\quad \sigma_1 \sim \sigma_2$

- Almost touching geometries




Golden

Remote sensing of sea ice

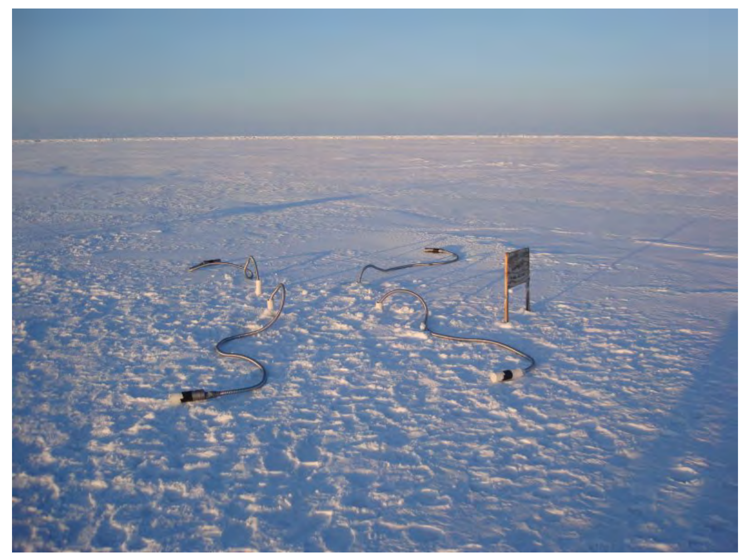
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

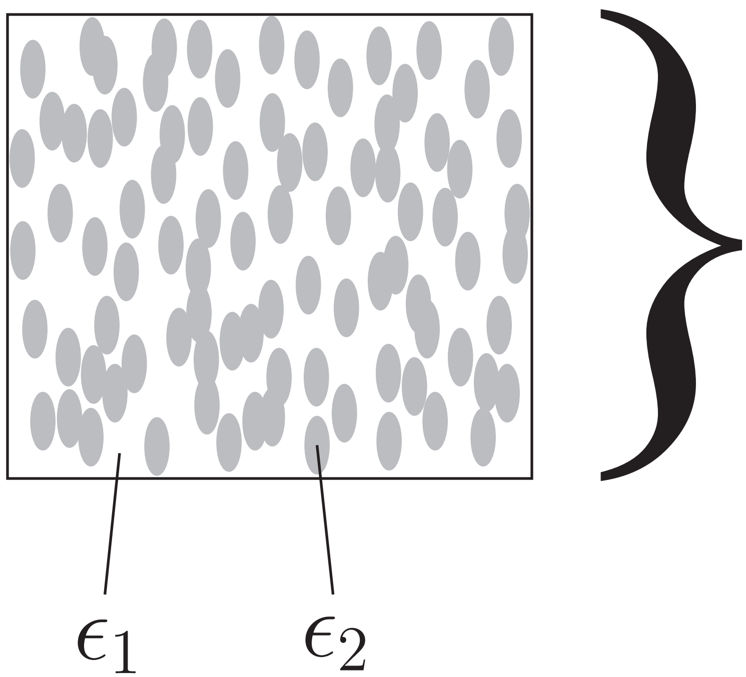
$$\varepsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \text{ , composite geometry} \right)$$

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method for Homogenization

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

separates geometry from parameters

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

geometry → $d\mu(z)$

material parameters → $s - z$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

μ
- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n-point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$$

$\chi =$ characteristic function of the brine phase

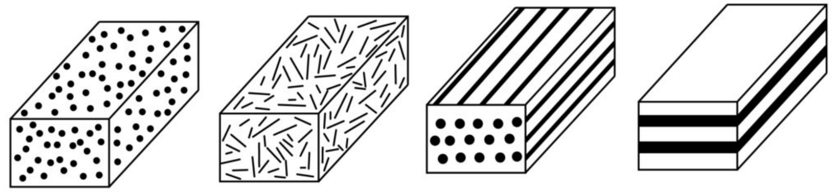
$$E = s\,(s + \Gamma\chi)^{-1} e_k$$

$\Gamma\chi$: microscale → macroscale

$\Gamma\chi$ *links scales*

Golden and Papanicolaou, Comm. Math. Phys. 1983

complexities of mixture geometry



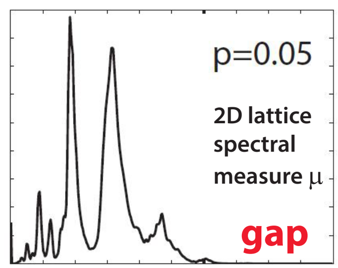
distilled

distilled

Analytic Continuation Method

Stieltjes Integral Representations
for Homogenized Parameters

Bergman 1978, Milton 1979
Golden & Papanicolaou 1983



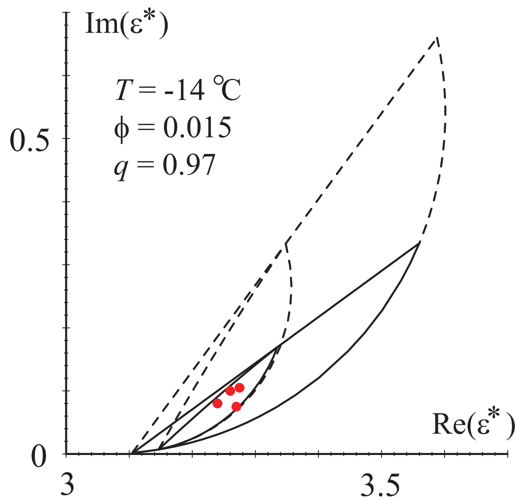
spectral properties of operator (matrix)
~ quantum states, energy levels for atoms

eigenvectors eigenvalues

EXTEND to: polycrystals, advection diffusion, waves through ice pack

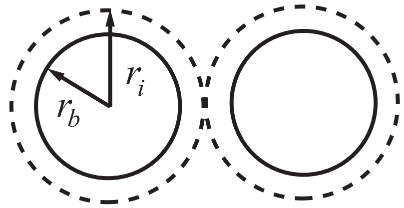
forward and inverse bounds on the complex permittivity of sea ice

forward bounds



4.75 GHz data

matrix particle

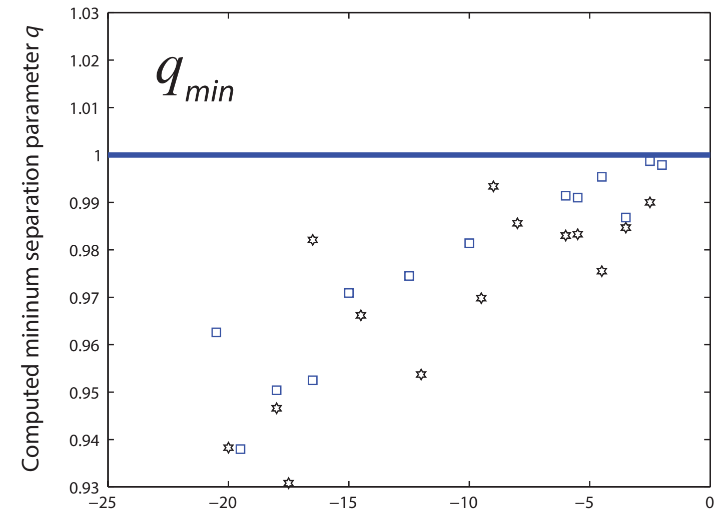


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



Slab temperature °C

Inverse Homogenization

Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001), McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)

ε^* → ***composite geometry*** **(spectral measure μ)**

inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

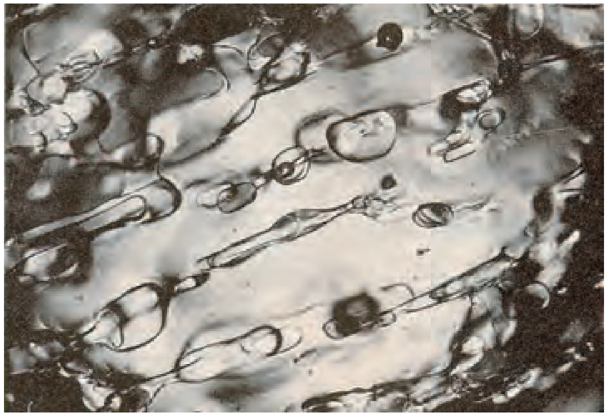
inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

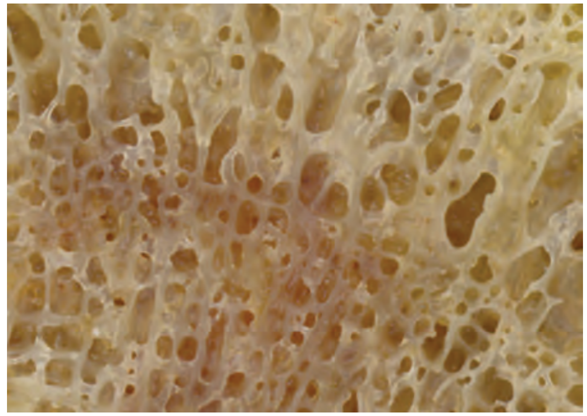
rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

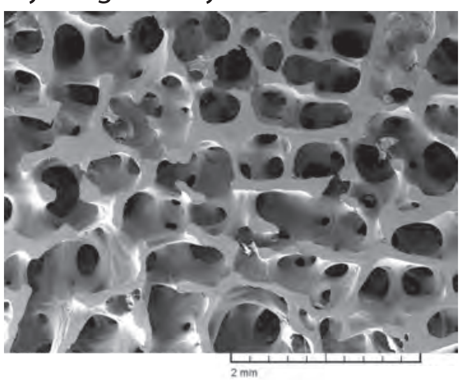
SEA ICE | **HUMAN BONE**



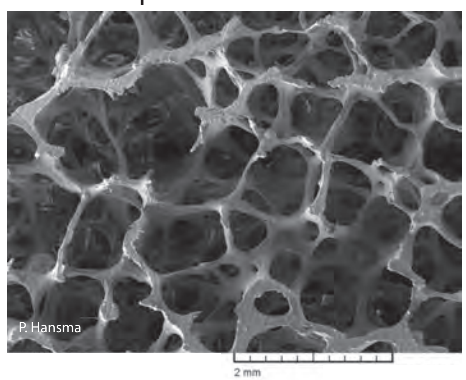


spectral characterization of porous microstructures in human bone

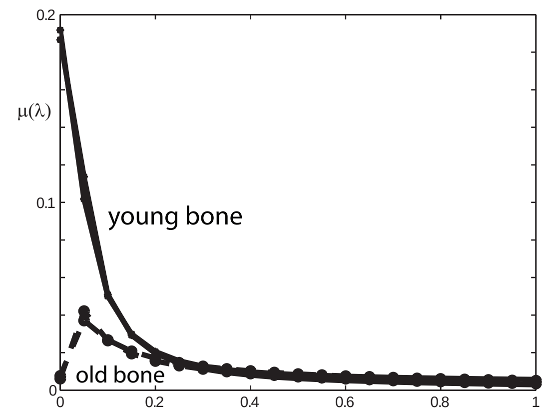
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures from complex permittivity data



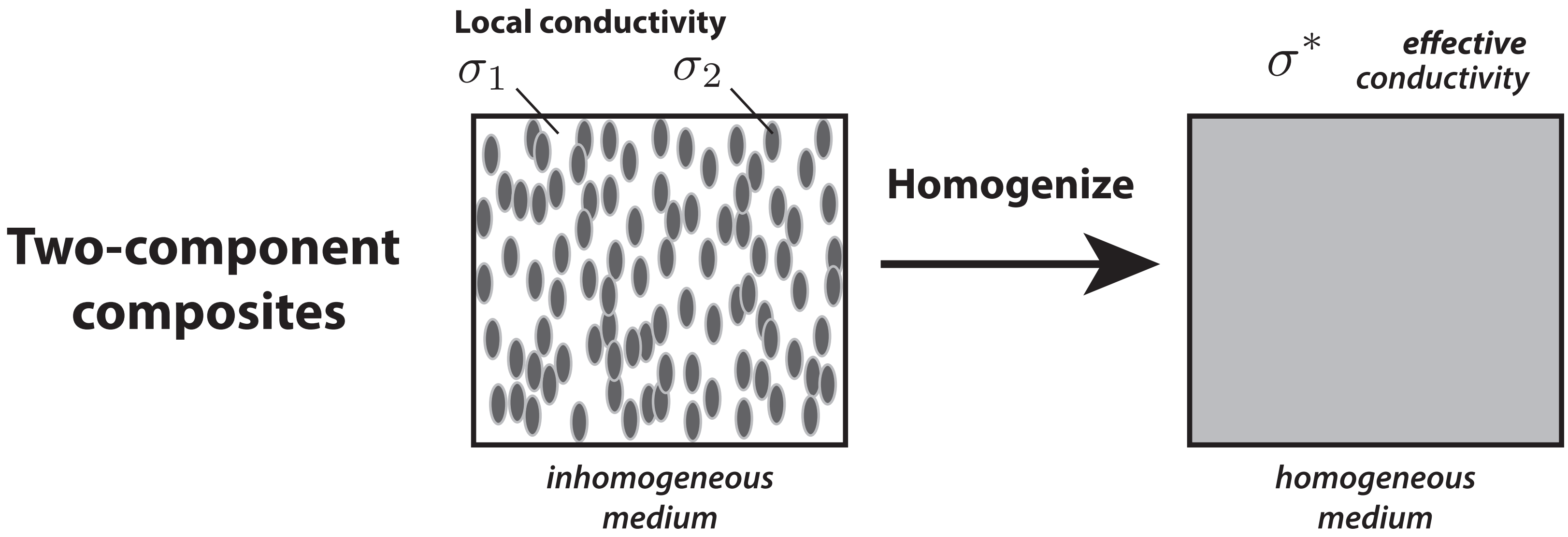
use regularized inversion scheme

apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

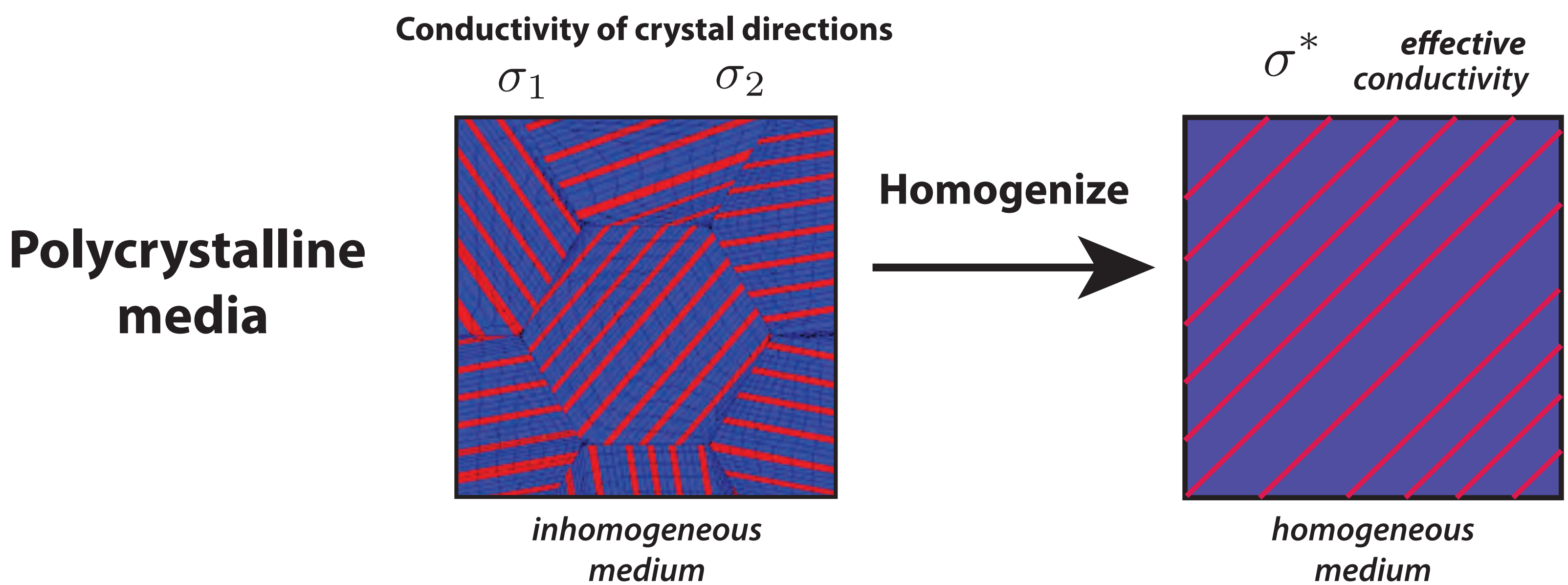
Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

Homogenization for polycrystalline materials



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium



Mathematical formulation for composite materials

Two-component material

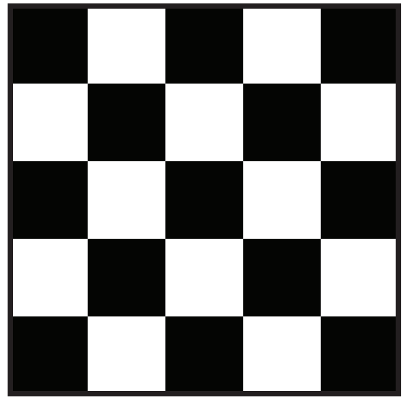
Local conductivity

$$\sigma = \sigma_1\chi_1 + \sigma_2\chi_2$$

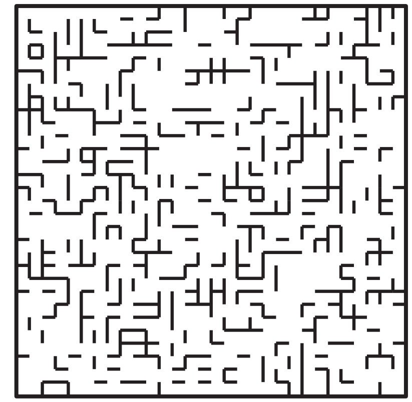
$$\chi_2 = 1 - \chi_1$$

$$\chi_i\chi_j = \delta_{ij}\chi_i$$

Continuum composite



Discrete composite



$$\vec{\nabla}\cdot\vec{J} = 0, \quad \vec{\nabla}\times\vec{E} = 0, \quad \vec{J} = \sigma\vec{E}, \quad \vec{E} = \vec{\nabla}\phi + \vec{e}_k, \quad \langle\vec{E}\rangle = \vec{e}_k$$

Polycrystalline material

Local conductivity

$$\sigma = R\,diag(\sigma_1, \sigma_2, \sigma_2)R^T$$
$$= \sigma_1 X_1 + \sigma_2 X_2$$

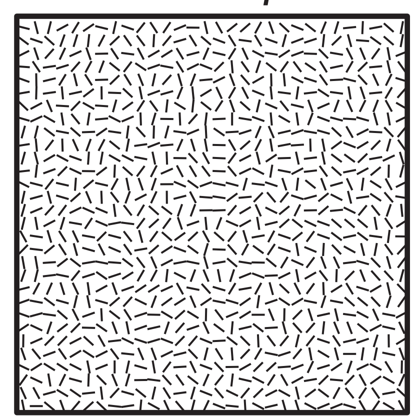
$$X_2 = I - X_1$$

$$X_i X_j = \delta_{ij} X_i$$

Random Rotation Matrix

Continuum composite

Discrete composite



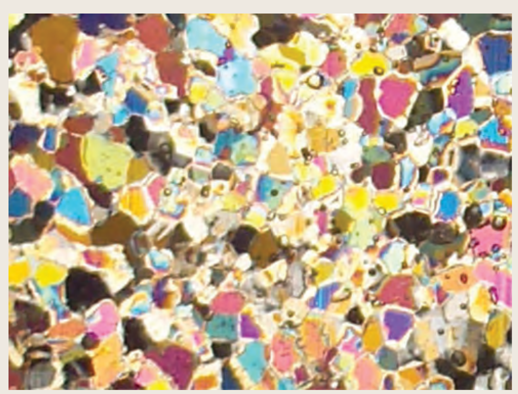
Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

  Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**

  orientation statistics
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



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An invited review commemorating 350 years of scientific publishing at the Royal Society

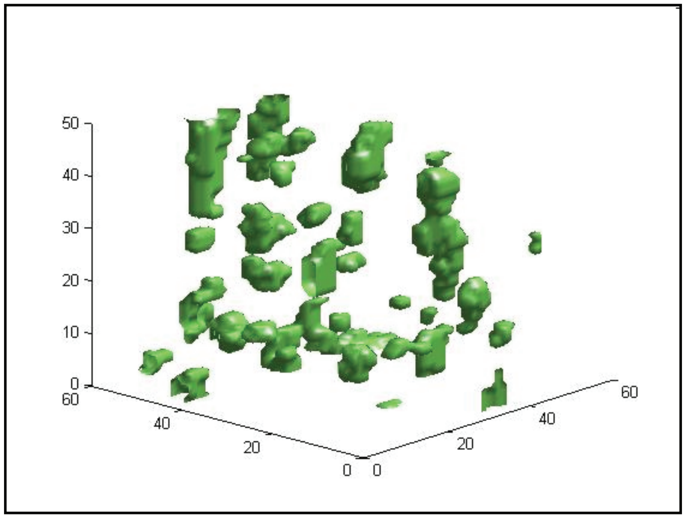
A method to distinguish between different types of sea ice using remote sensing techniques

A computer model to determine how a human should walk so as to expend the least energy

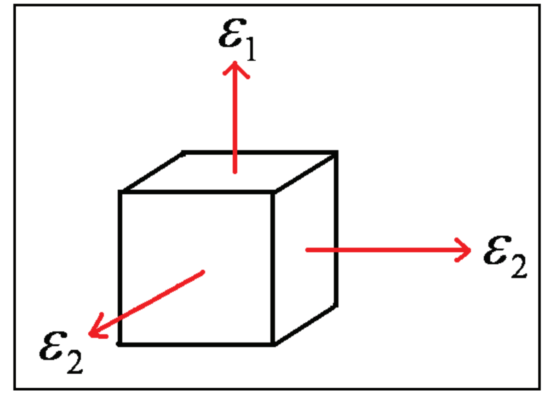
THE ROYAL SOCIETY PUBLISHING

21/01/15 1:47 PM

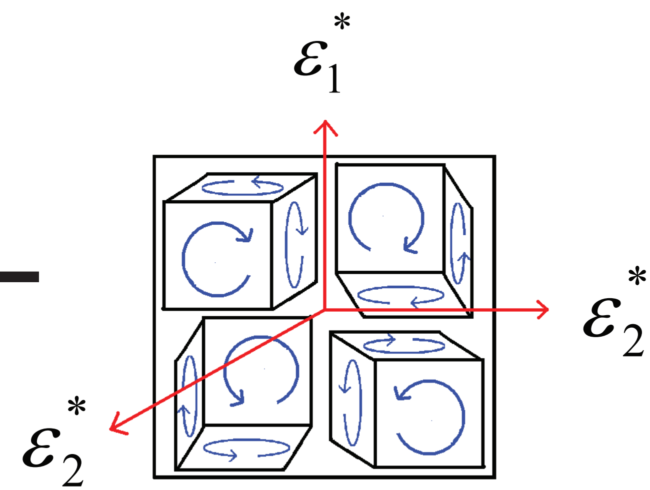
two scale homogenization for polycrystalline sea ice



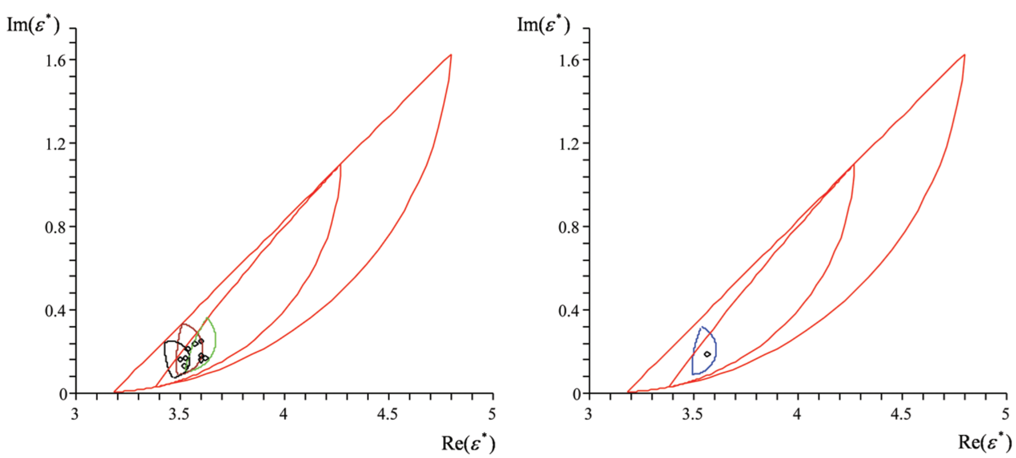
numerical homogenization for single crystal



analytic continuation for polycrystals



bounds



Gully, Lin, Cherkaev, Golden, Proc. Roy. Soc. A (and cover) 2015

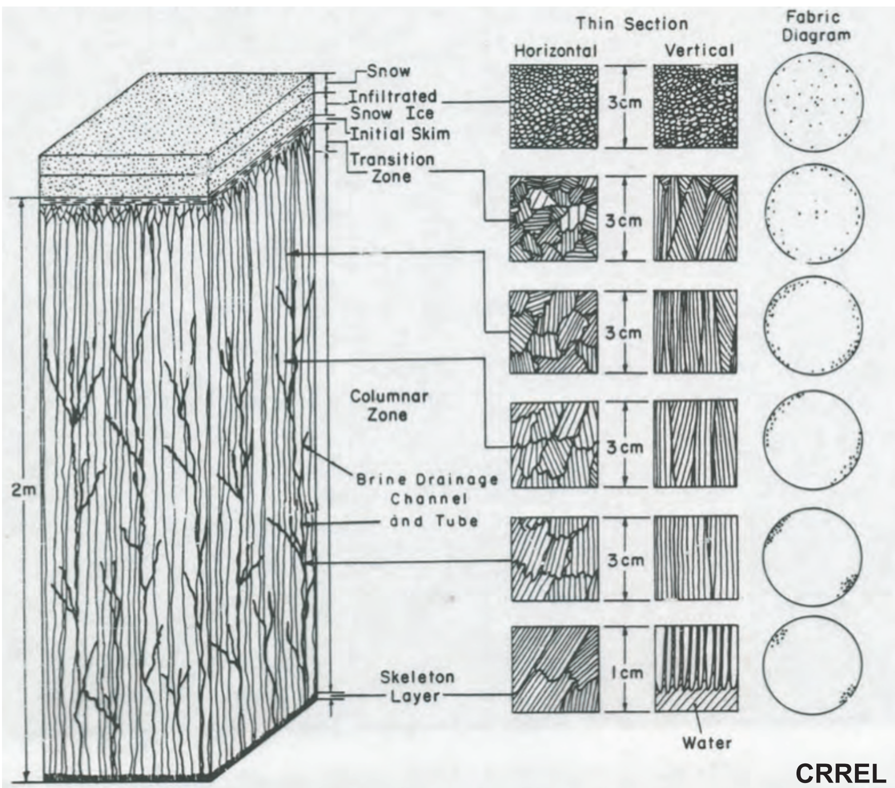
Rigorous bounds on the complex permittivity tensor of sea ice with polycrystalline anisotropy in the horizontal plane

Kenzie McLean, Elena Cherkaev, Ken Golden 2022

motivated by **Weeks and Gow, *JGR* 1979: c-axis alignment in Arctic fast ice off Barrow**

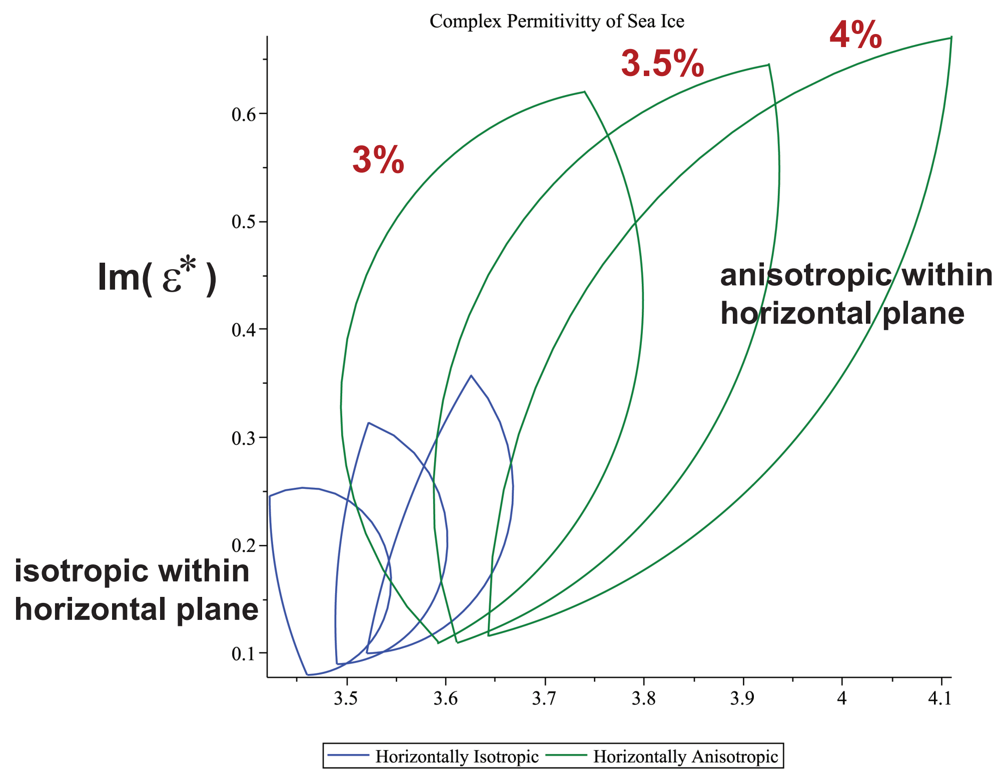
Golden and Ackley, *JGR* 1981: radar propagation model in aligned sea ice

input: orientation statistics



output: bounds

Im(ε^*)



Re(ε^*)

direct calculation of spectral measures

Murphy, Hohenegger, Cherkaev, Golden, *Comm. Math. Sci.* 2015

- depends only on the composite geometry
- discretization of microstructural image gives binary network
- fundamental operator becomes a random matrix
- spectral measure computed from eigenvalues and eigenvectors

once we have the spectral measure μ it can be used in Stieltjes integrals for other transport coefficients:

electrical and thermal conductivity, complex permittivity, magnetic permeability, diffusion, fluid flow properties

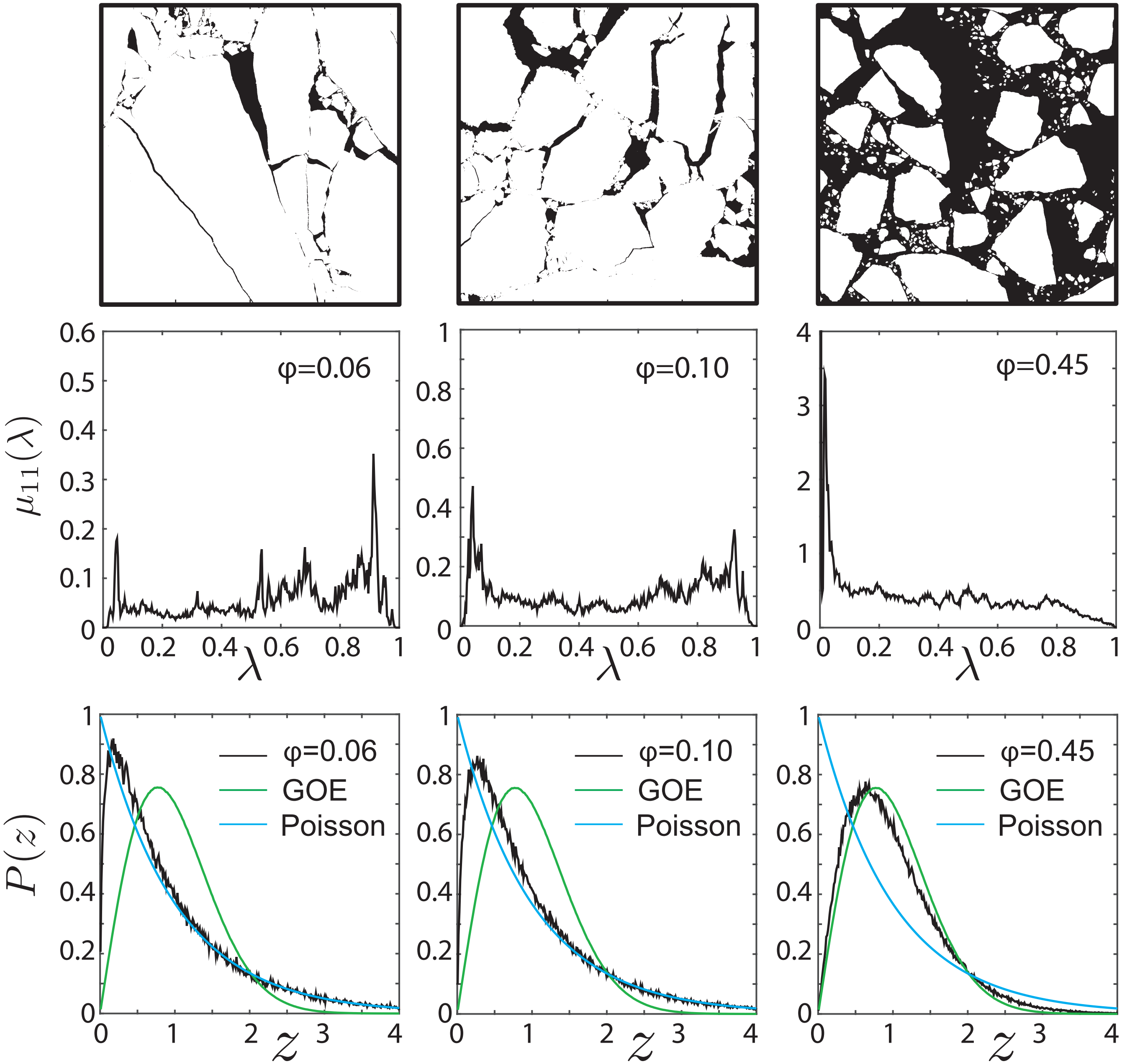
earlier studies of spectral measures

Day and Thorpe 1996
Helsing, McPhedran, Milton 2011

Spectral computations for sea ice floe configurations

spectral measures

eigenvalue spacing distributions



UNIVERSAL Wigner-Dyson distribution

uncorrelated → **level repulsion**

Murphy, Cherkaev, Golden, *Phys. Rev. Lett.* 2017

Eigenvalue Statistics of Random Matrix Theory

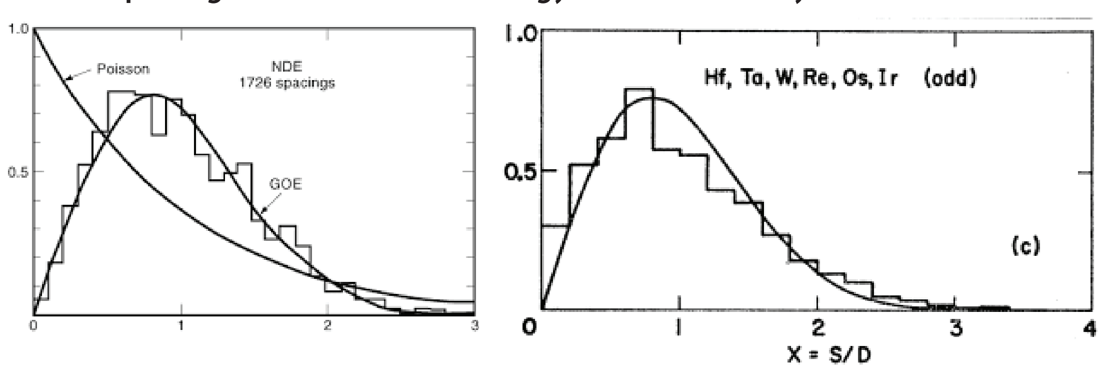
Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

$[\mathbf{N}]_{ij} \sim N(0,1)$, $\mathbf{A} = (\mathbf{N}+\mathbf{N}^T)/2$ ***Gaussian orthogonal ensemble (GOE)***

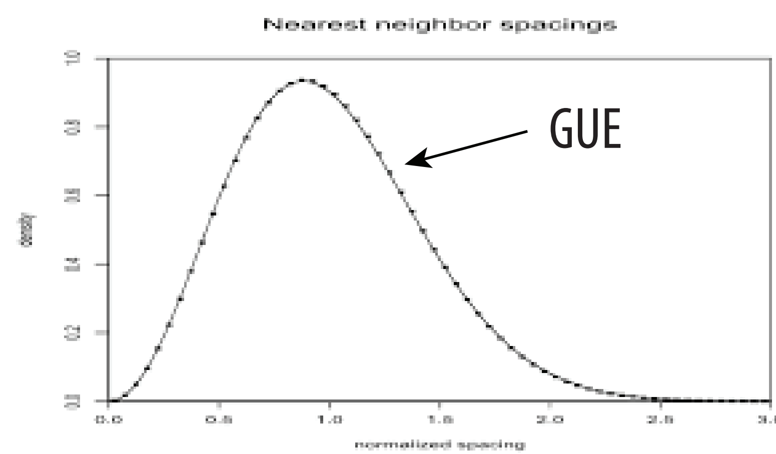
$[\mathbf{N}]_{ij} \sim N(0,1)+iN(0,1)$, $\mathbf{A} = (\mathbf{N}+\mathbf{N}^\dagger)/2$ ***Gaussian unitary ensemble (GUE)***

Short range and long range correlations of eigenvalues are measured by various eigenvalue statistics.

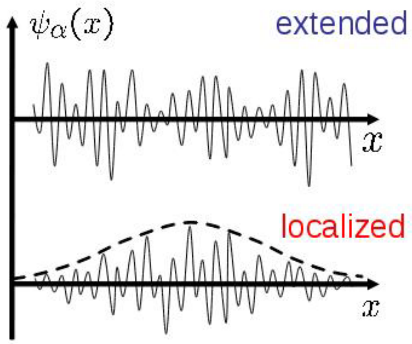
Spacing distributions of energy levels for heavy atomic nuclei



Spacing distributions of the first billion zeros of the Riemann zeta function



Universal eigenvalue statistics arise in a broad range of "unrelated" problems!



Anderson localization

disorder-driven

metal / insulator transition

Anderson 1958
Mott 1949
Evangelou 1992
Shklovshii et al 1993

Wave equations

propagation vs. localization in wave physics:
quantum, optics, acoustics, water waves

Laplace + Diffusion equations

we find percolation-driven

Anderson transition for classical transport in composites

mobility edges, localization, universal spectral statistics

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

but no wave interference or scattering effects at play!

Where to look to see this behavior exploited in tunable media that display rich transport properties?

Go back to the dawn of ordered, aperiodic materials -

quasicrystals.

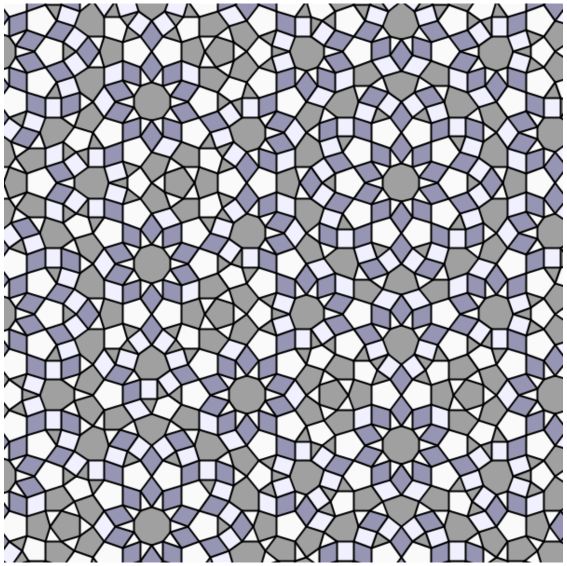
Shechtman et al. 1984
Levine & Steinhardt 1984

Order to Disorder in Quasiperiodic Composites

D. Morison (Physics), N. B. Murphy, E. Cherkaev, K. M. Golden, *Communications Physics* 2022

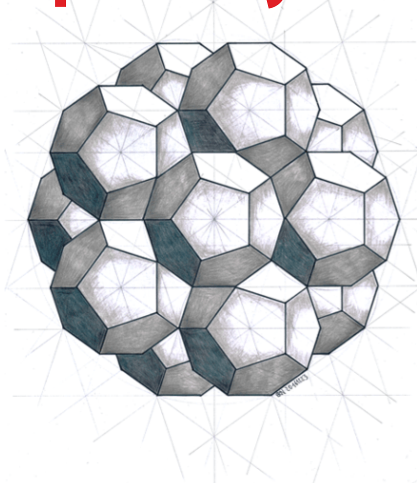
quasiperiodic crystal

quasicrystal



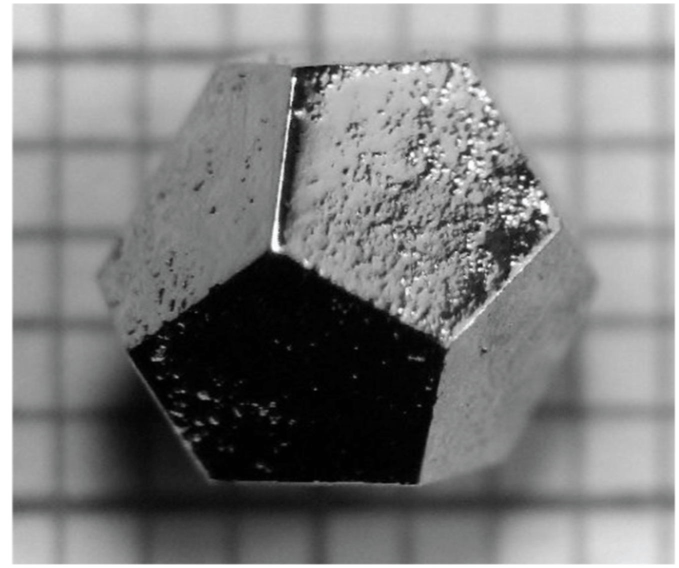
quasiperiodic checkerboard

Stampfli, 2013

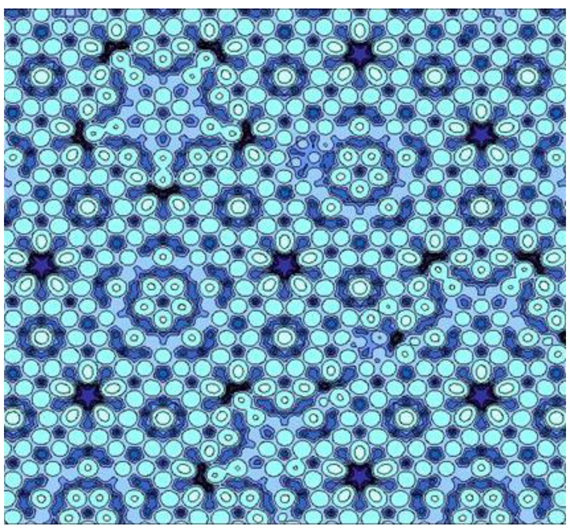


dense packing of dodecahedra

3D Penrose tiling Tripkovic, 2019



Holmium–magnesium–zinc quasicrystal



energy surface Al-Pd-Mn quasicrystal

Unal et al., 2007

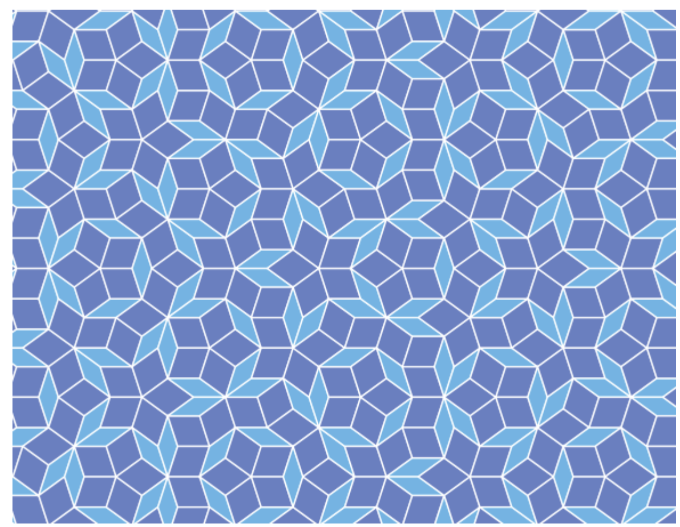
ordered but aperiodic

lacks translational symmetry

Shechtman et al., *Phys. Rev. Lett.*, 1984
Levine & Steinhardt, *Phys. Rev. Lett.*, 1984

classical transport in quasiperiodic media

Golden, Goldstein & Lebowitz, *Phys. Rev. Lett.*, 1985
Golden, Goldstein & Lebowitz, *J. Stat. Phys.*, 1990
⋮



aperiodic tiling of the plane - R. Penrose 1970s

1D, 2D inhomogeneous materials - quasiperiodic

$$\sigma(x) = 3 + \cos x + \cos kx$$

effective conductivity

$$\sigma^*(k) = \begin{cases} \text{constant} & k \ \text{irrational} \quad \text{quasiperiodic} \\ f(k) & k \ \text{rational} \quad \text{periodic} \end{cases}$$

Golden, Goldstein, Lebowitz
Classical transport in modulated structures, *Phys. Rev. Lett.* 1985

...

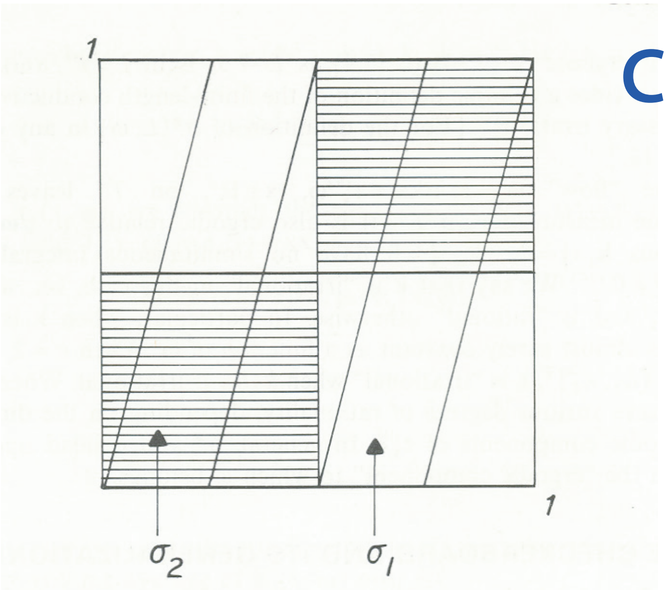
G. Bouchitté, S. Guenneau, F. Zolla, *SIAM Multiscale Modeling & Simulation*, 2010

E. Cherkaev, S. Guenneau, N. Wellander, *IEEE Metamaterials, 2017*

N. Wellander, S. Guenneau, E. Cherkaev, *Math. Methods in the Applied Sci.*, 2017

Classical transport in quasiperiodic media

Golden, Goldstein, and Lebowitz
Phys. Rev. Lett. 1985
J. Stat. Phys. 1990



line of slope k through an infinite checkerboard

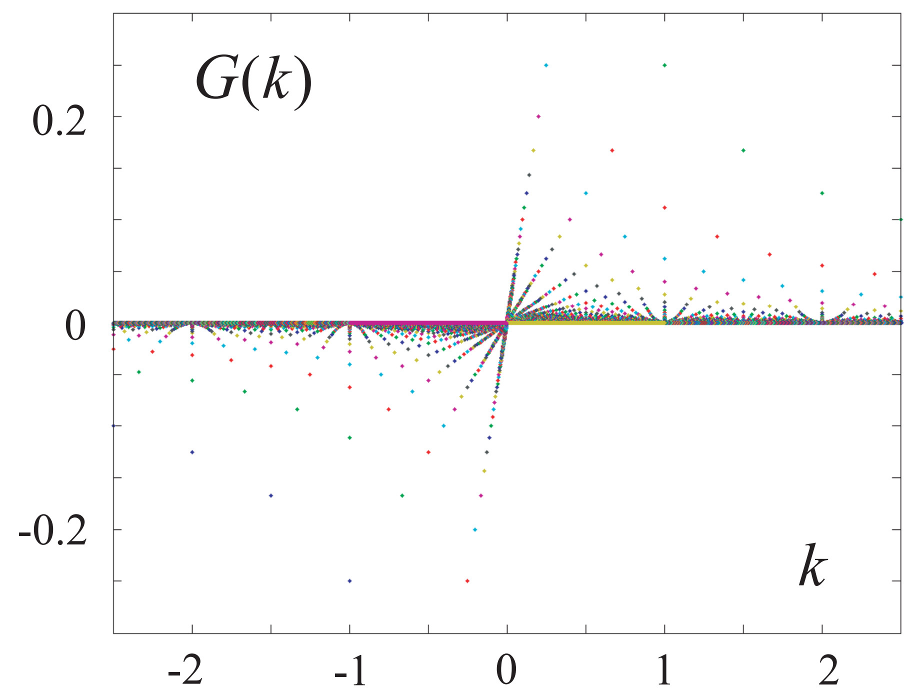
1D two component composite material

effective conductivity $\sigma^*(k)$

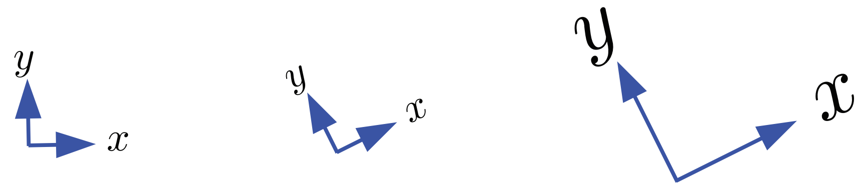
effective resistivity $1/\sigma^*(k) = 1 - G(k)$

$$G(k) = \begin{cases} 0, & k \text{ irrational} \\ 1/pq, & k = p/q \ \text{ rational} \end{cases}$$

continuous at *k* irrational
discontinuous at *k* rational

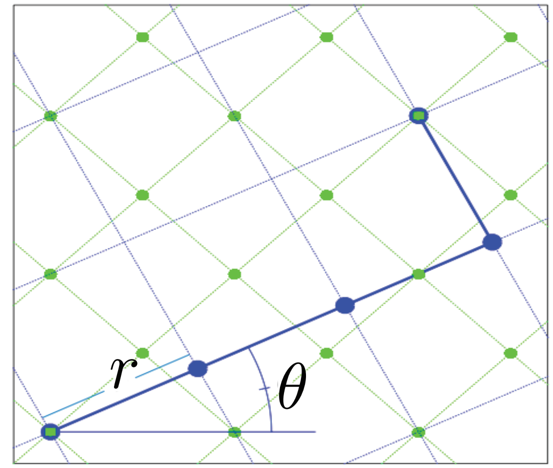


Moiré patterns generate two component composites on any scale



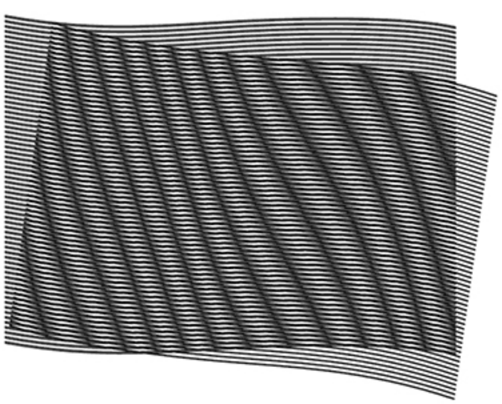
rotation
dilation

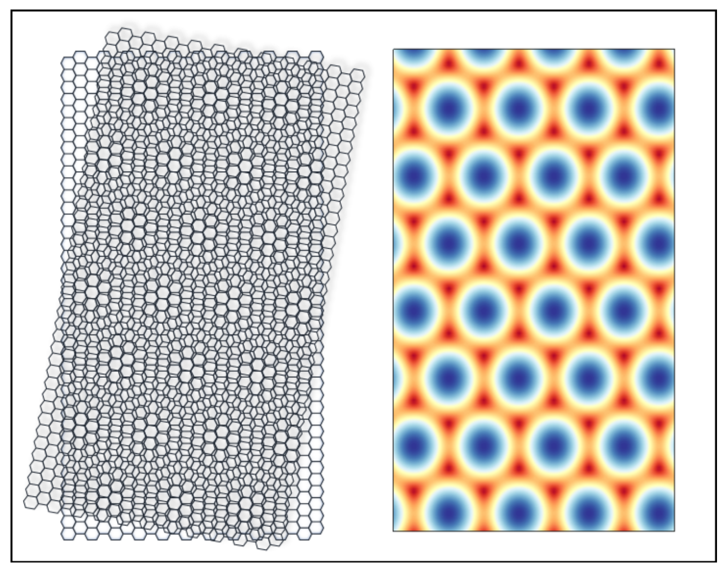
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = r \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$\psi\left(x', y'\right) = \cos 2\pi x' \cos 2\pi y'$$

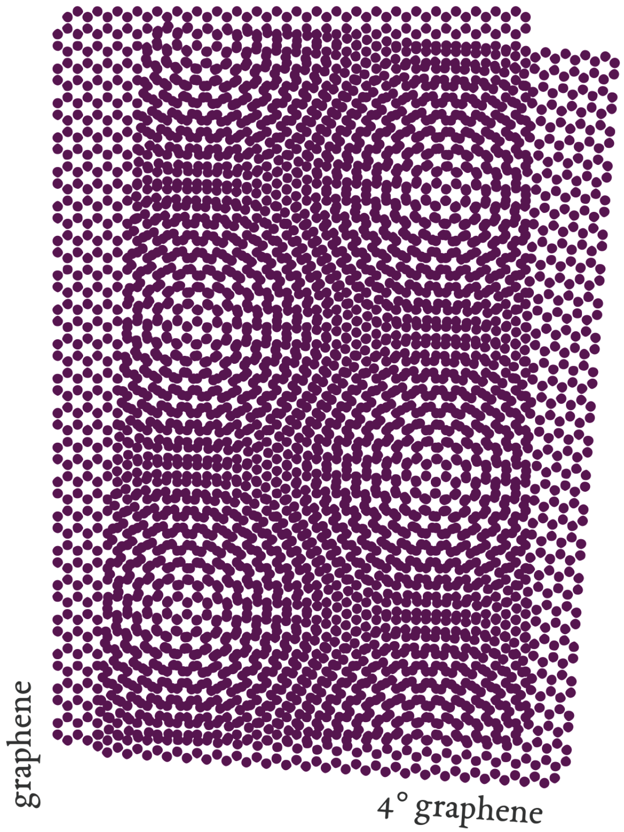
$$\chi = \begin{cases} 1, & \psi \geq 0 \\ 0, & \psi < 0 \end{cases}$$





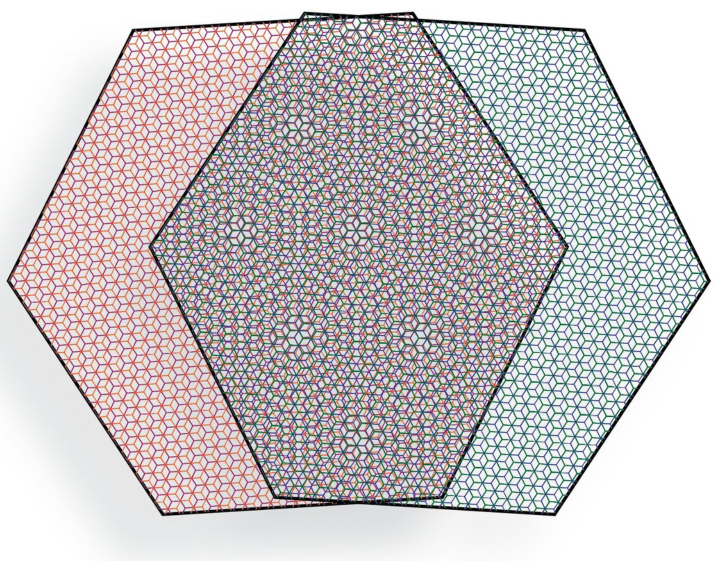
quantum dots
artificial atoms

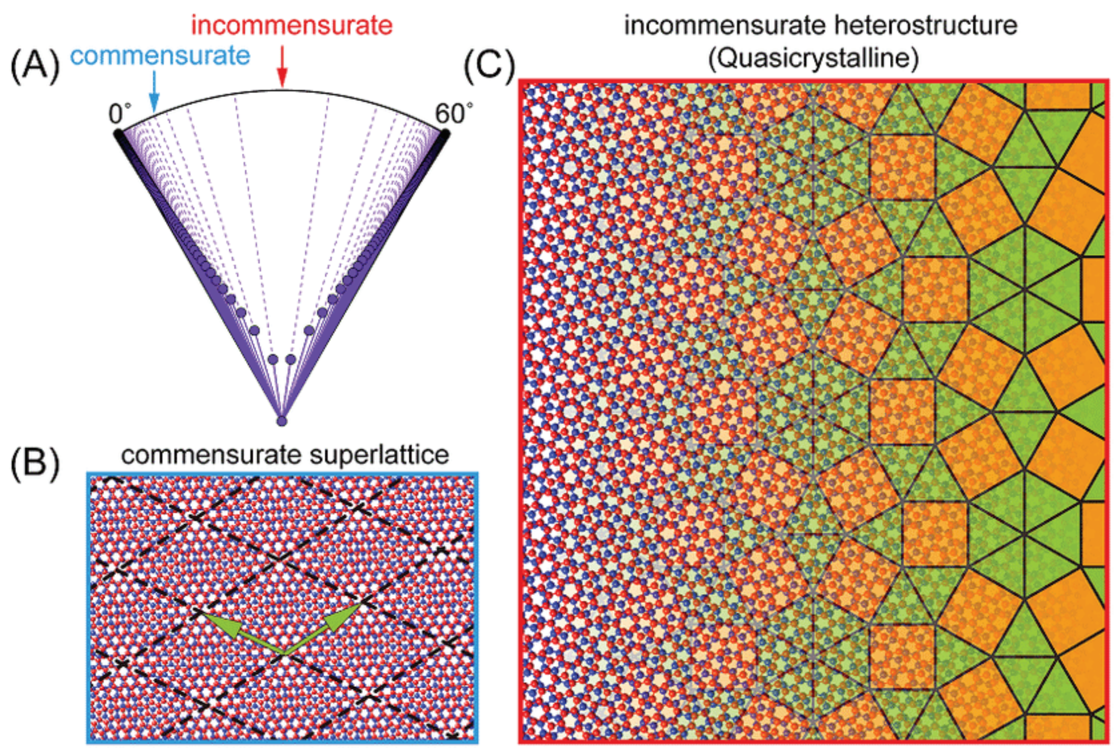
Tran et al.
Nature 2019



twisted bilayer graphene

superconducting
magic twist angle





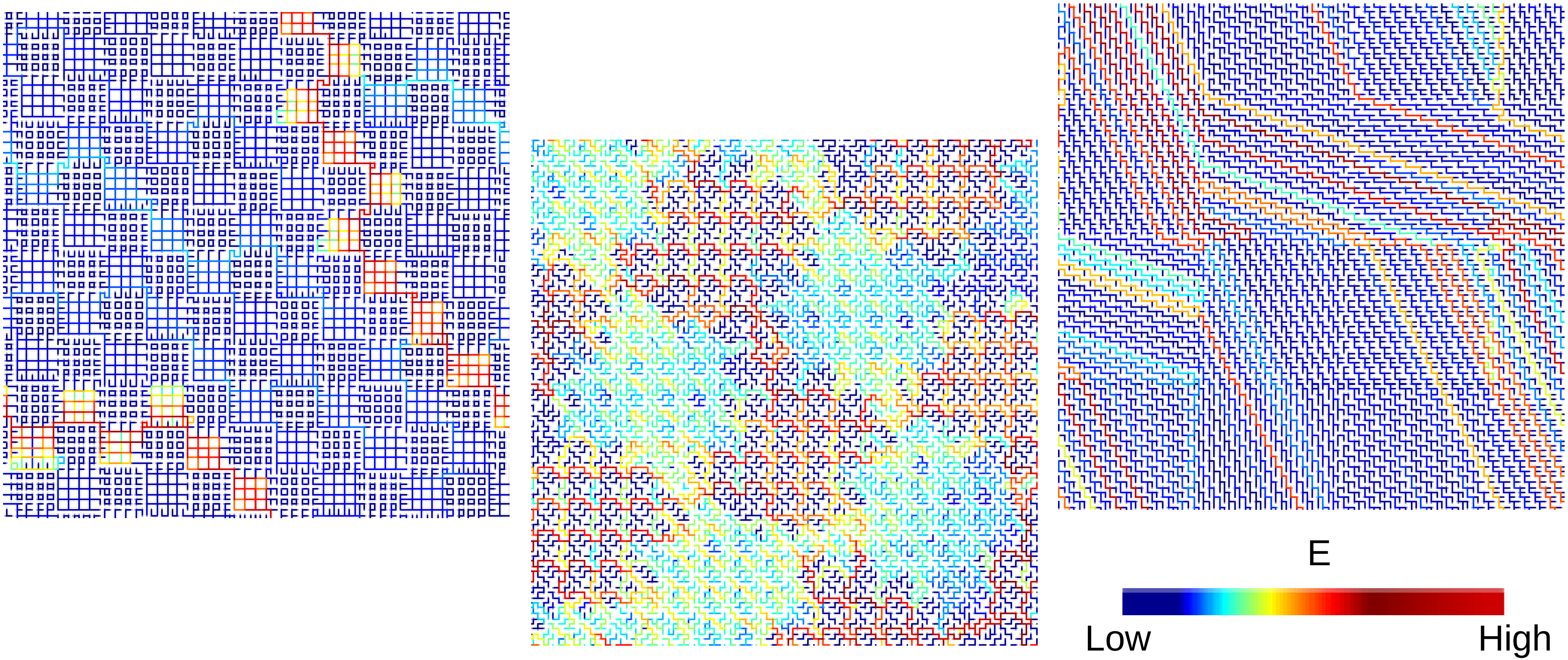
Yao et al., 2018

Small Difference in Moiré Parameters

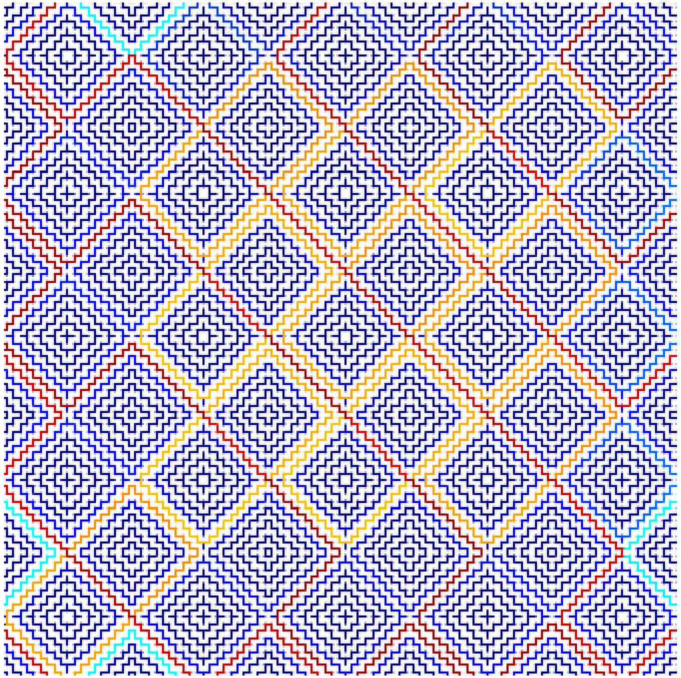
↓

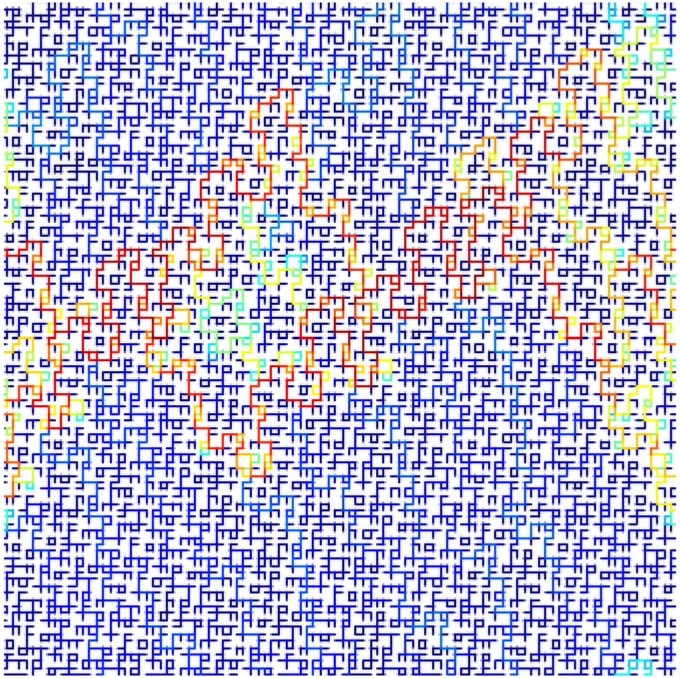
Big Difference in Material Properties

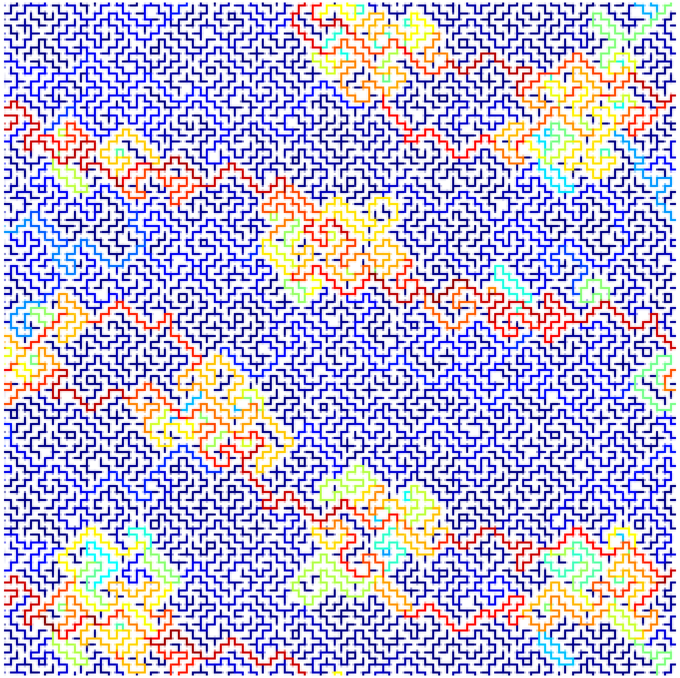
Wide Variety of Microgeometries



Wide Variety of Microgeometries





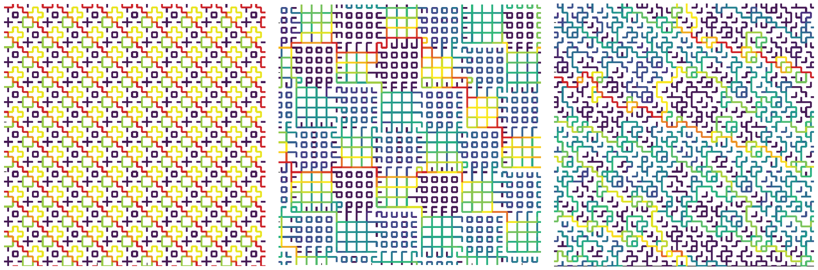


Order to disorder in quasiperiodic composites

Morison, Murphy, Cherkaev, Golden, Comm. Phys. 2022

twisted bilayer composites

Parameterized Moiré Pattern Creates Tunable Microgeometry

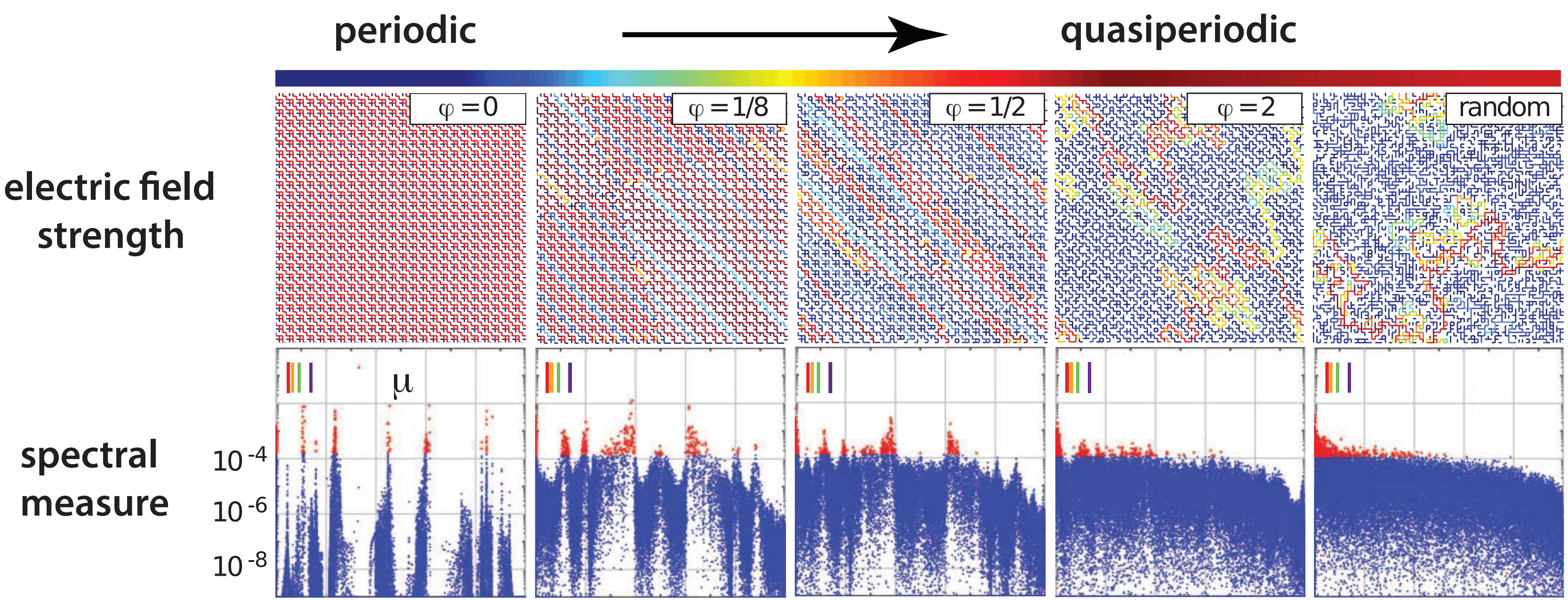


sea ice - inspired high tech spin off

tunable Moiré composites with exotic properties

(optical, electrical, thermal, ...), Anderson localization; our Moiré patterned geometries are similar to twisted bilayer graphene

but can be engineered on any scale!



RRN at percolation threshold

we bring the solid state physics framework for electronic transport and band gaps in semiconductors to classical transport in periodic and quasiperiodic composites

Anderson transition as twist angle is tuned

photonic crystals and quasicrystals

communications physics

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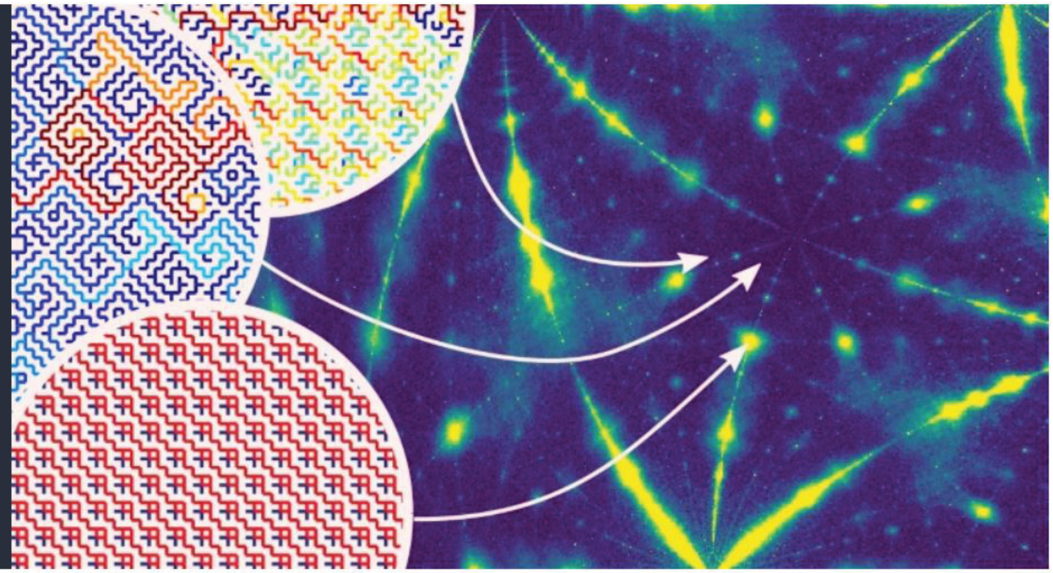
nature > communications physics

constellation of periodic systems in a sea of randomness

Order to disorder in quasiperiodic composites

David Morison, N. Benjamin Murphy ... Kenneth M. Golden

Article | 14 June 2022



Moiré parameter space

Featured

Article

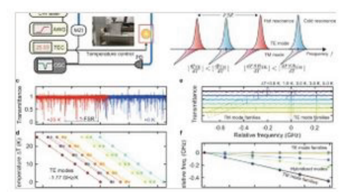
Open Access

10 Jan 2023

Versatile tuning of Kerr soliton microcombs in crystalline microresonators

High-repetition rate microresonator-based frequency combs offer powerful and compact optical frequency comb sources that are of great importance to various applications. Here, the authors extend the tunability of the Kerr soliton frequency combs by exploiting thermal effects and frequency stabilization techniques.

Shun Fujii, Koshiro Wada ... Takasumi Tanabe



Article

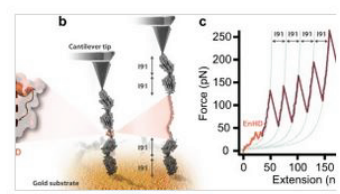
Open Access

12 Jan 2023

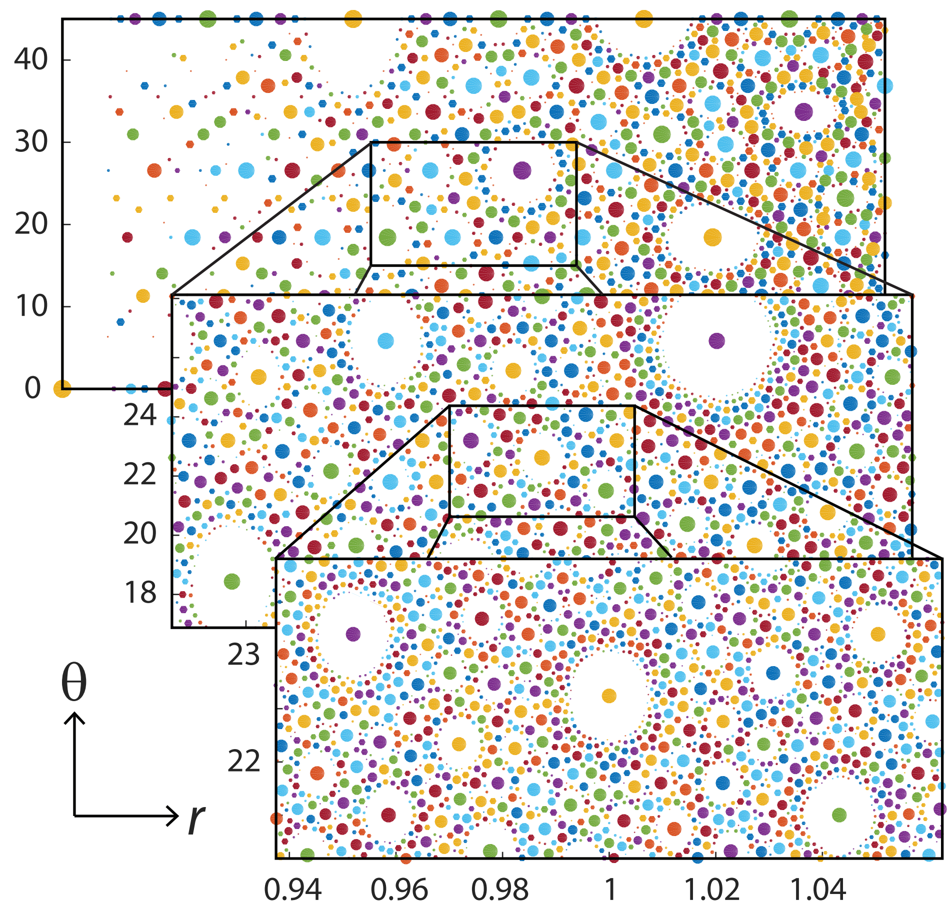
Compliant mechanical response of the ultrafast folding protein EnHD under force

Exhibiting low-energy (un)folding barriers and fast kinetics, ultrafast folding proteins are enticing models to study protein dynamics. The authors use single molecule force spectroscopy AFM to capture the compliant behaviour hallmarking the dynamics of ultrafast folding proteins under force.

Antonio Reifs, Irene Ruiz Ortiz ... Raul Perez-Jimenez



Fractal arrangement of periodic systems

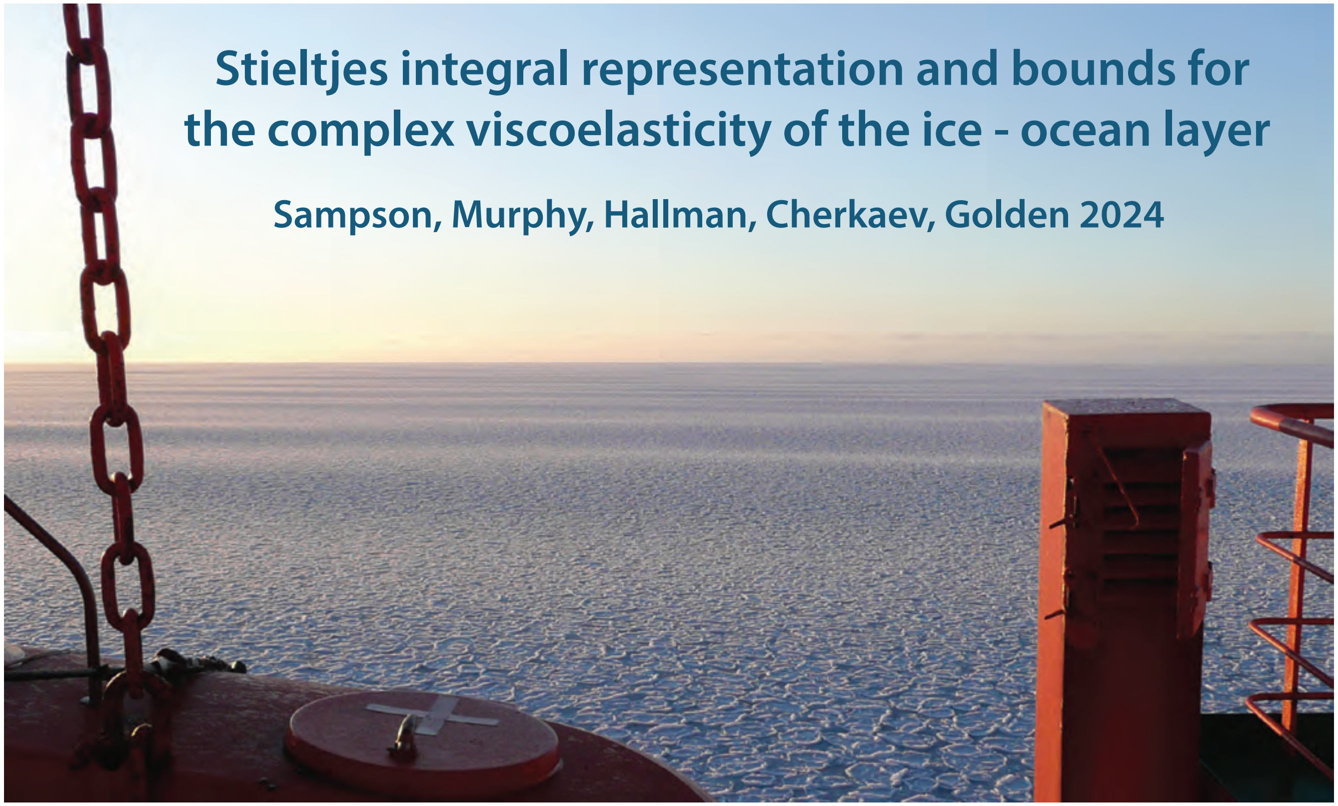


Sequential insets zooming into smaller regions of parameter space.

size of the dots ~ length of period

(large dot ~ small period; small dot ~ large period; white space ~ "infinite" period)

ocean wave propagation through the sea ice pack



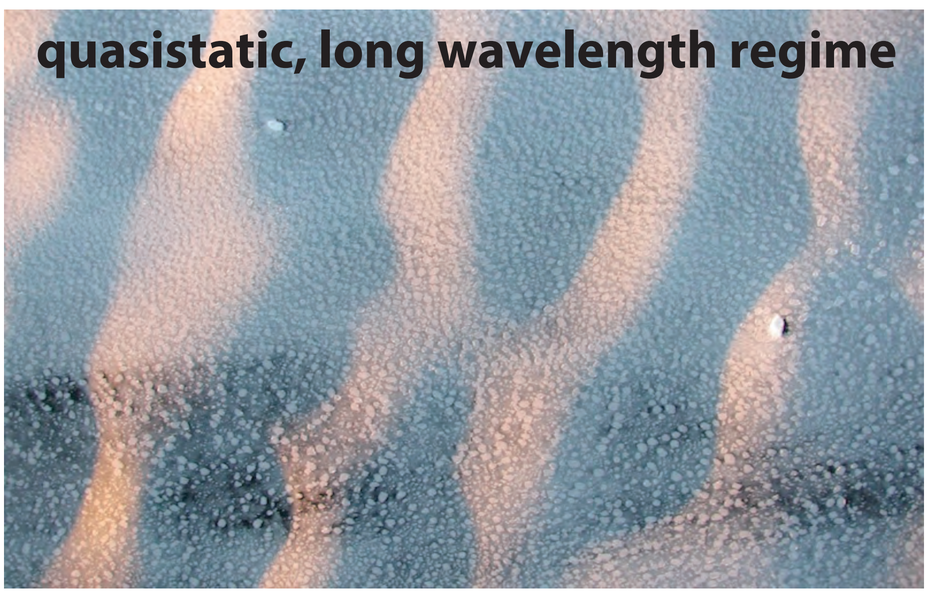
- wave-ice interactions critical to growth and melting processes
- break-up; pancake promotion floe size distribution

effective layer parameter previously fit to wave data

Keller 1998
Mosig, Montiel, Squire 2015
Wang, Shen 2012

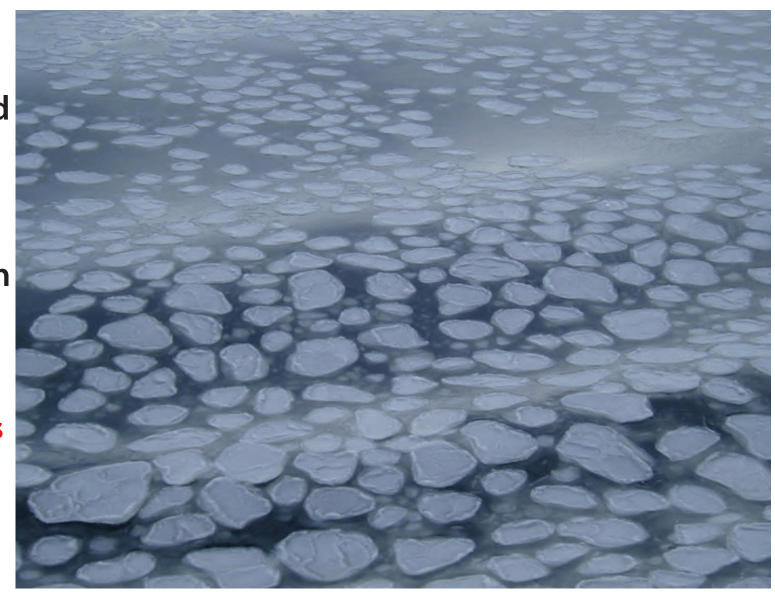
Analytic Continuation Method

Bergman 1978, Milton 1979
Golden and Papanicolaou 1983
Milton, *Theory of Composites* 2002



homogenized parameter depends on sea ice concentration and ice floe geometry

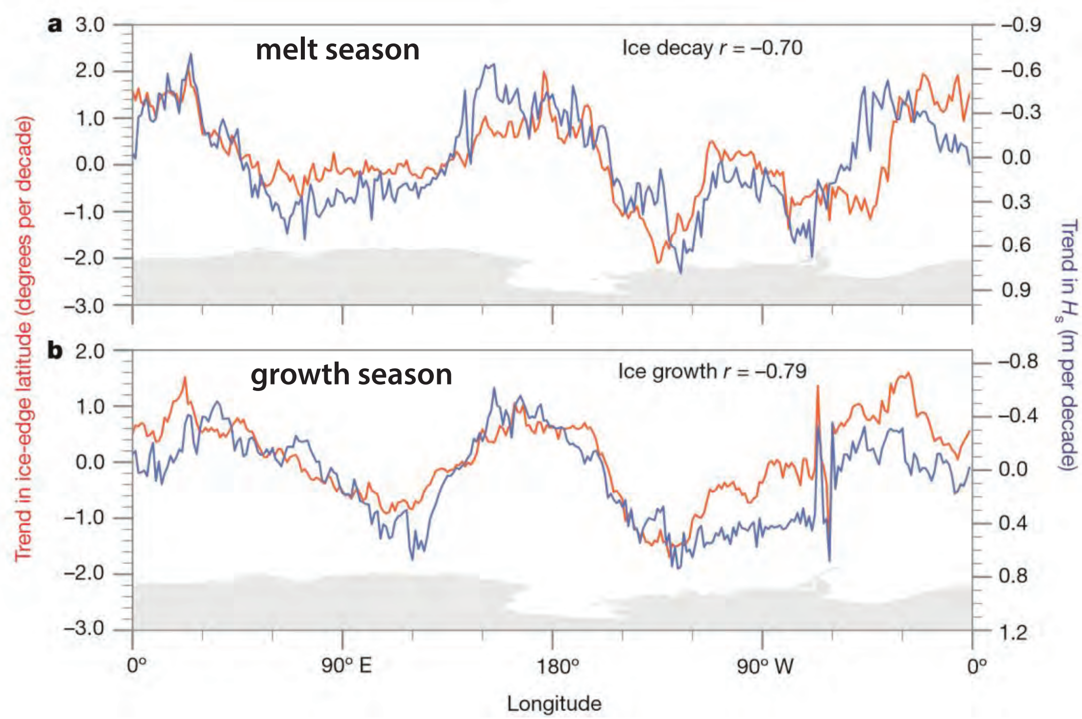
like EM waves



Storm-induced sea-ice breakup and the implications for ice extent

Kohout et al., *Nature* 2014

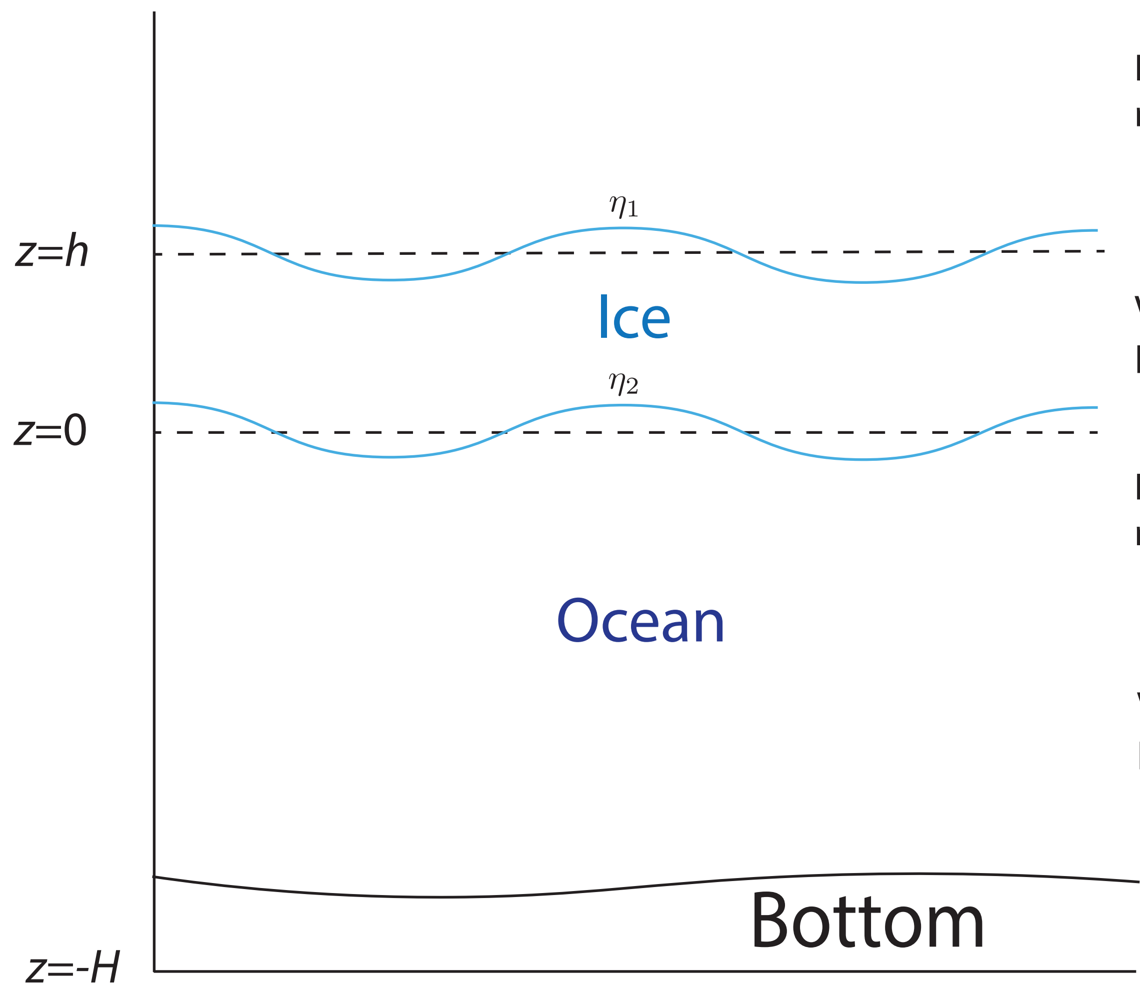
- during three large-wave events, significant wave heights did not decay exponentially, enabling large waves to persist deep into the pack ice.
- large waves break sea ice much farther from the ice edge than would be predicted by the commonly assumed exponential decay



ice extent compared with significant wave height

Waves have strong influence on both the floe size distribution and ice extent.

Two Layer Models and Effective Parameters



G shear modulus — P pressure — ω angular frequency — U velocity field

ν viscosity — λ Poission ratio — ρ density — g gravity

Viscous fluid layer (Keller 1998)
Effective Viscosity ν

Equations of motion:

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 U + g$$

Viscoelastic fluid layer (Wang-Shen 2010)
Effective Complex Viscosity $\nu_e = \nu + iG/\rho\omega$

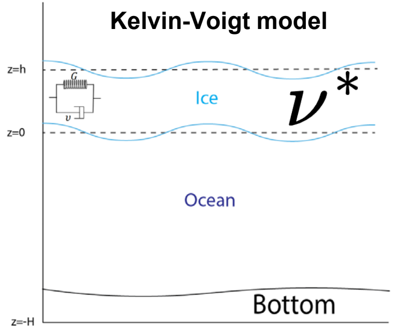
Equations of motion

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho}\nabla P + \nu_e \nabla^2 U + g$$

Viscoelastic thin beam (Mosig *et al.* 2015)
Effective Complex Shear Modulus $G_v = G - i\omega\rho\nu$

Stieltjes integral representation for effective complex viscoelastic parameter; bounds

Sampson, Murphy, Cherkaev, Golden 2017



Single effective rheological parameter (Mosig et al. 2015)

$$\nu^* = G - i\omega\rho v$$

Effective complex viscoelasticity

Integral representation

$$\frac{\nu^*}{\nu_2} = ||\epsilon_s^0||^2(1 - F(s))$$

$$F(s) = \int_0^1 \frac{d\mu(\lambda)}{s-\lambda}$$

divergence-free deviatoric stress

$$\nabla \cdot \sigma_s = 0$$

microscale

$$\sigma_s = 2\nu\epsilon_s$$

$$\nu(\vec{x}) = \chi_1\nu_1 + \chi_2\nu_2$$

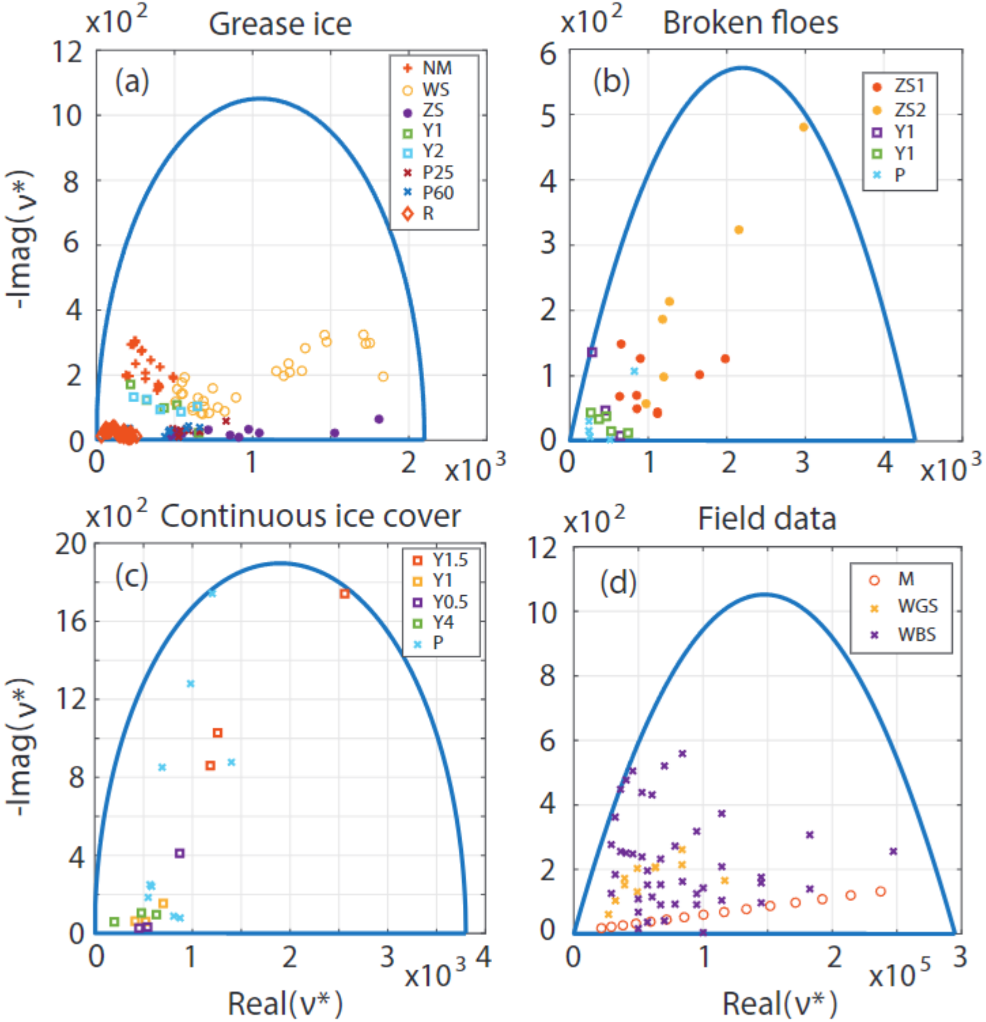
macroscale

$$\langle\sigma_s\rangle = 2\nu^*\epsilon_s^0$$

$$\langle\epsilon_s\rangle = \epsilon_s^0$$

Forward bounds for the effective viscoelasticity are fitted to well known wave-ice datasets, including *Wadhams et al. 1988, Newyear & Martin 1997, Wang & Shen 2010, Meylan et al. 2014*, and several others!

G	P	C	ρ	v	g	u
shear modulus	pressure	elasticity	density	kinematic viscosity	gravity	displacement



Waves in sea ice and solid state physics

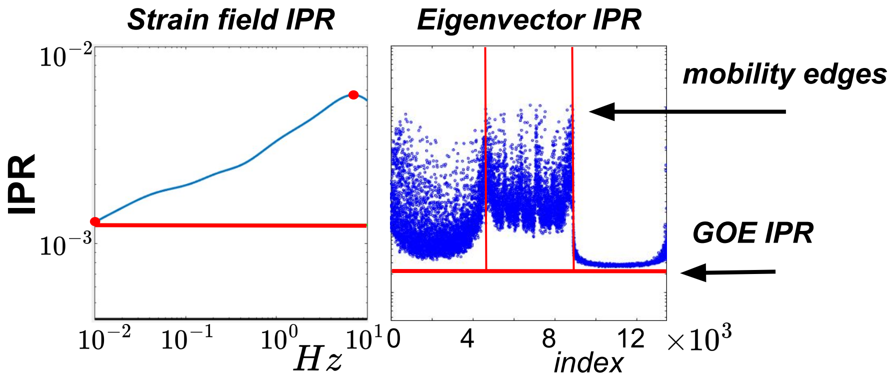
Resolvent representation of the deviatoric strain field

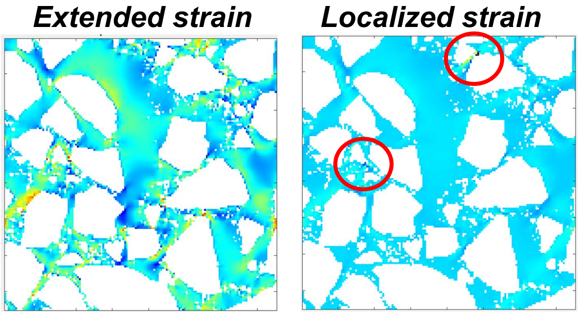
$$\chi\epsilon_s = s\left(sI - \chi\Gamma^S\chi\right)^{-1}\chi\epsilon_s^0$$

Stieltjes integral representation of effective complex viscoelasticity

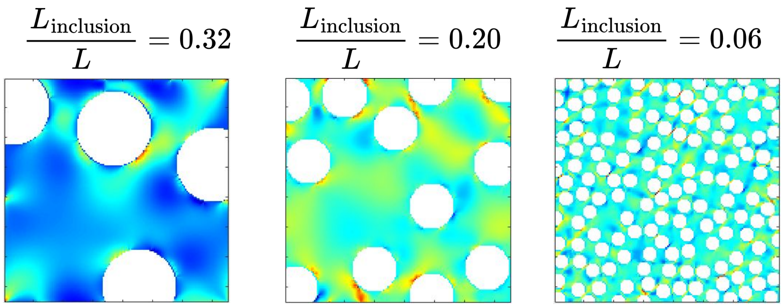
$$\frac{\nu^*}{\nu_2} = \int_0^1 \frac{d\mu(\lambda)}{s-\lambda}$$

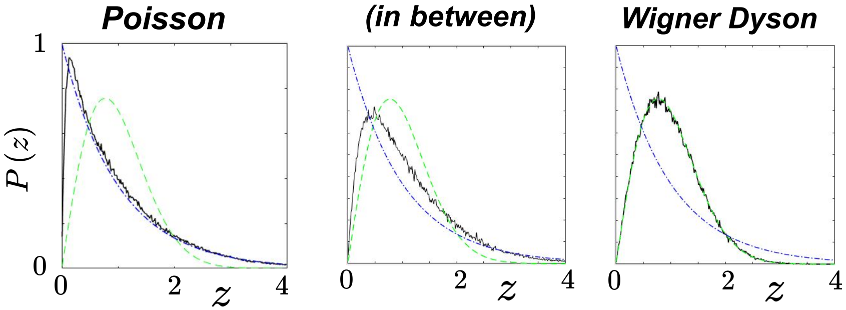
Increasing geometric order →





$f = 0.01Hz$ $f = 7.079Hz$





Transition in the eigenvalue spacing distribution of $\chi\Gamma^S\chi$

Large inclusions → Small inclusions

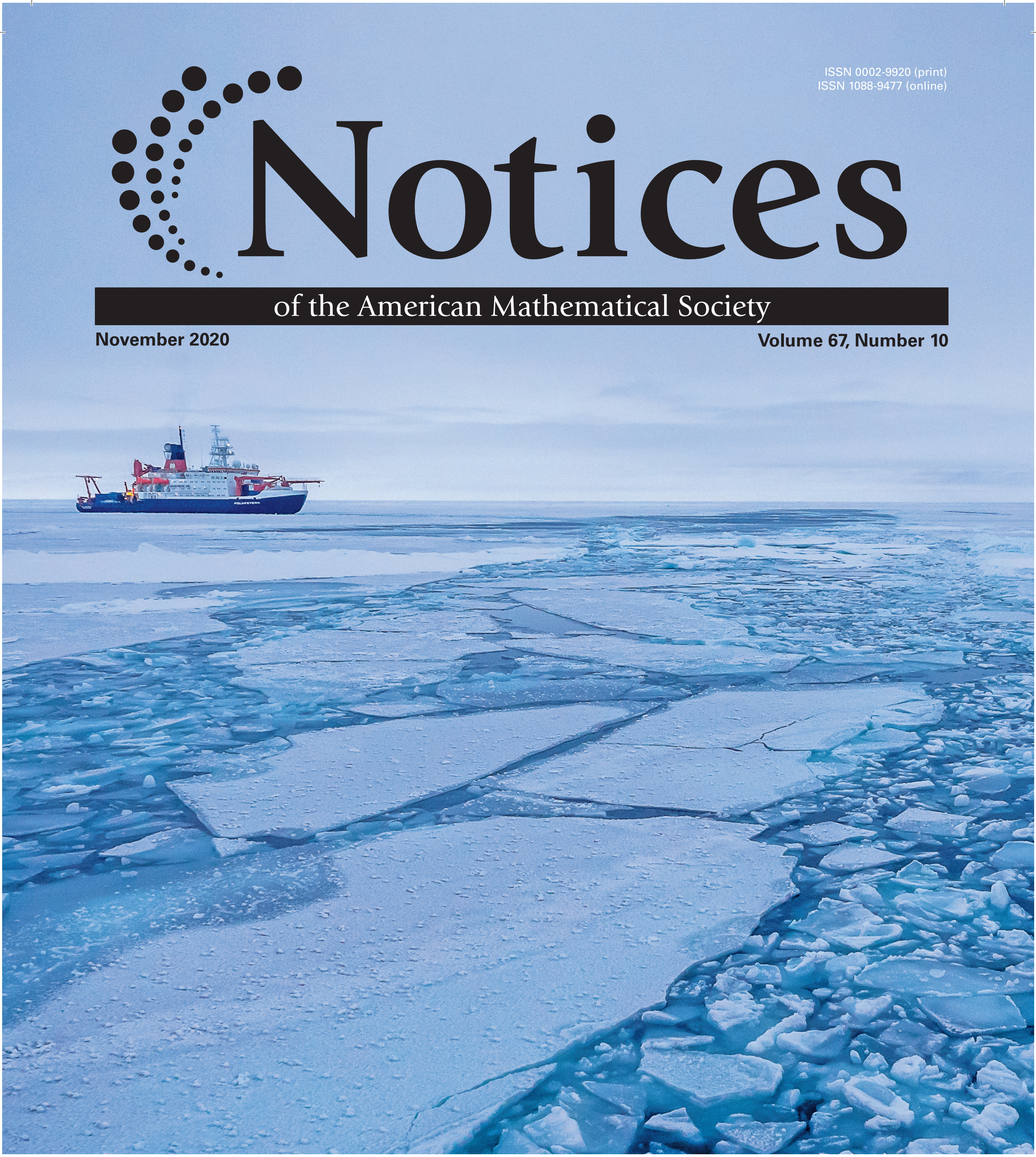
"Local to collective deformation transition"

ISSN 0002-9920 (print)
ISSN 1088-9477 (online)

Notices

of the American Mathematical Society

November 2020 Volume 67, Number 10



AMERICAN MATHEMATICAL SOCIETY

Advancing research. Creating connections.

The cover is based on "Modeling Sea Ice," page 1535.

NSF Research Training Grant (RTG) with 15 Applied Math faculty:

optimization and inverse problems

July 2022 - June 2027

Overall goal: Build an advanced, competitive U.S. STEM workforce.

- Strengthen our graduate and postdoctoral programs in applied math to attract top students in the nation, and place them in top jobs.
- Provide transformative experiences that draw students into math.

Arctic Mathpeditions - May 2024 & 2026

OPEN POSITIONS:
Postdoctoral, Ph.D., Undergraduate

Arctic Mathpedition 2024

NSF RTG Arctic Mathpedition, May 2024

on the frozen Arctic Ocean north of Utqiaġvik, AK

We took 7 math students working on sea ice models to the Arctic to do *experiments* on the physics and biology of sea ice.

Jody Reimer, Ken Golden

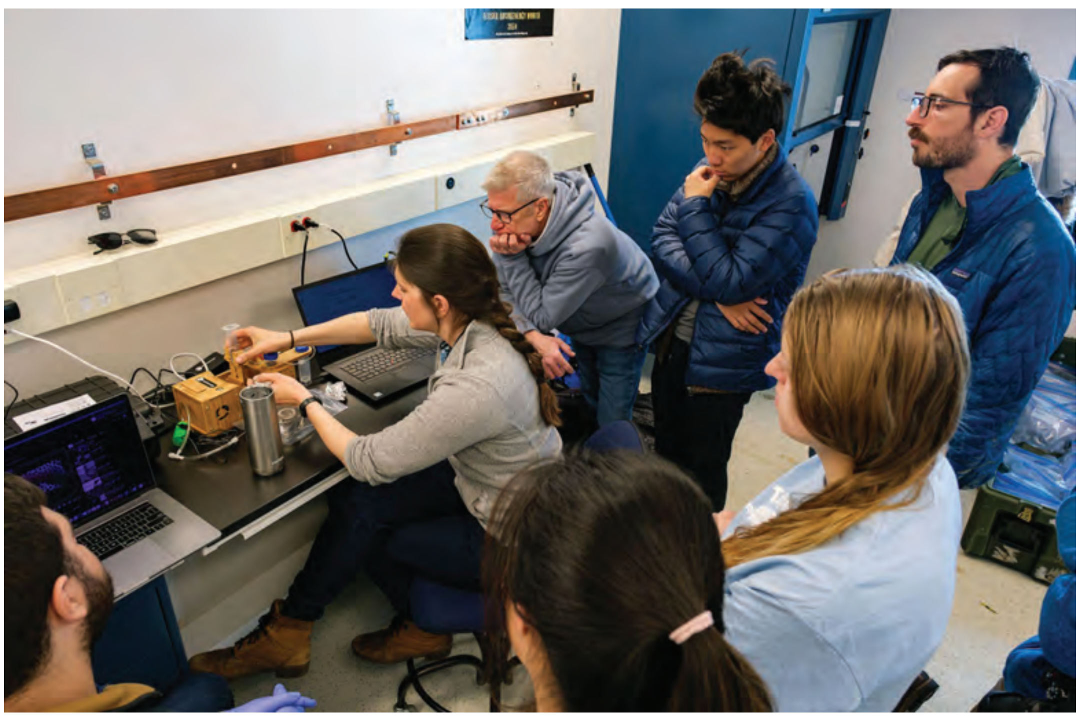
[Seth & Tarn]

Anthony Lee	High School
David Gluckman	Undergraduate
Kathy Lin	Undergraduate
Nash Ward	Undergraduate
Daniel Hallman	Graduate Student
Anthony Jajeh	Graduate Student
Delaney Mosier	Graduate Student
Marco Lozzi	Student Photojournalist

see what you're modeling; close the gap between theory and experiment; connect physics & bio; experience climate change first-hand; math outreach to locals

Math Dept Colloquium, Nov 21

NSF RTG Arctic Mathpedition 2, May 2026

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bottom of a sea ice core