





Earthrise

December 24, 1968

William Anders

NASA

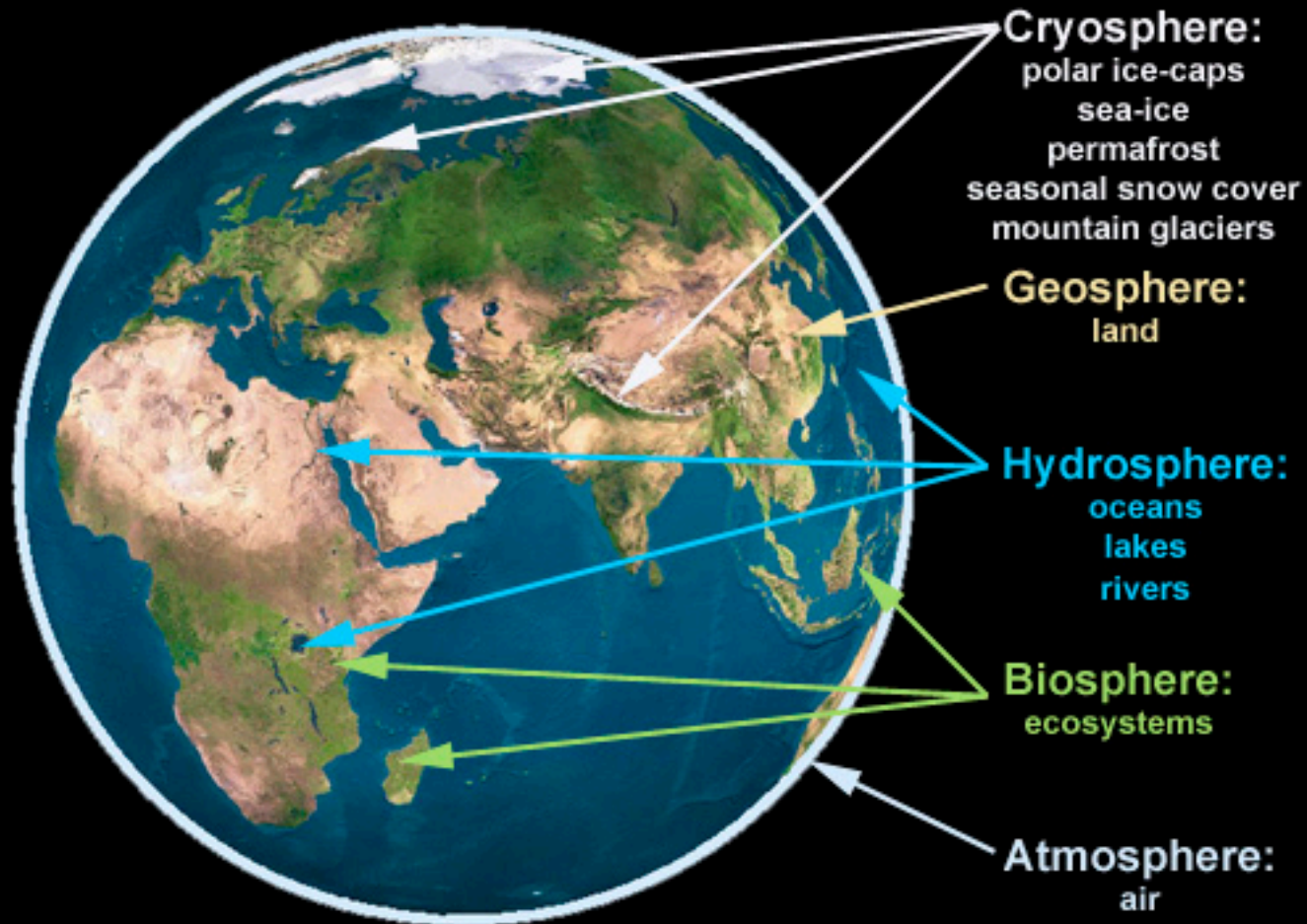


# **Introduction to Modeling Earth's Climate System: Energy Balance Models**

**Ken Golden**  
SLMath UAF, 25 June 2025

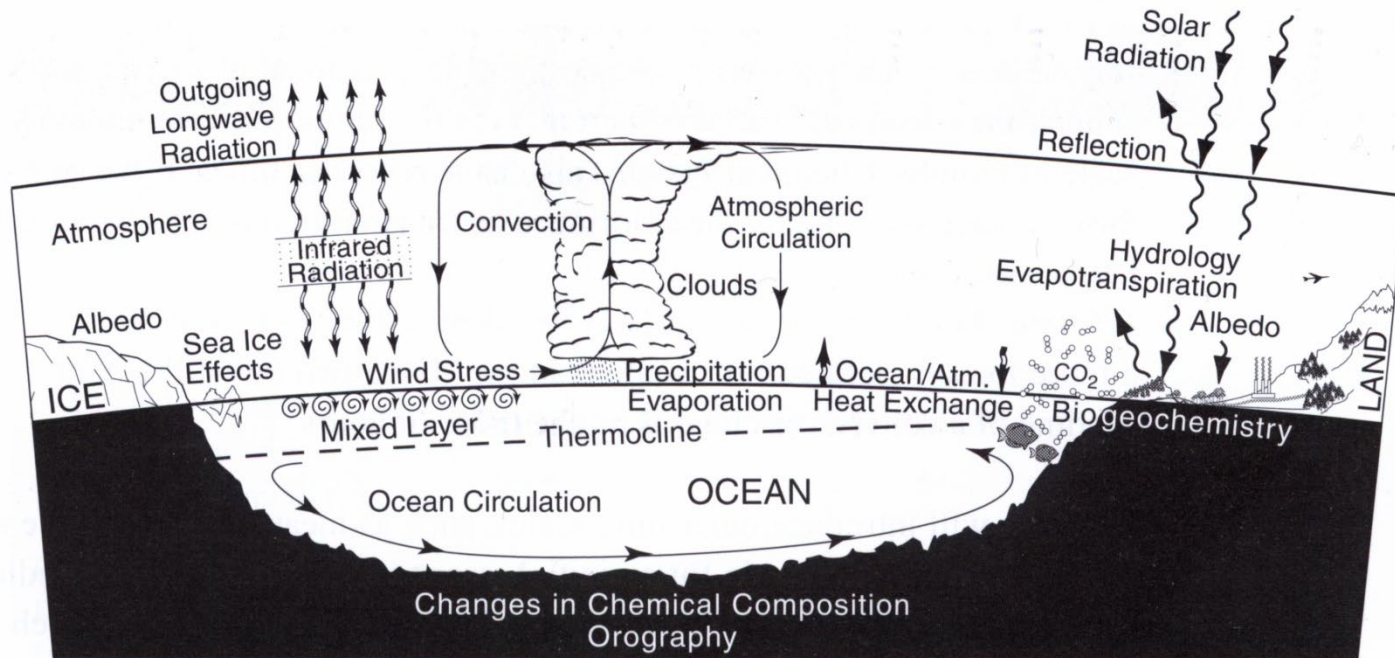
**William Anders**  
Earthrise, NASA  
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# The components of Earth's Climate System





# Basics of global climate



Schematic of some important processes within each component of the climate system.

Basic components: **cryosphere** – land ice, snow, sea ice, ice shelves, permafrost  
**biosphere** – sum total of all living things  
**lithosphere** – solid Earth – ocean basins, mountain ranges  
**atmosphere, hydrosphere** (oceans, seas, rivers, ...)

# *sea ice and global ocean circulation*

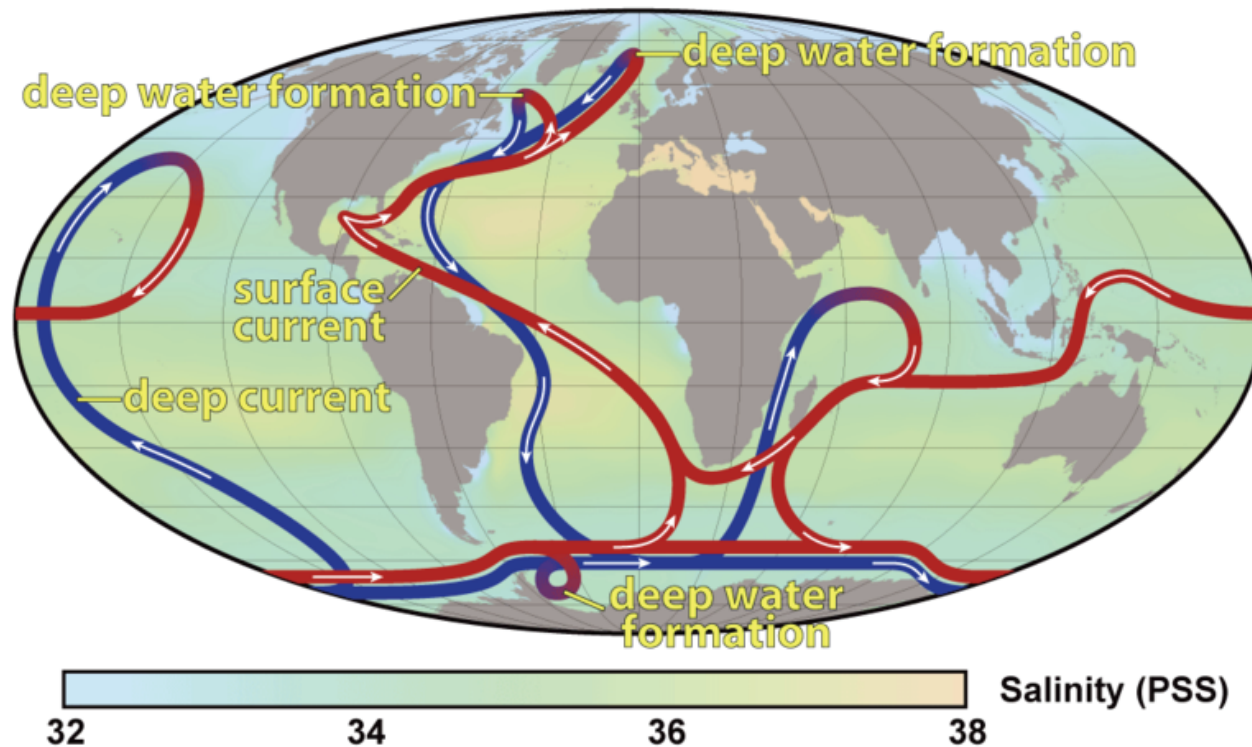
*brine expulsion from freezing sea water* results  
in sinking of dense, saltier water to great depths

*melting sea ice inputs fresher water  
into the upper ocean*



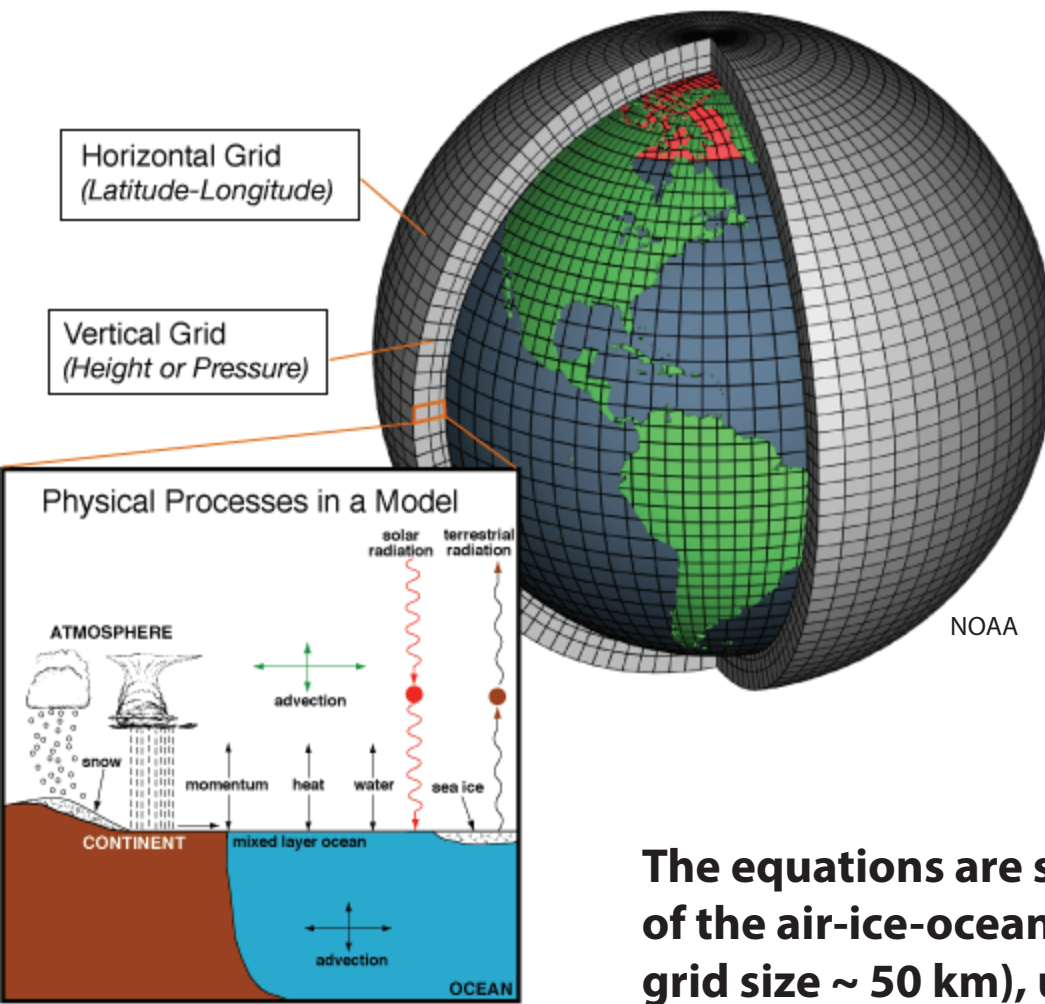
**deep-water formation drives  
circulation in the world's oceans**

## **Thermohaline Circulation**



## **GLOBAL THERMOHALINE CONVEYOR BELT**

# Global Climate Models



Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

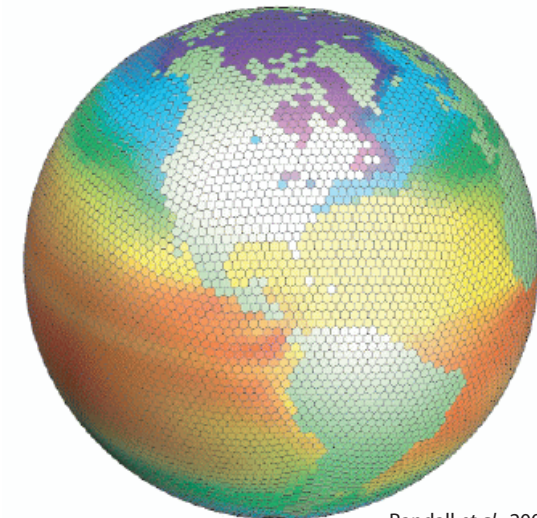
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 50 km), using very powerful computers.

key challenge :

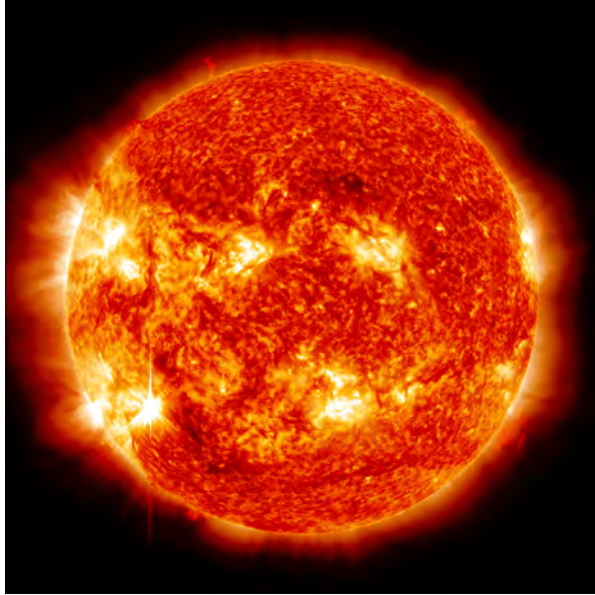
*incorporating sub - grid scale processes*

linkage of scales





# climate system parameters



$$S_0 = 1,368 \text{ W m}^{-2}$$

solar energy flux per unit area

**solar “constant”**

depends on sun's strength  
and average distance from  
Earth to Sun

**MODEL INPUT**



$$T(t) = \langle \text{Temperature}(x, t) \rangle$$

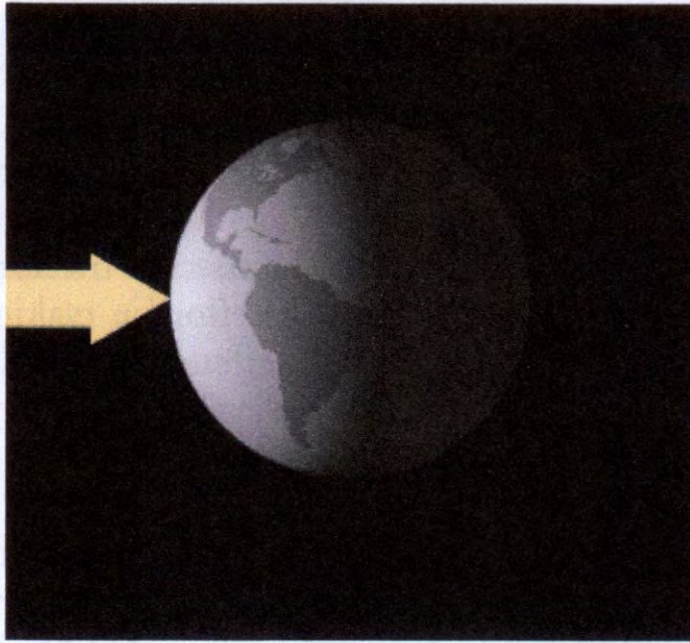
global mean  
surface temperature

**How to predict ??**

(How to define and measure??)

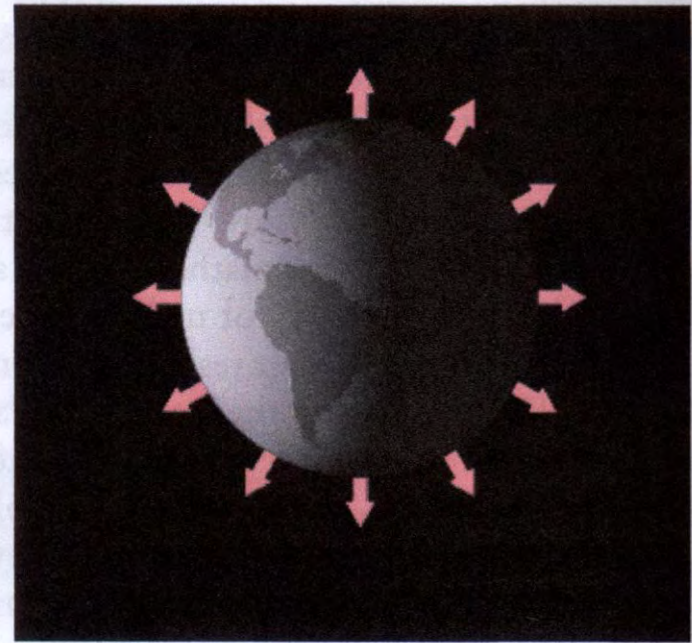
**MODEL OUTPUT**

# Earth's energy balance



$$E_{in}$$

incoming shortwave radiation  
sunlight



$$E_{out}$$

outgoing longwave radiation  
heat

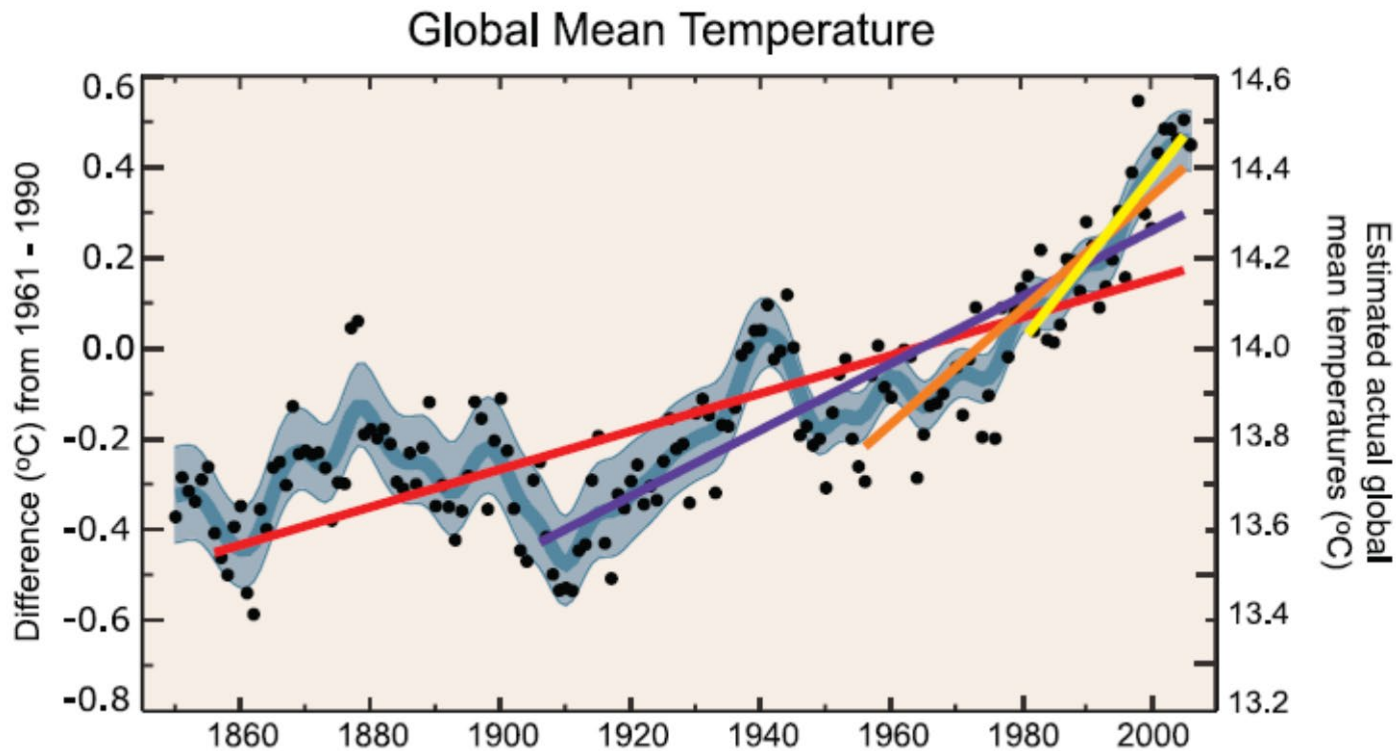
if  $E_{out} > E_{in}$  planet **cools**

if  $E_{out} < E_{in}$  planet **warms**



# Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):  
Warming is “unequivocal”



Dots: yearly average

Curve: decadal average

Blue: uncertainty interval





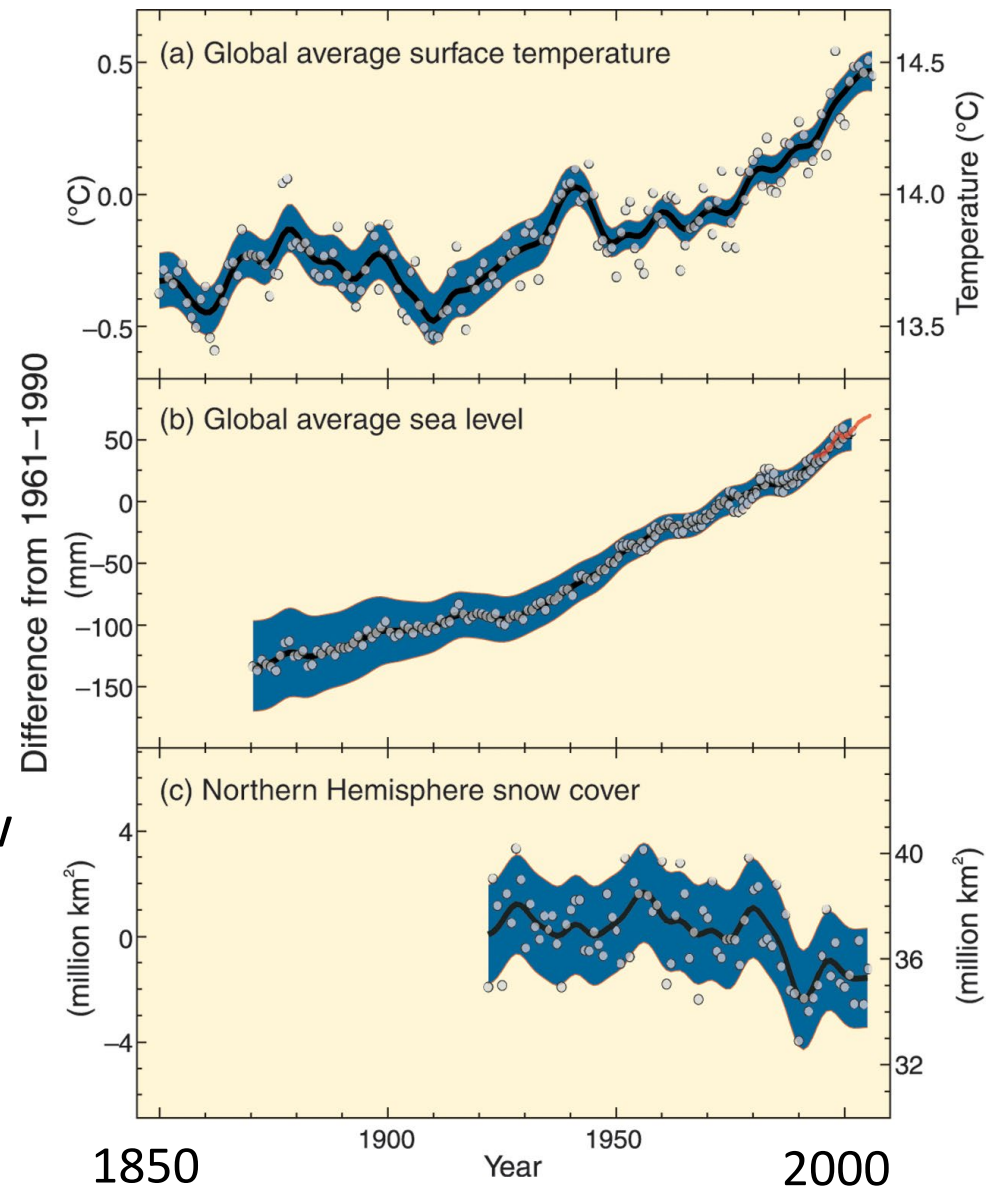
# IPCC: Warming is “unequivocal”

Global mean surface temp

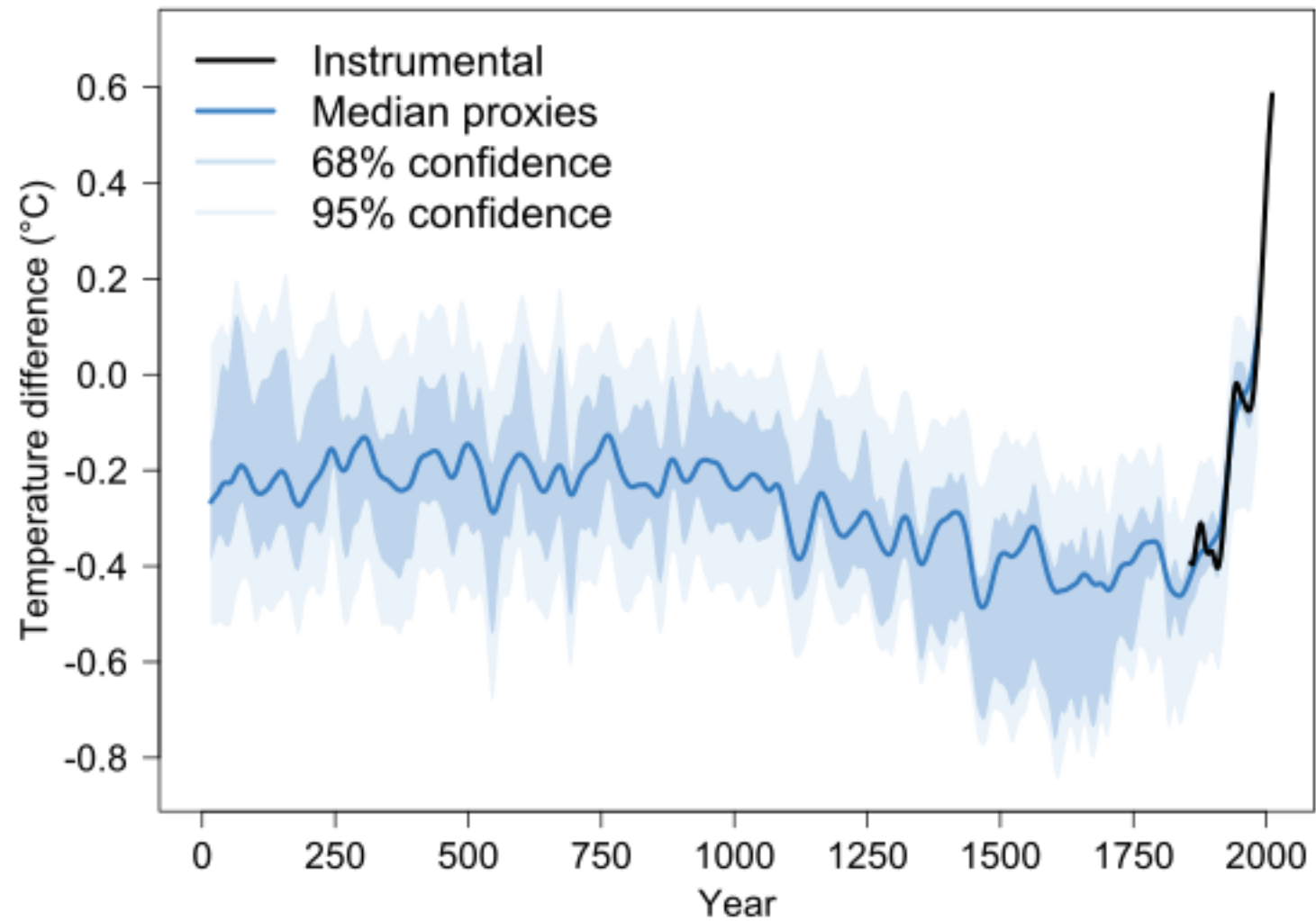
Global mean sea level

Northern Hemisphere snow  
cover (March-April)

Dots: yearly average  
Curve: decadal average  
Blue: uncertainty interval



## Mean Global Temperature over the past 2000 years



# Energy Balance Model

$$C \frac{dT}{dt} = E_{in} - E_{out}$$



***rate of change with time of  
mean global temperature***

*C = heat capacity  
= energy needed to  
raise temperature  
by one Kelvin*

***$E_{in} = ?$***

***$E_{out} = ?$***

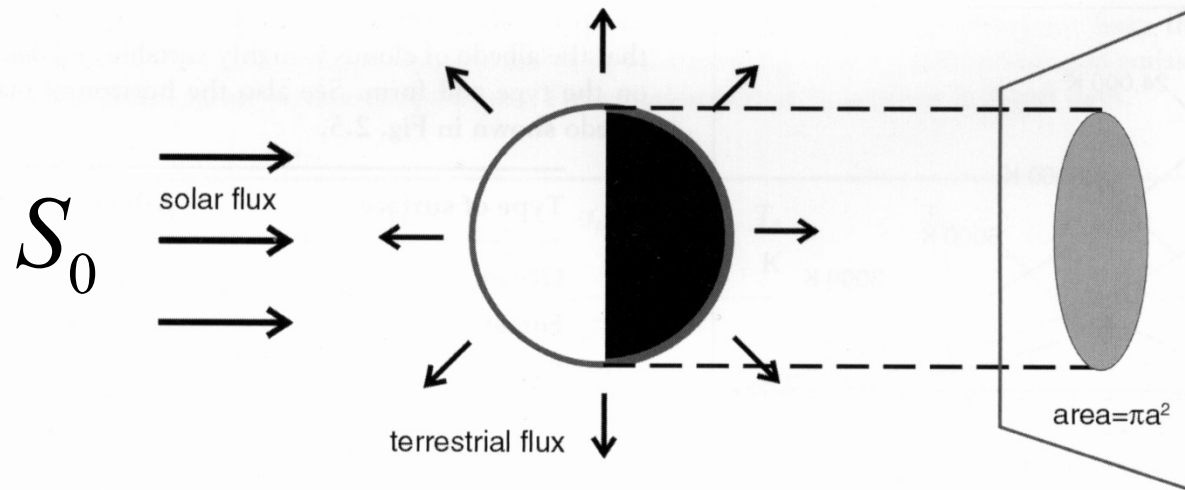
$E_{in}$ 

FIGURE 2.4. The spinning Earth is imagined to intercept solar energy over a disk of radius  $a$  and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

solar power incident on Earth

$$S_0 \pi a^2 = 1.74 \times 10^{17} \text{ W}$$

most important climate parameter:  
how much energy reflected vs. how much energy absorbed

*reflection*      **albedo**  $\alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$

*absorption*      **co-albedo**  $1 - \alpha$       *fraction of energy that reaches Earth's surface*

amount of solar energy  
intercepted by Earth per  
unit time (as a flat disk)

$$\pi R^2 S_0$$

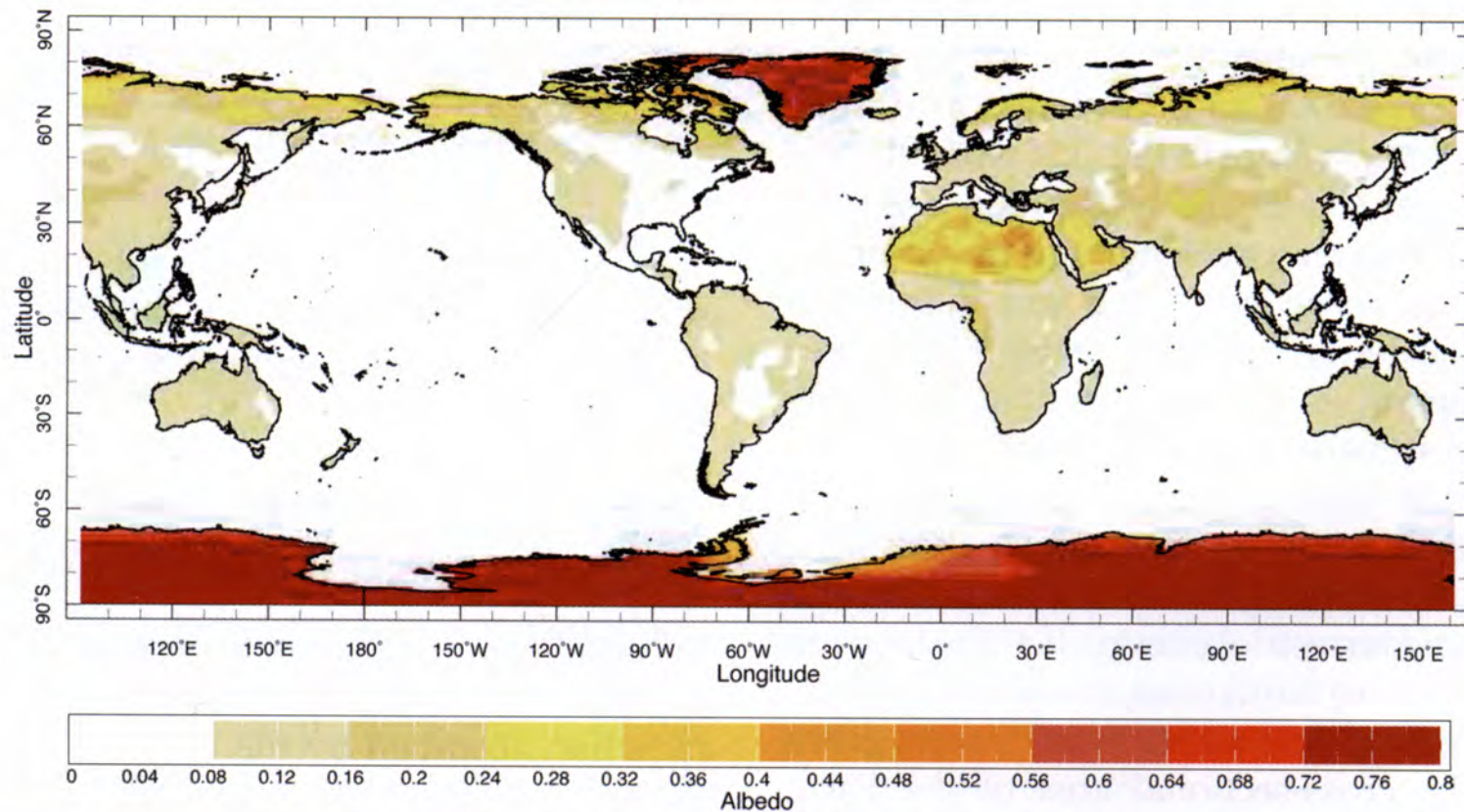
amount of solar energy  
reaching Earth's surface  
per unit time

$$(1 - \alpha) \pi R^2 S_0$$

amount of solar energy  
reaching Earth's surface  
per unit time / unit area

$$E_{in} = \frac{(1 - \alpha) \pi R^2 S_0}{4 \pi R^2} = \frac{(1 - \alpha) S_0}{4} = (1 - \alpha) Q$$

# albedo of Earth's surface



**FIGURE 2.5.** The albedo of the Earth's surface. Over the ocean the albedo is small (2–10%). It is larger over the land (typically 35–45% over desert regions) and is particularly high over snow and ice (~80%) (see Table 2.2).

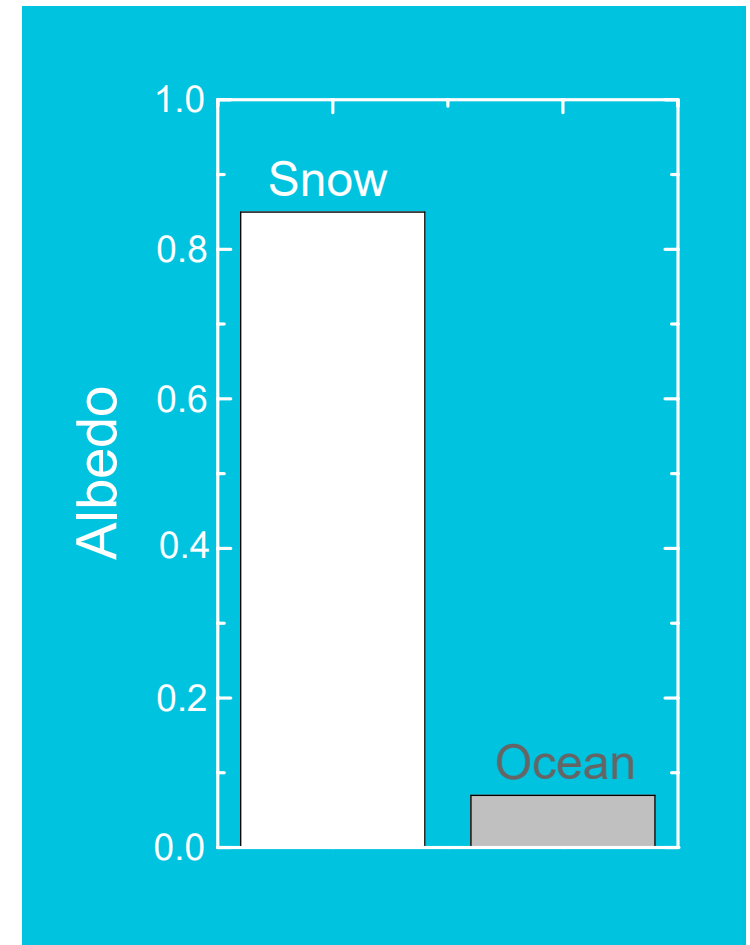
**average albedo ~ 0.3**



# polar ice caps critical to global climate in reflecting incoming solar radiation



white snow and ice  
reflect



dark water and land  
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

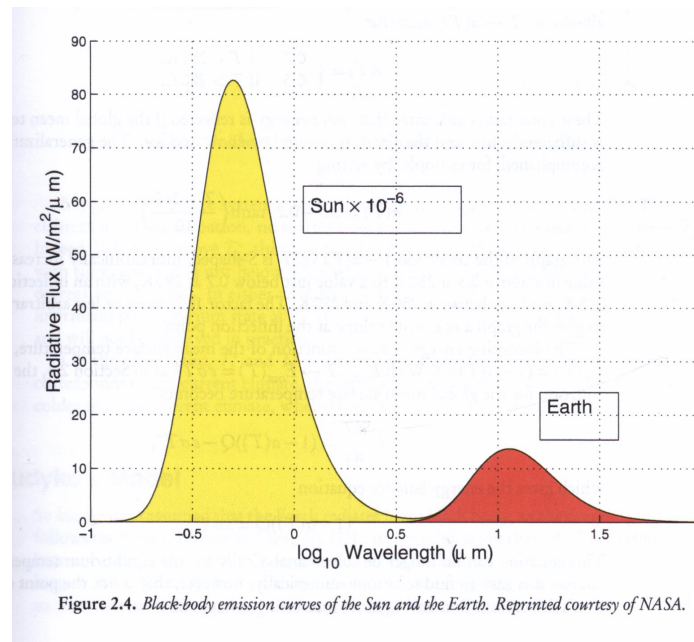
# How much energy radiates out into space from a ball at temperature T?

Earth radiates mostly in infrared spectrum, energy radiated depends on T  
(approximate as a “black body”)

$$E_{out} = \sigma T^4$$

$$\sigma = 5.67 * 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

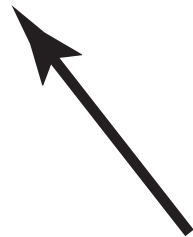
Stefan - Boltzmann constant



# Energy Balance Model

$$C \frac{dT}{dt} = E_{in} - E_{out}$$

*C* = heat capacity  
= energy needed to  
raise temperature  
by one Kelvin

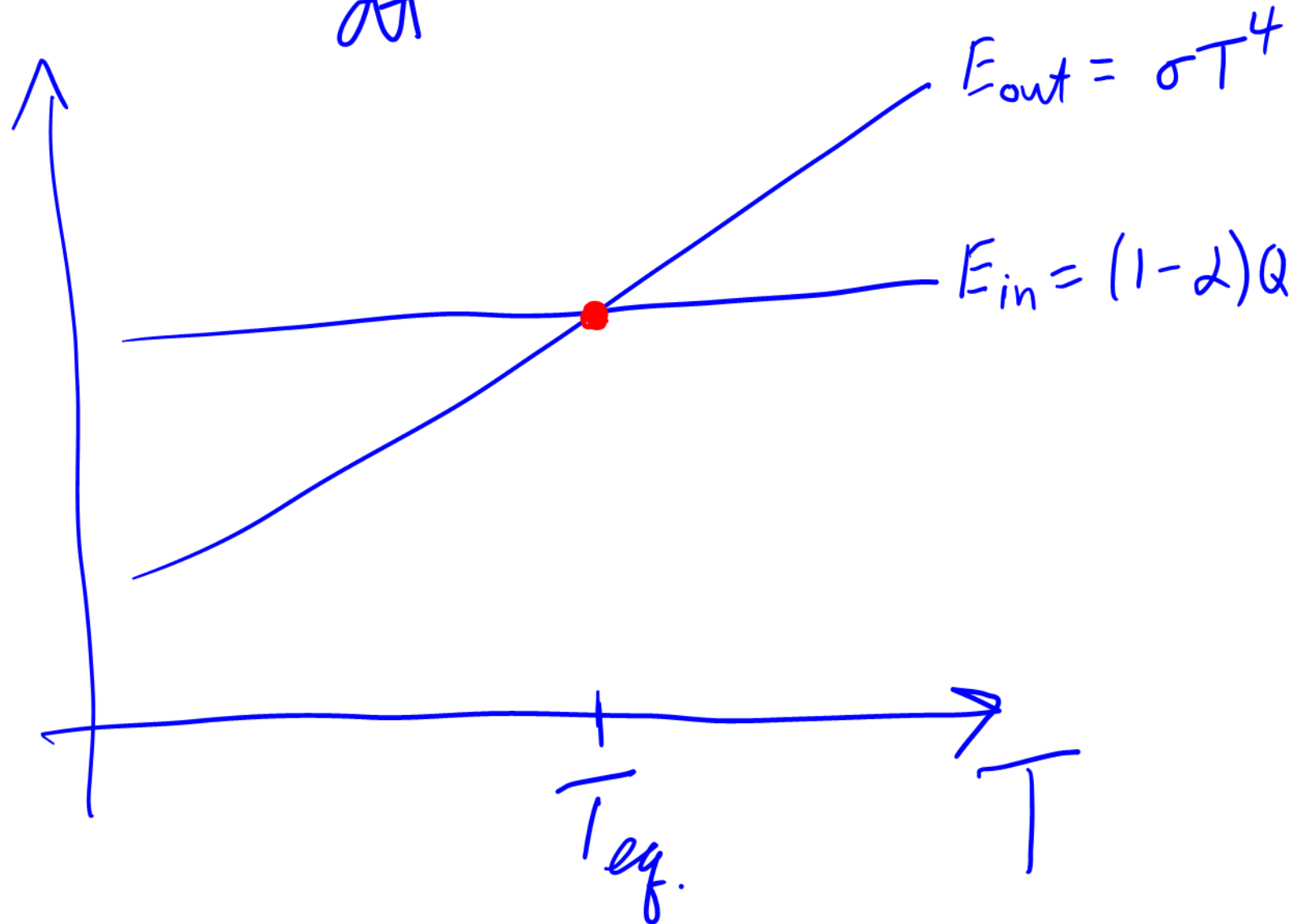


***rate of change of  
mean global temperature***

$$C \frac{dT}{dt} = (1 - \alpha)Q - \sigma T^4$$

$$C \frac{dT}{dt} = E_{in} - E_{out}$$

At equilibrium  $\frac{dT}{dt} = 0 \Rightarrow E_{in} = E_{out}$



Find steady-state **equilibrium** temperature:  $C \frac{dT}{dt} = E_{in} - E_{out}$

$$\frac{dT}{dt} = 0 \quad \text{constant temperature} \\ \text{rate of change} = 0$$

$$E_{in} = E_{out}$$

solve  $\sigma T^4 = (1 - \alpha)Q$

$$T_{eq} = \left( \frac{(1 - \alpha) Q}{\sigma} \right)^{1/4}$$

$$T_{eq} \approx 254.8 \text{ K} = -1.03^\circ \text{F}, \text{ which is quite cold!}$$

The actual measured temperature of the surface of the Earth is  $287.89 \text{ K} = 58.5^\circ \text{F}$

# Basics of global warming date back to the 1800's



1. Jean Baptiste Joseph Fourier (1768-1830), French mathematician and natural philosopher, did groundbreaking work in mathematics and the theory of heat. He was the first to propose that the Earth's atmosphere acts to raise the planet's temperature.

“As a dam built across a river causes a local deepening of the stream, so our atmosphere, thrown as a barrier across the terrestrial rays, produces a local heightening of the temperature at the Earth’s surface.”

**HEAT EQUATION  
FOURIER SERIES**

atmosphere traps **heat** escaping  
from Earth - acts like a **greenhouse**

**Fourier, 1827**



2. 1861 John Tyndall discovers in his laboratory that certain gases - water vapor and CO<sub>2</sub> are opaque to heat rays. He understood that such **gases high in the air help keep our planet warm by interfering with escaping radiation.**

1856 Eunice Newton Foote gives paper at AAAS conference on *her* experiments

3. 1896 Svante Arrhenius (**1903 Nobel Prize in Chemistry**) proposed:  
relationship between atmospheric CO<sub>2</sub> concentrations and temperature;  
**global warming may result from fossil fuel combustion (burning coal).**

He and Thomas Chamberlin calculated that human activities could warm the earth by adding carbon dioxide to the atmosphere.

*In a nutshell:*

**Scientific basis of CO<sub>2</sub> greenhouse effect was given by Tyndall (1861); while the first extensive calculations of its magnitude were made by Arrhenius (1896).**

# Pretty good .... but what are we missing?

## greenhouse effect

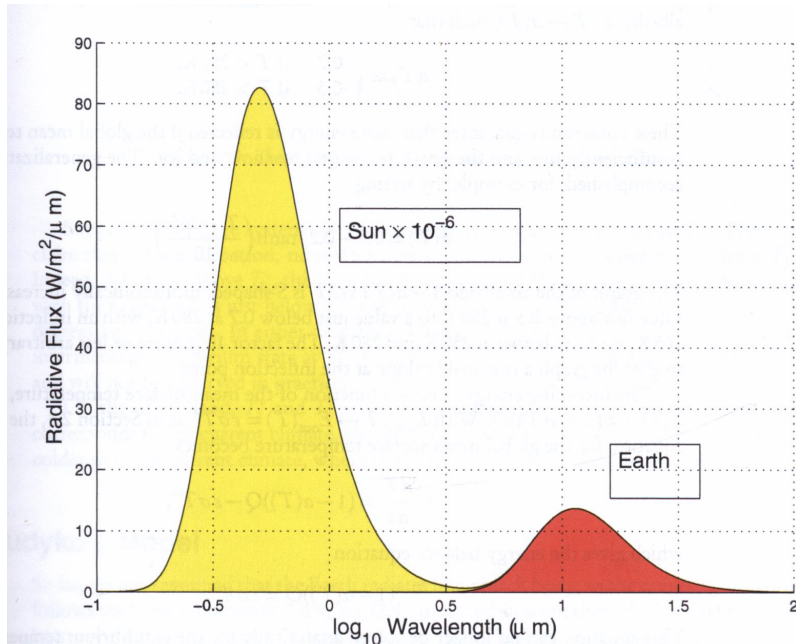
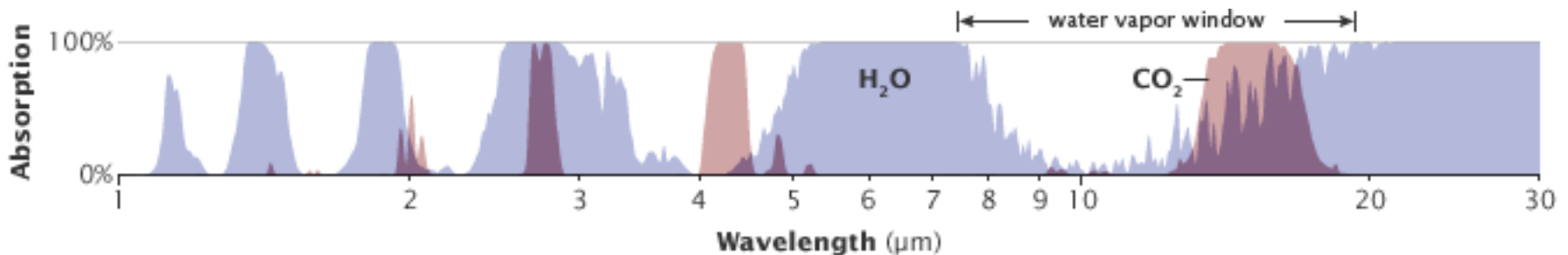
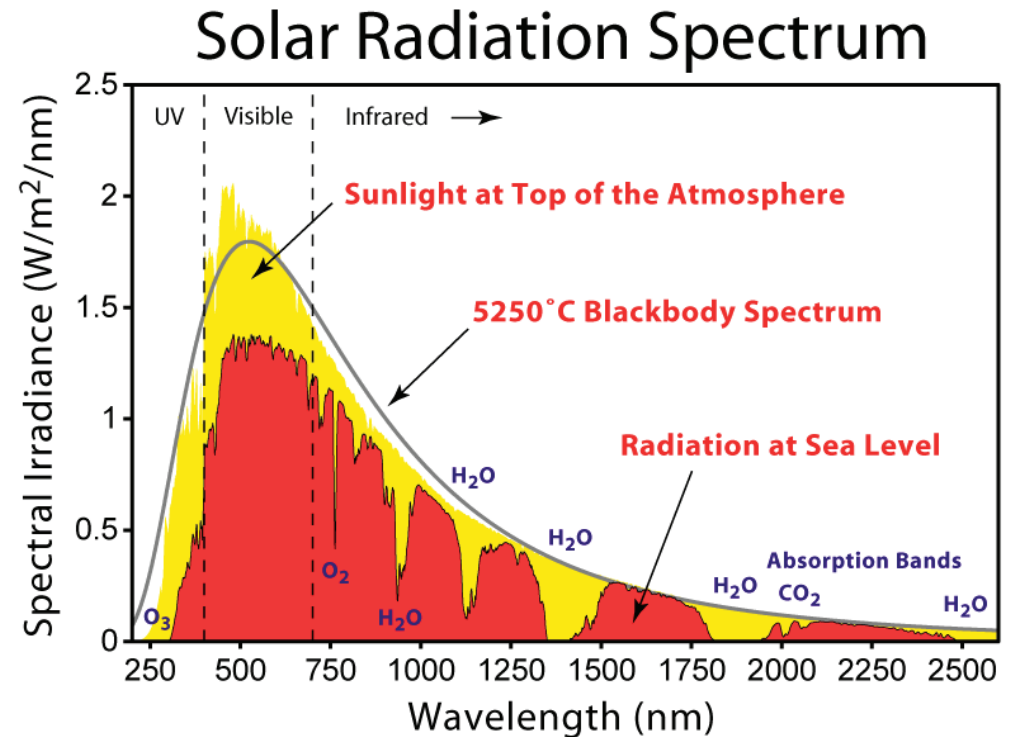


Figure 2.4. Black-body emission curves of the Sun and the Earth. Reprinted courtesy of NASA.

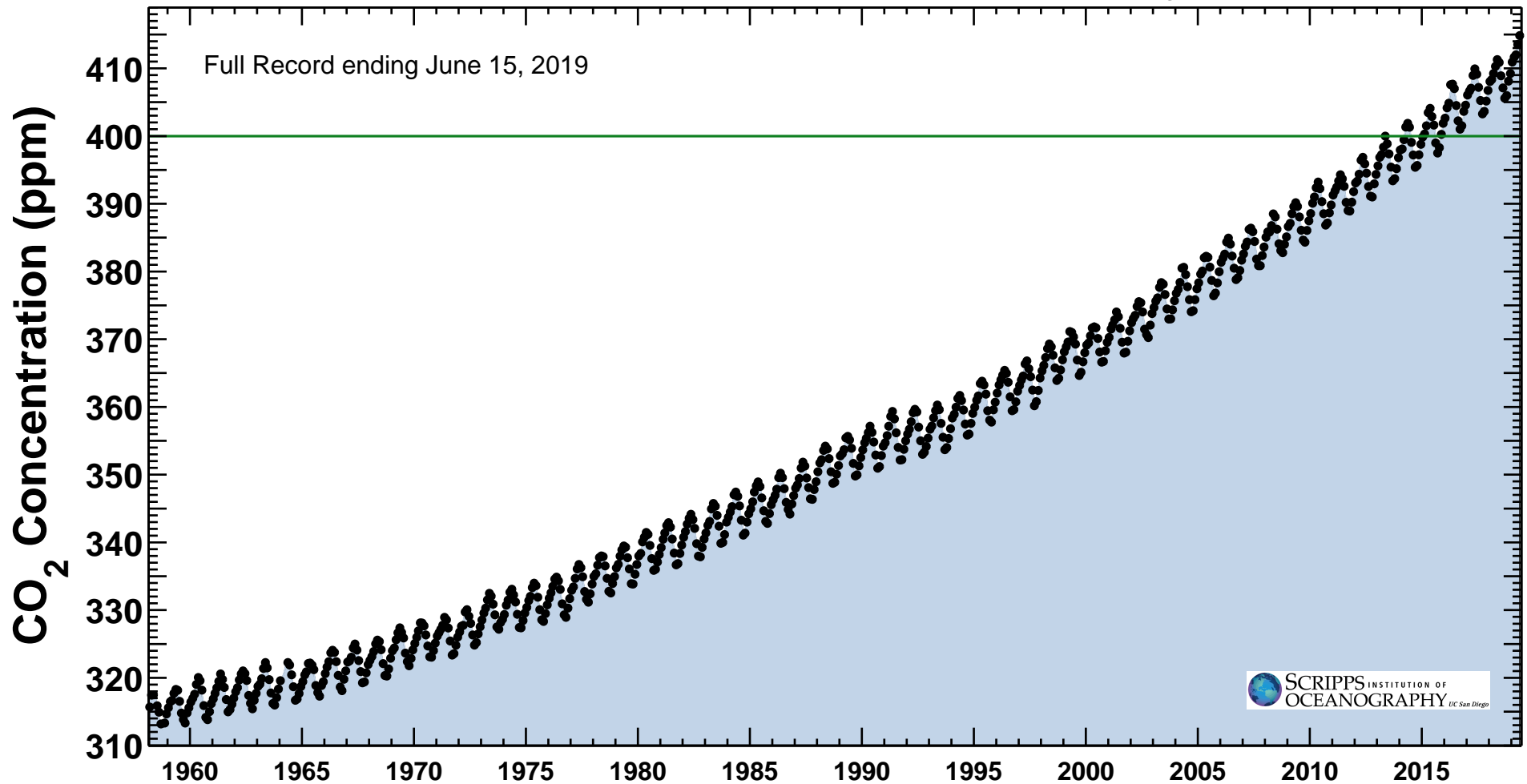


Earth's emission spectrum in far infrared > 5 μm

Latest CO<sub>2</sub> reading  
June 13, 2019

414.71 ppm

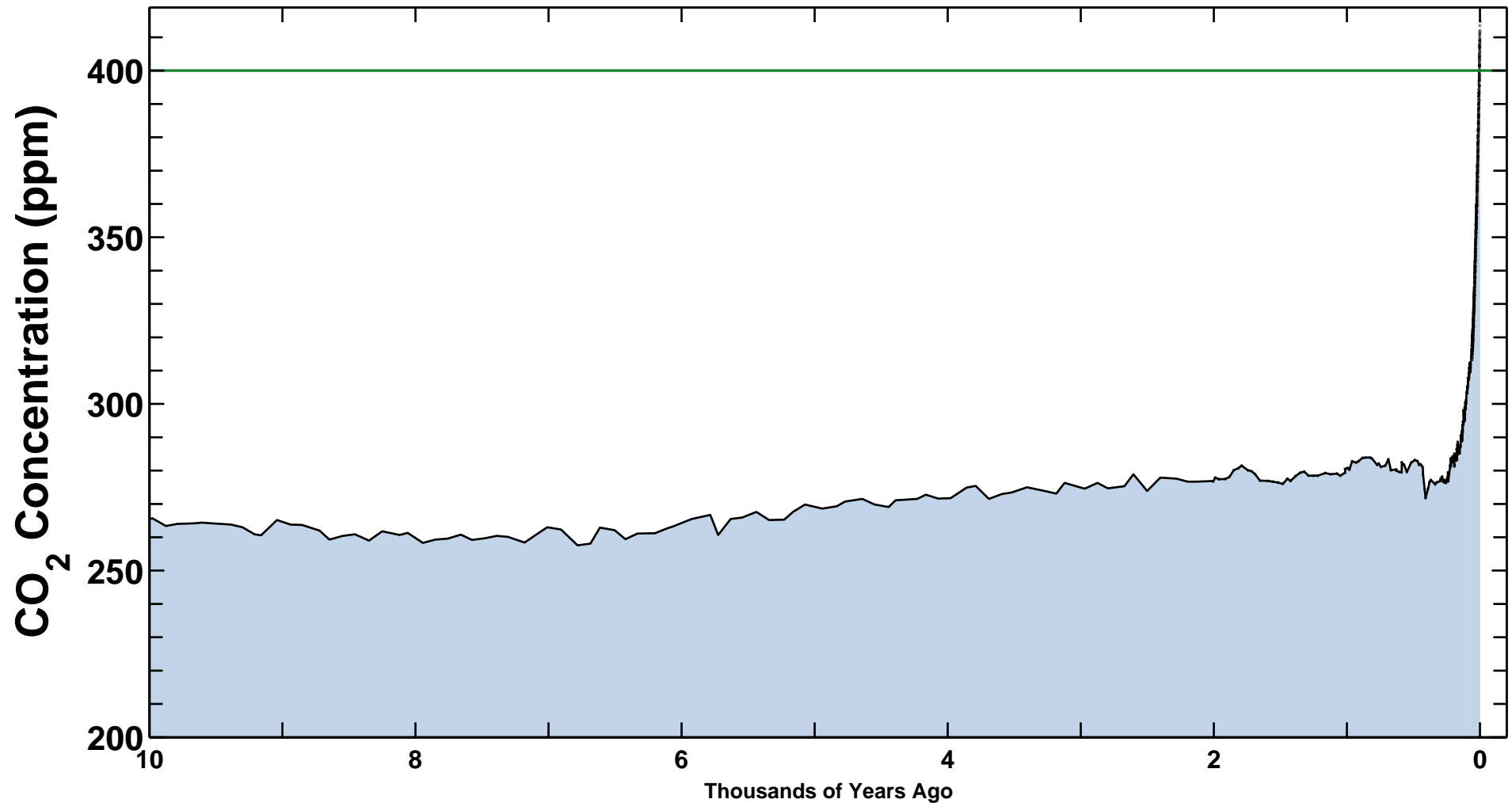
## Carbon dioxide concentration at Mauna Loa Observatory



Latest CO<sub>2</sub> reading  
June 13, 2019

414.71 ppm

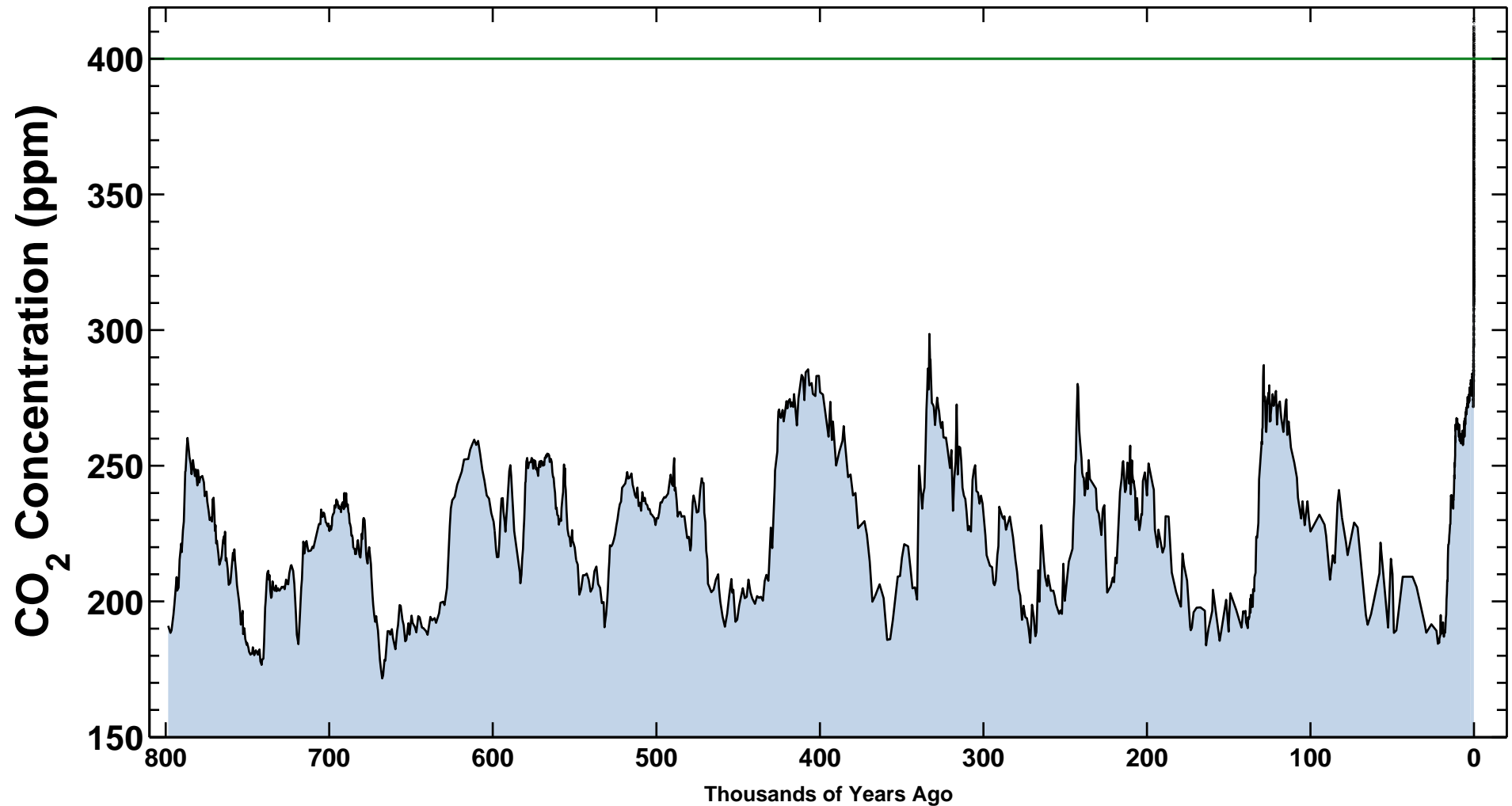
Ice-core data before 1958. Mauna Loa data after 1958.



Latest CO<sub>2</sub> reading  
June 13, 2019

414.71 ppm

Ice-core data before 1958. Mauna Loa data after 1958.



# Parameterize the Greenhouse Effect

reduce how much energy  
can leave the Earth system

$$E_{out} \rightarrow \varepsilon E_{out} \quad 0 < \varepsilon < 1$$

$$E_{in} = E_{out} \Rightarrow (1 - \alpha) Q = \varepsilon \sigma T^4$$

Equilibrium

$$T_{eq} = \left( \frac{(1 - \alpha) Q}{\varepsilon \sigma} \right)^{1/4}$$

With  $\alpha = 0.30$ ,  $S_0 = 1,368 \text{ W/m}^2$

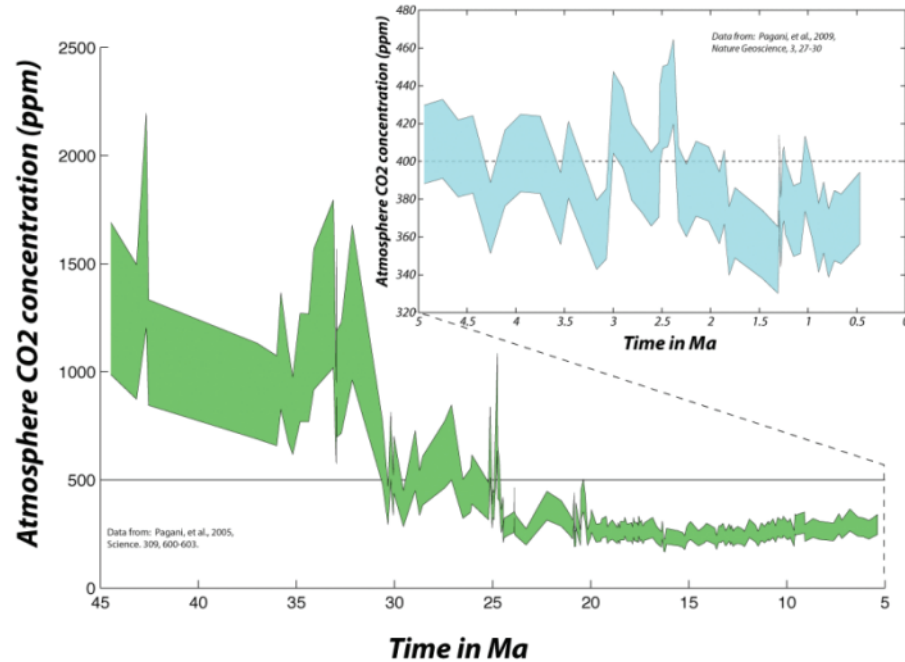
we get  $T_{eq} = 287.7 \text{ K}$   $58.2^\circ \text{ F}$

IF we adjust  $\varepsilon$  to be 0.62

So with 38% reduction in  $E_{out}$  we can  
obtain 2010 avg. global temp.



# When was the last time CO<sub>2</sub> was above 400 ppm for a sustained period?



**The PLIOCENE Era ~ 2.6 - 5.3 M years ago**

Giant camels roamed the forested high Arctic.

Sea levels were at least 30 feet higher.

Global temperatures were ~ 3°C higher.



The atmosphere warms (on geophysical time scales) much faster than the great land ice sheets can melt.

The atmosphere and ice sheets are out of equilibrium.

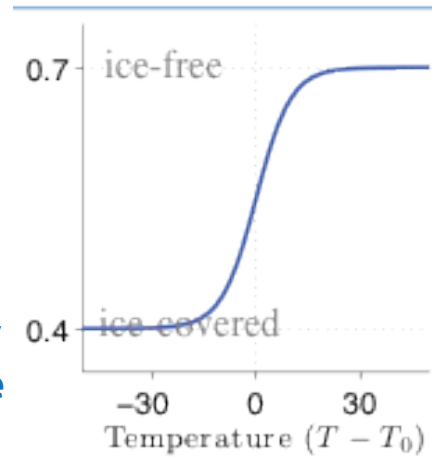
**We've just put a quart of ice cream in the oven.  
It's still frozen, but after a while it will melt!**

Assuming albedo independent of Earth's temperature too simplified....

planetary  
co-albedo

$$1 - \alpha$$

cold  
icy  
reflective



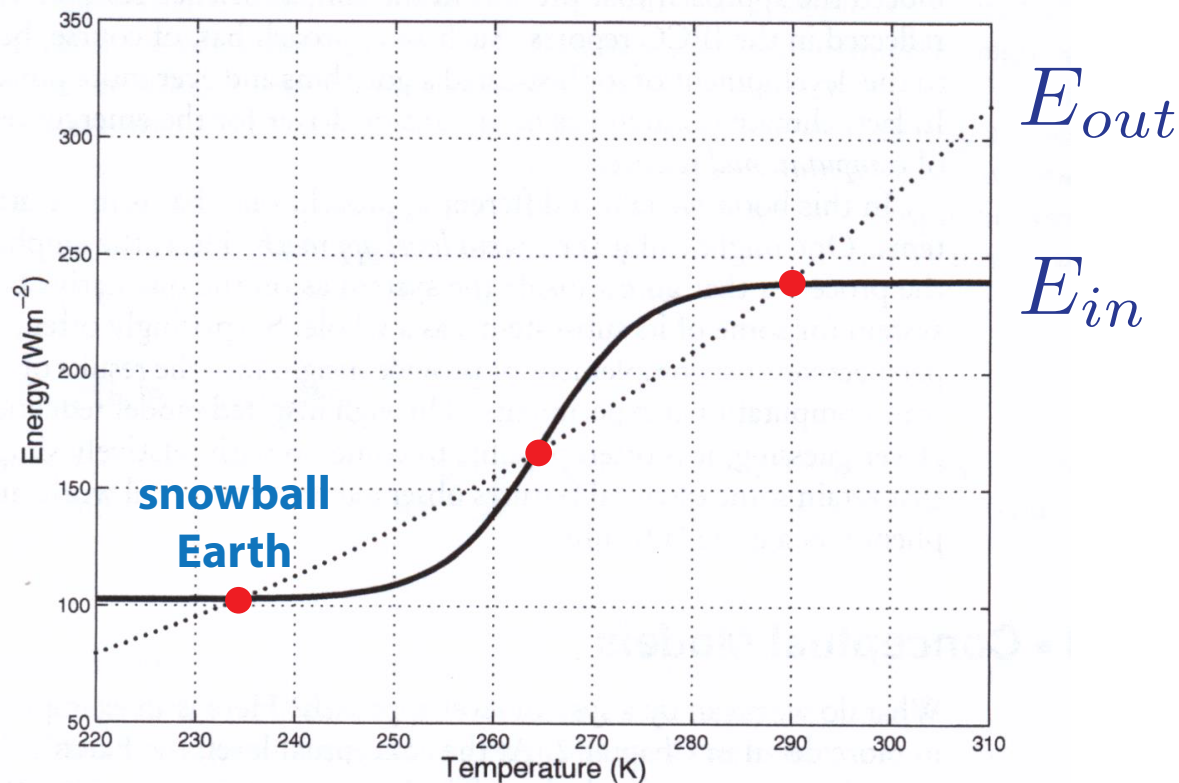
warm  
watery  
absorptive

albedo nonlinear  
in temperature

Now solve for equilibrium temperature  $E_{in} = E_{out}$

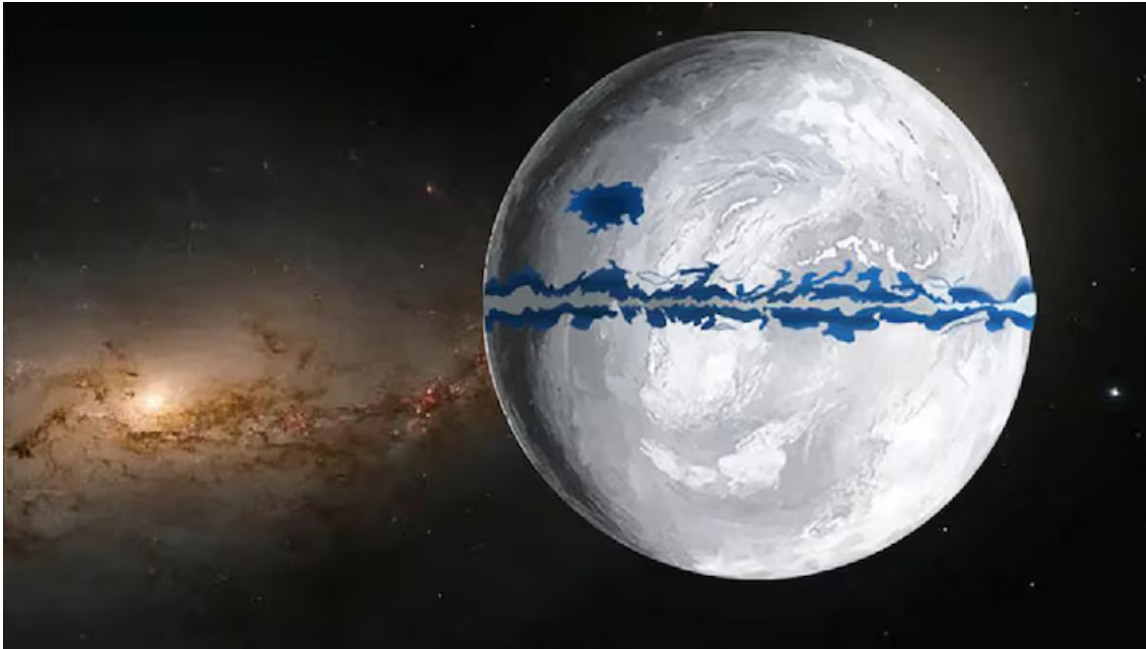
$$\sigma T^4 = (1 - \alpha(T))Q$$

**3 solutions!**  
**multiple equilibria**



**stable vs. unstable equilibria**

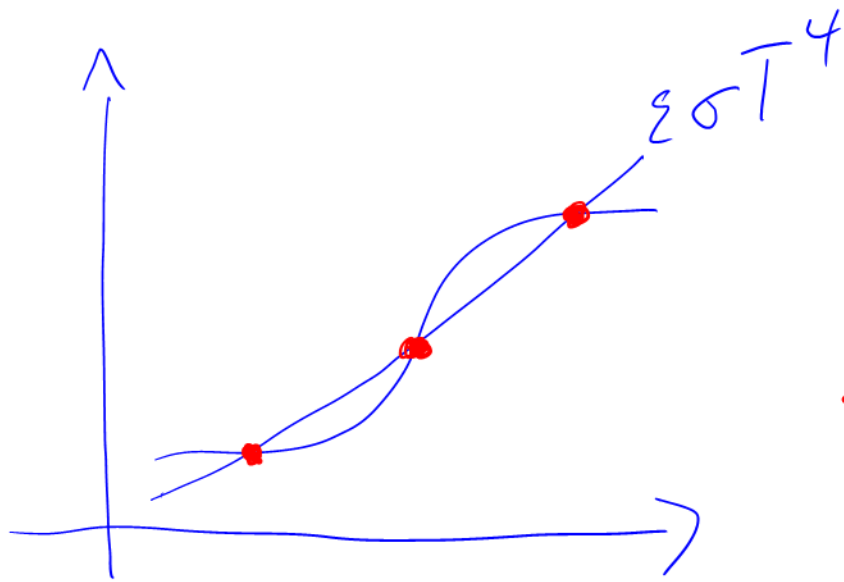
# Snowball Earth



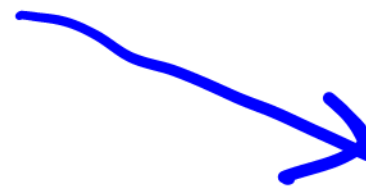
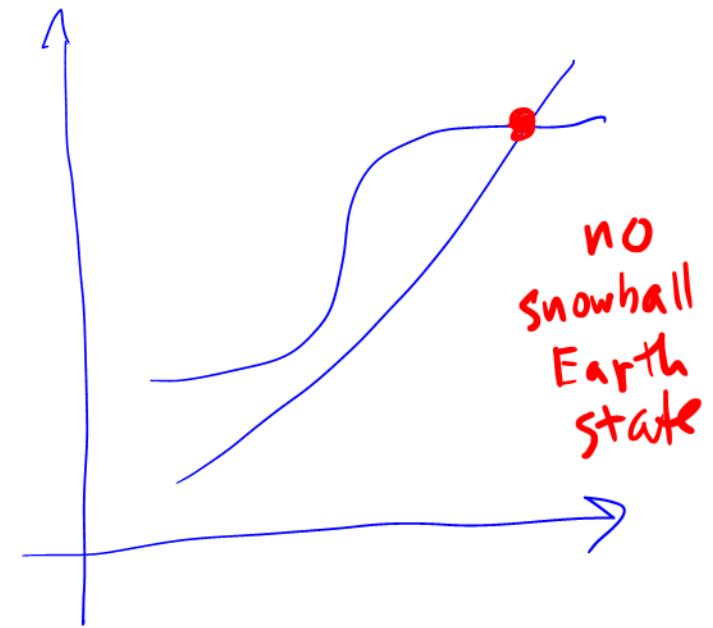
~ 4 times in stable snowball state  
during Neoproerozoic age  
750 - 580 M years ago  
~ 50 degrees colder  
“complete” glaciation of Earth

geological deposits found  
only during glaciation found  
in tropical areas @ sea level

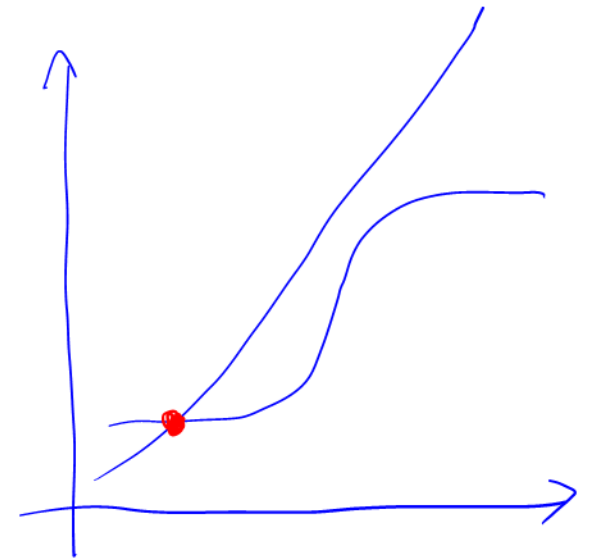
build-up of CO<sub>2</sub> from volcanoes  
normally checked by erosion of  
continents (rocks) but everything  
covered by ice! Until tipping point  
from HUGE greenhouse effect



decrease  $\epsilon$   
 $\uparrow$  greenhouse



decrease  $Q$   
 increase  $\epsilon$



$$\frac{dT}{dt} = (1 - \alpha(T))Q - \epsilon \sigma T^4$$

# Bifurcation Diagram

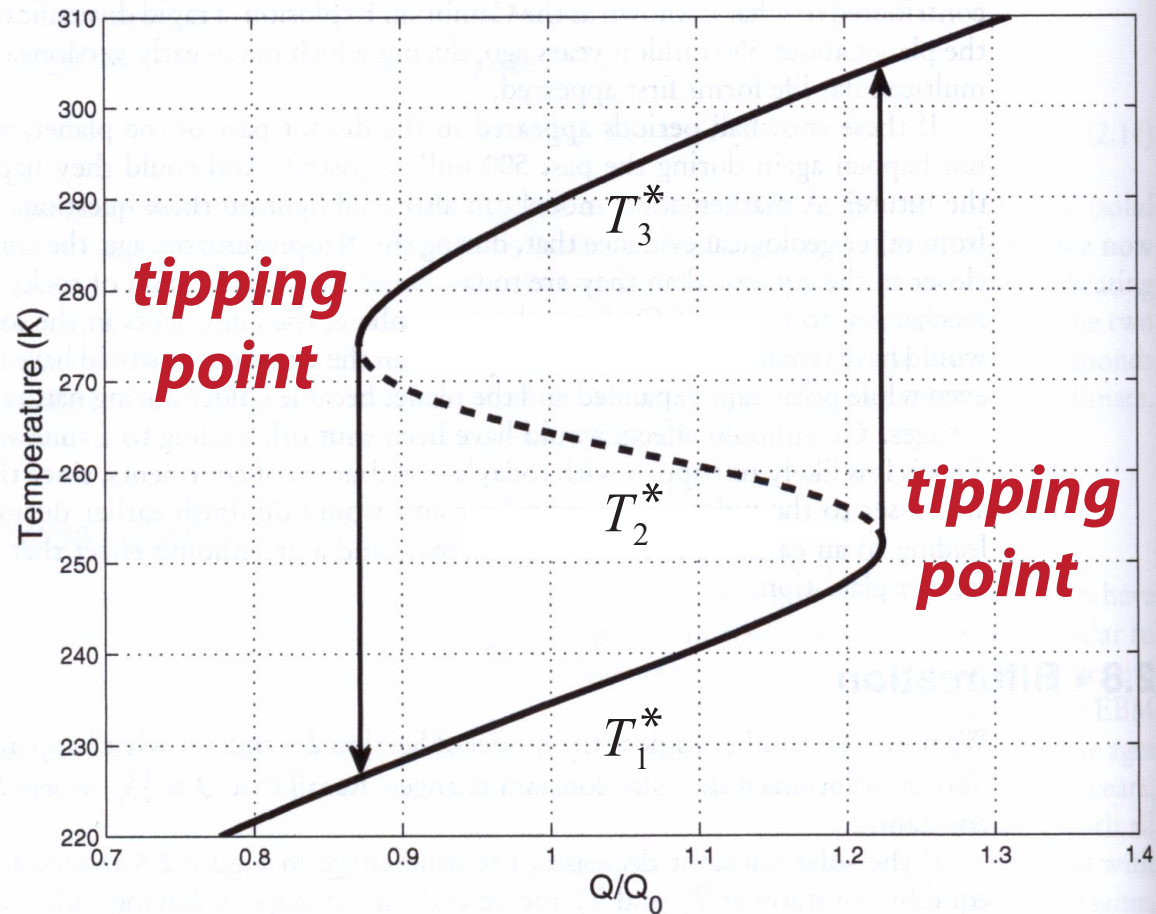
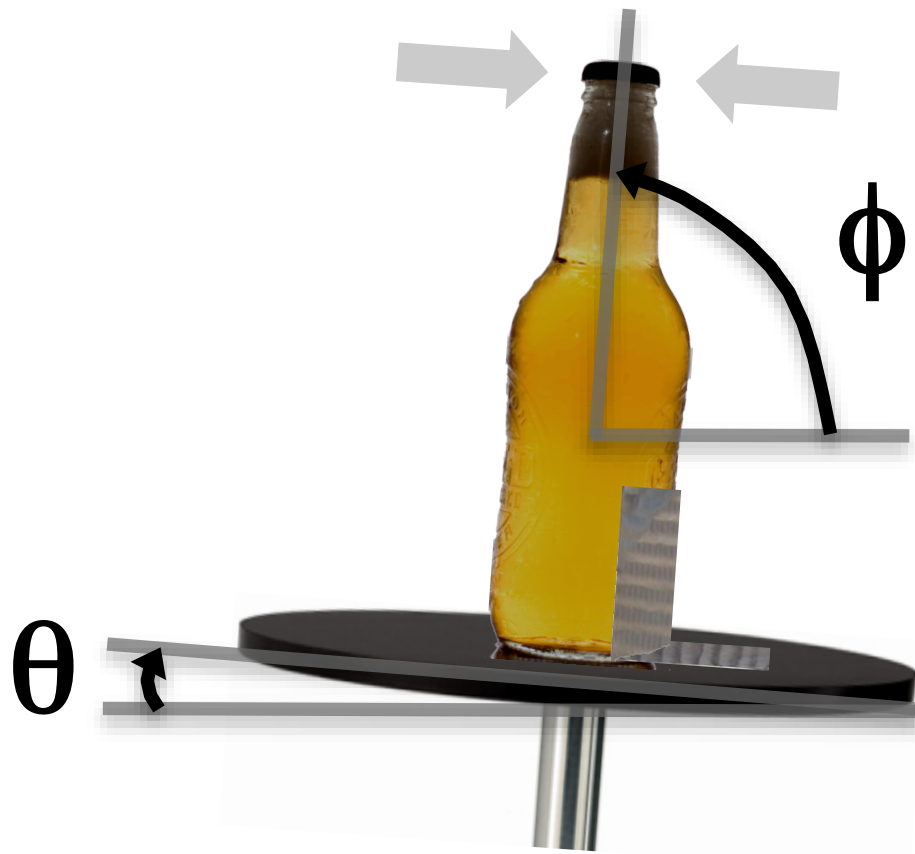


Figure 2.6. Mean surface temperatures at equilibrium as a function of the solar constant (in units of its present value).





***“tipping point”***

Tape a bottle to a tilted table, vary  $\theta$  and observe  $\phi$ .

Tilt the table until bottle tips -- irreversible process since bottle doesn't tip back until table tilted back very far.

***hysteresis***

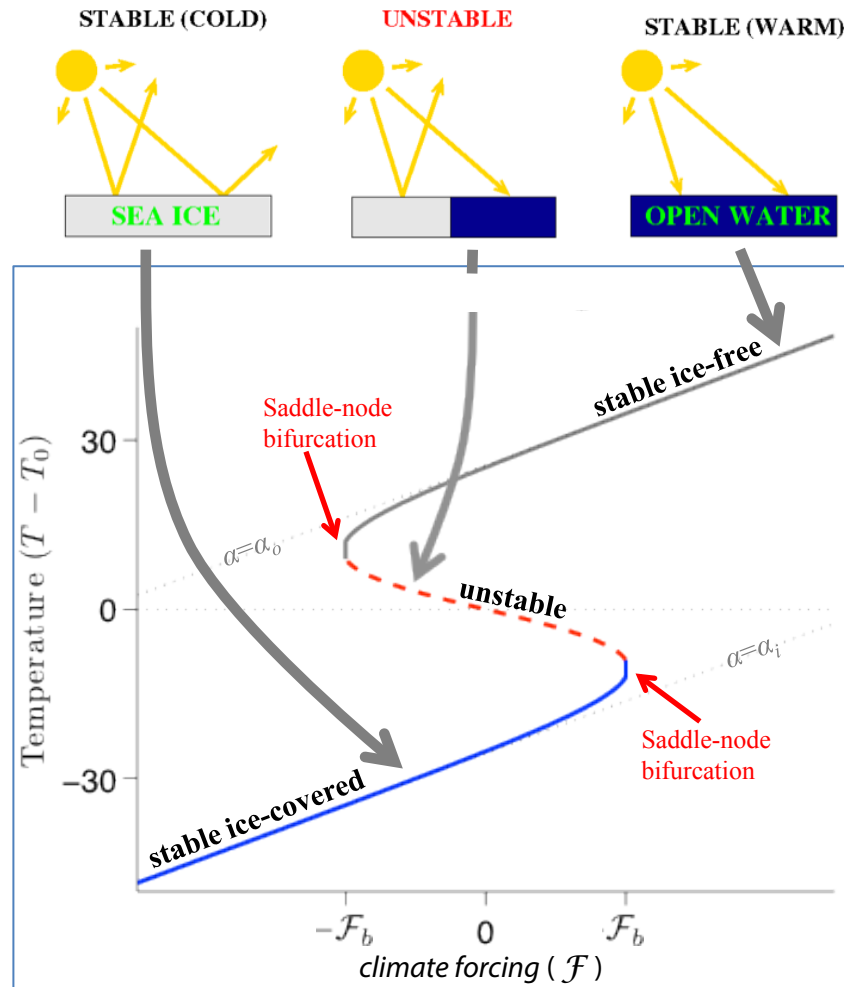
# cryospheric tipping point

*a tipping point exists if the ice does not recover from loss caused by climatic warming even if the climatic forcing returns to the colder conditions that existed before the loss*

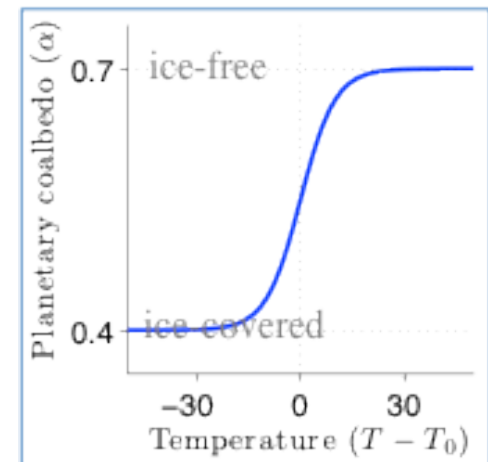
## hysteresis

# What would a tipping point in sea ice cover look like?

## *Bifurcation Diagram*



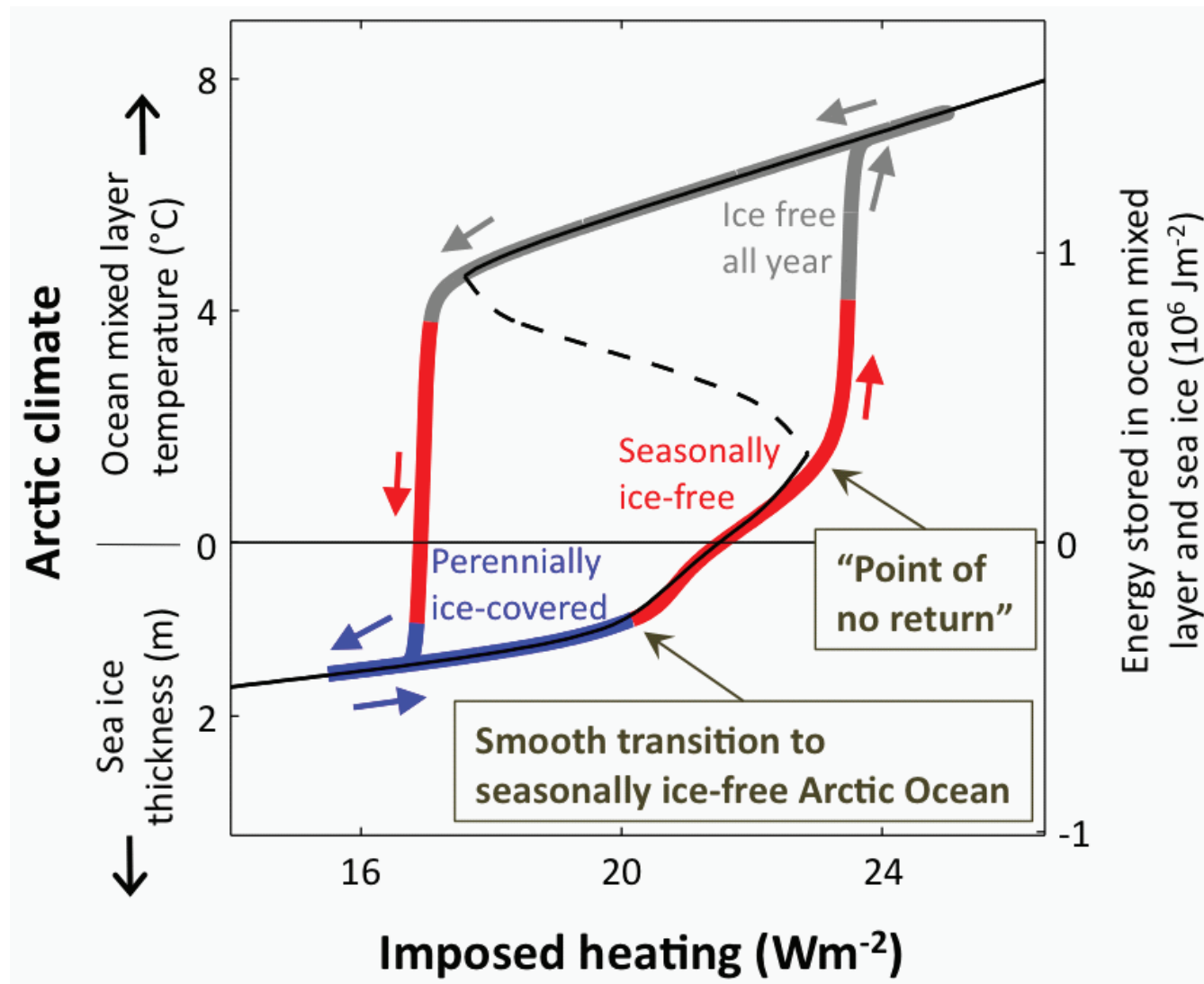
nonlinear  
ice-albedo  
feedback



loss of summer Arctic sea ice    *smooth transition*

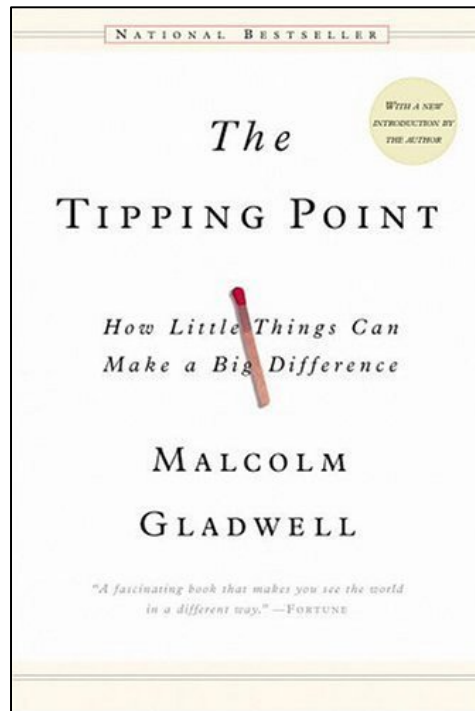
loss of winter Arctic sea ice    *critical threshold*

tipping point



# ***tipping points in the mainstream***

Increasing emphasis in recent years on idea of **climate tipping points**, with September Arctic sea ice cover receiving much of the attention.



**Melting of the Greenland ice sheet**

**Melting of the West Antarctic ice sheet**

**Permafrost and tundra loss, leading to the release of methane**

**Formation of Atlantic deep water near the Arctic ocean ●●●**

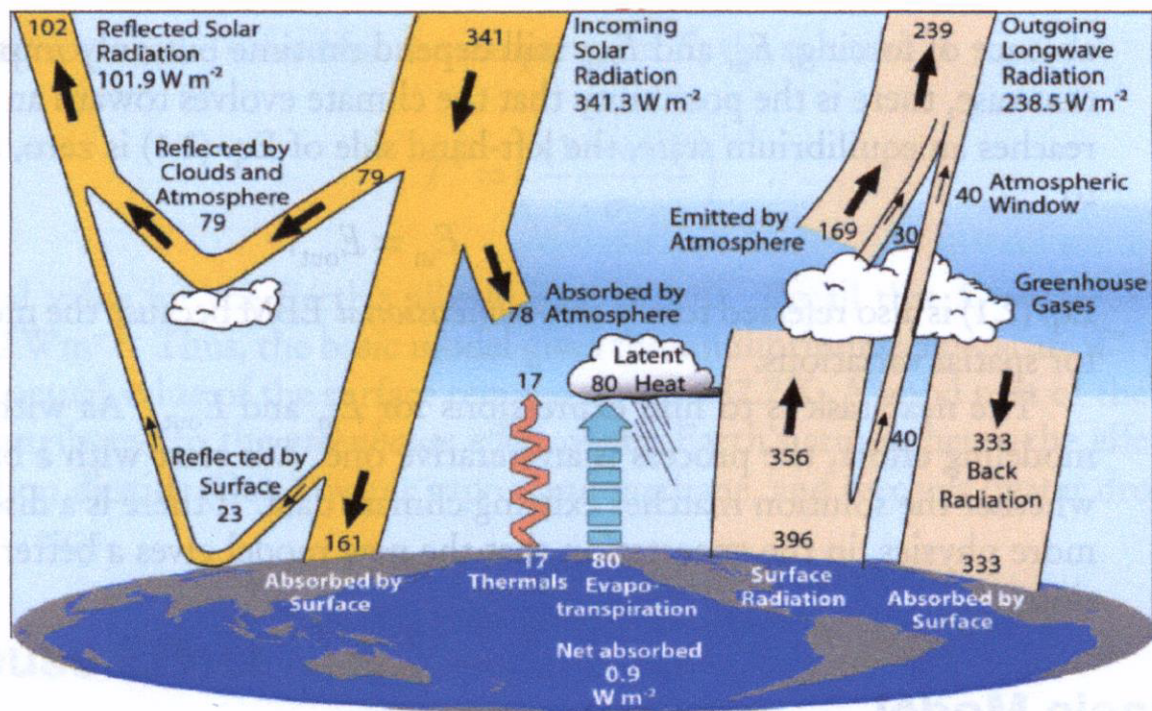


Figure 2.3. Detailed radiative energy balance [112].



# Global Climate Models

*climate fueled by the nonuniform spatial distribution of incoming solar radiation.*

*Stute et al., PNAS 2001*

**Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.**

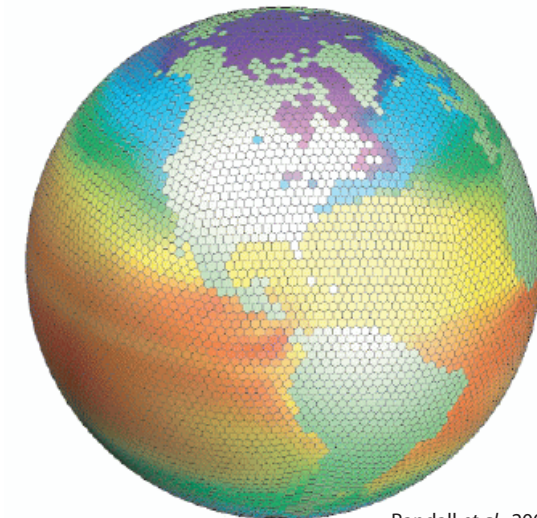
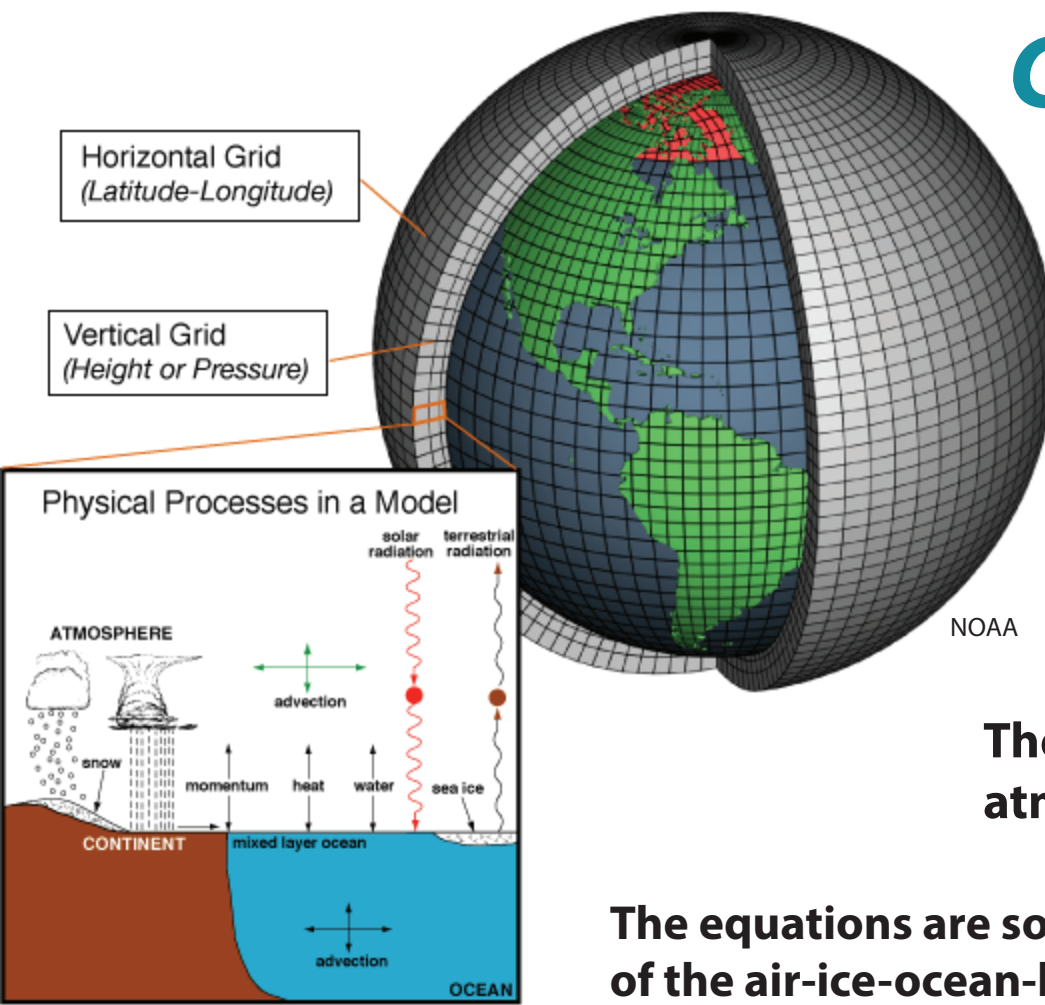
**They describe the state of the ocean, land, ice atmosphere, biosphere, and their interactions.**

**The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.**

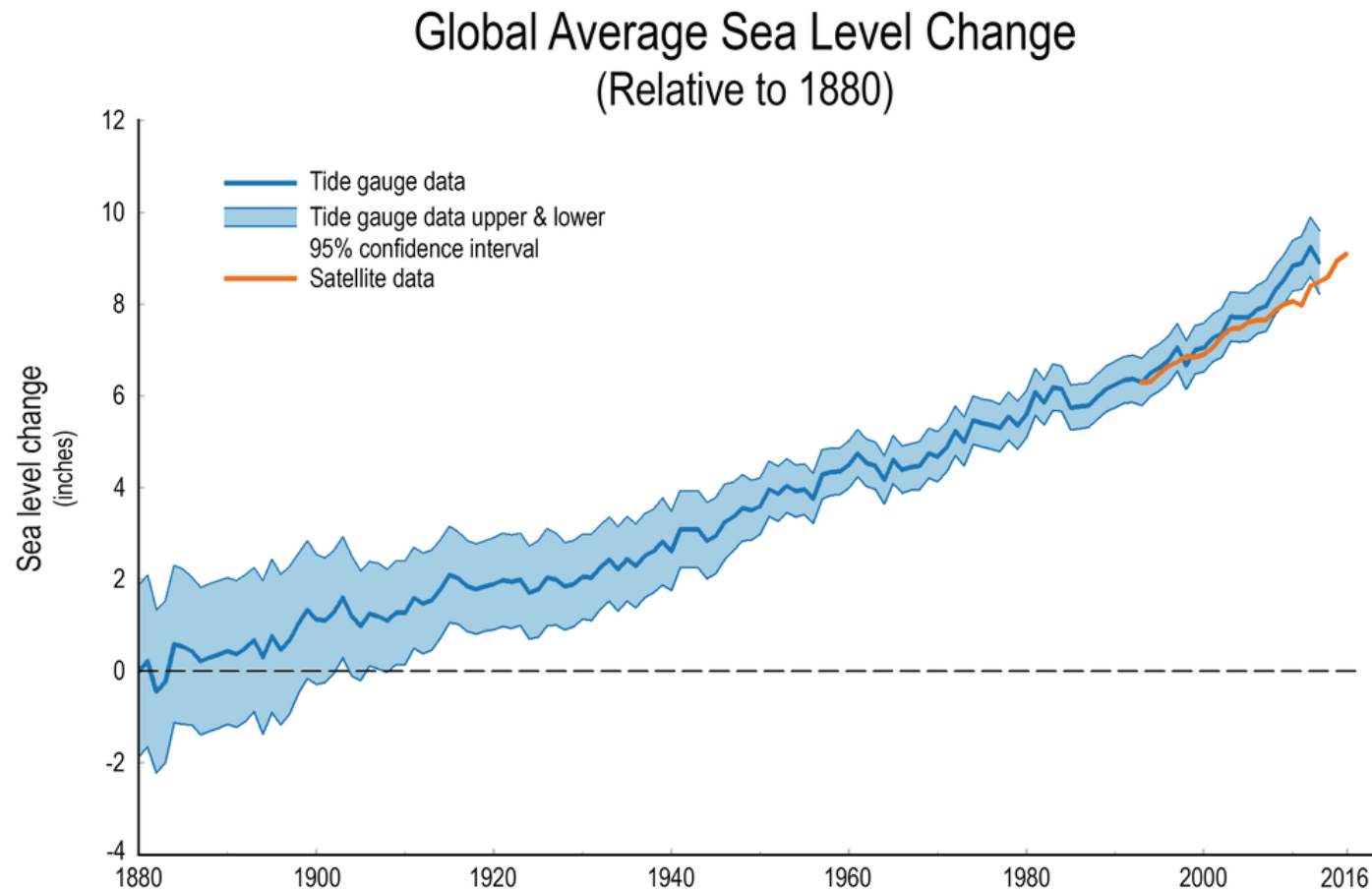
**key challenge :**

***incorporating sub - grid scale processes***

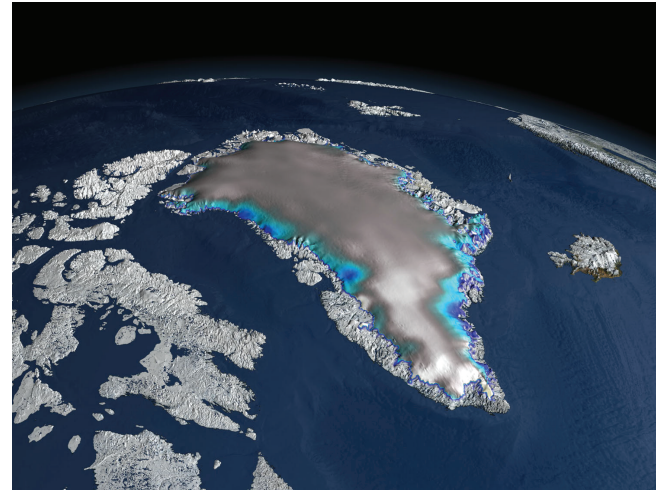
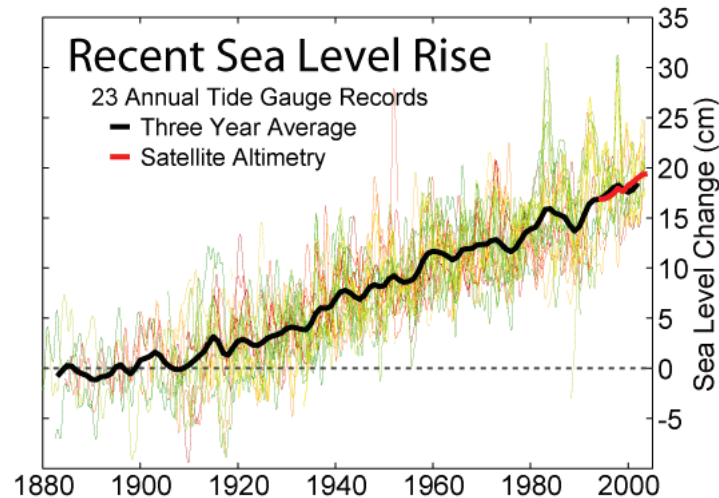
**linking scales**



# Why is sea level rising?



# *As Earth's climate warms, why does sea level rise?*



- ***melting land ice: Antarctica, Greenland, mountain glaciers***  
(if all melted : 70 m or ~230 ft of sea level rise)

- ***thermal expansion of warming ocean***

*accounts for about 25% of rise in last half of 20th century, rate ~ tripled in 21st century*

- ***continental rebound***

*rise of land masses that were depressed by the huge weight of ice sheets*



***The interaction of warm waters with the periphery of the large ice sheets represents one of the most significant possibilities for abrupt change in the climate system.***

***no coupling yet of ice sheets and ocean in climate models - no feedback effects***





