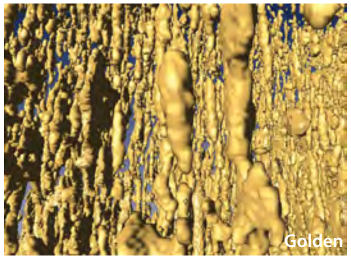
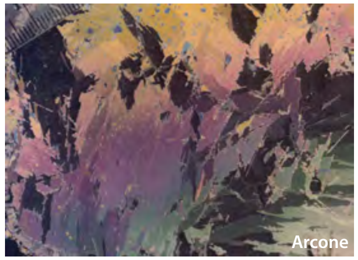
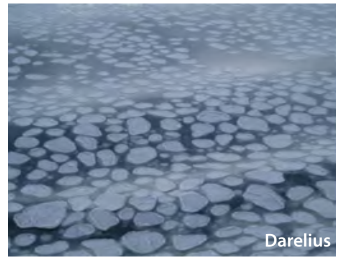
millimeters



centimeters



meters



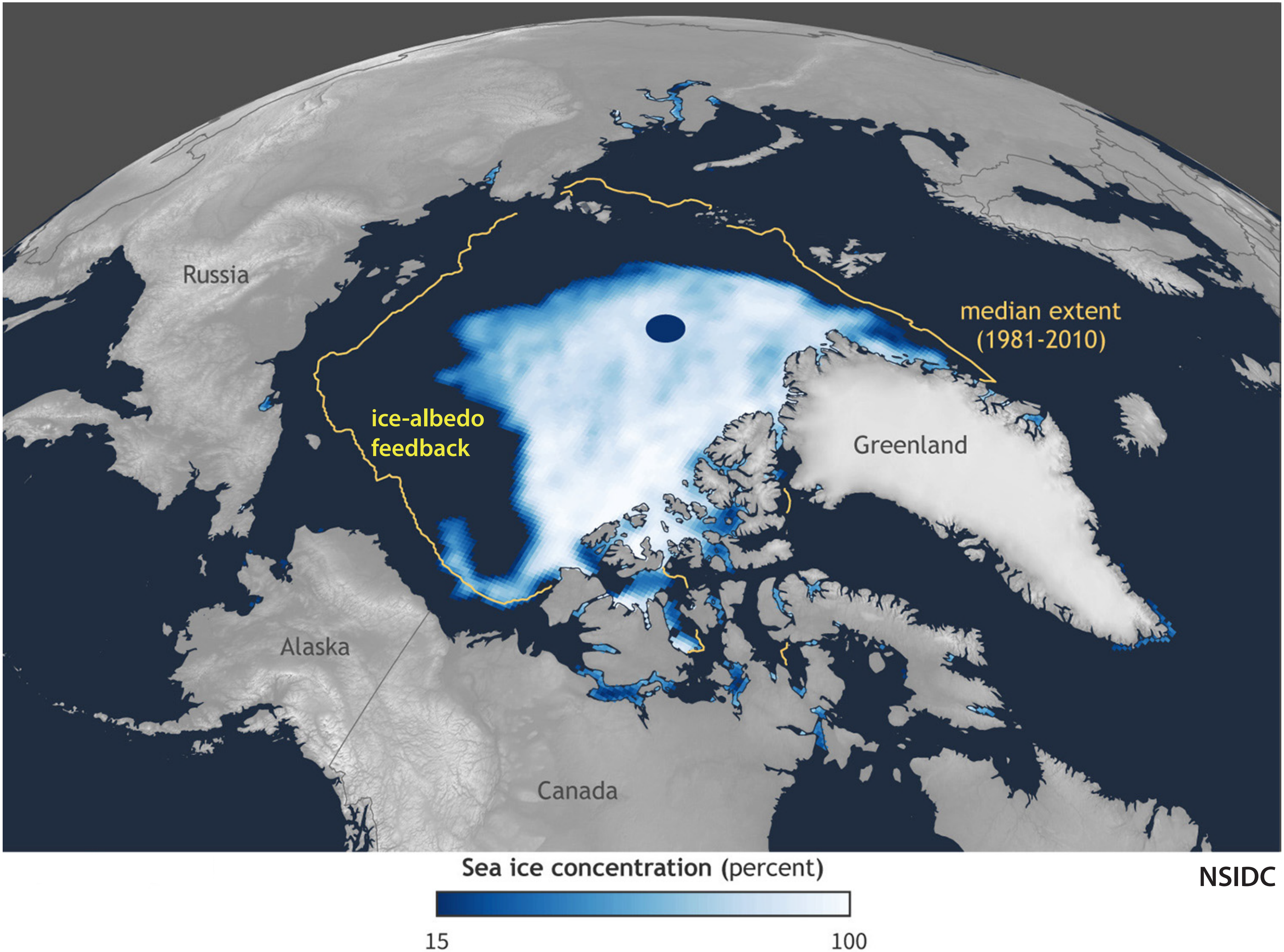
kilometers

10^3 kilometers

From Micro to Macro in the Physics and Ecology of Sea Ice

Ken Golden, University of Utah

Arctic sea ice extent September 15, 2020

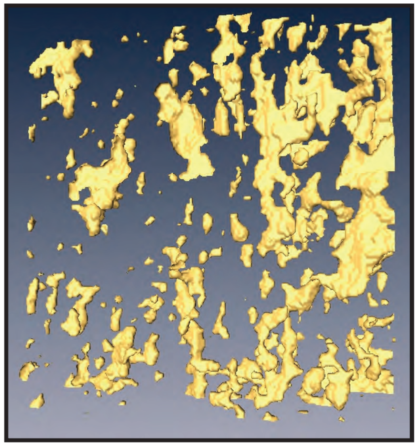


Sea Ice is a Multiscale Composite Material

microscale

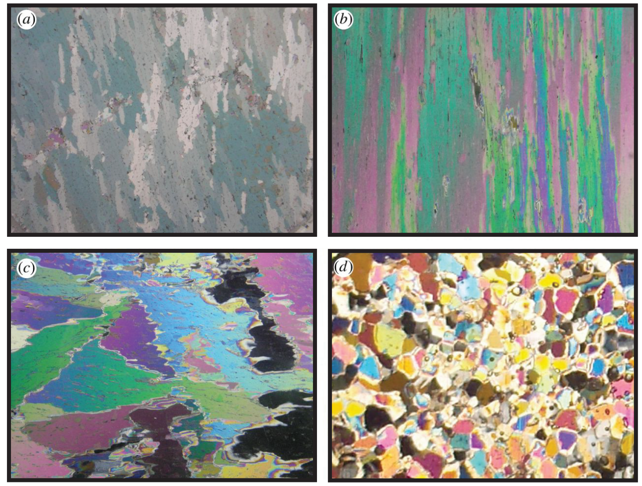
brine inclusions

Weeks & Assur 1969



H. Eicken
Golden *et al. GRL* 2007

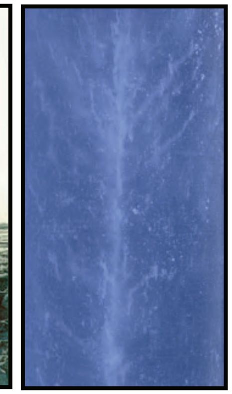
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters **centimeters**

---

mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges

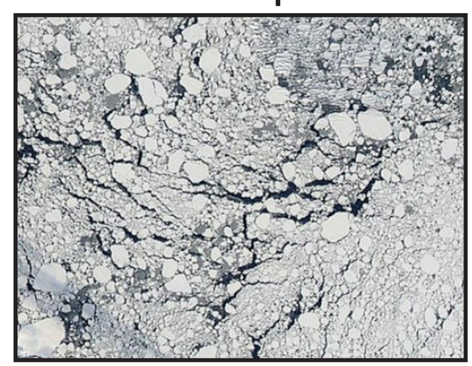
K. Golden

macroscale

sea ice floes

J. Weller

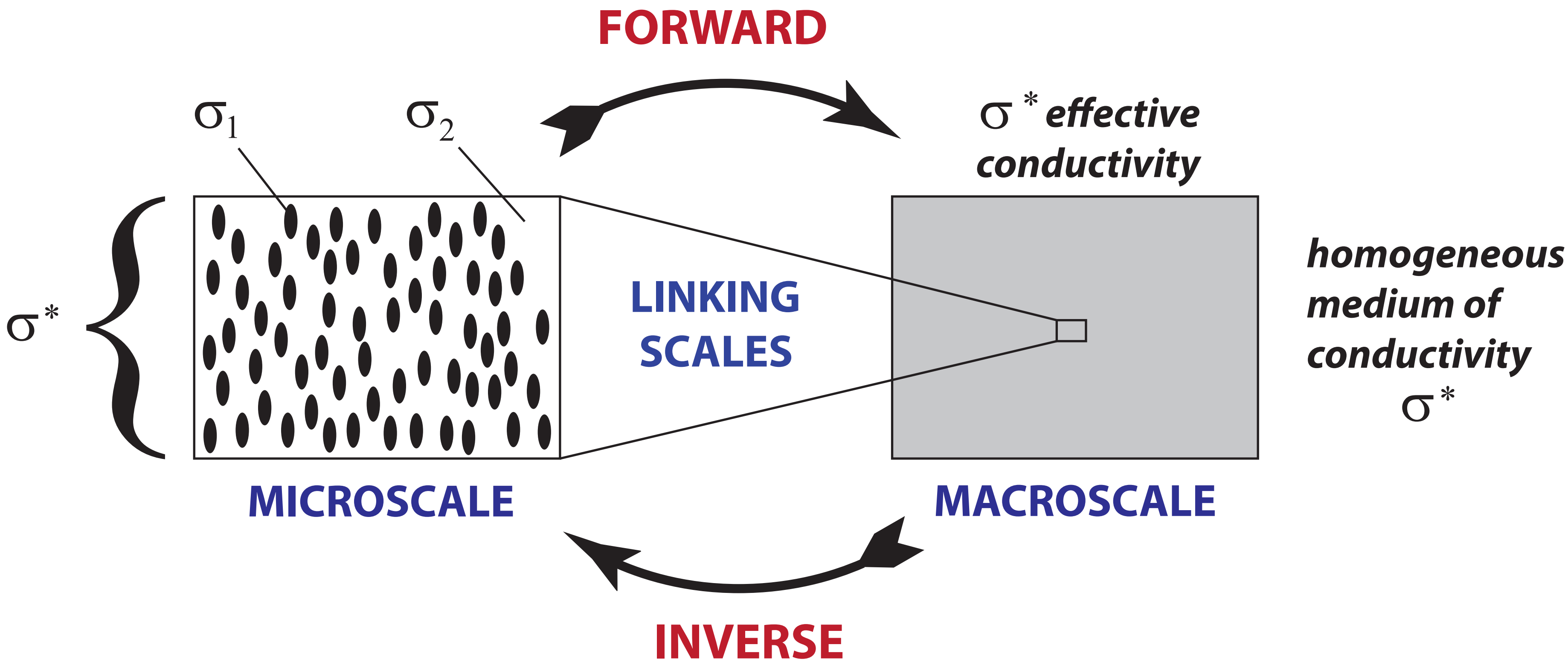
sea ice pack



NASA

meters **kilometers**

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about?

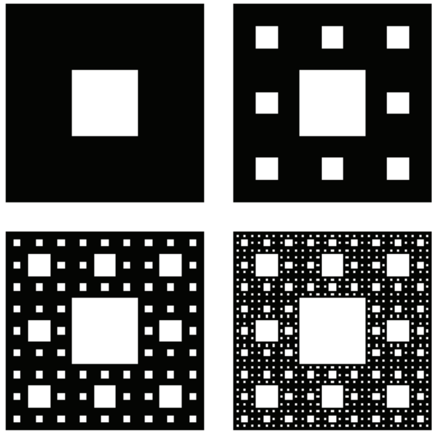
A brief tour of recent results on multiscale modeling of physical and ecological processes in the sea ice system.

microscale

mesoscale

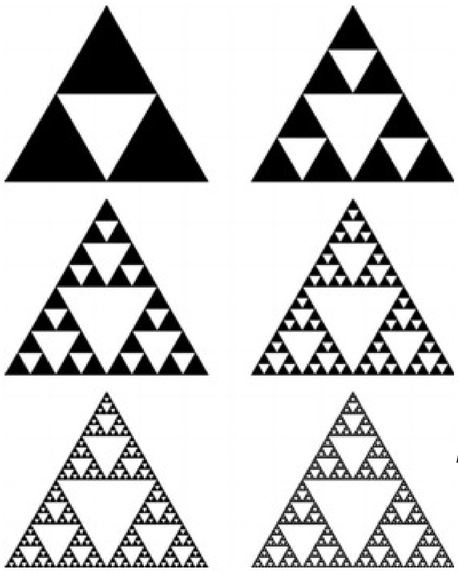
macroscale

through the lens of fractal geometry

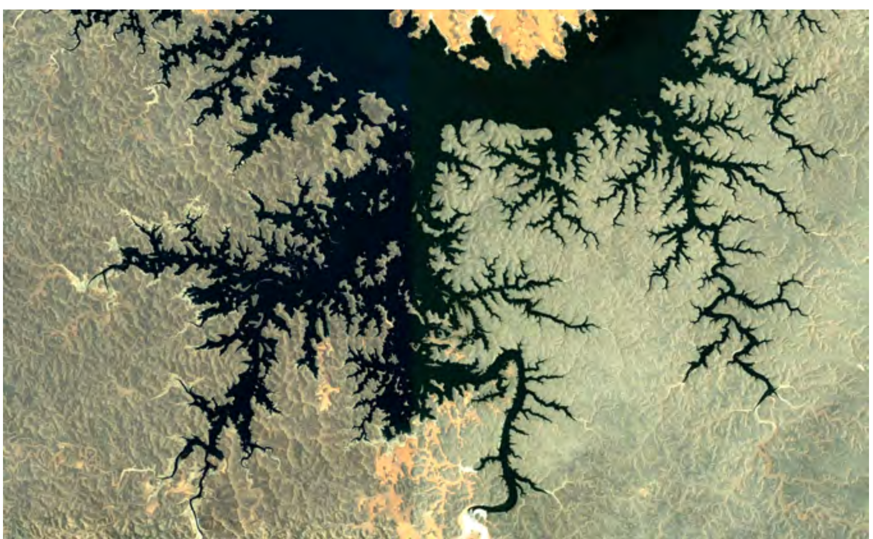


fractals

self-similar structure
non-integer dimension

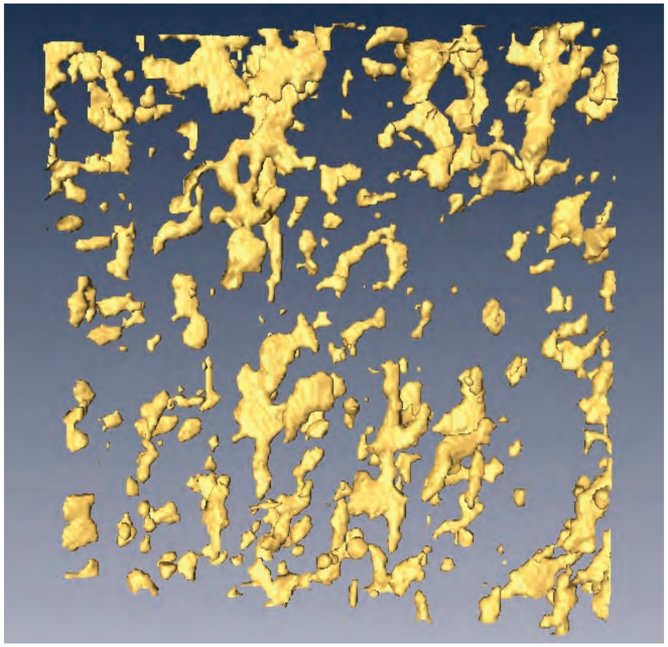


$$D = \frac{\log 3}{\log 2} = 1.585...$$

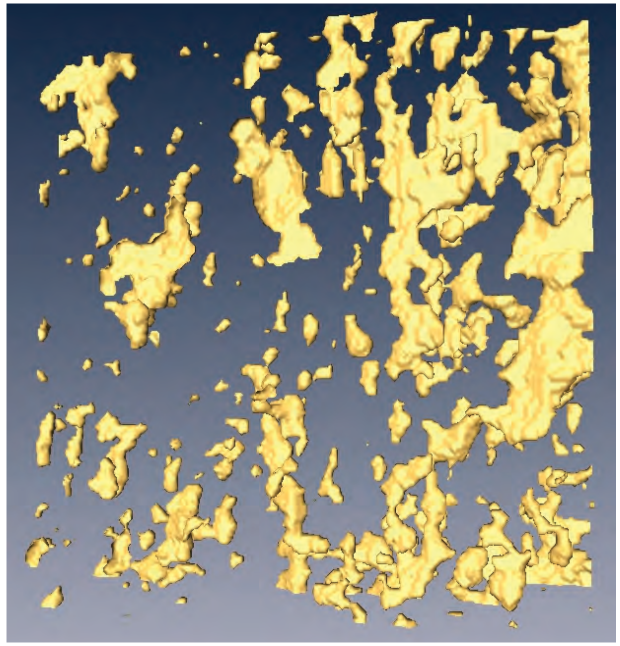


microscale

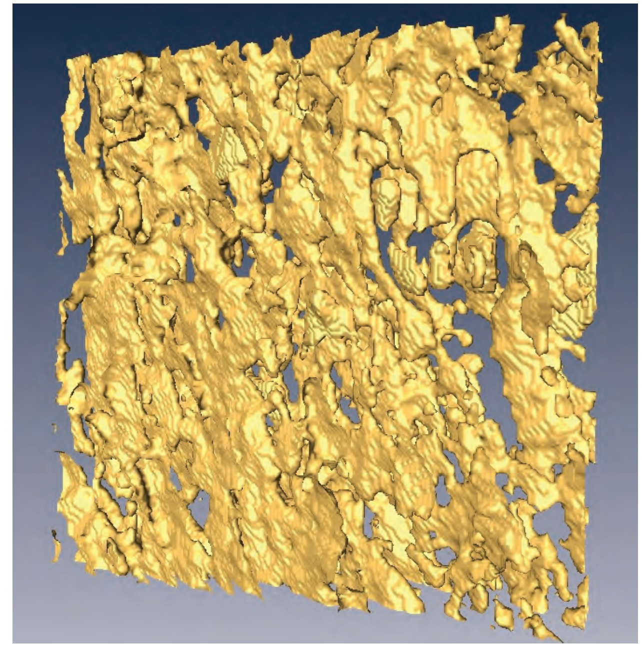
brine volume fraction and ***connectivity*** increase with temperature



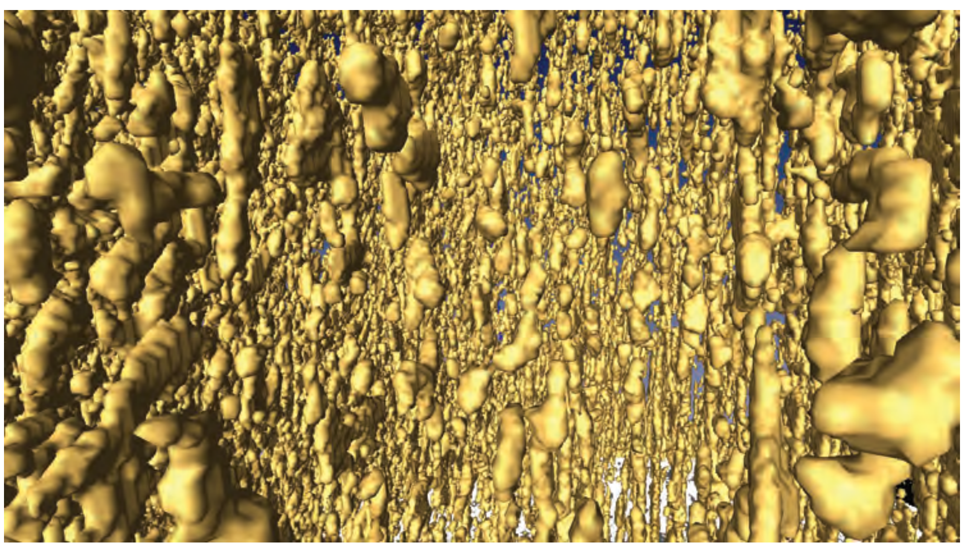
T = -15 °C, $\phi = 0.033$



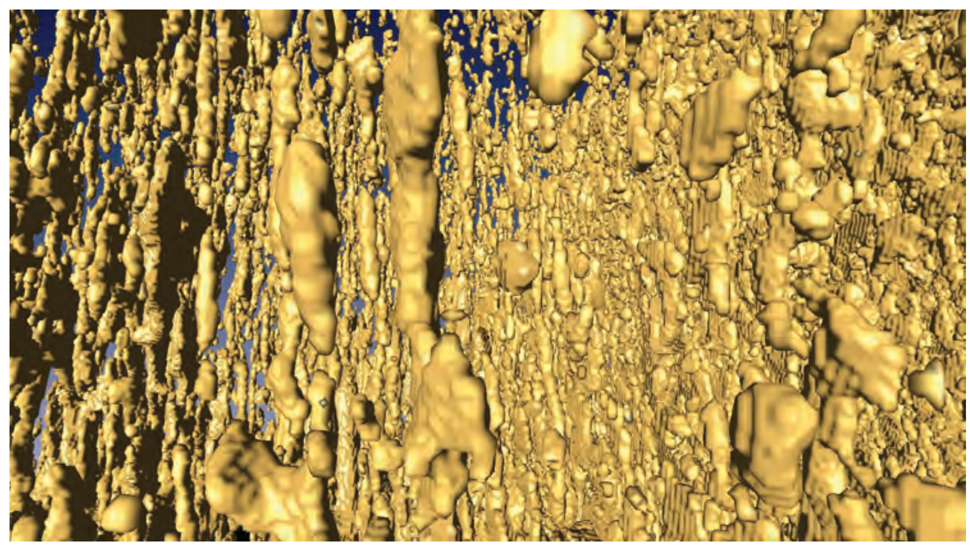
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8° C, $\phi = 0.057$



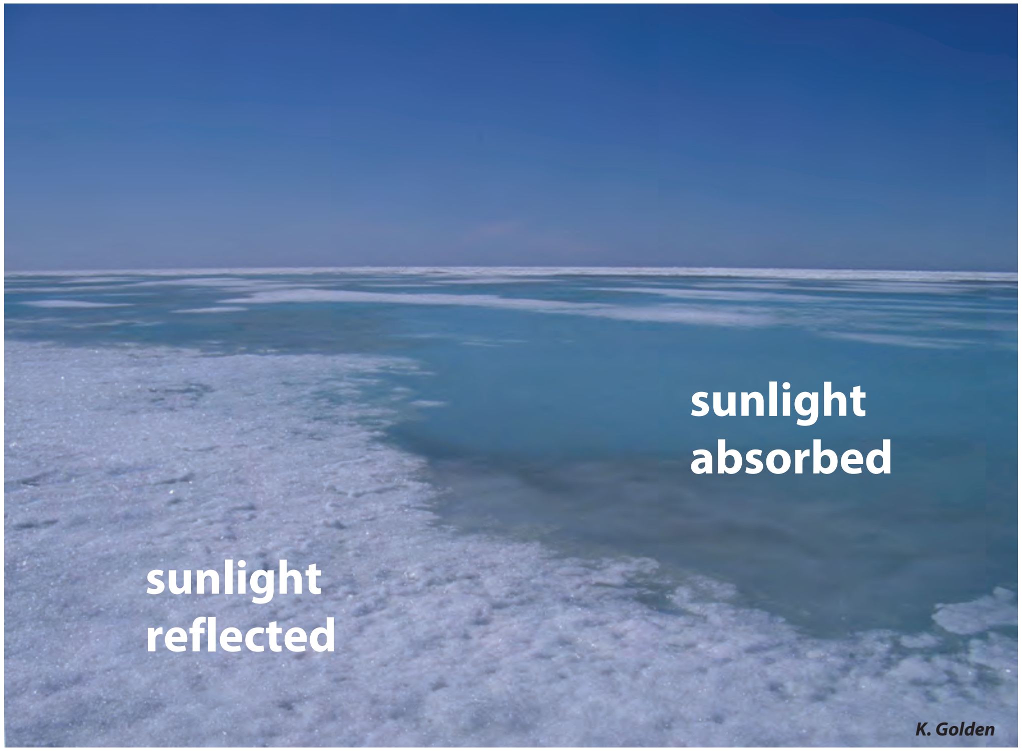
T = -4° C, $\phi = 0.113$

X-ray tomography for brine in sea ice

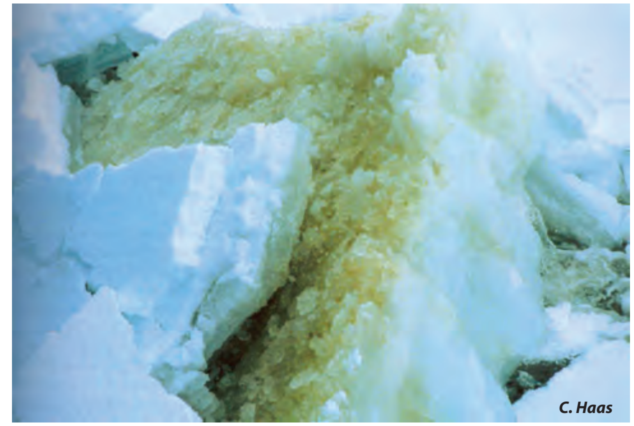
Golden *et al.*, *Geophysical Research Letters*, 2007

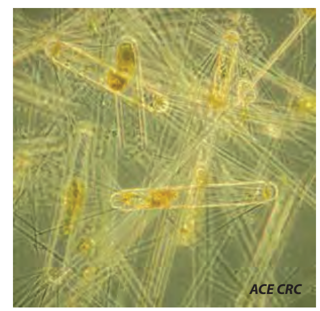
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

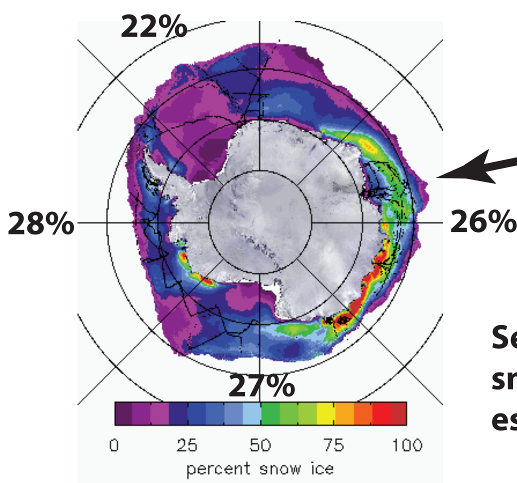
evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO_2*

Critical behavior of fluid transport in sea ice

Arctic field data

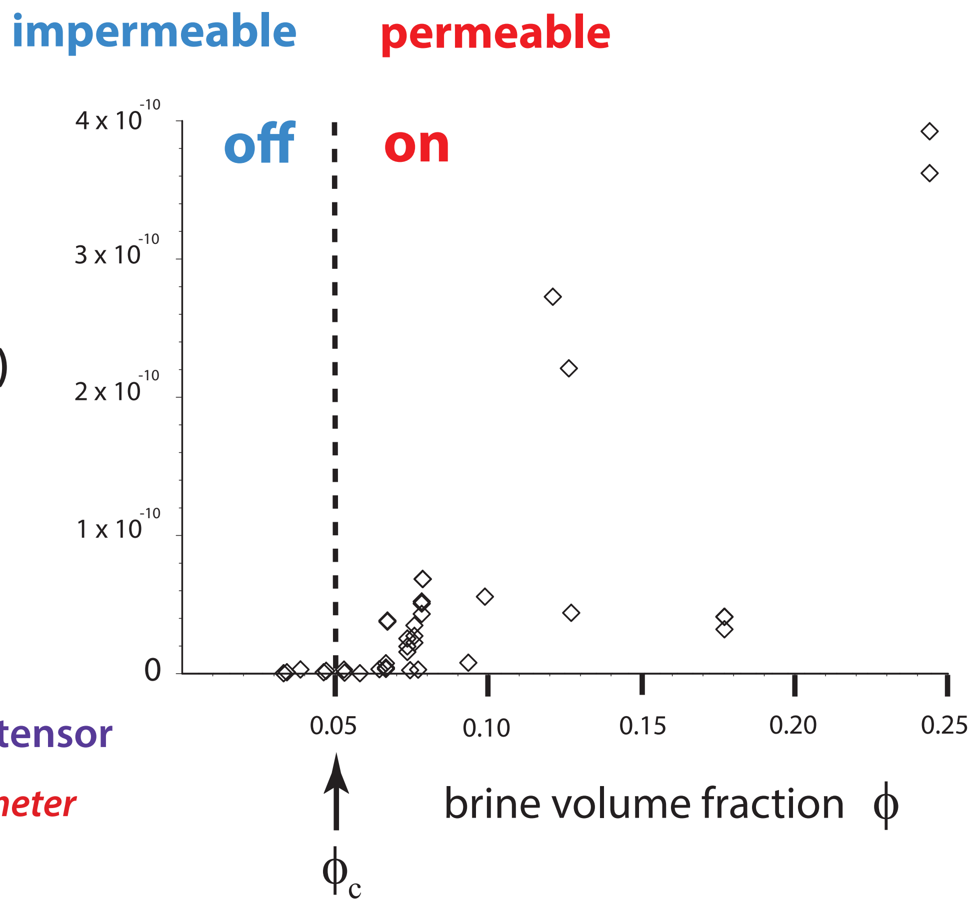
vertical fluid permeability k (m^2)

Darcy's Law

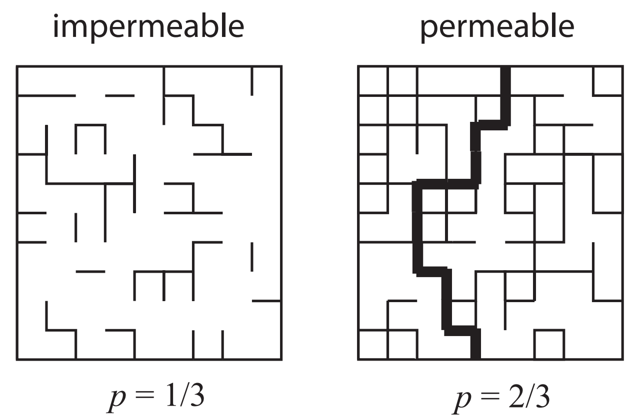
$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

$\mathbf{k}$ = fluid permeability tensor

homogenized parameter



"on - off" switch for bulk fluid flow



lattice percolation

FRACTAL percolation clusters

PERCOLATION THRESHOLD $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C,\ S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

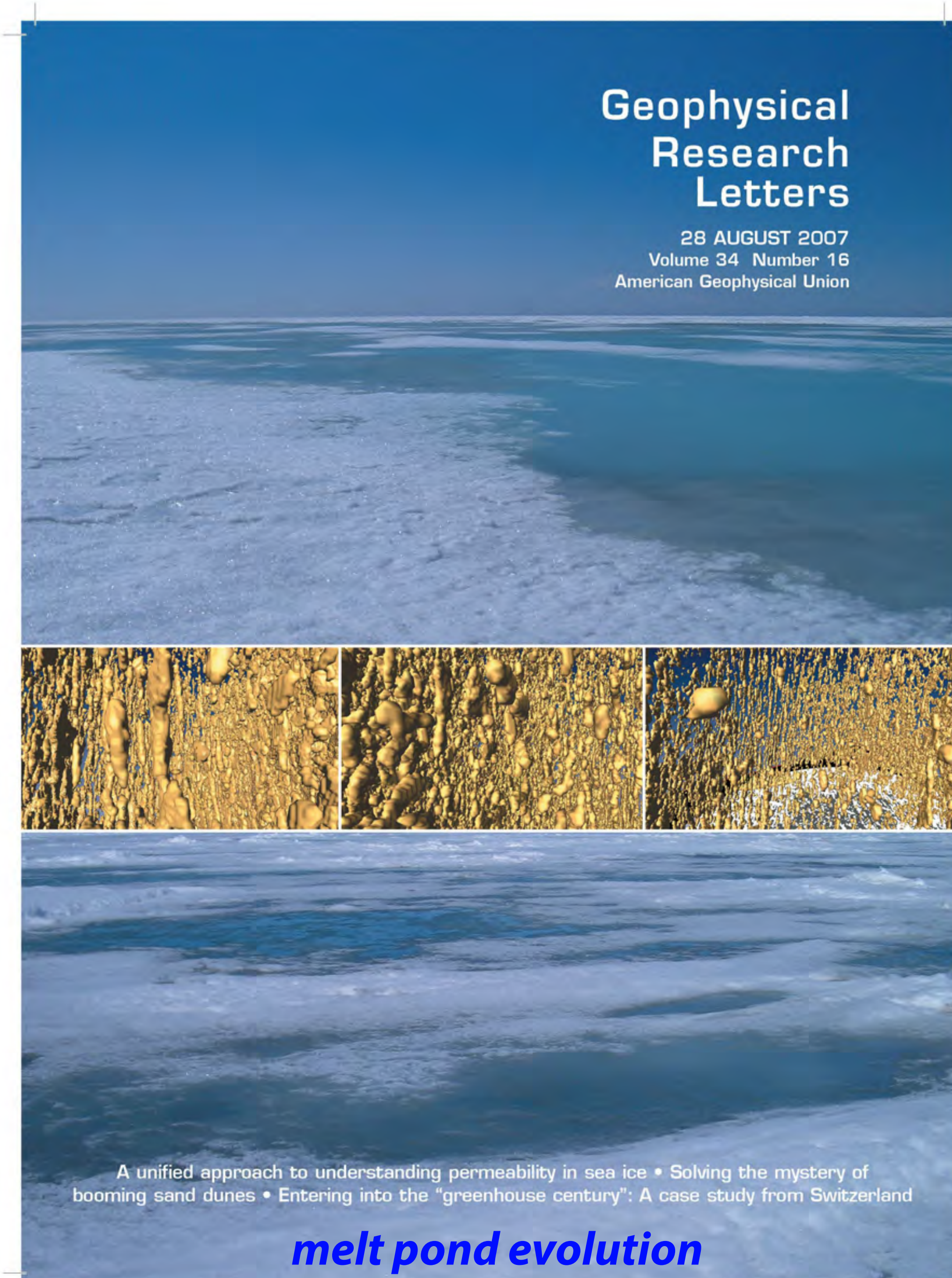
Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007

microscale

governs

mesoscale processes



melt pond evolution

percolation theory for fluid permeability

$$k(\phi) = k_0 \, (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8} \ \mathrm{m}^2$$

from critical path analysis in hopping conduction

hierarchical model

rock physics

network model

rigorous bounds

X-ray tomography for brine inclusions

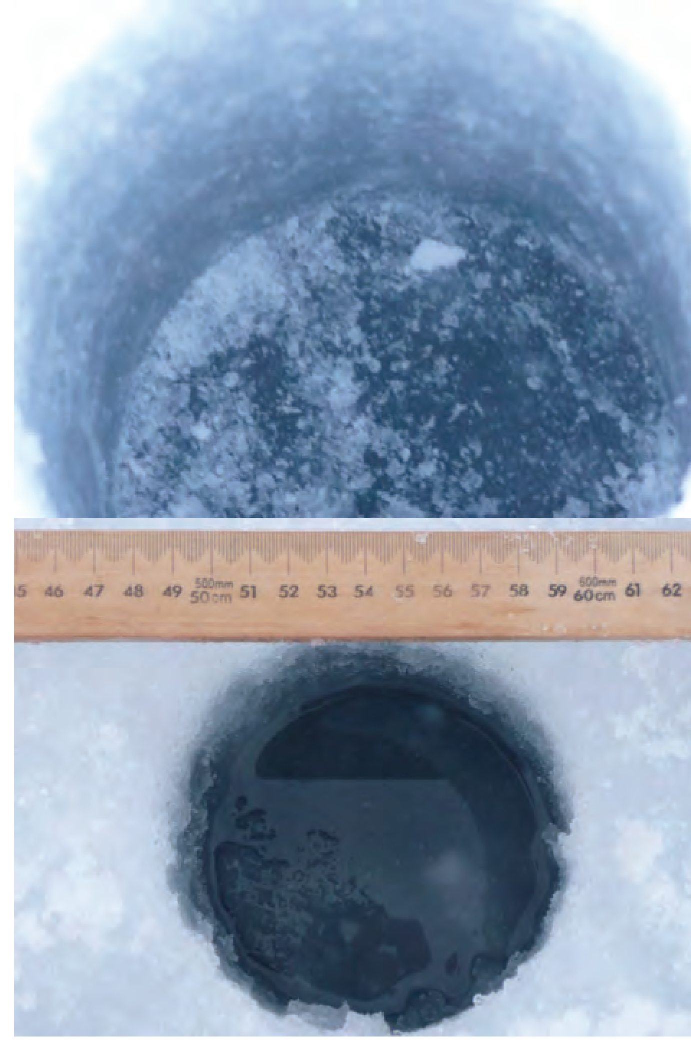
confirms rule of fives

brine percolation threshold of $\phi = 5\%$ for bulk fluid flow

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

theories agree closely with field data

photo by Jan Lieser



measuring
fluid permeability
of Antarctic sea ice

SIPEX 2007

Arctic and Antarctic field experiments

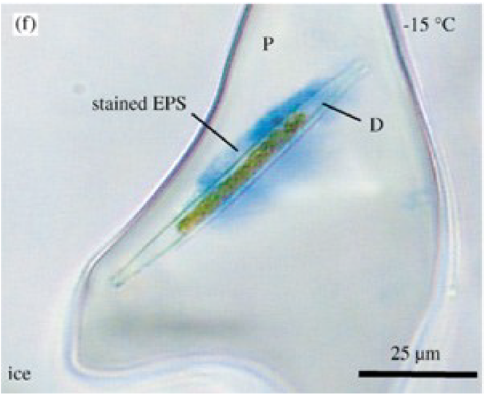
develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	***Antarctic***	***SIPEX***
2010	***Antarctic***	***McMurdo Sound***
2011	***Arctic***	***Barrow AK***
2012	***Arctic***	***Barrow AK***
2012	***Antarctic***	***SIPEX II***
2013	***Arctic***	***Barrow AK***
2014	***Arctic***	***Chukchi Sea***

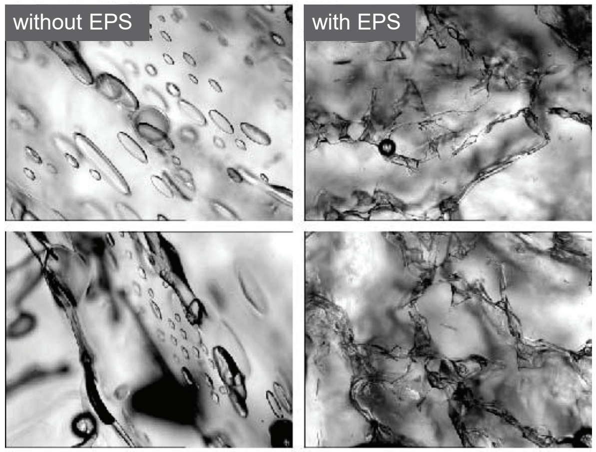
Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport? How does the biology affect the physics?

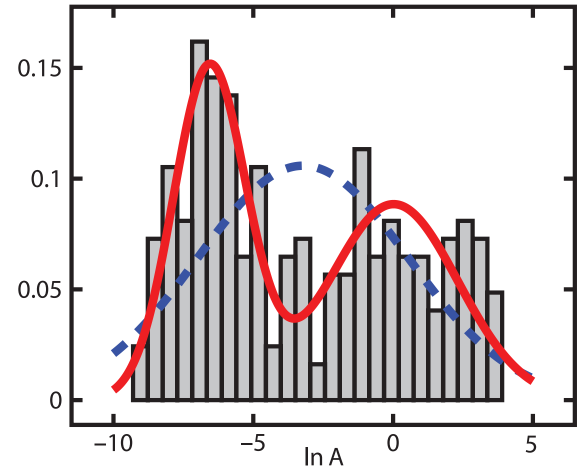


Krembs

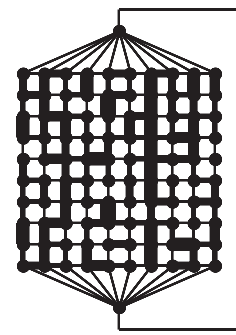
FRACTAL

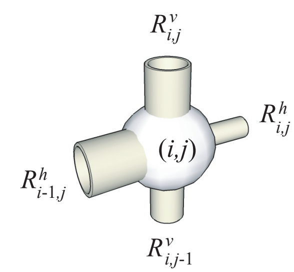


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

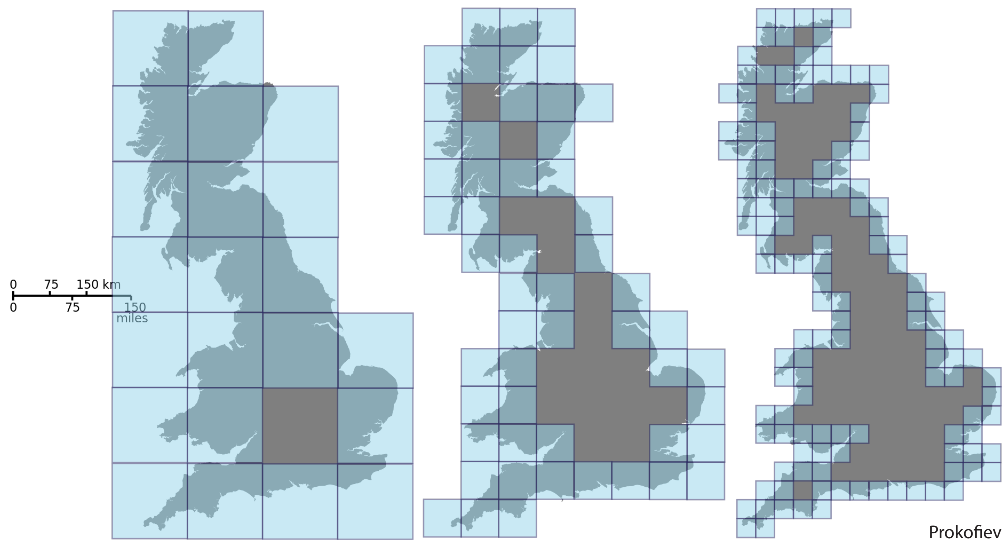
- 2D random pipe model with bimodal distribution of pipe radii

- Rigorous bound on permeability *k;* results predict observed drop in *k*

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
***Multiscale Modeling and Simulation*, 2018**

Thermal Evolution of Brine Fractal Geometry in Sea Ice

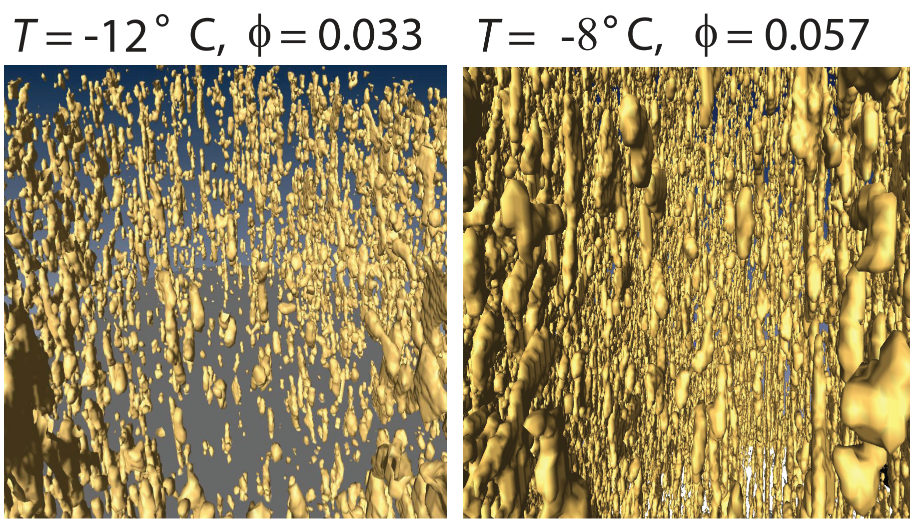
Nash Ward, Daniel Hallman, Benjamin Murphy, Jody Reimer,
Marc Oggier, Megan O'Sadnick, Elena Cherkaev and Kenneth Golden, 2023



fractal dimension of the coastline of Great Britain by box counting

$$N(\epsilon) \sim \epsilon^{-D}$$

brine channels and inclusions "look" like fractals (from 30 yrs ago)

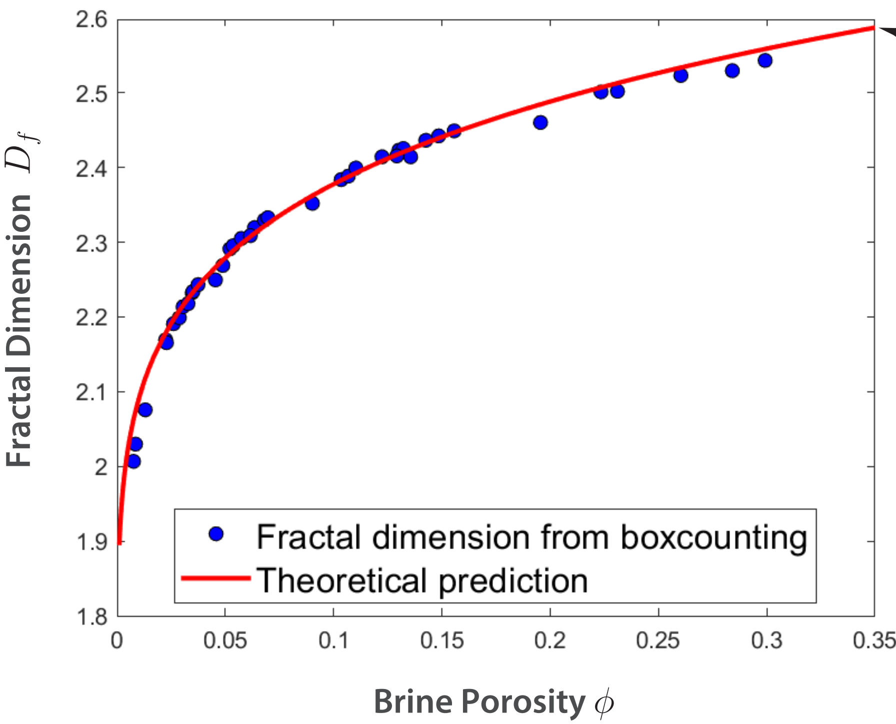


Golden, Eicken, et al. *GRL*, 2007

X-ray computed tomography of brine in sea ice

columnar and granular

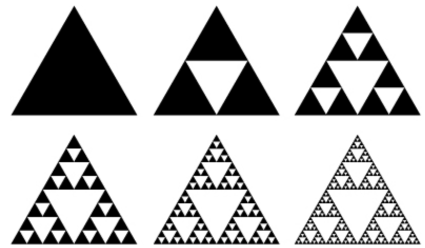
The first comprehensive, quantitative study of the fractal dimension of brine in sea ice and its strong dependence on temperature and porosity.

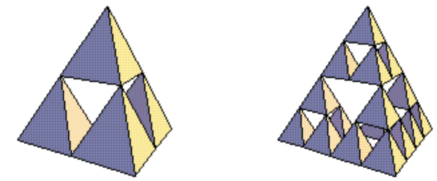


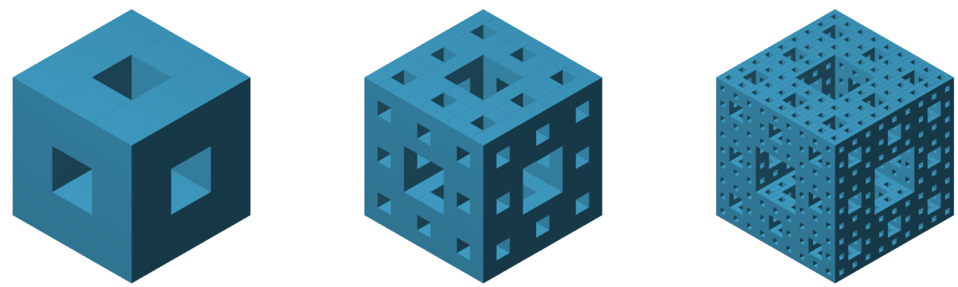
$$D_f = 3 - \frac{\ln \phi}{\ln(\lambda_{min}/\lambda_{max})}$$

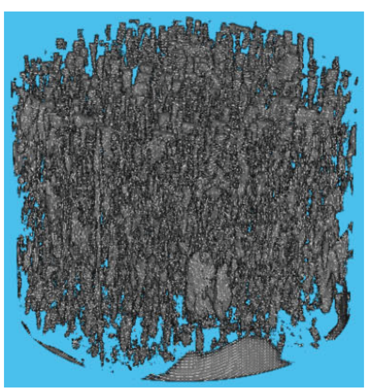
The red curve is exact for the Sierpinski pyramid (an exactly self-similar geometry); discovered for sandstones - statistically self-similar porous media like sea ice.

Katz and Thompson, 1985; Yu and Li, 2001

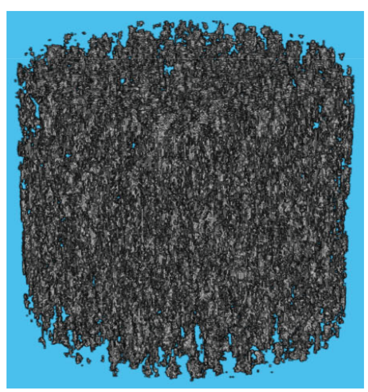




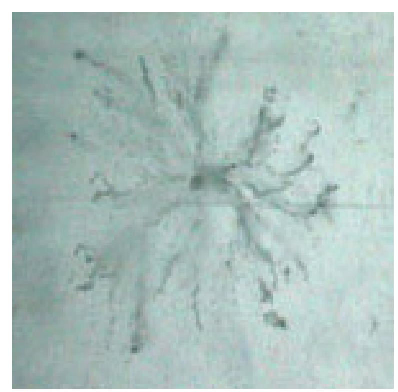




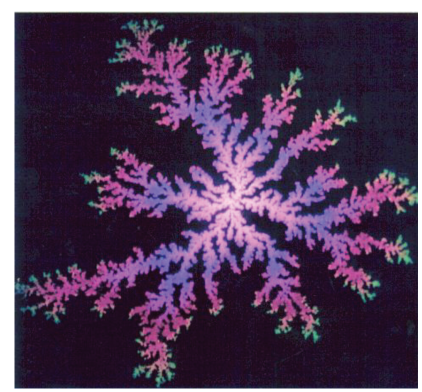
X-ray tomography



DLA model

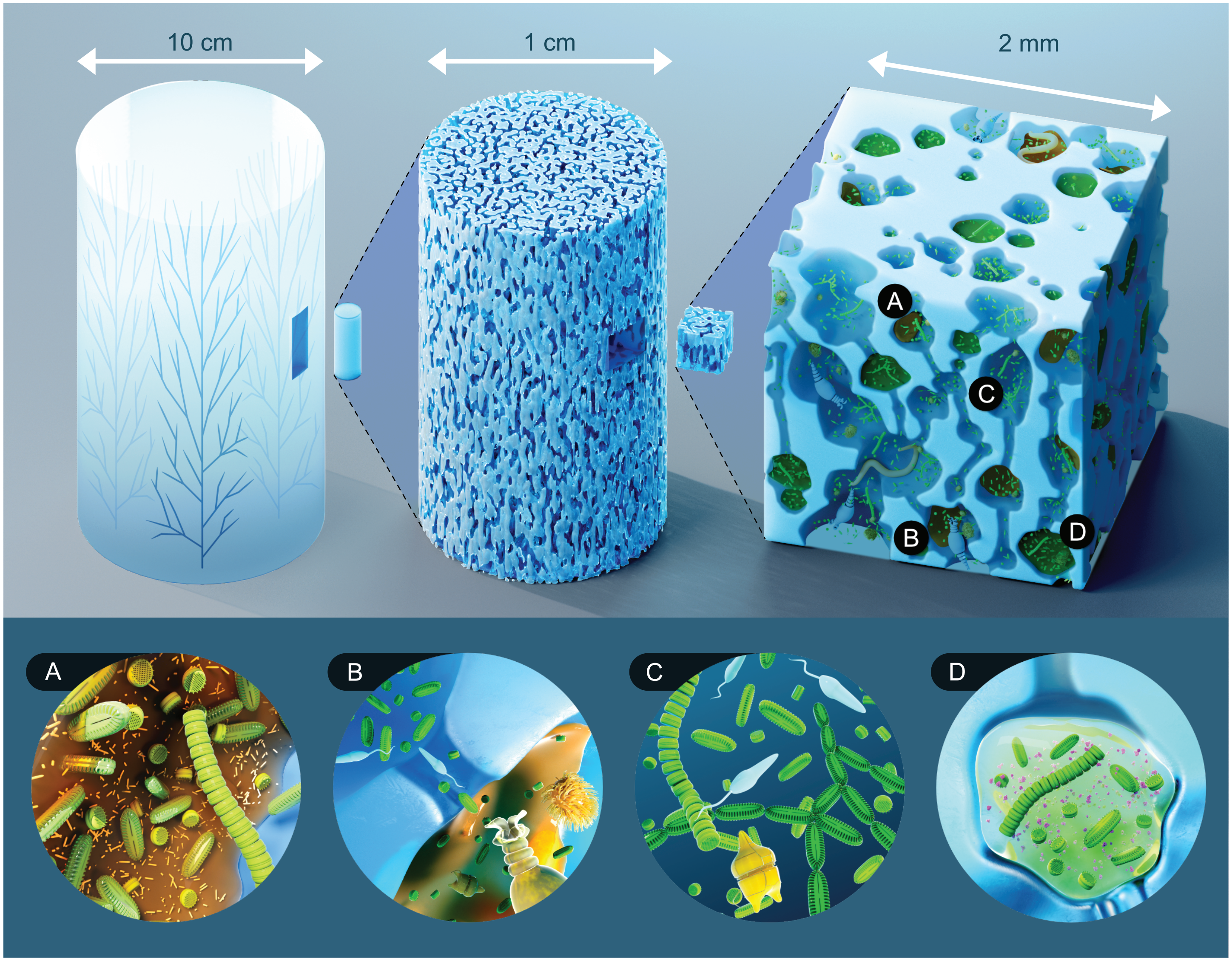


brine channel in sea ice



diffusion limited aggregation

Implications of brine fractal geometry on sea ice ecology and biogeochemistry



Brine inclusions are home to ice endemic organisms, e.g., bacteria, diatoms, flagellates, rotifers, nematodes.

The habitability of sea ice for these organisms is inextricably linked to its complex brine geometry.

(A) Many sea ice organisms attach themselves to inclusion walls; inclusions with a higher fractal dimension have greater surface area for colonization.

(B) Narrow channels prevent the passage of larger organisms, leading to refuges where smaller organisms can multiply without being grazed, as in (C).

(D) Ice algae secrete extracellular polymeric substances (EPS) which alter incusion geometry and may further increase the fractal dimension.

Remote sensing of sea ice

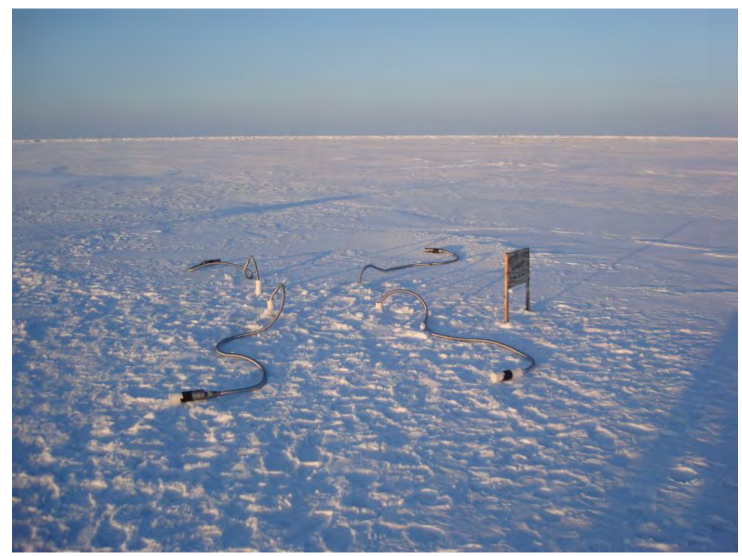
INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

ε^*

effective complex permittivity
(dielectric constant, conductivity)

sea ice thickness
ice concentration



brine volume fraction
brine inclusion connectivity


Golden

Theory of Effective Electromagnetic Behavior of Composites

analytic continuation method

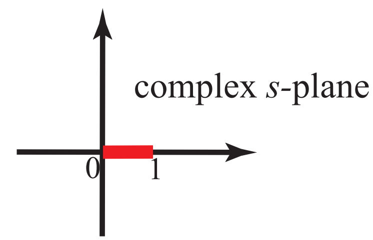
Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)
Theory of Composites, Milton (2002)

composite geometry **(spectral measure μ)** $\longrightarrow$ ε^*

integral representation, rigorous bounds, approximations, etc.

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

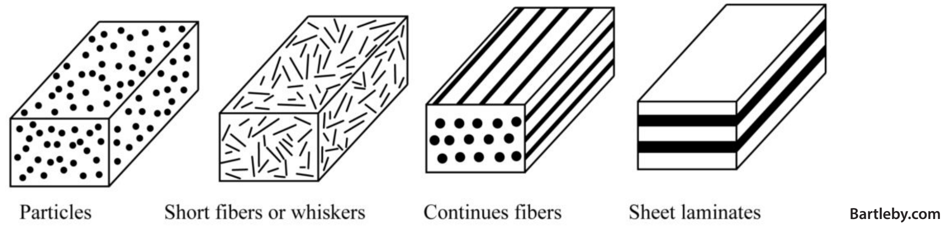


Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001)
McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)

ε^* $\longrightarrow$ ***composite geometry*** **(spectral measure μ)**

recover brine volume fraction, connectivity, etc.

complexities of mixture geometry



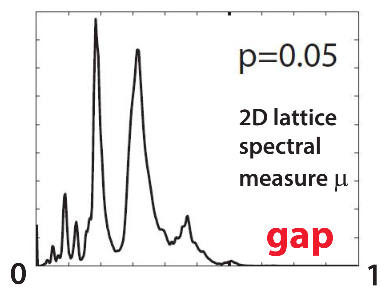
distilled

distilled

Analytic Continuation Method

Stieltjes Integral Representations
for Homogenized Parameters

Bergman 1978, Milton 1979
Golden & Papanicolaou 1983



spectral properties of operator (matrix)
~ quantum states, energy levels for atoms

eigenvectors eigenvalues

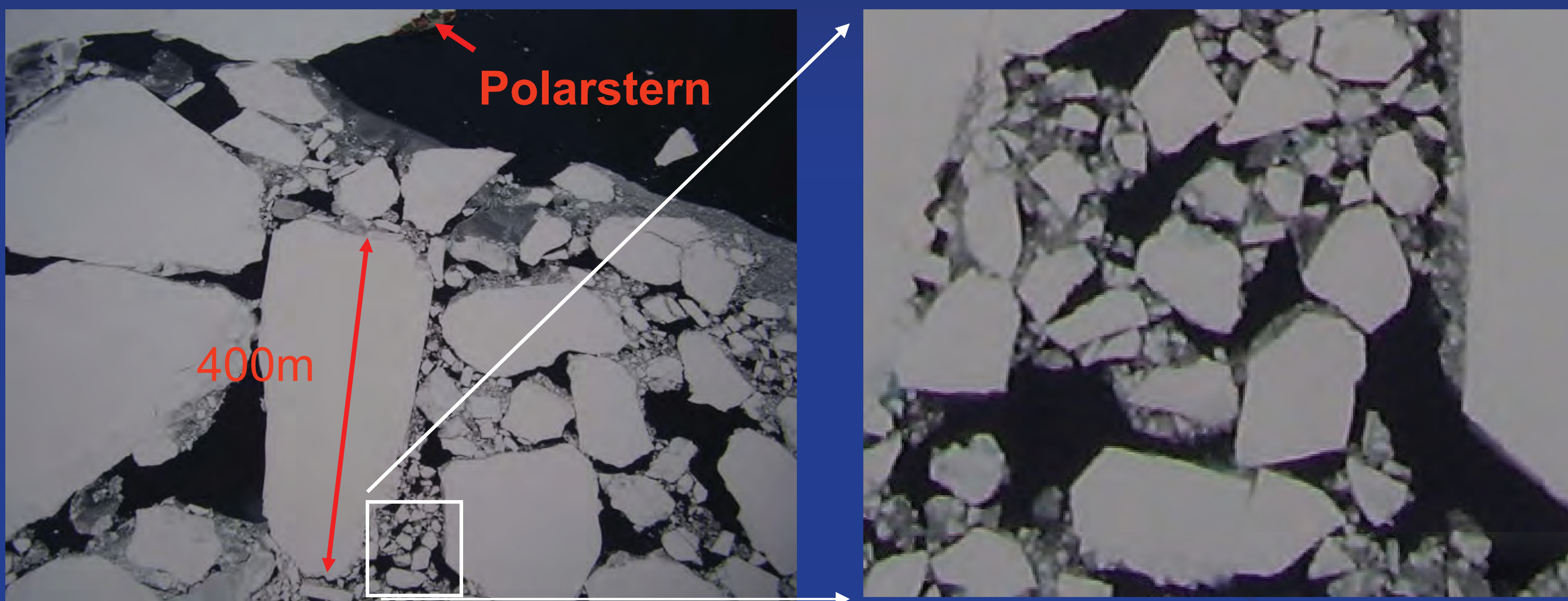
EXTEND to: polycrystals, advection diffusion, waves through ice pack

mesoscale

The sea ice pack has fractal structure.

Self-similarity of sea ice floes

Weddell Sea, Antarctica

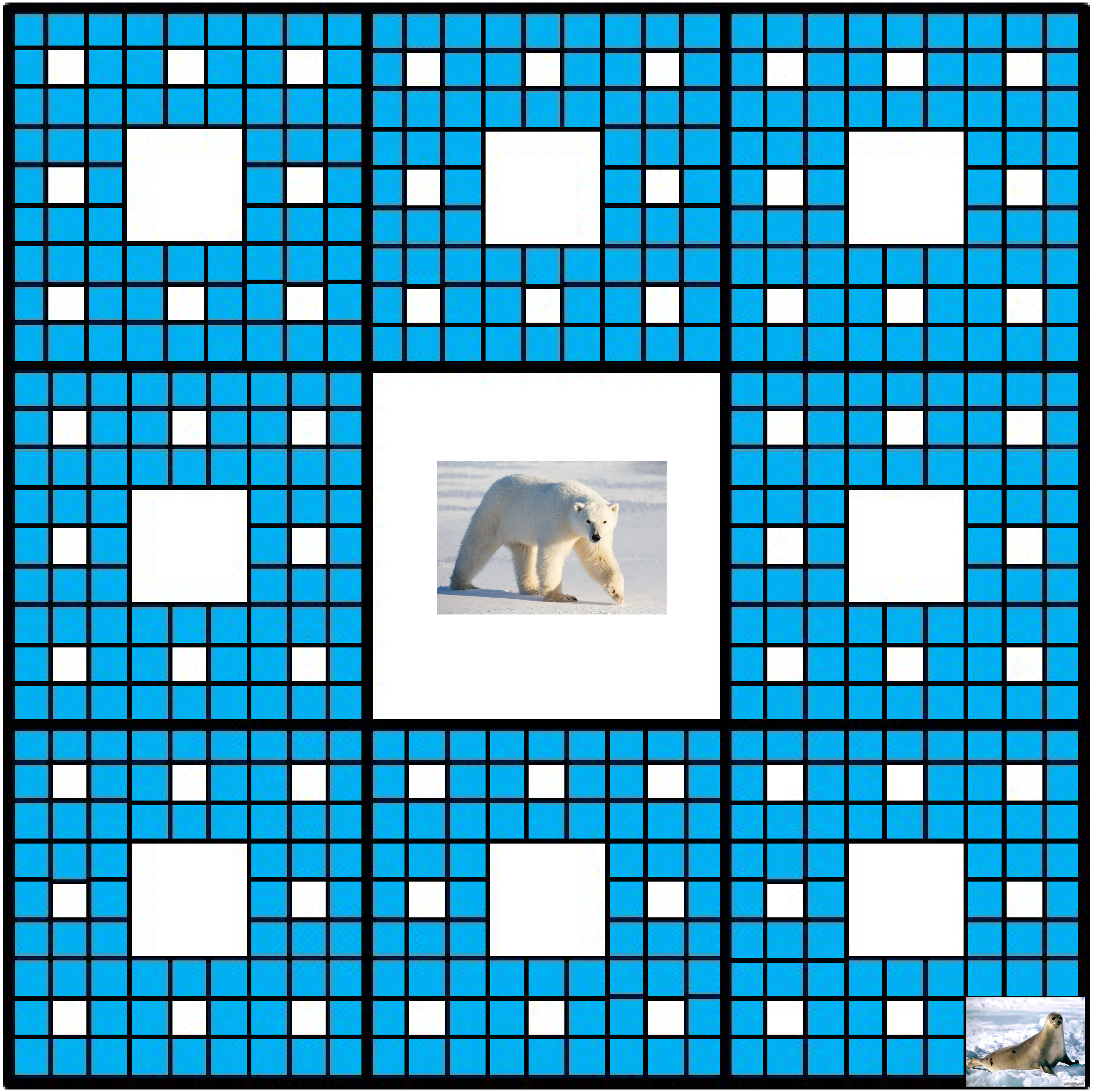


fractal dimensions of Okhotsk Sea ice pack
smaller scales D~1.2, larger scales D~1.9

Toyota, *et al. Geophys. Res. Lett.* 2006
Rothrock and Thorndike, *J. Geophys. Res. 1984*

polar bear foraging in a fractal icescape

Nicole Forrester
Jody Reimer
Ken Golden



It costs the polar bear 5 times the energy to swim through water than to walk on sea ice.

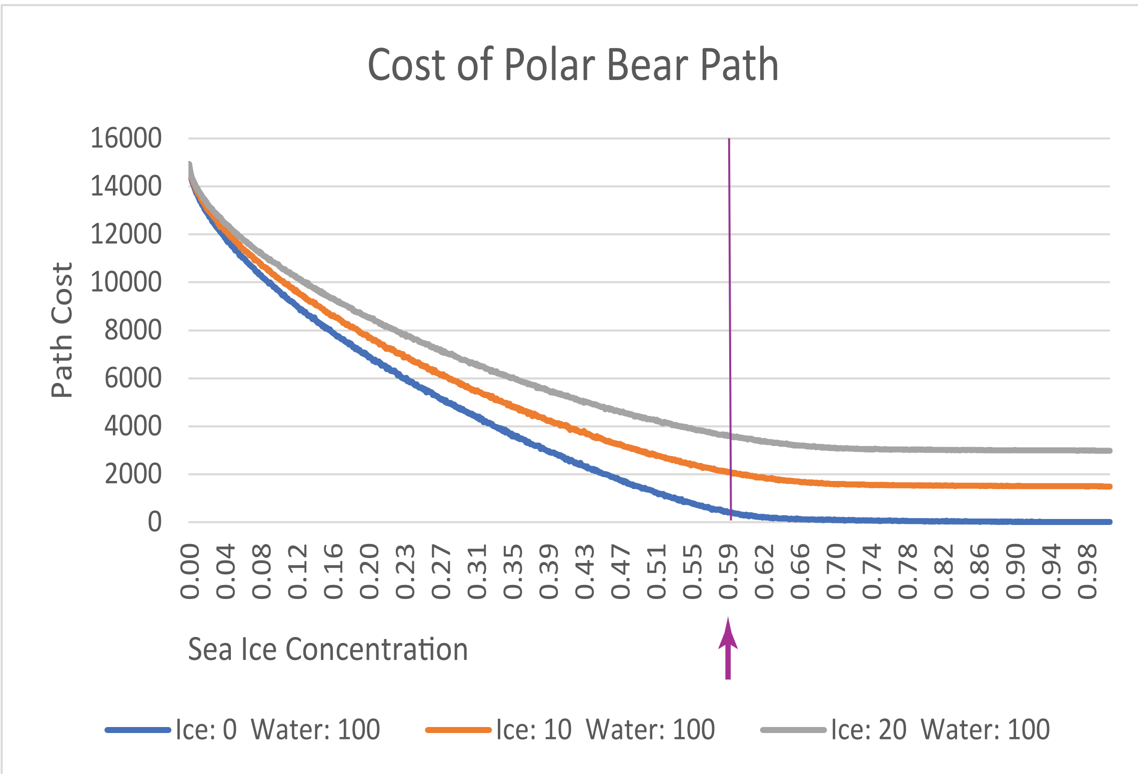
1/5, 1/10, 1/100, ...

What pathway to a seal minimizes energy spent?

Polar Bear Percolation

Optimal Movement of a Polar Bear in a Heterogenous Icescape

$C(p)$



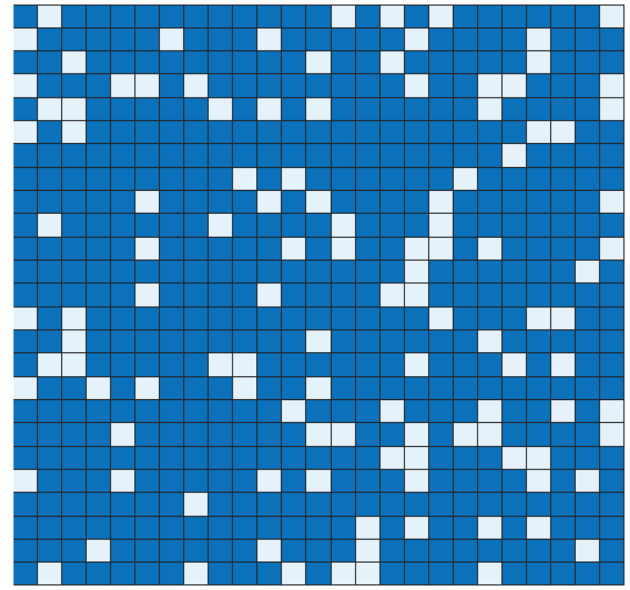
$\leftarrow h = 0.2$

$\leftarrow h = 0.1$

$\leftarrow h = 0$

$$h = \frac{c_i}{c_w}$$

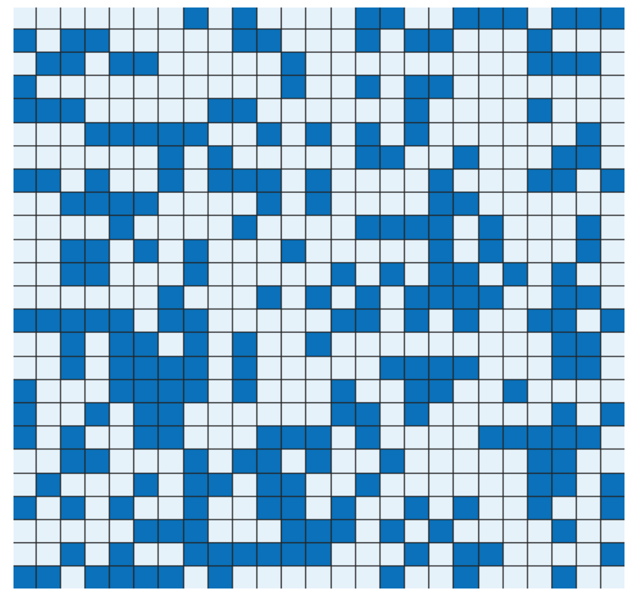
ratio of local "conductivities"



20% Ice

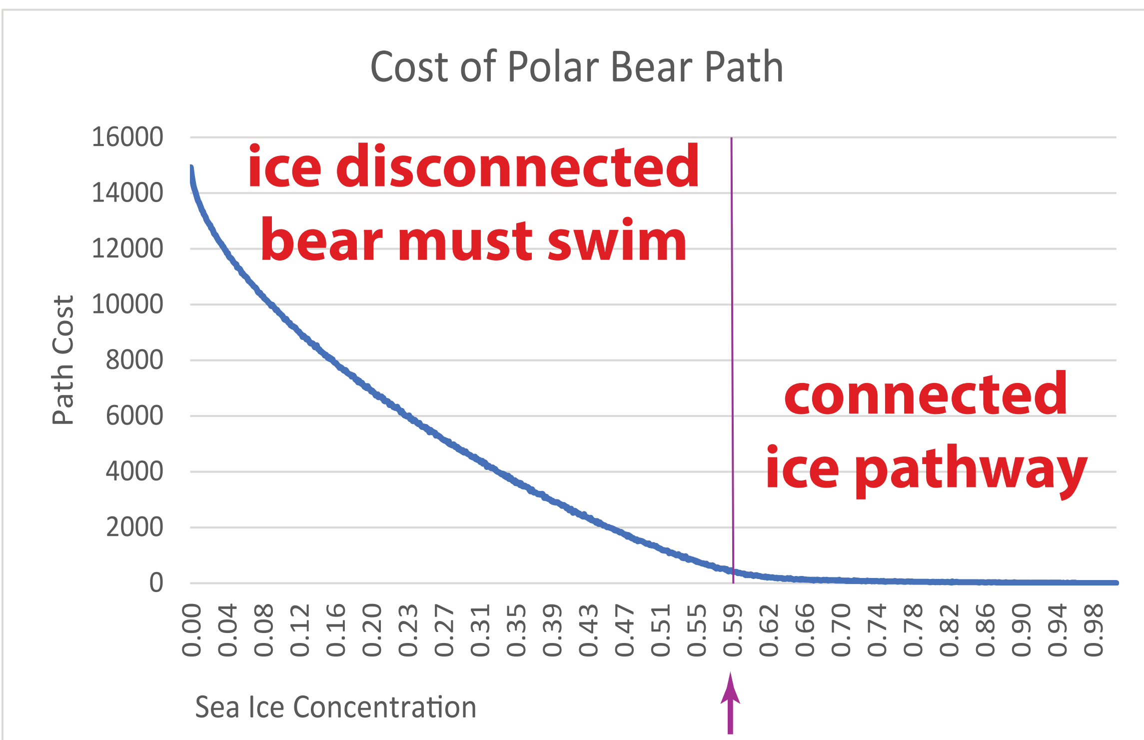
site percolation threshold

$p_c = 0.59$ *for* $d = 2$

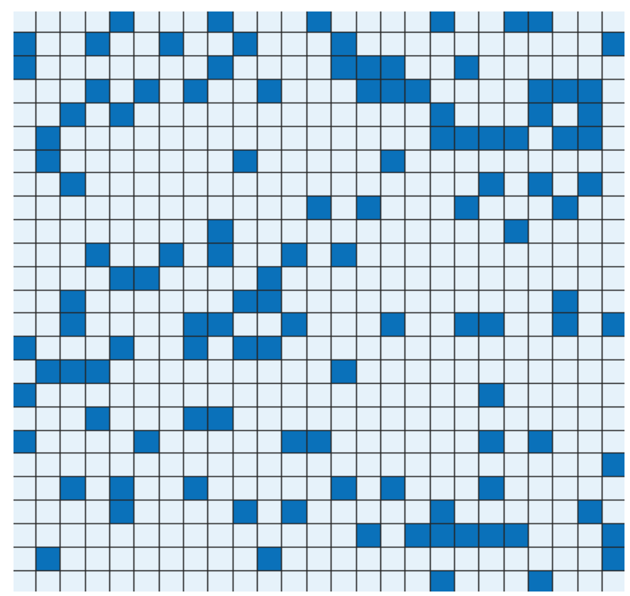


60% Ice

$C(p)$



$\leftarrow h = 0$



80% Ice

wave propagation in the marginal ice zone (MIZ)

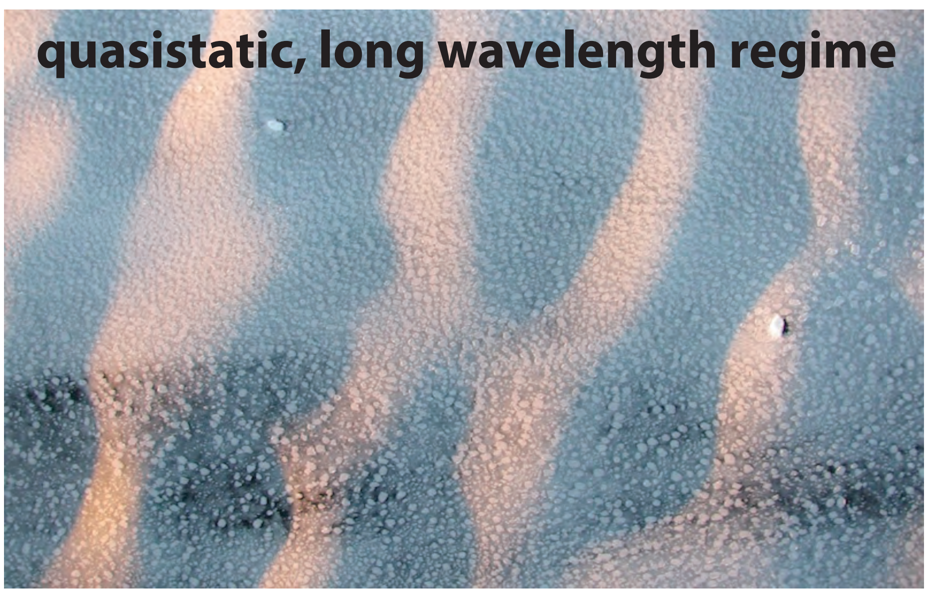
first theory of key parameter in wave-ice interactions only fitted to wave data before

Keller, 1998
Mosig, Montiel, Squire, 2015
Wang, Shen, 2012

Analytic Continuation Method

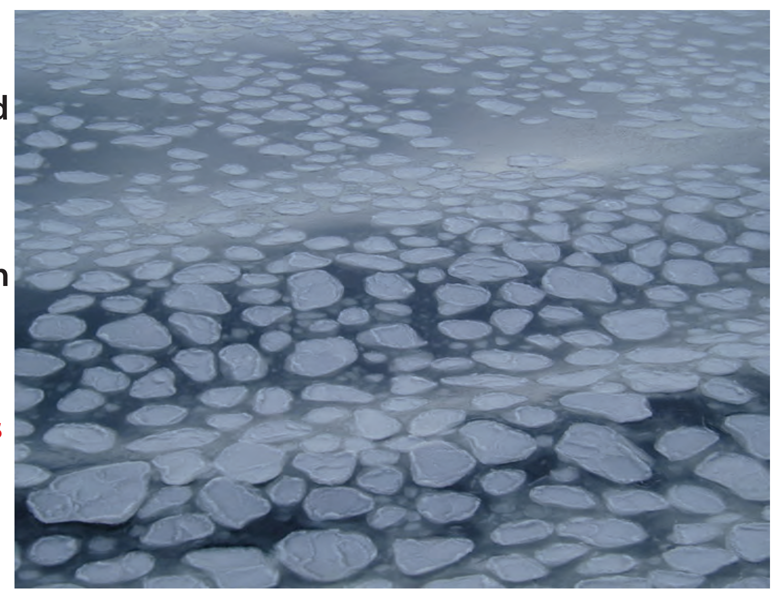
Bergman (78) - Milton (79)
integral representation for ε^*
Golden and Papanicolaou (83)

Milton, *Theory of Composites* (02)



homogenized parameter depends on sea ice concentration and ice floe geometry

like EM waves



melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

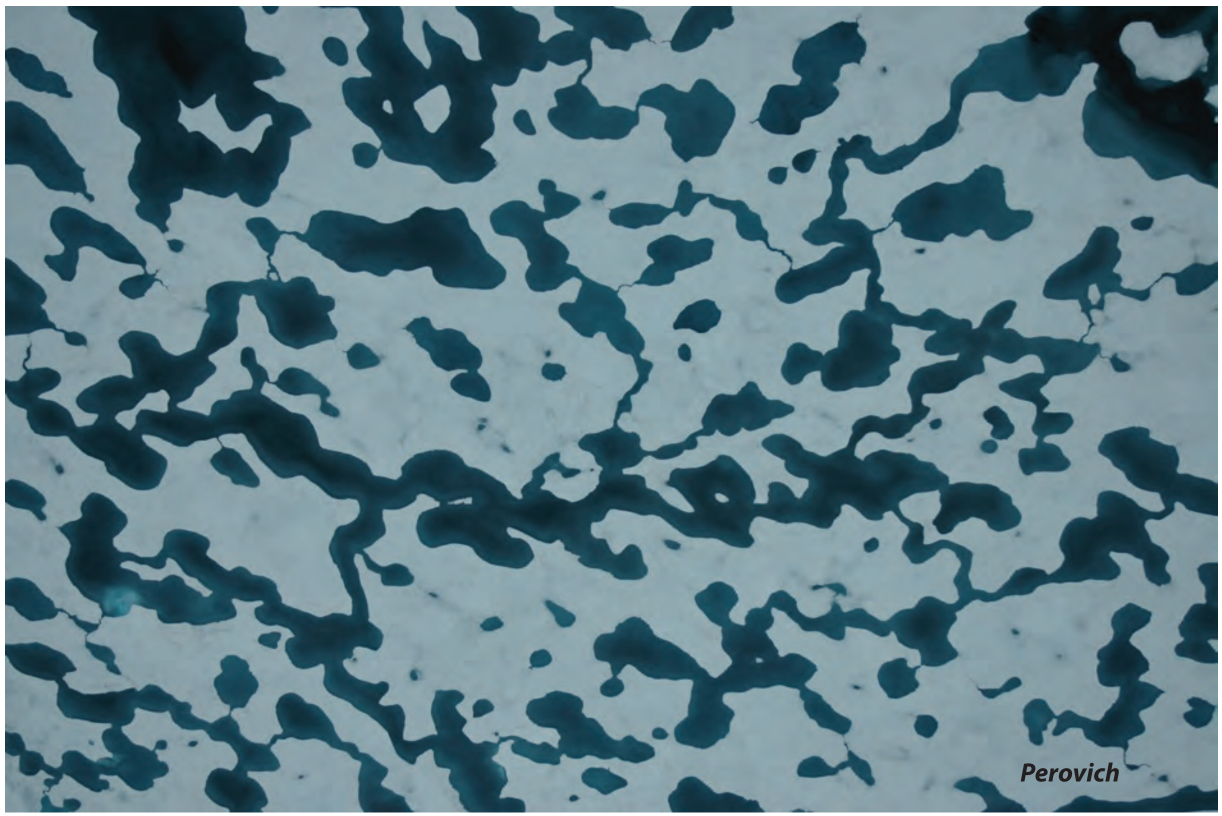
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

Flocco, Feltham, Hunke 2012



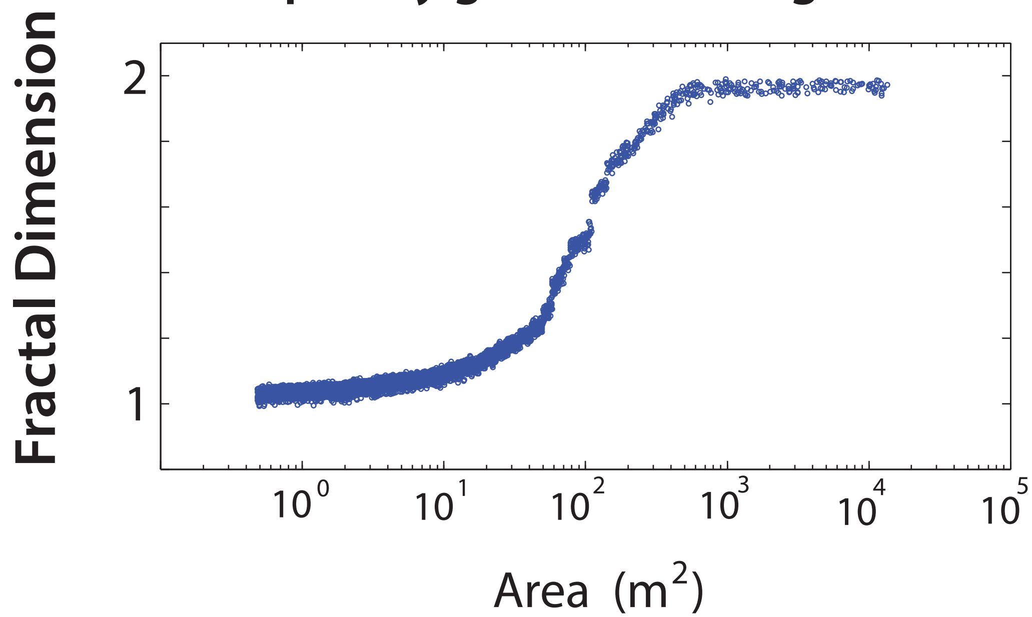
Are there universal features of the evolution similar to phase transitions in statistical physics?

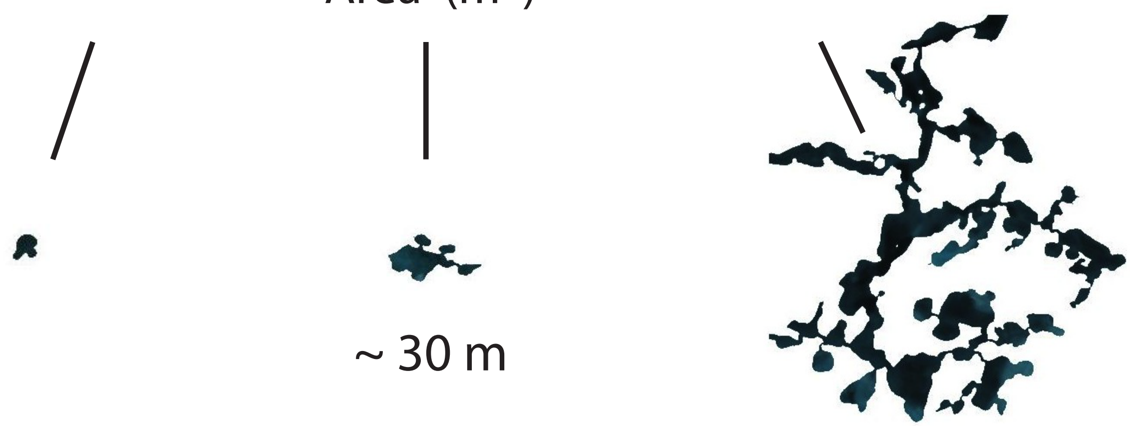
Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale



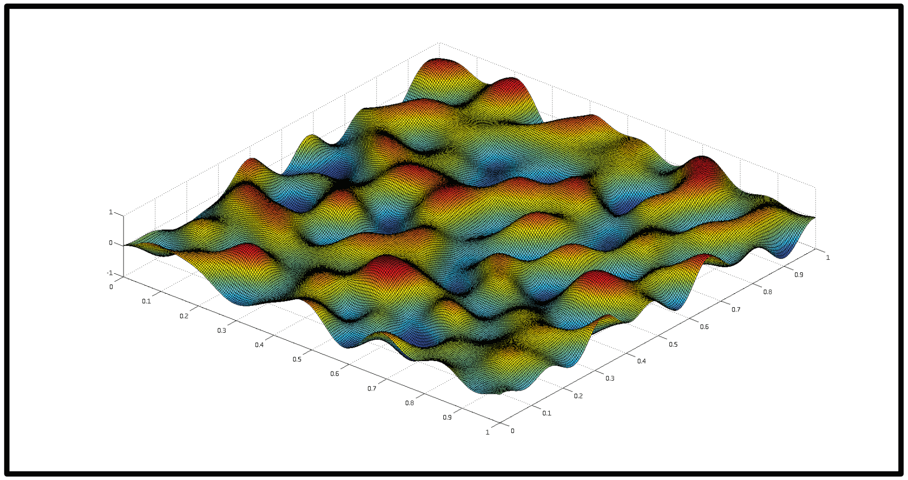


simple pond ***transitional pond*** ***complex pond***

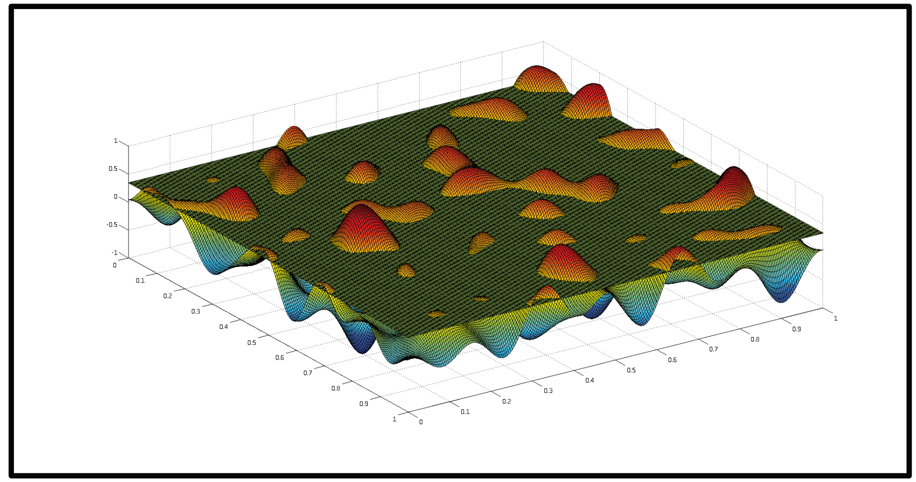
Continuum percolation model for melt pond evolution

level sets of random surfaces

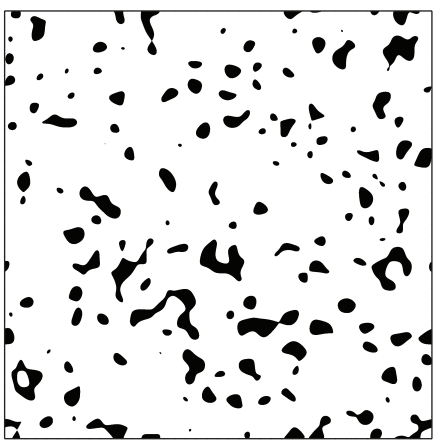
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

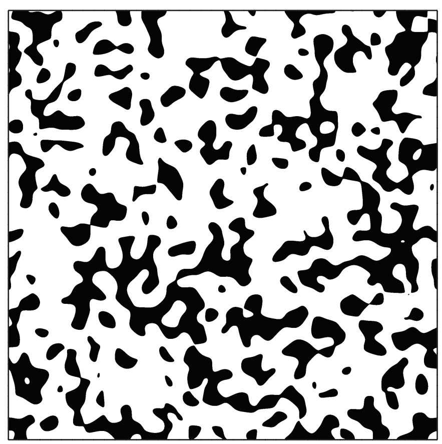


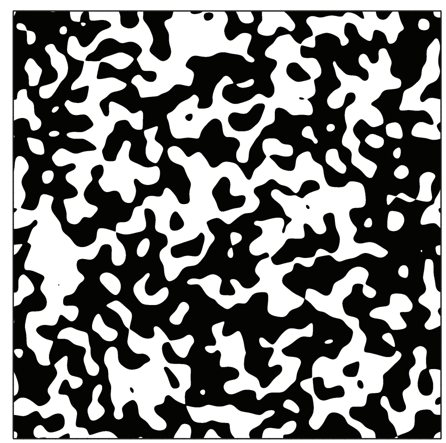
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds

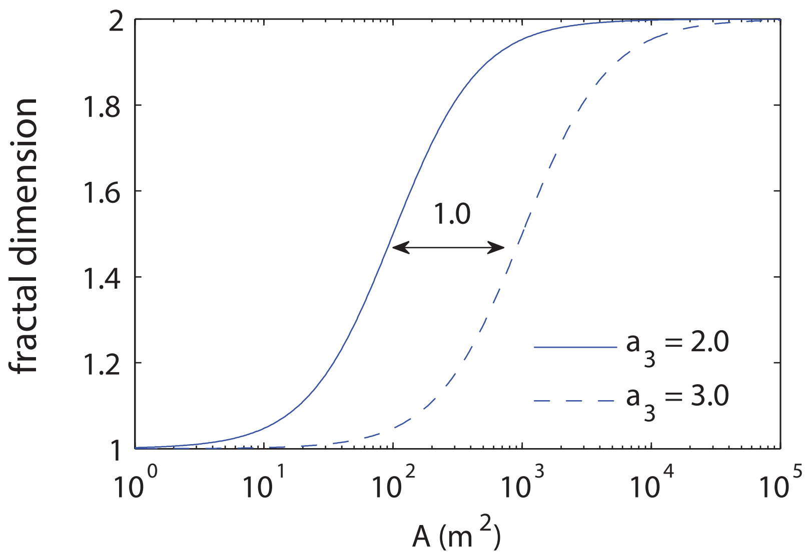


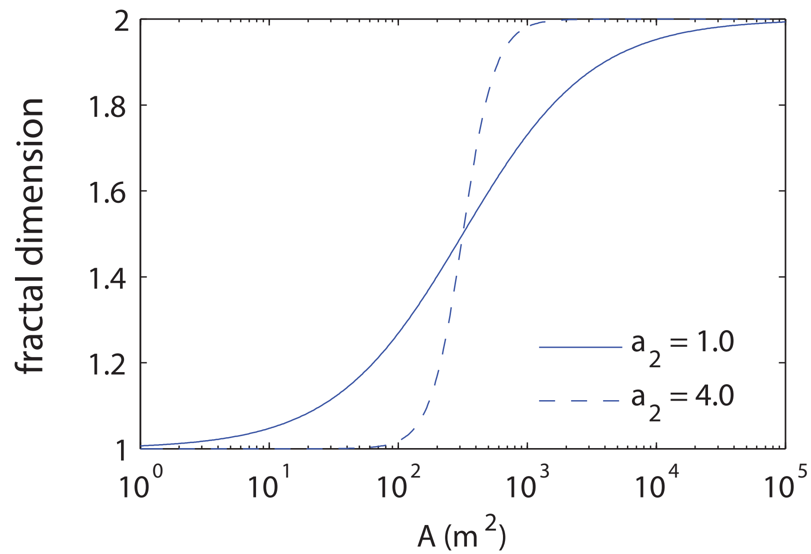




electronic transport in disordered media *diffusion in turbulent plasmas* *Isichenko, Rev. Mod. Phys., 1992*

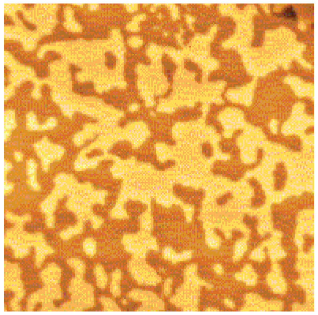
fractal dimension curves depend on statistical parameters defining random surface



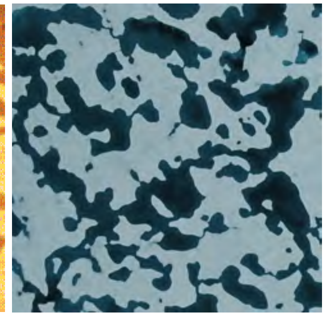


From magnets to melt ponds

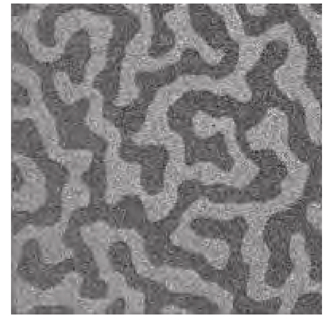
100 year old model for magnetic materials used to explain melt pond geometry



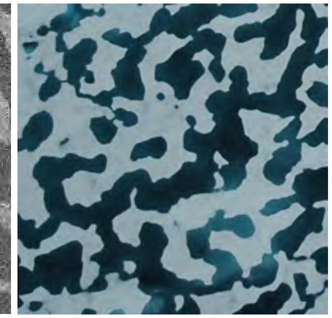
magnetic domains in cobalt



Arctic melt ponds

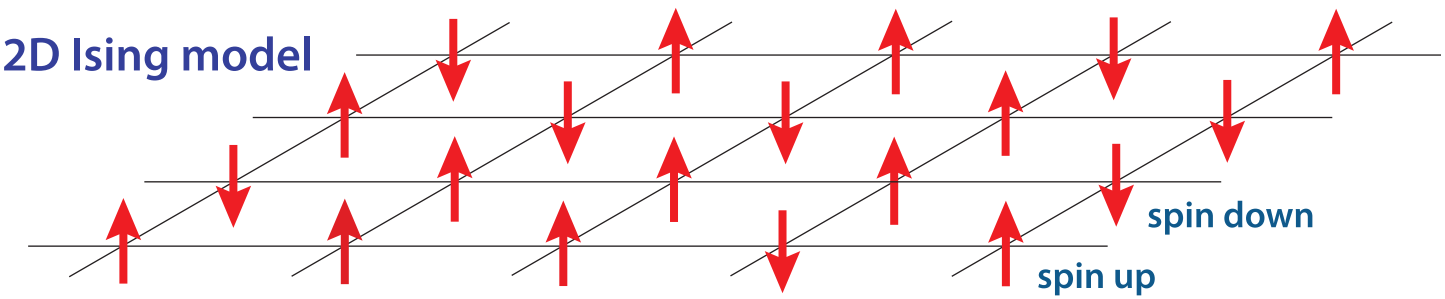


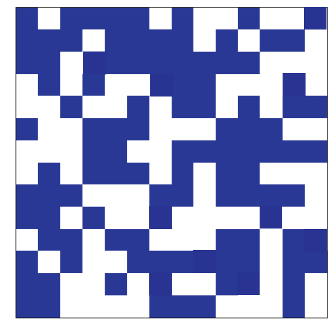
magnetic domains in cobalt-iron-boron



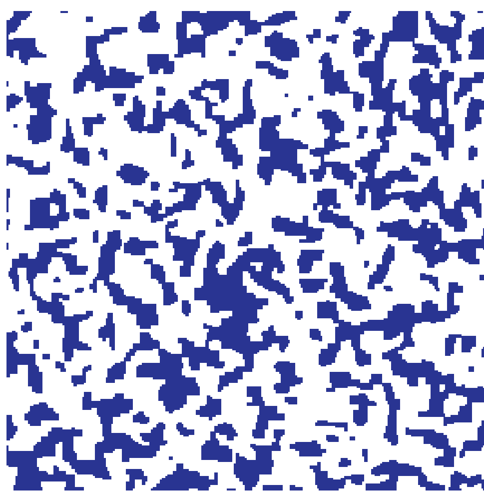
Arctic melt ponds

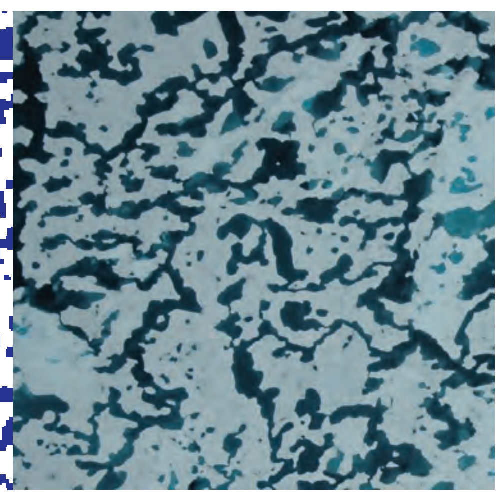
2D Ising model





model





real ponds (Perovich)

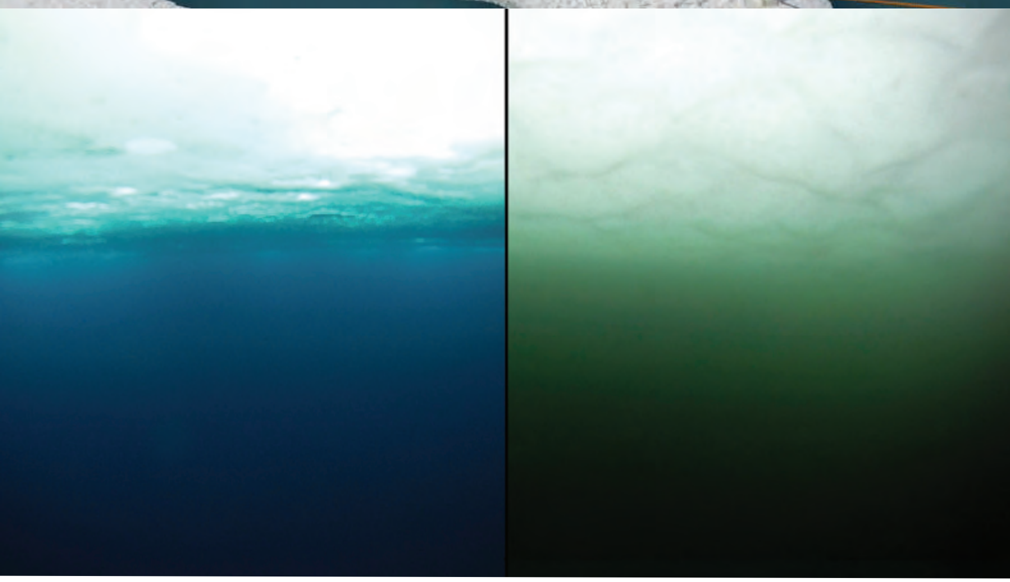
Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Scientific American, EOS, PhysicsWorld, ...

Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom bloom

massive under-ice algal bloom

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances*** 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden

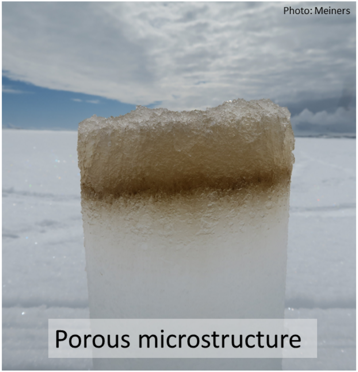
Geophys. Res. Lett. 2019

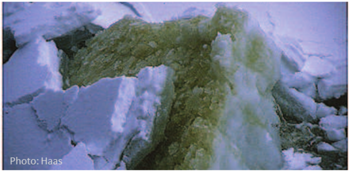
FRACTAL MELT POND STRUCTURE

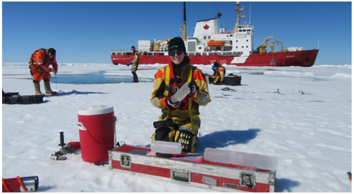
(2015 AMS MRC)

Sea ice algae

Uncertainty quantification for ecological models with random parameters, Reimer, Adler, Golden, Naryan, Ecol. Lett. 2022







Can we improve agreement between algae models and data?

80% of polar bear diet can be traced to ice algae*.

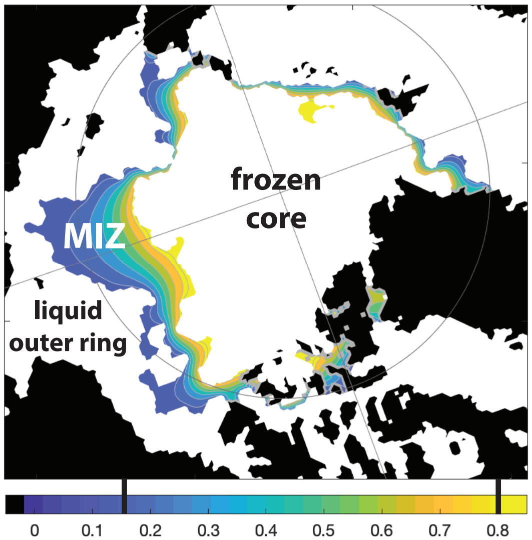
---

* Brown TA, et al. (2018). *PloS one*, 13(1), e0191631

macroscale

Model larger scale effective behavior with partial differential equations that *homogenize* complex local structure and dynamics.

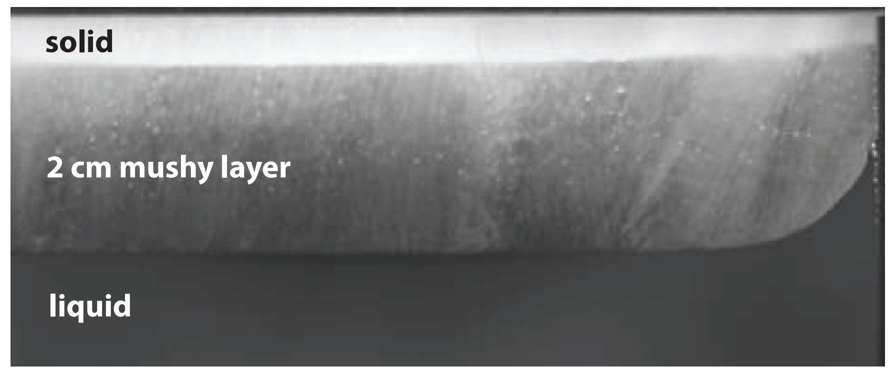
Arctic MIZ



sea ice concentration ψ

Predict MIZ width and location with basin-scale phase change model.

seasonal and long term trends



NaCl-H2O in lab
(Peppin et al., 2007;, J. Fluid Mech.)

Partial differential equation models and deep learning for the sea ice concentration field, 2023

Delaney Mosier, Eric Brown, Court Strong,
Jingyi Zhu, Bao Wang, Ken Golden

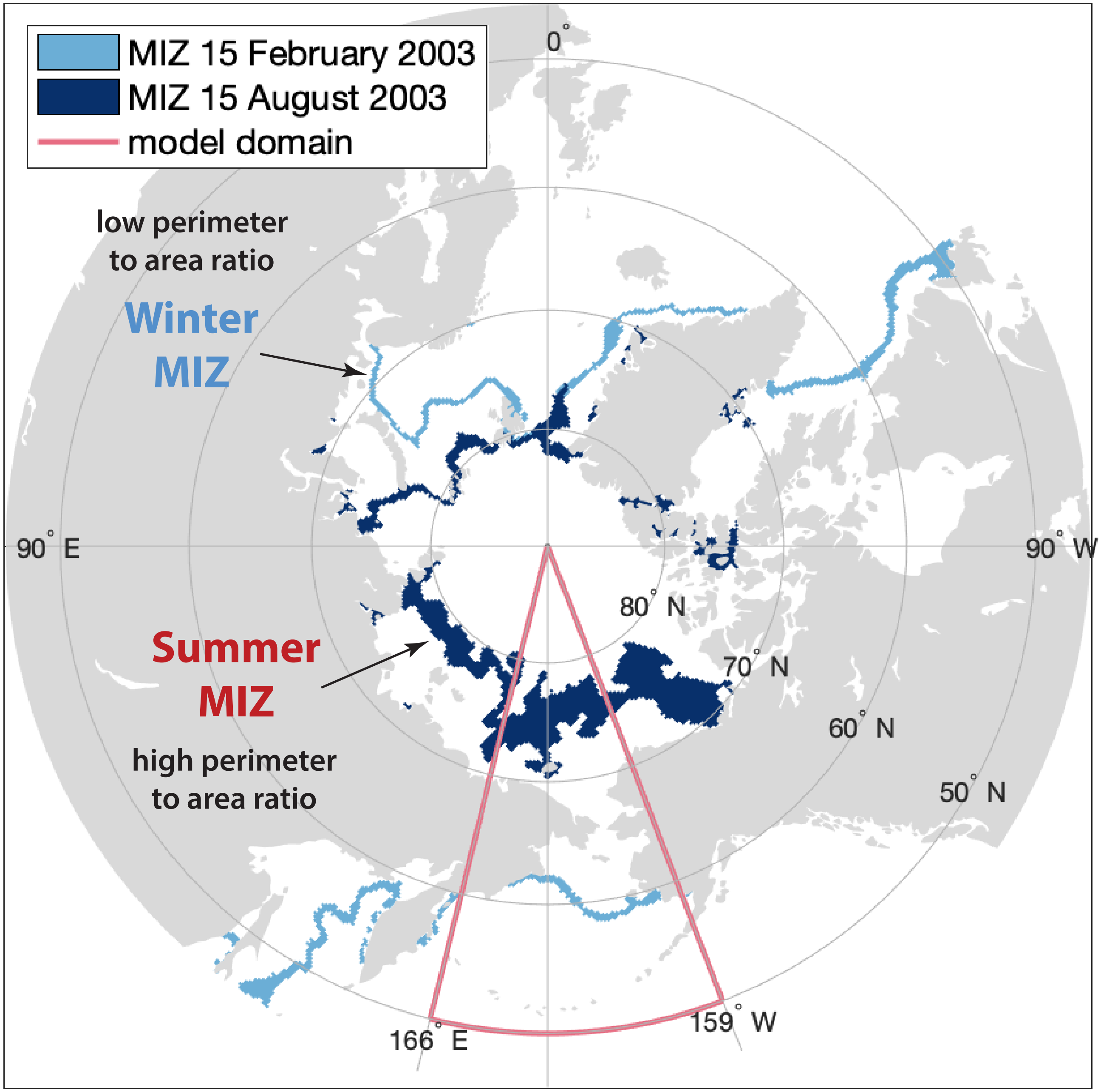
Annual cycle of Arctic marginal ice zone location and width explained by macroscale mushy layer model, 2023

C. Strong, E. Cherkaev, and K. M. Golden

Observed Arctic MIZ

low fractal dimension

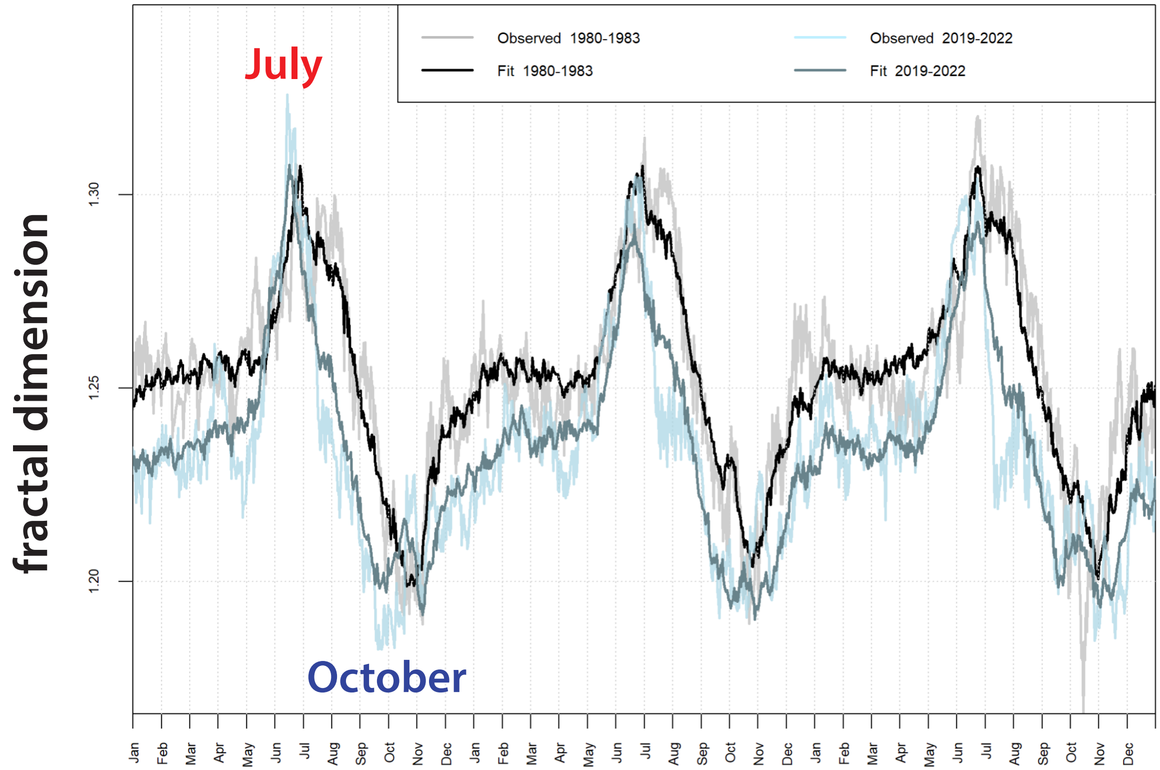
high fractal dimension

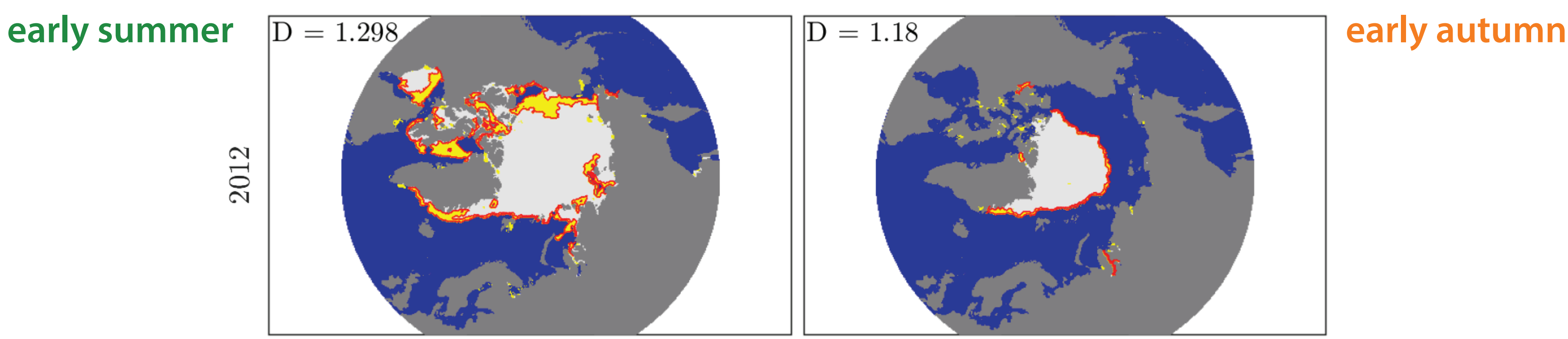


Evolution of the Fractal Geometry of the Arctic Marginal Ice Zone

Julie Sherman, Court Strong, Ken Golden, submitted 2023

Compute the fractal dimension of the boundary of the Arctic MIZ by boxcounting methods; analyze seasonal cycle and long term trends.





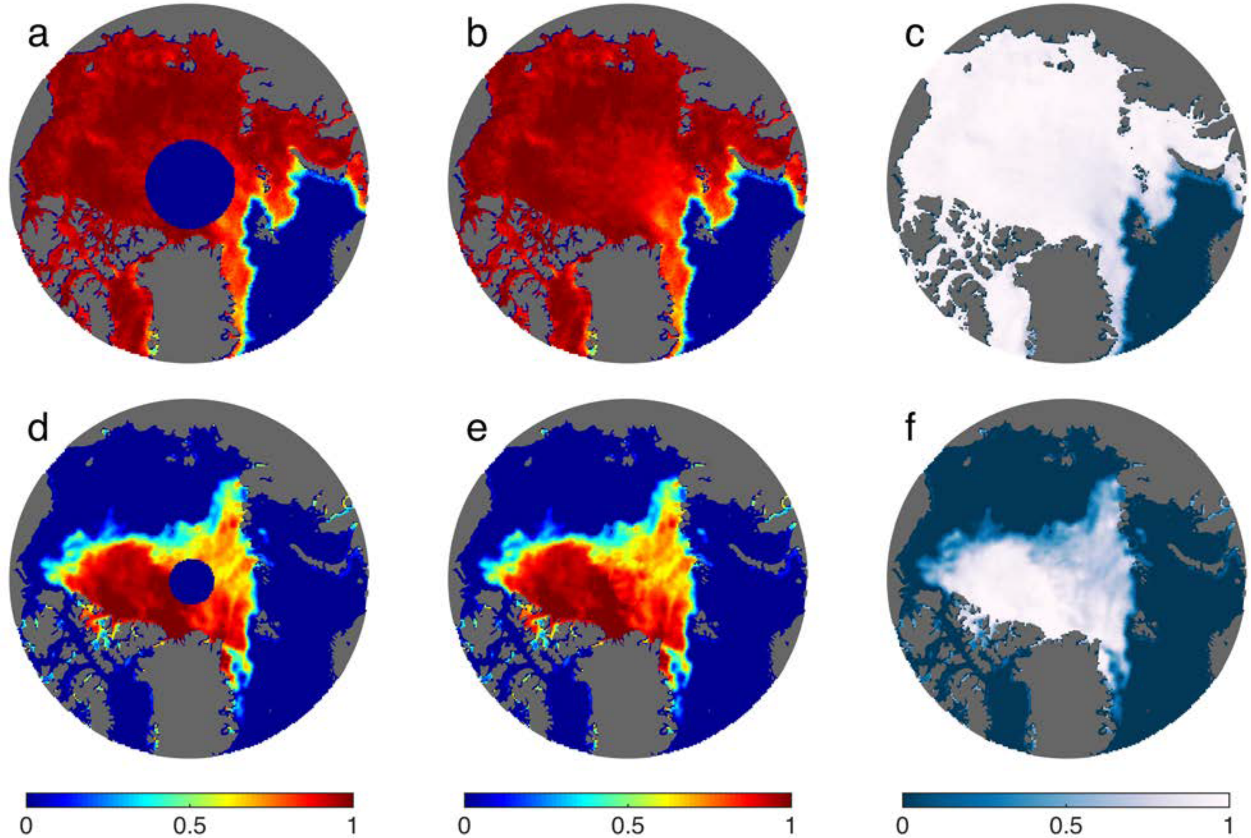
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

fill = harmonic function satisfying satellite BC's plus learned stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

Global Sea Ice Concentration Climate Data Records, 2022
Lavergne, Sorensen, et al., Norwegian Met. Inst., ... OSI SAF

Conclusions

Our research is helping to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

Mathematics for sea ice advances the theory of composites, inverse problems, and other areas of science and engineering.

Modeling sea ice leads to unexpected areas of math and physics.

University of Utah Sea Ice Modeling Group (2017-2023)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Professor of Atmospheric Sciences
Ben Murphy, Adjunct Assistant Professor of Mathematics

Postdoctoral Researchers: **Noa Kraitzman, Jody Reimer, Bohyun Kim, Debdeep Bhattacharya**

Graduate Students: Kyle Steffen (now at UT Austin)
Christian Sampson (now at NCAR)
Huy Dinh (MURI sea ice Postdoc at NYU/Courant)
Rebecca Hardenbrook (-> Dartmouth Postdoc)
David Morison (Physics Department)
Ryleigh Moore, Delaney Mosier, Daniel Hallman,
Julie Sherman, Anthony Jajeh

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich,
Dane Gollero, Samir Suthar, Anna Hyde, Kitsel
Lusted, Ruby Bowers, Kimball Johnston, Jerry
Zhang, Nash Ward, David Gluckman, Kayla
Stewart, Nicole Forrester, Jaskehar Sekhon

High School Students: J. Chapman, T. Quah, D. Webb, A. Lee, A. Dorsky

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The cover is based on "Modeling Sea Ice," page 1535.

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THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999