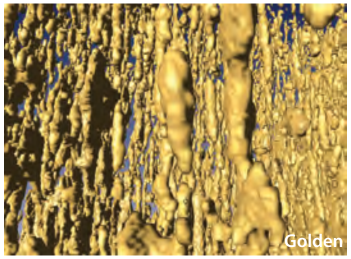
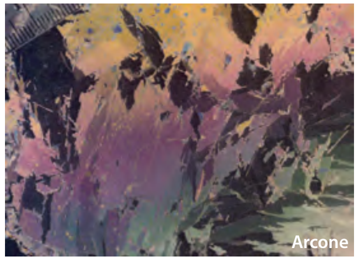
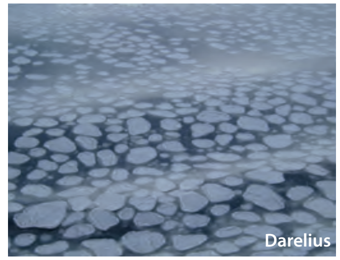
millimeters



centimeters

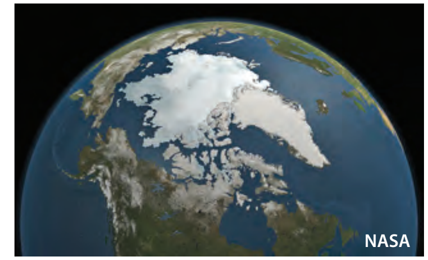


meters



kilometers

10^3 kilometers

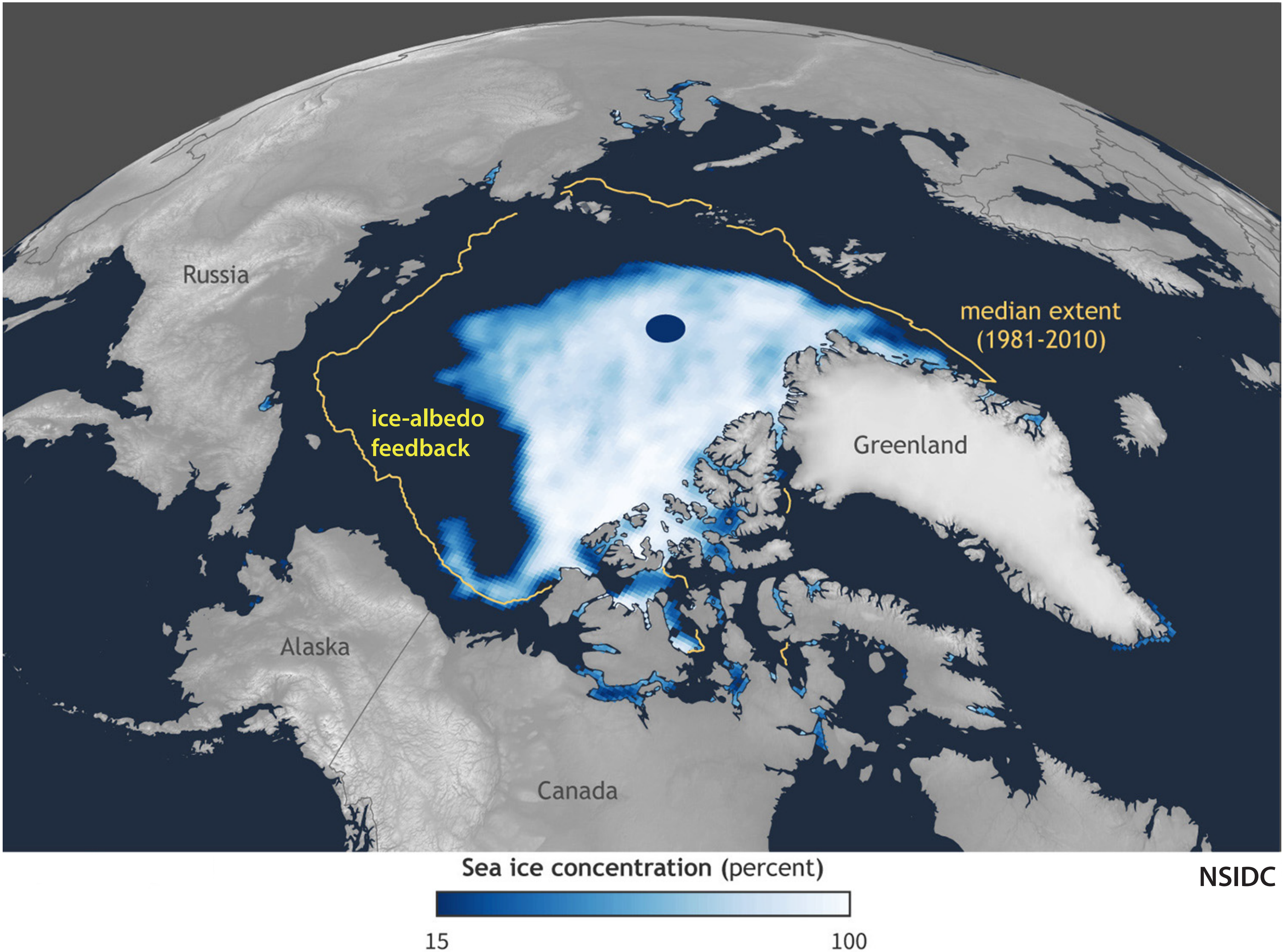


Modeling the Melt

What math tells us about disappearing polar sea ice

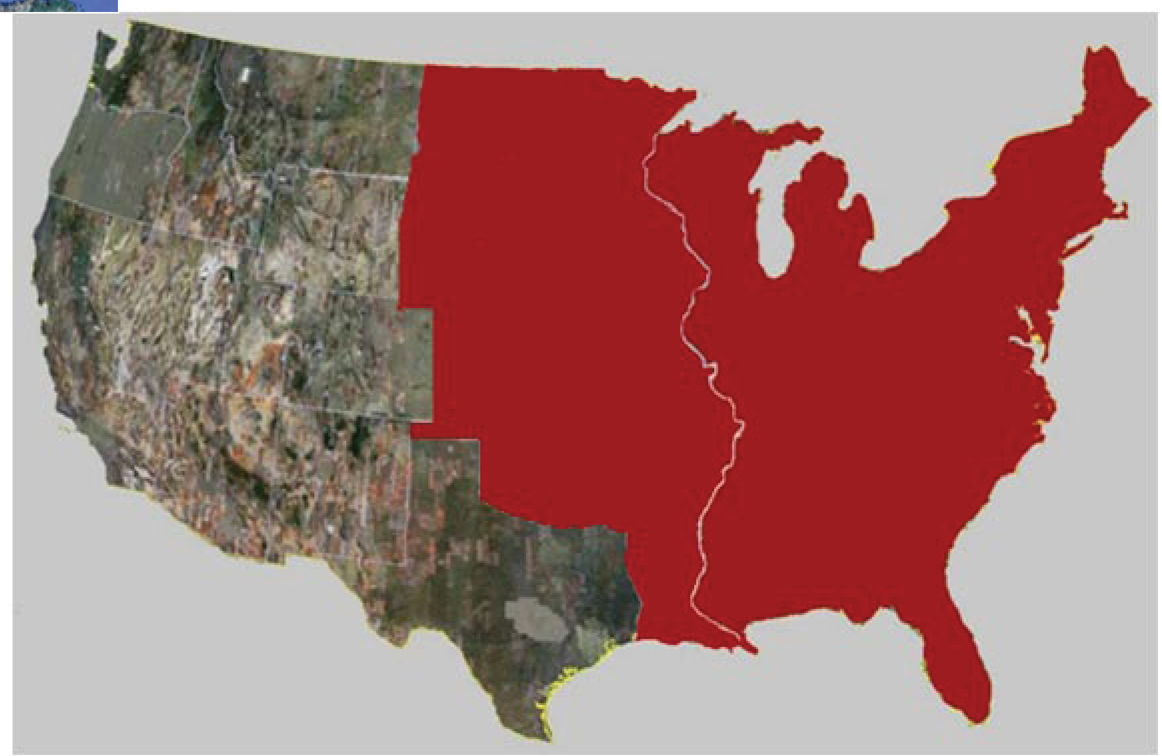
Ken Golden, University of Utah

Arctic sea ice extent September 15, 2020



NSIDC

recent losses
in comparison to
the United States

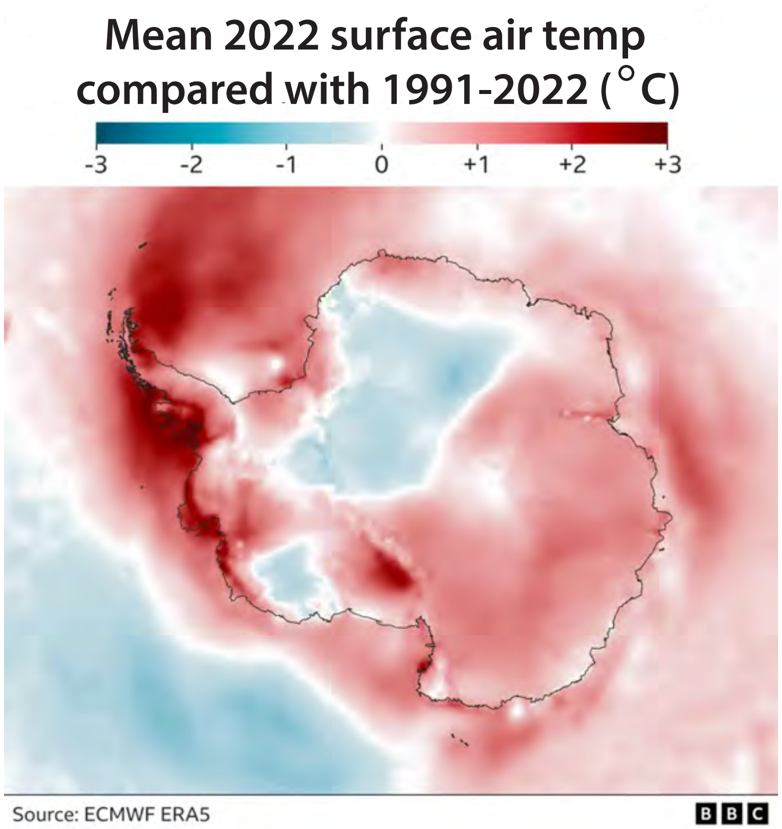


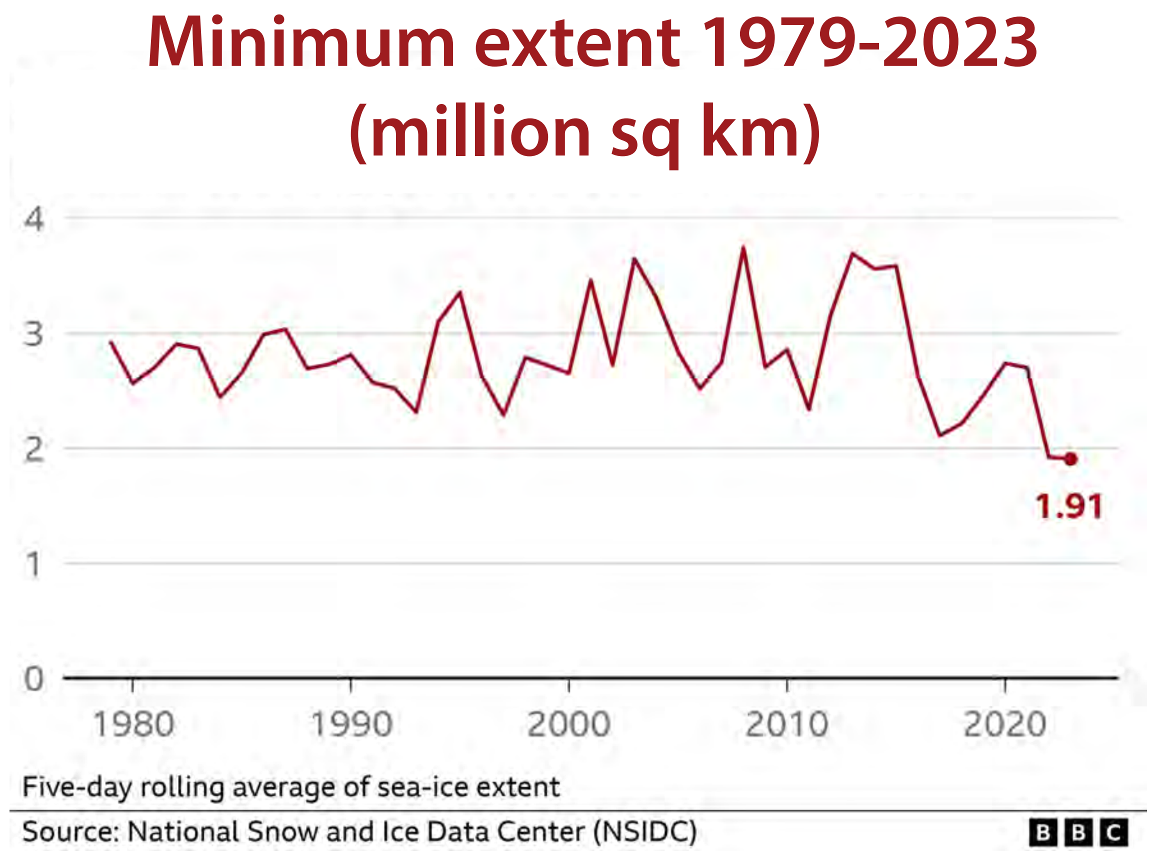
Perovich

New Record Low for Antarctic Sea Ice

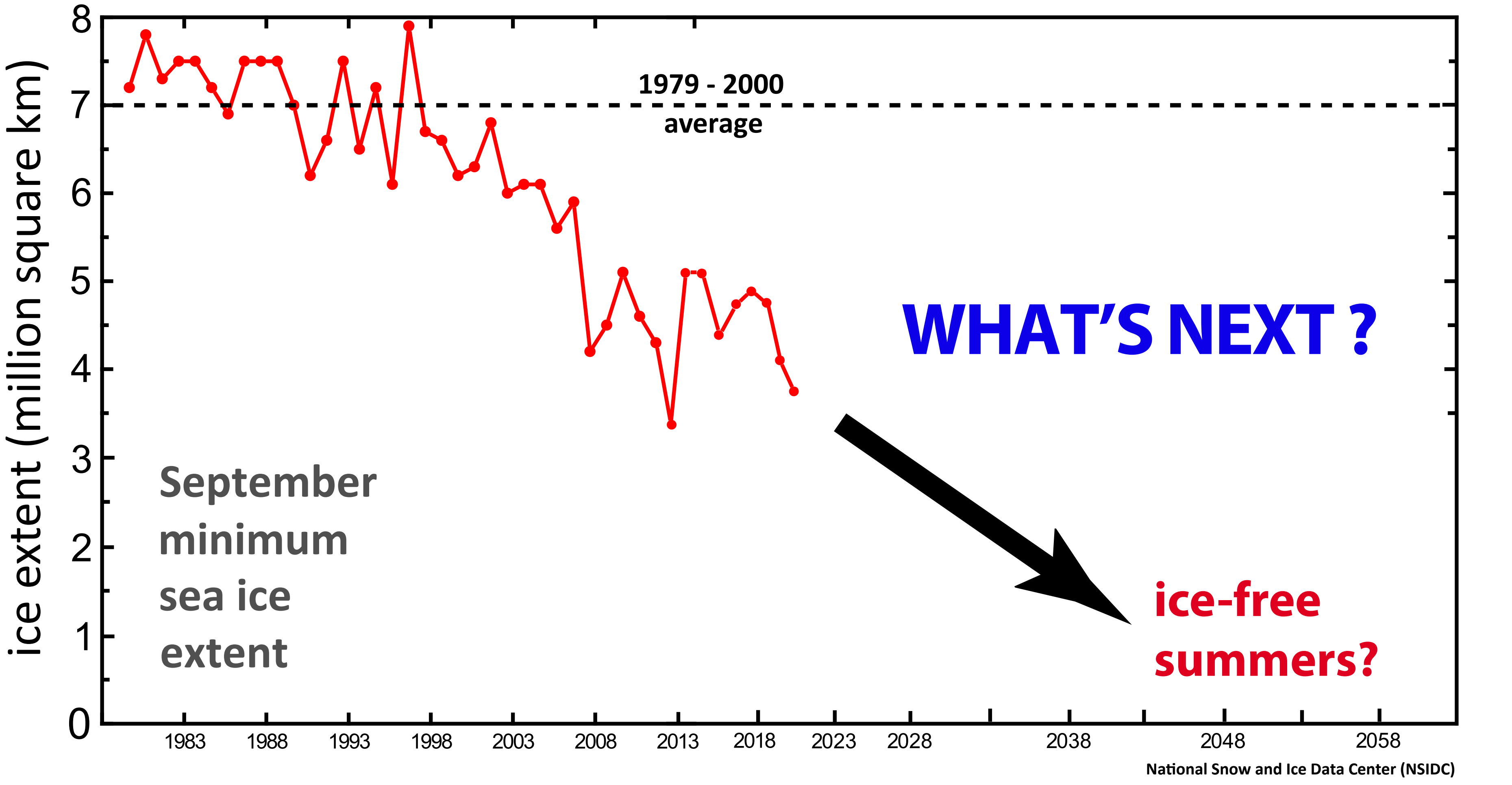
February 13, 2023

Much of Antarctica warmer than average





ARCTIC summer sea ice loss



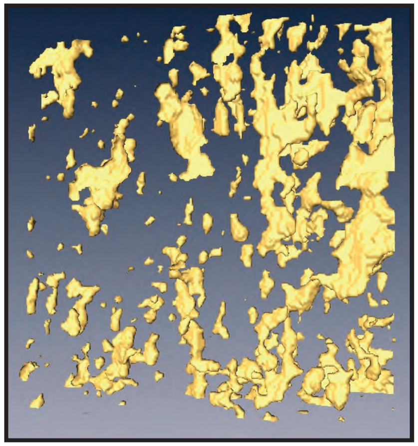
predictions require lots of math modeling

Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

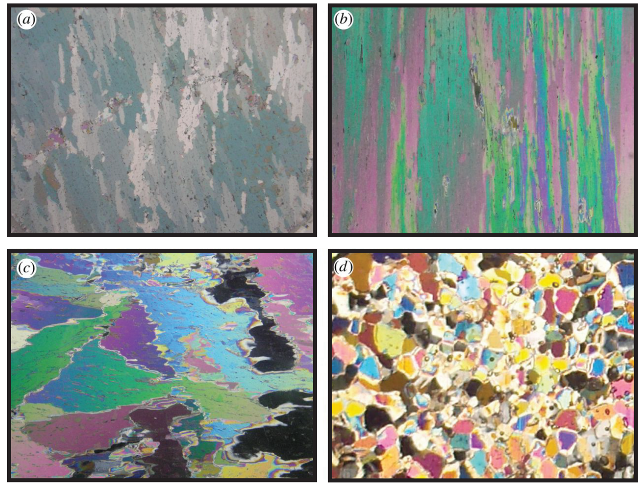
Weeks & Assur 1969



H. Eicken

Golden *et al. GRL* 2007

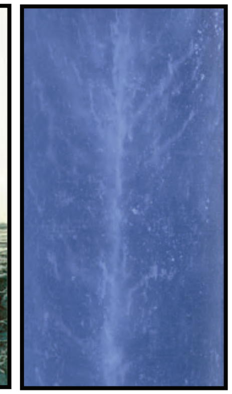
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

millimeters

centimeters

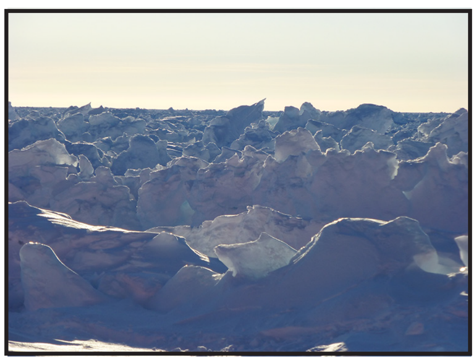
---

mesoscale

Arctic melt ponds

K. Frey

Antarctic pressure ridges



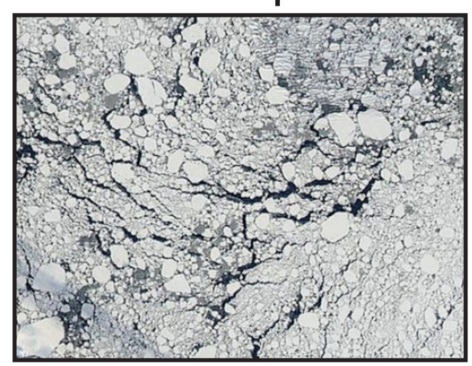
K. Golden

macroscale

sea ice floes

J. Weller

sea ice pack

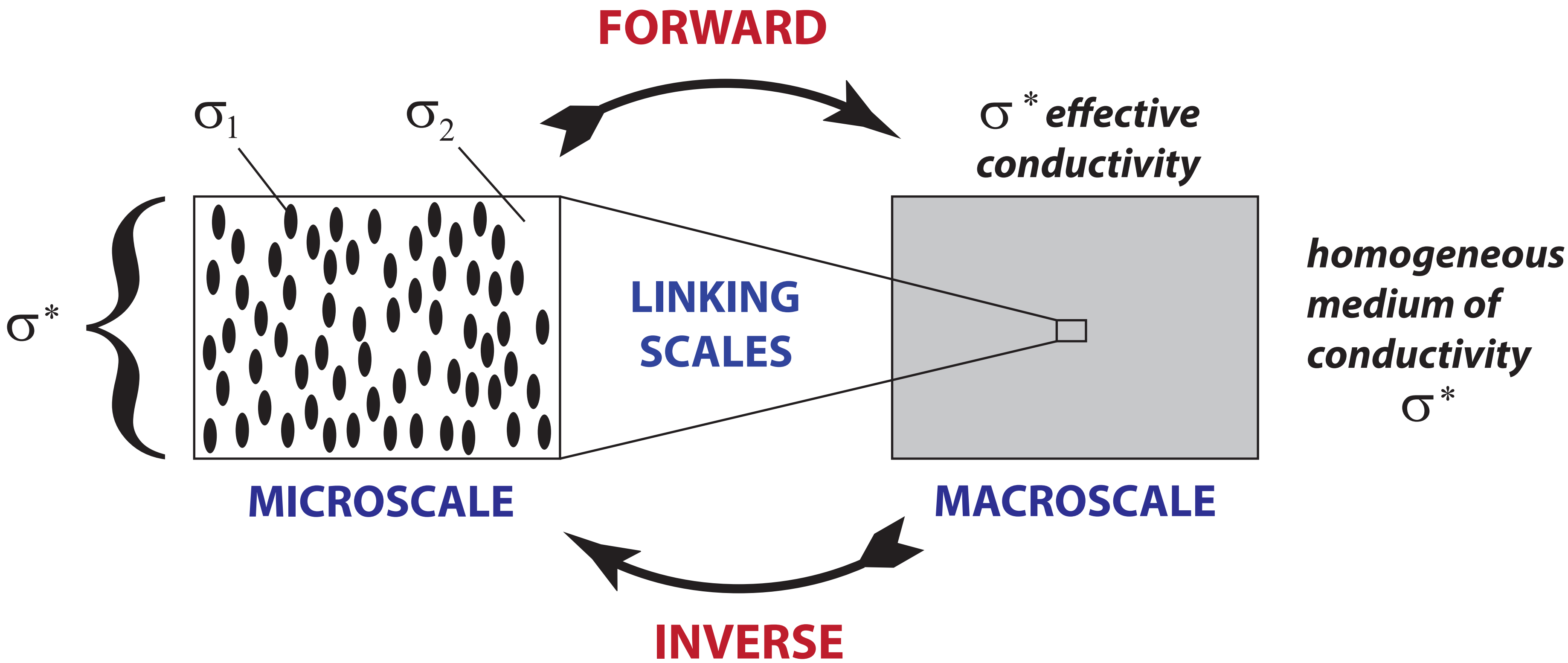


NASA

meters

kilometers

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

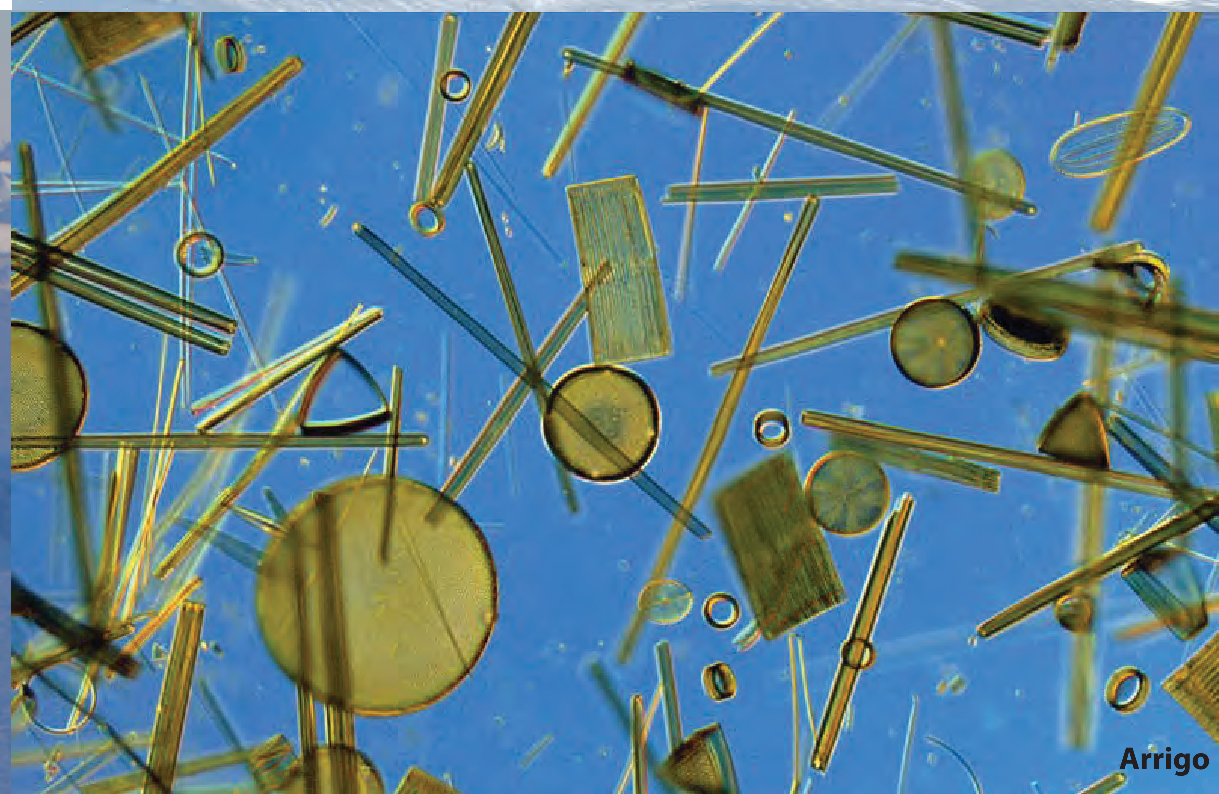
widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

microbes, megafauna, and the physics of sea ice

How do sea ice properties affect the life it hosts?

How does life in and on sea ice affect its physical properties?

Golden



Arrigo

What is our research about?

Using math, physics & engineering to model sea ice and its ecosystems, advance representation of sea ice in climate models & IMPROVE PROJECTIONS.

Inputs

THEORY of COMPOSITES

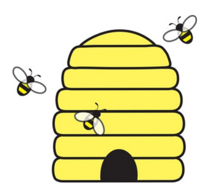
- electrical engineering
- porous media, rocks
- physics of magnets & semiconductors
- fractal geometry
- data science

MODELING SEA ICE & ECOSTEMS

microscale

mesoscale

macroscale

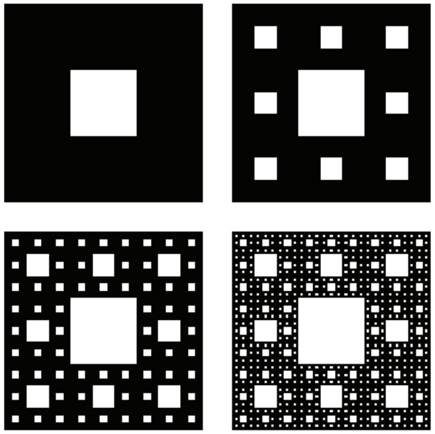


Outputs

CLIMATE MODELING

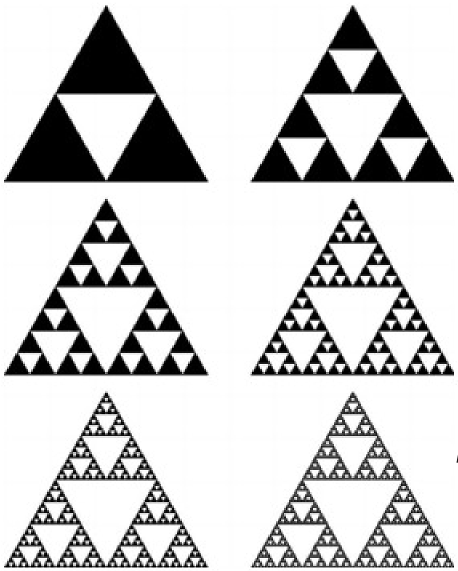
- sea ice physics & biology
- novel composites, polycrystals
- remote sensing
- biomedical imaging & biomaterials

cross pollination

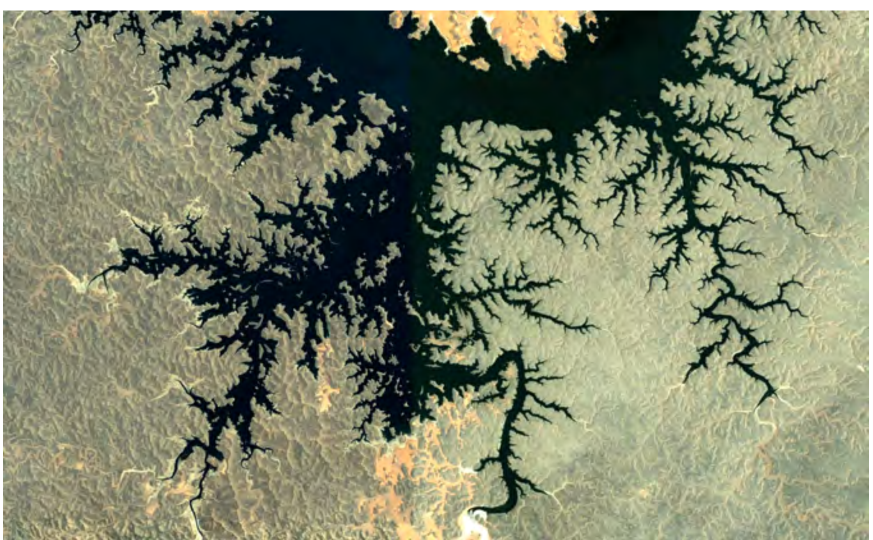


fractals

self-similar structure
non-integer dimension



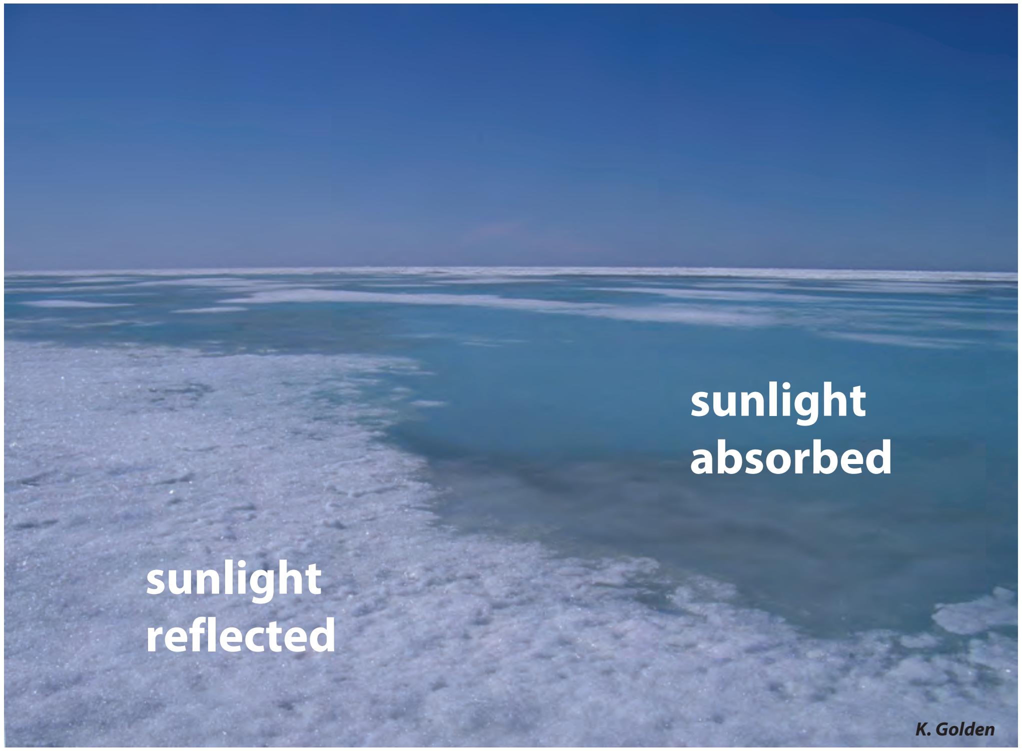
$$D = \frac{\log 3}{\log 2} = 1.585...$$



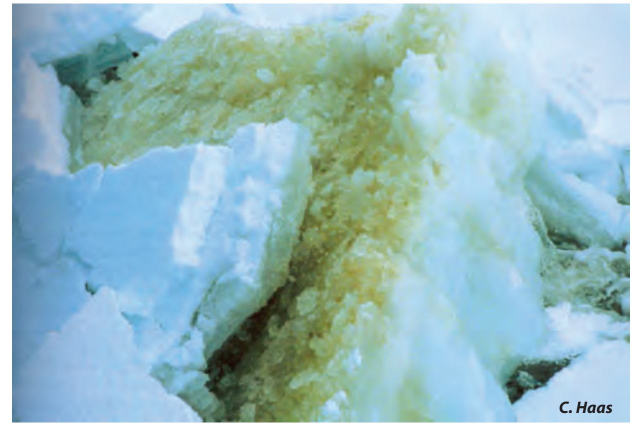
microscale

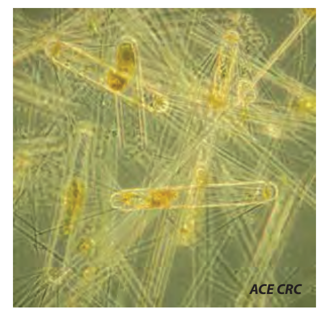
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

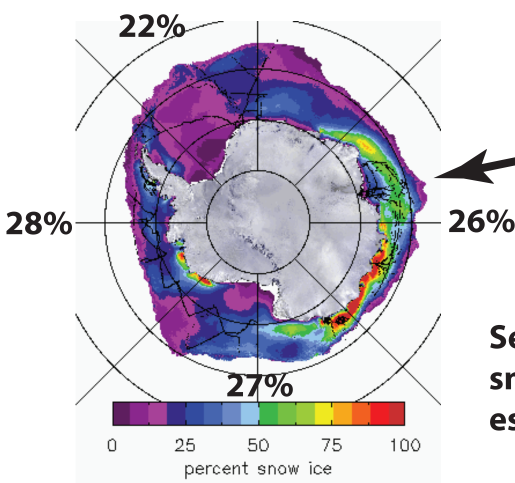
evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







T. Maksym and T. Markus, 2008

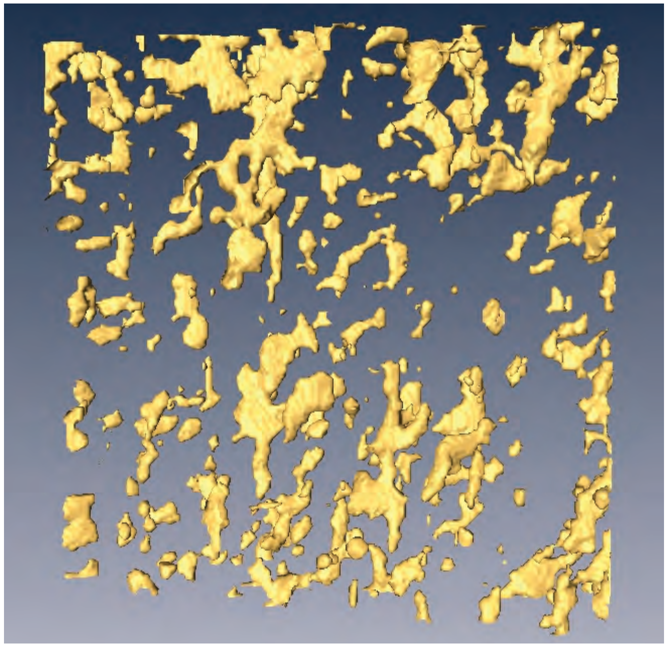
September snow-ice estimates

Antarctic surface flooding and snow-ice formation

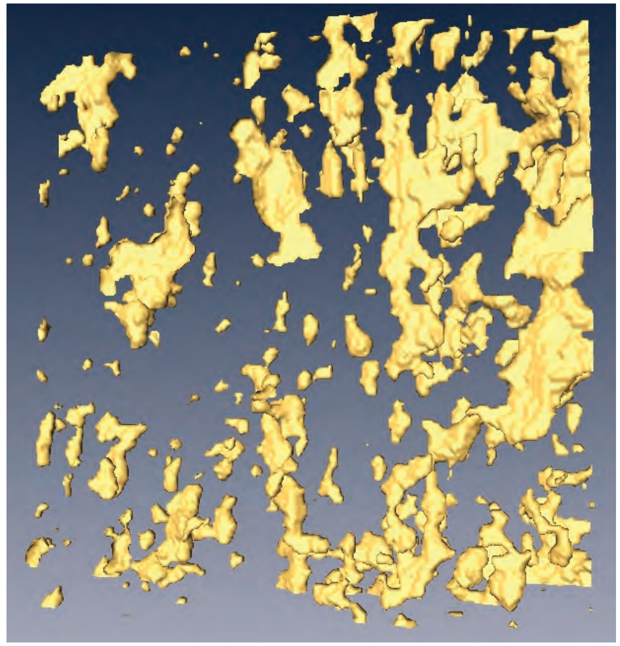
- evolution of salinity profiles

- ocean-ice-air exchanges of heat, CO_2

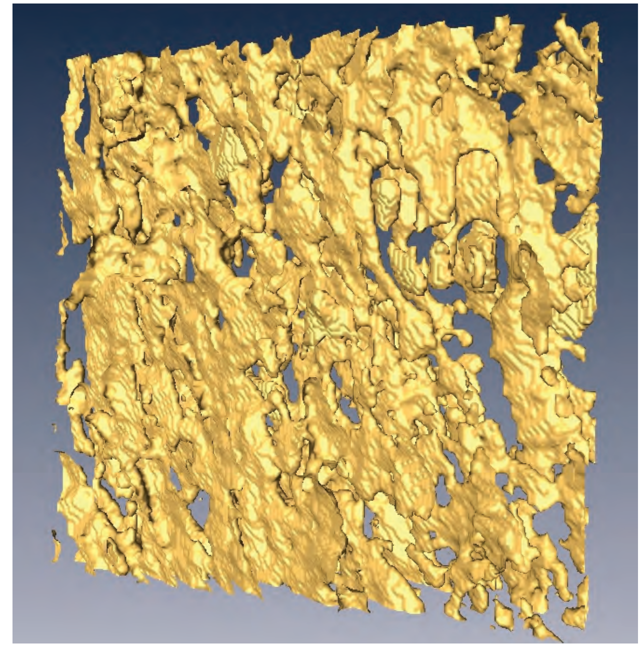
brine volume fraction and ***connectivity*** increase with temperature



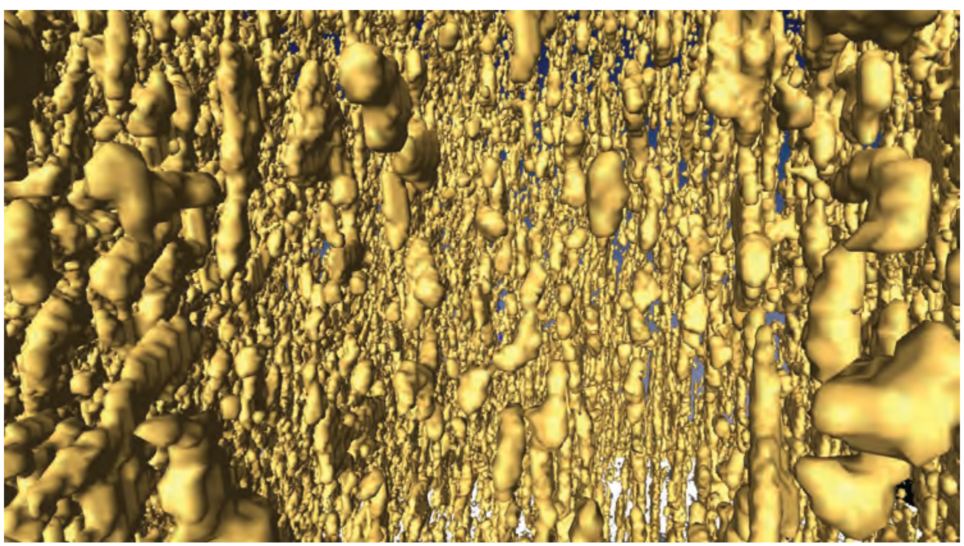
T = -15 °C, $\phi = 0.033$



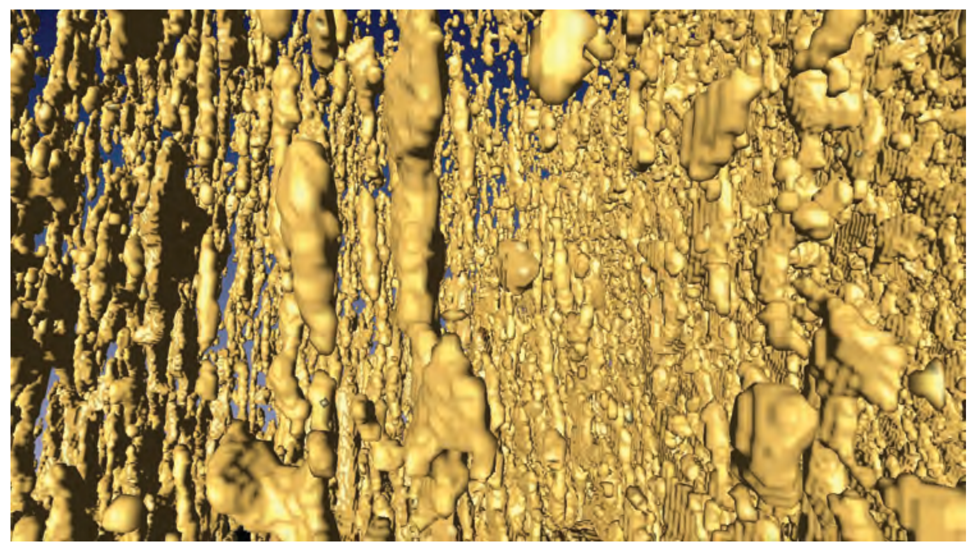
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$



T = -8 °C, $\phi = 0.057$



T = -4 °C, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

Critical behavior of fluid transport in sea ice

Arctic field data

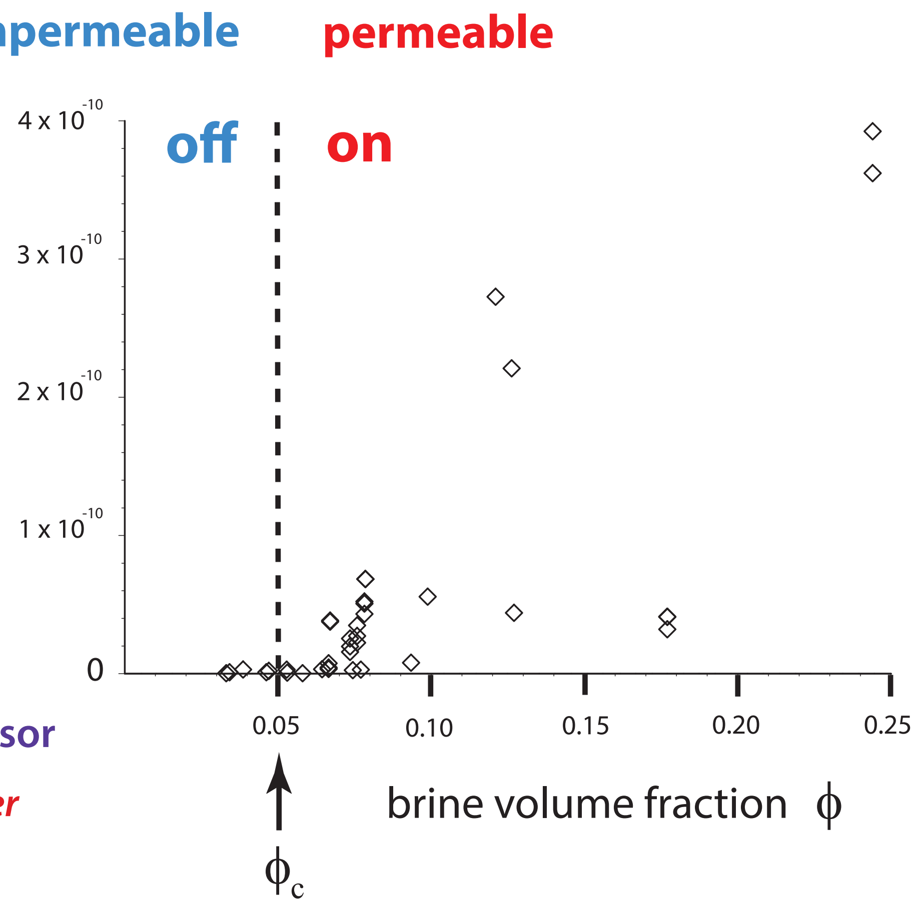
vertical fluid permeability k (m^2)

Darcy's Law

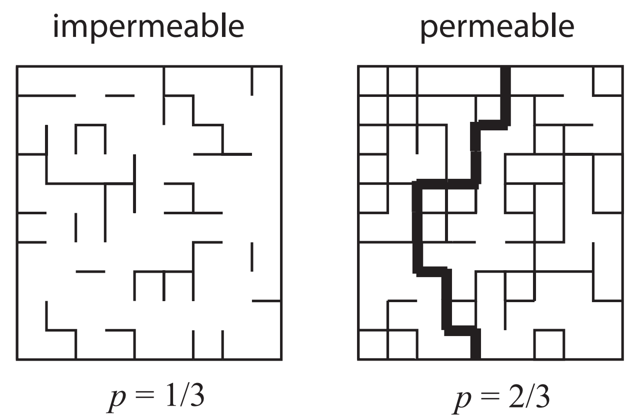
$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

$\mathbf{k}$ = fluid permeability tensor

homogenized parameter



"on - off" switch for bulk fluid flow



lattice percolation

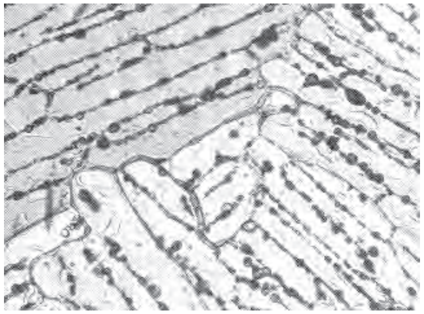
FRACTAL percolation clusters

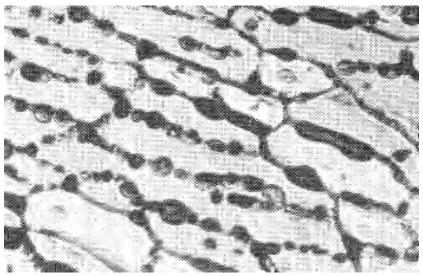
PERCOLATION THRESHOLD $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C, \; S \approx 5$ ppt

RULE OF FIVES

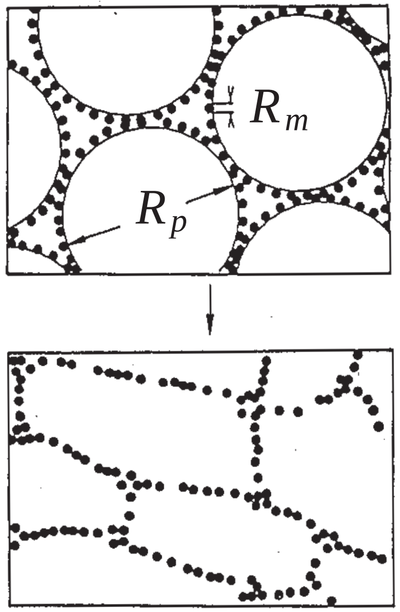
Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

cross pollination

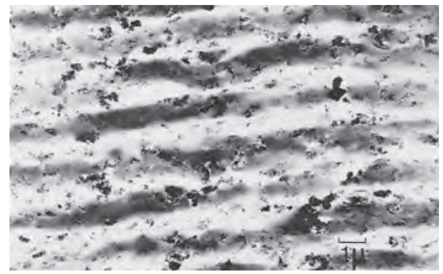




sea ice



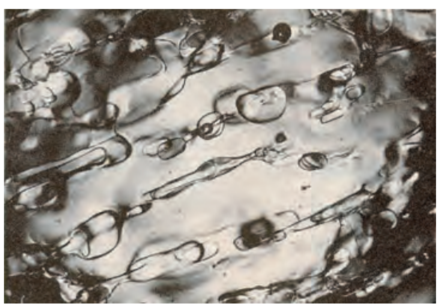
compressed powder



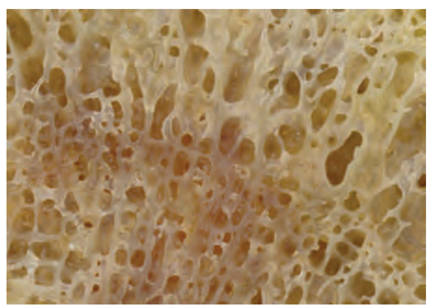
radar absorbing composite

use stealth technology to predict 5%

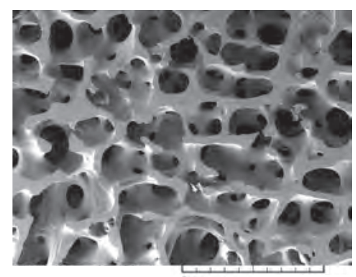
sea ice is a radar absorbing composite!



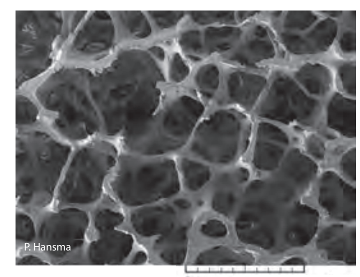
sea ice



human bone



young healthy trabecular bone



old osteoporotic trabecular bone

the math doesn't care if it's sea ice or bone!

new method of monitoring osteoporosis from sea ice

Golden, Murphy, Cherkaev, J. Biomech. 2011 mesoscale

melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

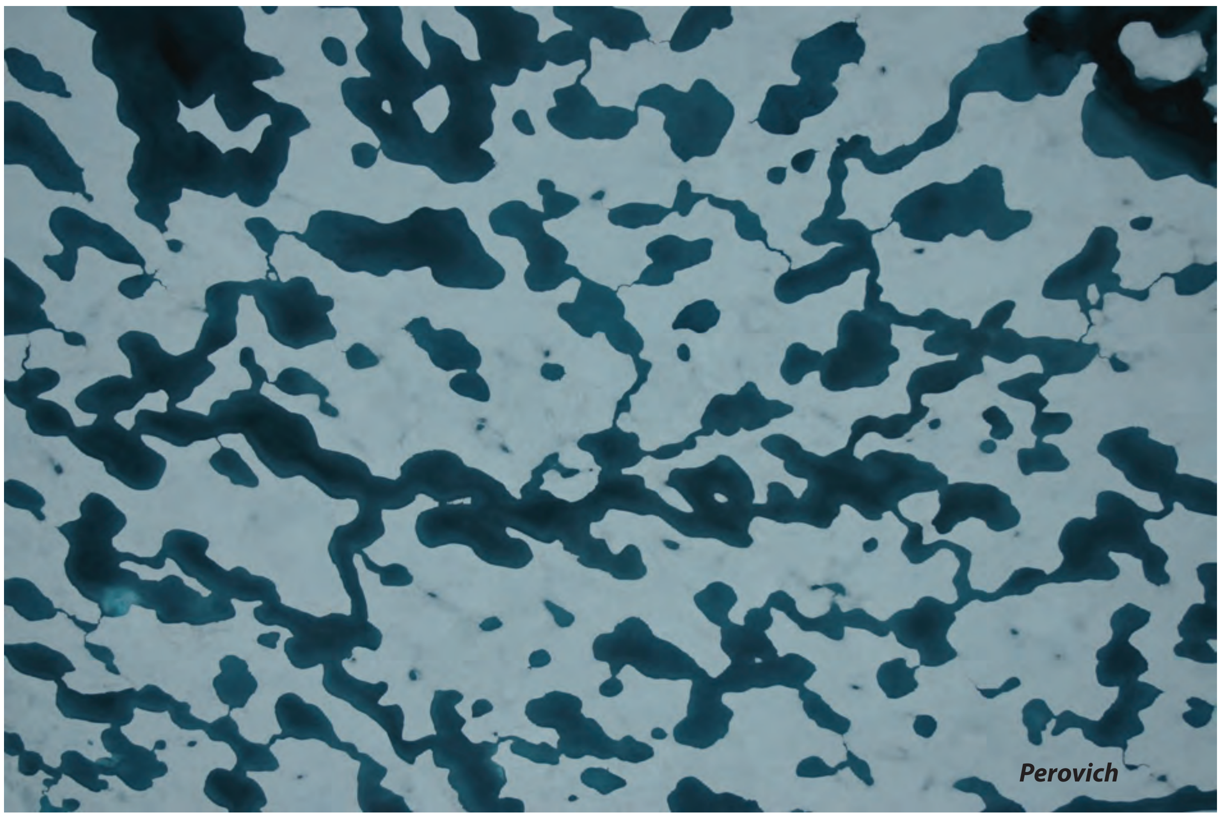
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

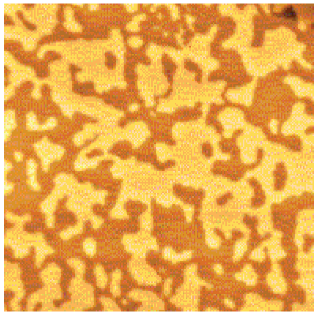
Flocco, Feltham, Hunke 2012

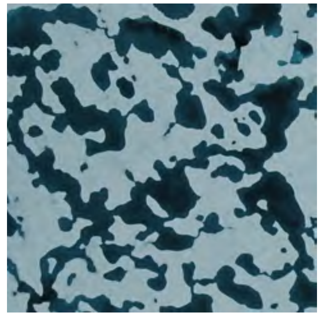


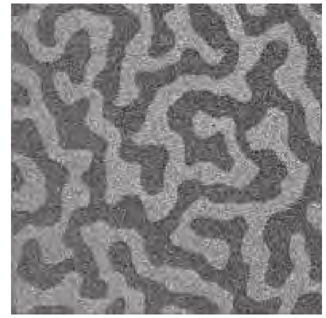
Are there universal features of the evolution similar to phase transitions in statistical physics?

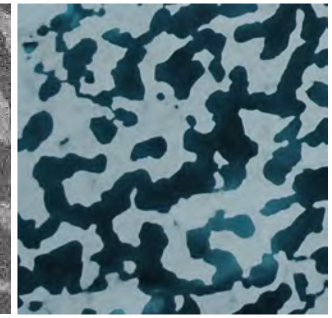
From magnets to melt ponds

100 year old model for magnetic materials used to explain melt pond geometry

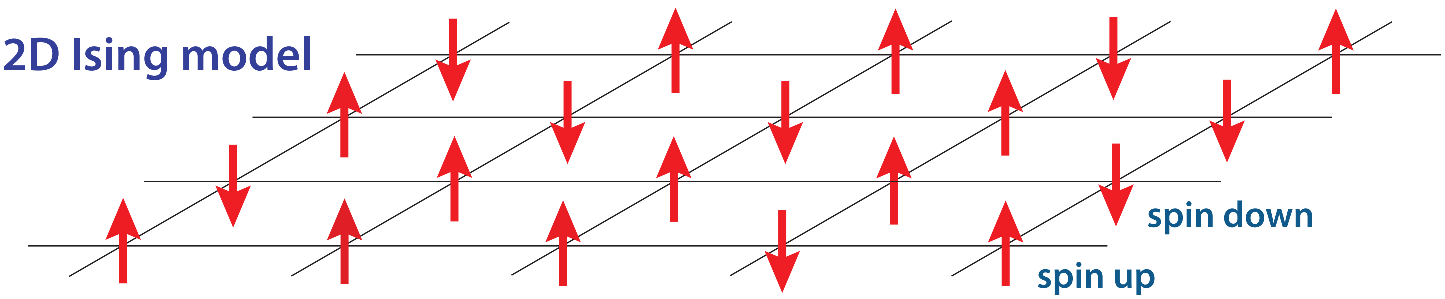

magnetic domains in cobalt

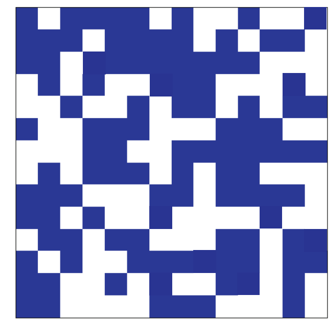

Arctic melt ponds


magnetic domains in cobalt-iron-boron

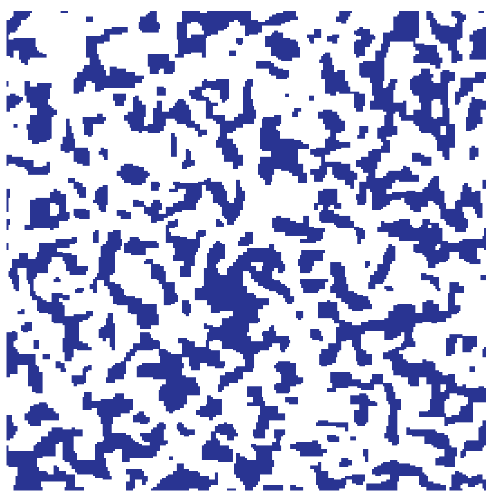

Arctic melt ponds

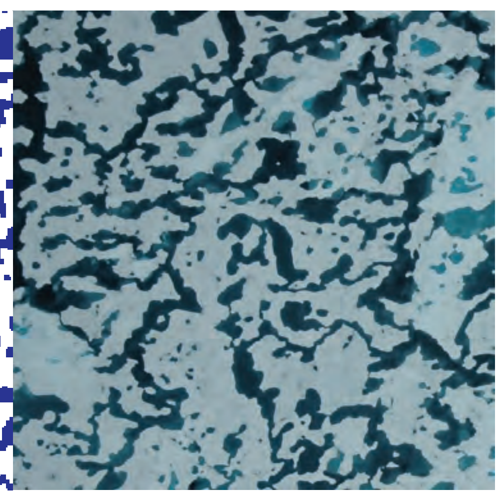
2D Ising model





model





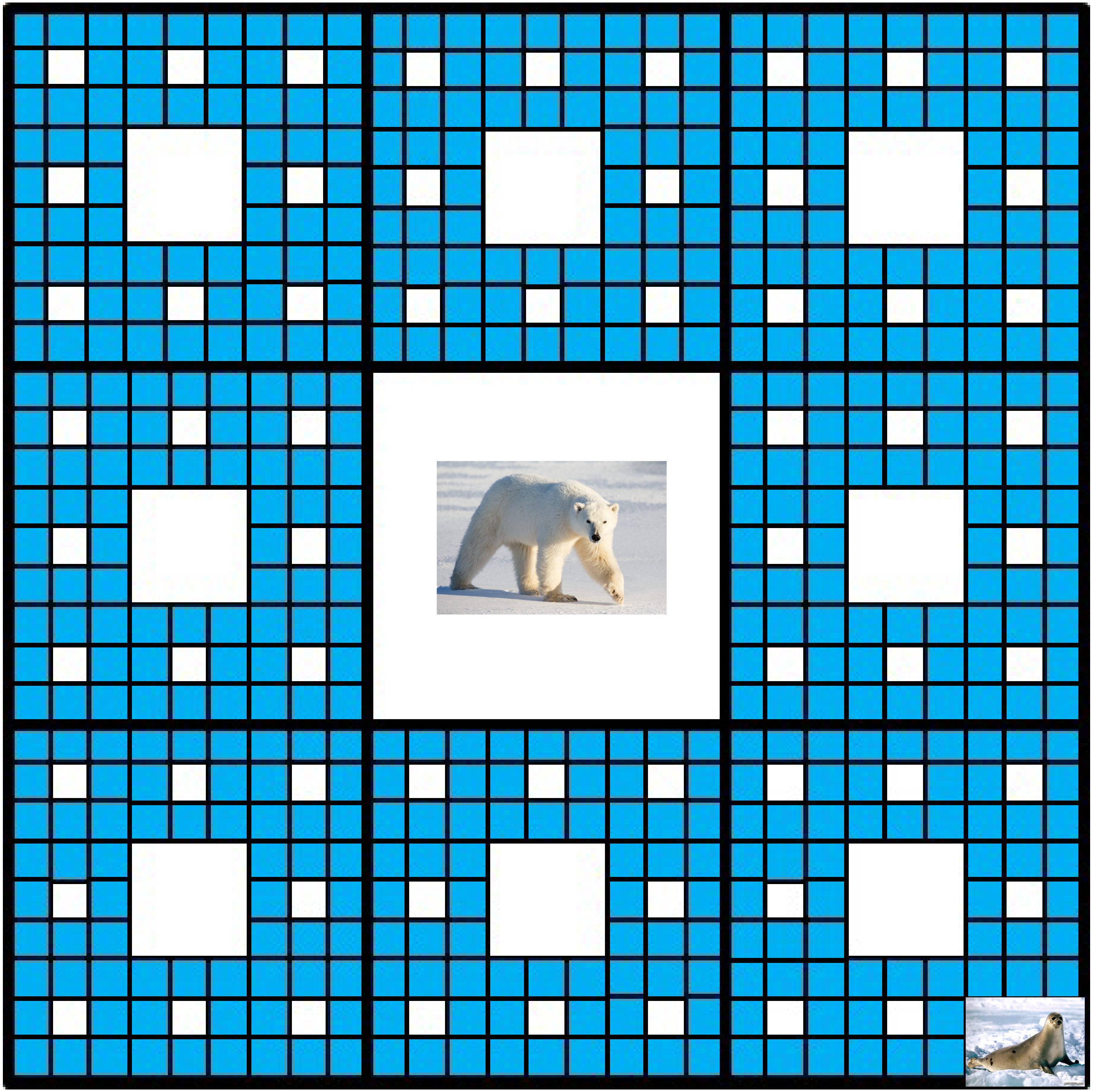
real ponds (Perovich)

Ma, Sudakov, Strong, Golden, *New J. Phys.* 2019

Scientific American, EOS, PhysicsWorld, ...

polar bear foraging in a fractal icescape

Nicole Forrester
Jody Reimer
Ken Golden



It costs the polar bear 5 times the energy to swim through water than to walk on sea ice.

What pathway to a seal minimizes energy spent?

macroscale

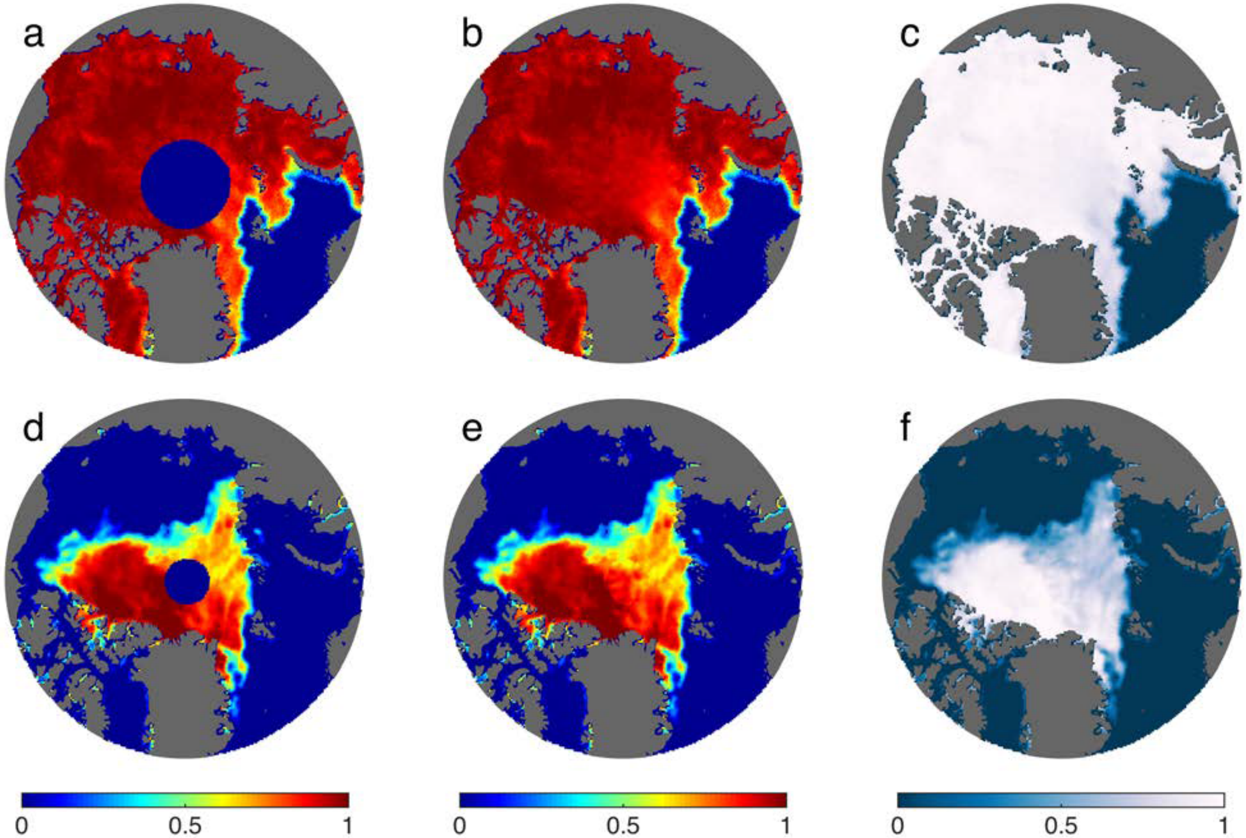
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

fill = harmonic function satisfying satellite BC's plus learned stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

Global Sea Ice Concentration Climate Data Records, 2022
Lavergne, Sorensen, et al., Norwegian Met. Inst., ... OSI SAF

Conclusions

Our research is helping to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

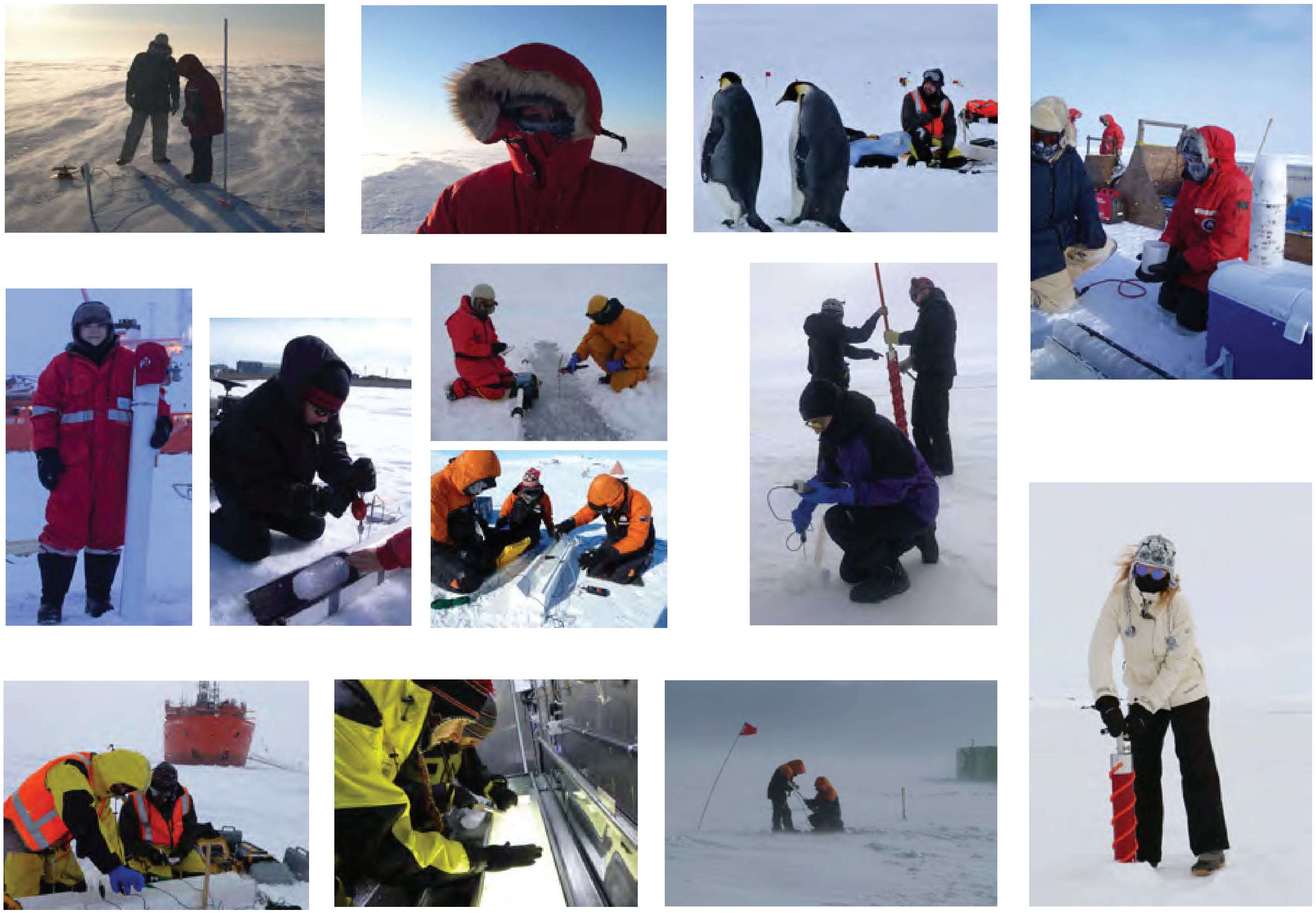
Developing MATH for sea ice advances theories of composites, imaging, climate, computation, data science and other areas.

MATH enables CROSS POLLINATION.

The math doesn't care if it's sea ice, magnets, bone, or a stealthy coating!

... leading us to unexpected areas of math, physics & engineering.

Thank you to so many postdocs, graduate students, undergraduates, high school students and colleagues who contributed to this work!



U. of Utah students in the Arctic and Antarctic (2003-2022): closing the gap between theory and observation - making math models come alive and experiencing climate change firsthand.

Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999