On Thinning Ice: Modeling Sea Ice in a Warming Climate

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ANTARCTICA

southern cryosphere

Weddell Sea

East Antarctic Ice Sheet

West Antarctic Ice Sheet

Ross Sea

sea ice

northern cryosphere



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

white snow and ice reflect







dark water and land absorb

albedo
$$\alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

the summer Arctic sea ice pack is melting



National Snow and Ice Data Center (NSIDC)

Arctic sea ice extent

September 15, 2020





recent losses in comparison to the United States



Perovich

thicker multiyear ice being replaced by thinner first year ice



... and sea ice volume is declining

1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012

Version:

IC-SST 2.0

Predicting what may come next requires lots of math modeling.



National Snow and Ice Data Center (NSIDC)

challenge:

Represent sea ice more realistically in climate models to improve projections.



How do patterns of dark and light evolve?



Account for key processes

e.g. melt pond evolution

Including PONDS in simulations LOWERS predicted sea ice volume over time by 40%.

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

... and other sub-grid scale structures and processes. *linkage of scales*

Sea Ice is a Multiscale Composite Material *microscale*

brine inclusions



H. Eicken

Golden et al. GRL 2007

Weeks & Assur 1969

millimeters

polycrystals



Gully et al. Proc. Roy. Soc. A 2015

centimeters

brine channels



D. Cole

K. Golden

mesoscale

macroscale

Arctic melt ponds



Antarctic pressure ridges





sea ice floes

sea ice pack





K. Golden

J. Weller

kilometers

NASA

meters

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...



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Physics of sea ice drives advances in many areas of science and engineering.

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Physics of sea ice drives advances in many areas of science and engineering.

How do scales interact in the sea ice system?



basin scale grid scale albedo

Linking Scales

km scale melt ponds





Perovich

Scales







meter scale snow topography

sea ice microphysics

fluid transport





sea ice may appear to be a barren, impermeable cap ...



brine inclusions in sea ice (mm)



micro - brine channel (SEM)

brine channels (cm)

sea ice is a porous composite

pure ice with brine, air, and salt inclusions





horizontal section

vertical section

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







Antarctic surface flooding and snow-ice formation

evolution of salinity profiles
ocean-ice-air exchanges of heat, CO₂

sea ice ecosystem



sea ice algae support life in the polar oceans

brine volume fraction and *connectivity* increase with temperature



$T = -15 \,^{\circ}\text{C}, \ \phi = 0.033$ $T = -6 \,^{\circ}\text{C}, \ \phi = 0.075$ $T = -3 \,^{\circ}\text{C}, \ \phi = 0.143$



 $T = -8^{\circ} C, \phi = 0.057$

X-ray tomography for brine in sea ice



 $T = -4^{\circ} C, \phi = 0.113$

Golden et al., Geophysical Research Letters, 2007

Critical behavior of fluid transport in sea ice



critical brine volume fraction $\phi_c \approx 5\%$ \checkmark $T_c \approx -5^{\circ}C, S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998 Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007 Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

nutrient replenishment controlled by ice permeability

biological activity turns on or off according to *rule of fives*

Golden, Ackley, Lytle

Science 1998

Fritsen, Lytle, Ackley, Sullivan Science 1994

critical behavior of microbial activity



percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open with probability p closed with probability 1-p*

percolation threshold $p_c = 1/2$ for d = 2

smallest *p* for which there is an infinite open cluster

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

 $\phi_c \approx 5\%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton^{*}, Miner, Pringle, Zhu, Geophysical Research Letters 2007



from critical path analysis in hopping conduction

critical

exponent

hierarchical model rock physics network model rigorous bounds

X-ray tomography for

confirms rule of fives

Pringle, Miner, Eicken, Golden

theories agree closely with field data

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport? How does the biology affect the physics?



- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability k; results predict observed drop in k

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden Multiscale Modeling and Simulation, 2018



Zhu, Jabini, Golden, Eicken, Morris *Ann. Glac.* 2006

tracers flowing through inverted sea ice blocks







Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	Antarctic	SIPEX
2010	Antarctic	McMurdo Sound
2011	Arctic	Barrow AK
2012	Arctic	Barrow AK
2012	Antarctic	SIPEX II
2013	Arctic	Barrow AK
2014	Arctic	Chukchi Sea



Remote sensing of sea ice



sea ice thickness ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

8*

effective complex permittivity (dielectric constant, conductivity)



brine volume fraction brine inclusion connectivity Bounds on the complex permittivity of polycrystalline materials by analytic continuation

> Adam Gully, Joyce Lin, Elena Cherkaev, Ken Golden

 Stieltjes integral representation for effective complex permittivity

Milton (1981, 2002), Barabash and Stroud (1999), ...

- Forward and inverse bounds orientation statistics
- Applied to sea ice using two-scale homogenization
- Inverse bounds give method for distinguishing ice types using remote sensing techniques





Proc. Roy. Soc. A 8 Feb 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A



An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques A computer model to determine how a human should walk so as to expend the least energy



higher threshold for fluid flow in granular sea ice

granular

microscale details impact "mesoscale" processes

5%

columnar

nutrient fluxes for microbes melt pond drainage snow-ice formation

10%

Golden, Sampson, Gully, Lubbers, Tison 2021

electromagnetically distinguishing ice types Kitsel Lusted, Elena Cherkaev, Ken Golden

wave propagation in the marginal ice zone (MIZ)



Sampson, Murphy, Cherkaev, Golden 2021

first theory of key parameter in wave-ice interactions only fitted to wave data before

Analytic Continuation Method

Bergman (78) - Milton (79) integral representation for ε^{*} Golden and Papanicolaou (83) Milton, *Theory of Composites* (02)



homogenized parameter depends on sea ice concentration and ice floe geometry

like EM waves



advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice heat transport in sea ice with convection sea ice floes in winds and ocean currents tracers, buoys diffusing in ocean eddies diffusion of pollutants in atmosphere

advection diffusion equation with a velocity field $ec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$
$$\vec{\nabla} \cdot \vec{u} = 0$$
$$homogenize$$
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, Ann. Math. Sci. Appl. 2017 Murphy, Cherkaev, Zhu, Xin, Golden, J. Math. Phys. 2020









melt pond formation and albedo evolution:

- major drivers in polar climate
- key challenge for global climate models

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006 Flocco, Feltham 2007 Skyllingstad, Paulson, Perovich 2009 Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

fractal curves in the plane

they wiggle so much that their dimension is >1



Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012



complexity grows with length scale



Ising model for ferromagnets —> Ising model for melt ponds

Ma, Sudakov, Strong, Golden, New J. Phys., 2019

 $\mathcal{H} = -\sum_{i}^{N} H_{i} s_{i} - J \sum_{\langle i,j \rangle}^{N} s_{i} s_{j} \qquad s_{i} = \begin{cases} \uparrow & +1 & \text{water (spin up)} \\ \downarrow & -1 & \text{ice (spin down)} \end{cases}$

random magnetic field represents snow topography

magnetization M

pond area fraction $F = \frac{(M+1)}{2}$

only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system "flows" toward metastable equilibria.



ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data

Order from Disorder



Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom bloom massive under-ice algal bloom

Arrigo et al., Science 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances* 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden Geophys. Res. Lett. 2019

(2015 AMS MRC)

Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km 06 January 1985

Gap radius: 311 km 30 August 2007

 $\Delta \psi = 0$



Strong and Golden, *Remote Sensing* 2016 Strong and Golden, *SIAM News* 2017 NOAA/NSIDC Sea Ice Concentration CDR product update will use our PDE method.

Conclusions

- 1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.
- 2. Mathematical methods developed for sea ice advance the theory of composites and other areas of science and engineering.
- 3. Homogenization and statistical physics help *link scales in sea ice and composites*; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.
- 4. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.
- 5. Field experiments are essential to developing relevant mathematics.
- 6. Our research is helping to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

University of Utah Sea Ice Modeling Group (2017-2021)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics Elena Cherkaev, Professor of Mathematics Court Strong, Associate Professor of Atmospheric Sciences Ben Murphy, Adjunct Assistant Professor of Mathematics

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Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)

Christian Sampson (now at UNC Chapel Hill with Chris Jones) Huy Dinh (now a sea ice MURI Postdoc at NYU/Courant) Rebecca Hardenbrook David Morison (Physics Department) Ryleigh Moore Delaney Mosier Daniel Hallman

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THANK YOU

Office of Naval Research

Applied and Computational Analysis Program Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences Division of Polar Programs











Australian Government

Department of the Environment and Water Resources Australian Antarctic Division











Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Thursday, July 23, 1998

Australia

Hobart

Macquarie

Island in

Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship Aurora Australis without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast. The extent of the damage is

not known. Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an

inert gas. ... The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known The 25 crew and 54 expedi-

tioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year rut at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the Aurora Australis in Antarctica. ision for about \$11 million year.

P&0

ision's Hobart office. He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available." Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The Aurora Australis is owned by P&O Australia and charted by the Antarctic Div-

director Richard Hein said yes-Casev terday the company was assessing the situation and a number of rescue options were being Scale considered. It was too early to say whether P&O would be liable for the cost of the aborted

Australia managing

mission. The vessel left Hobart last

Wednesday for a seven-week voyage mainly to study a polyn-ya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.

Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

Antarctica

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

THE ADVERTISER (Adelaide) Thurs 23 July 1998

Fire strands Antarctic ship in sea ice

AN engine room fire has Australian Anteretic Div- arctic continent and return disabled the icebreaker Aurora Australis in sea ico, deep in Antarotic waters. Incre were no injuries and

the ship was not in danger after Tuesday night's fire,

Moncur said. But Mr. Moncur said he expected it would have to abandon its

islon director Mr Rex to Hobart for repairs.

Page 14

The cause of the fire was not known but the engines would have to abandon its have been turned off, with pioneering mid-winter voy- the ship 100 neutron miles age to the edge of the Ant- from the Antarctic coast.

THE CANBERRA TIMES Thursday 23 July 1998 Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected Aurora Australis would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs. The fire had been extin-

guished and the engines were turned off. leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good. Crew had to wear breathing

The Aurora, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause beavy, sait-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully accessed.





scrahped following an engine-room firscripting. Aurora Australis yesterday. The 54 people on board. Weis (occid on decivin the

2:45 am July 22, 1998

``Please don't be alarmed but we have an uncontrolled fire in the engine room"

about 10 minutes later ...

``Please don't be alarmed but we're lowering the lifeboats"



of the American Mathematical Society

November 2020

Volume 67, Number 10







The cover is based on "Modeling Sea Ice," page 1535.

Modeling Sea Ice



Kenneth M. Golden, Luke G. Bennetts, Elena Cherkaev, Ian Eisenman, Daniel Feltham, Christopher Horvat, Elizabeth Hunke, Christopher Jones, Donald K. Perovich, Pedro Ponte-Castañeda, Courtenay Strong, Deborah Sulsky, and Andrew J. Wells

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