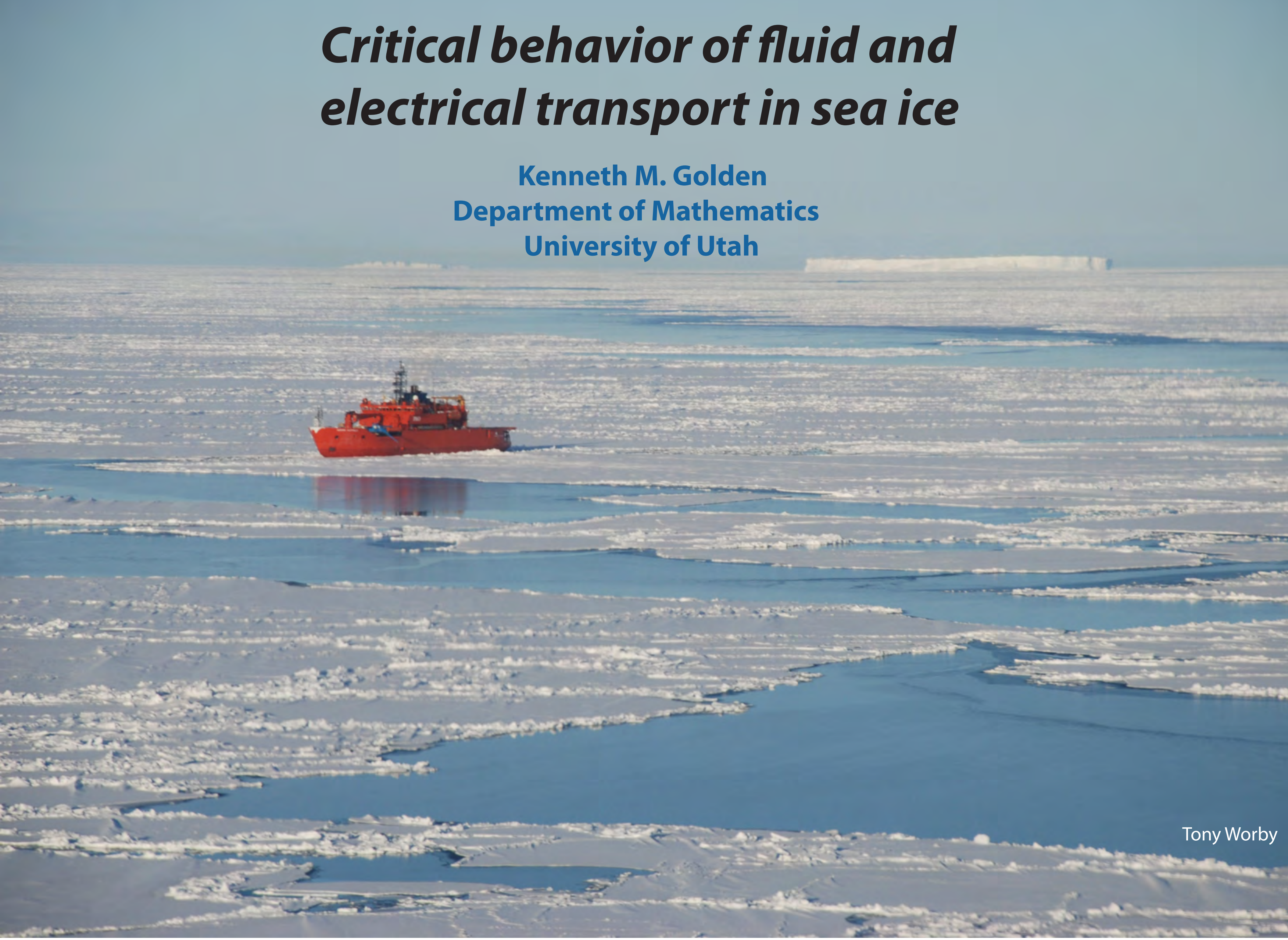
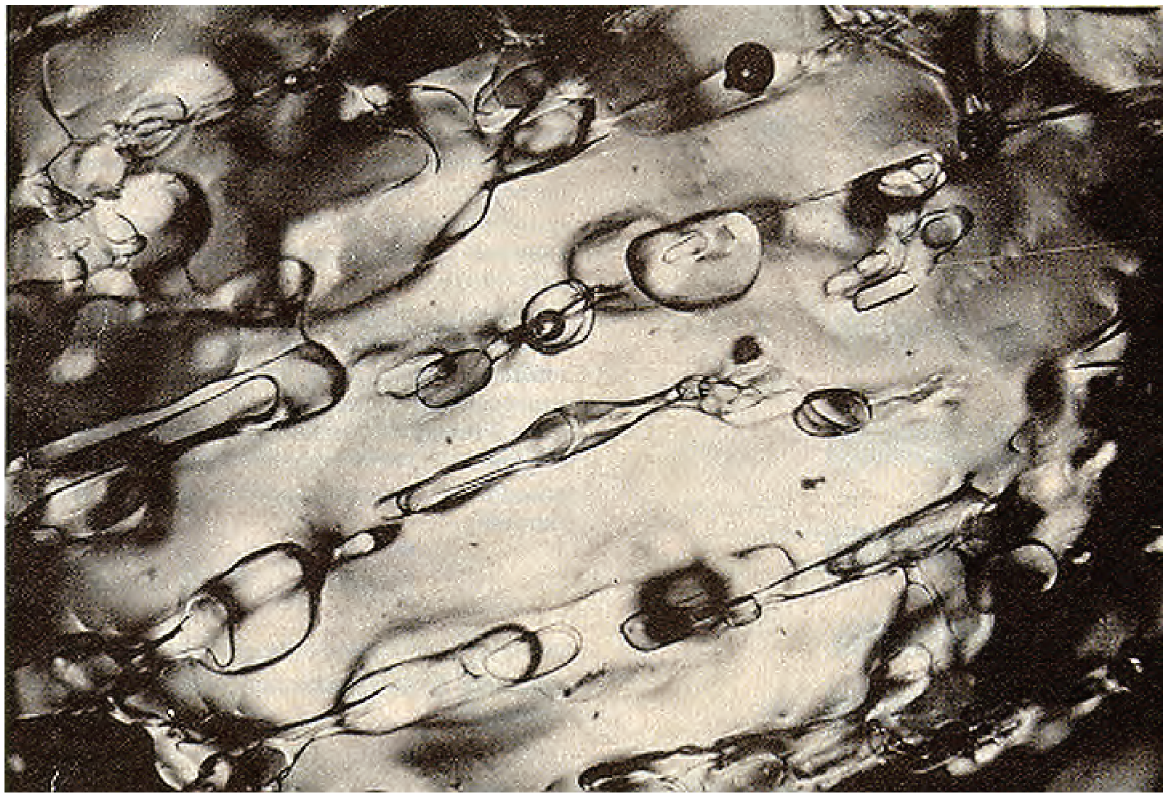
Critical behavior of fluid and electrical transport in sea ice

Kenneth M. Golden
Department of Mathematics
University of Utah



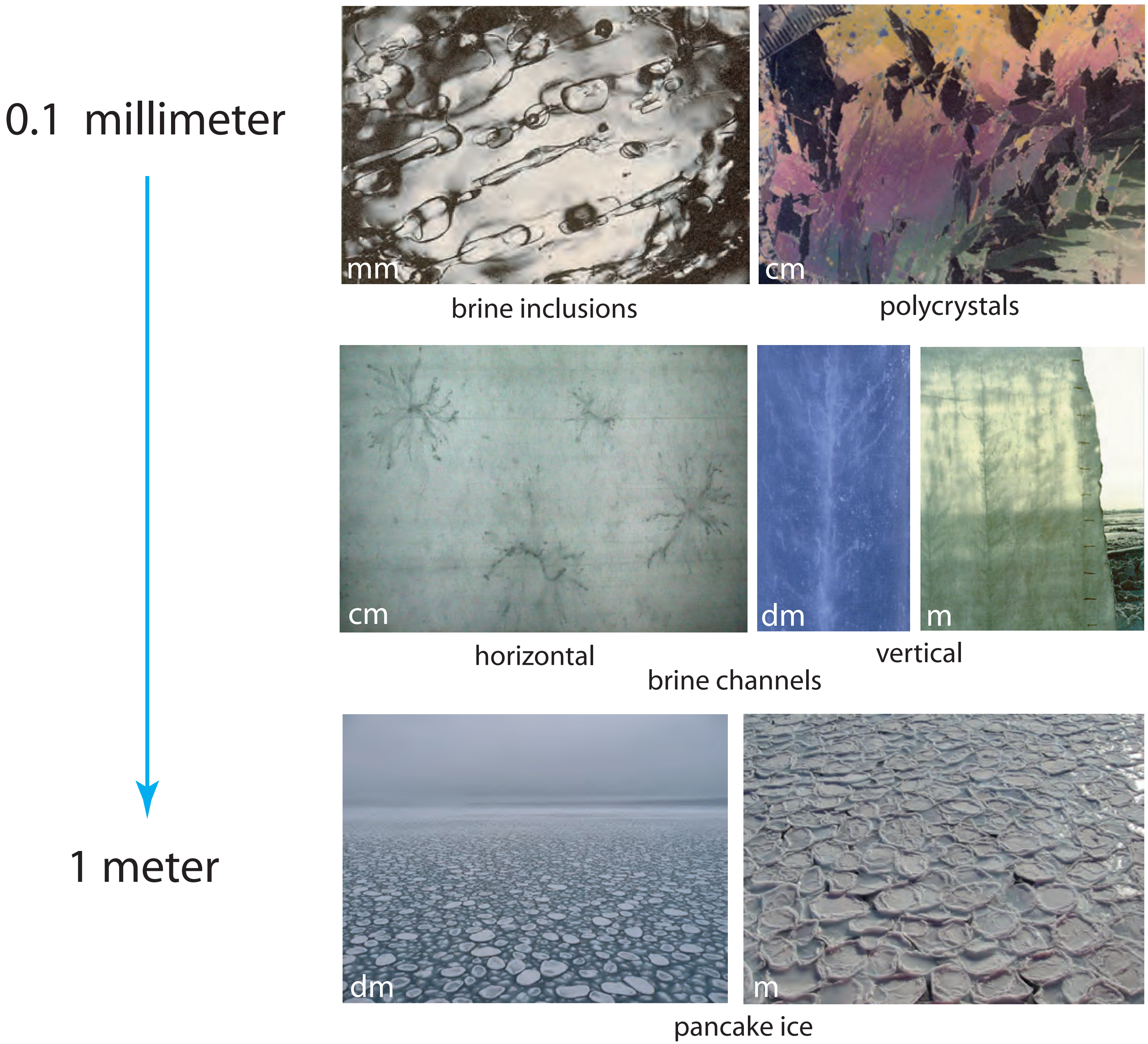
Workshop on the Multi-Phase Physics of Sea Ice   Santa Fe   8 September 2010

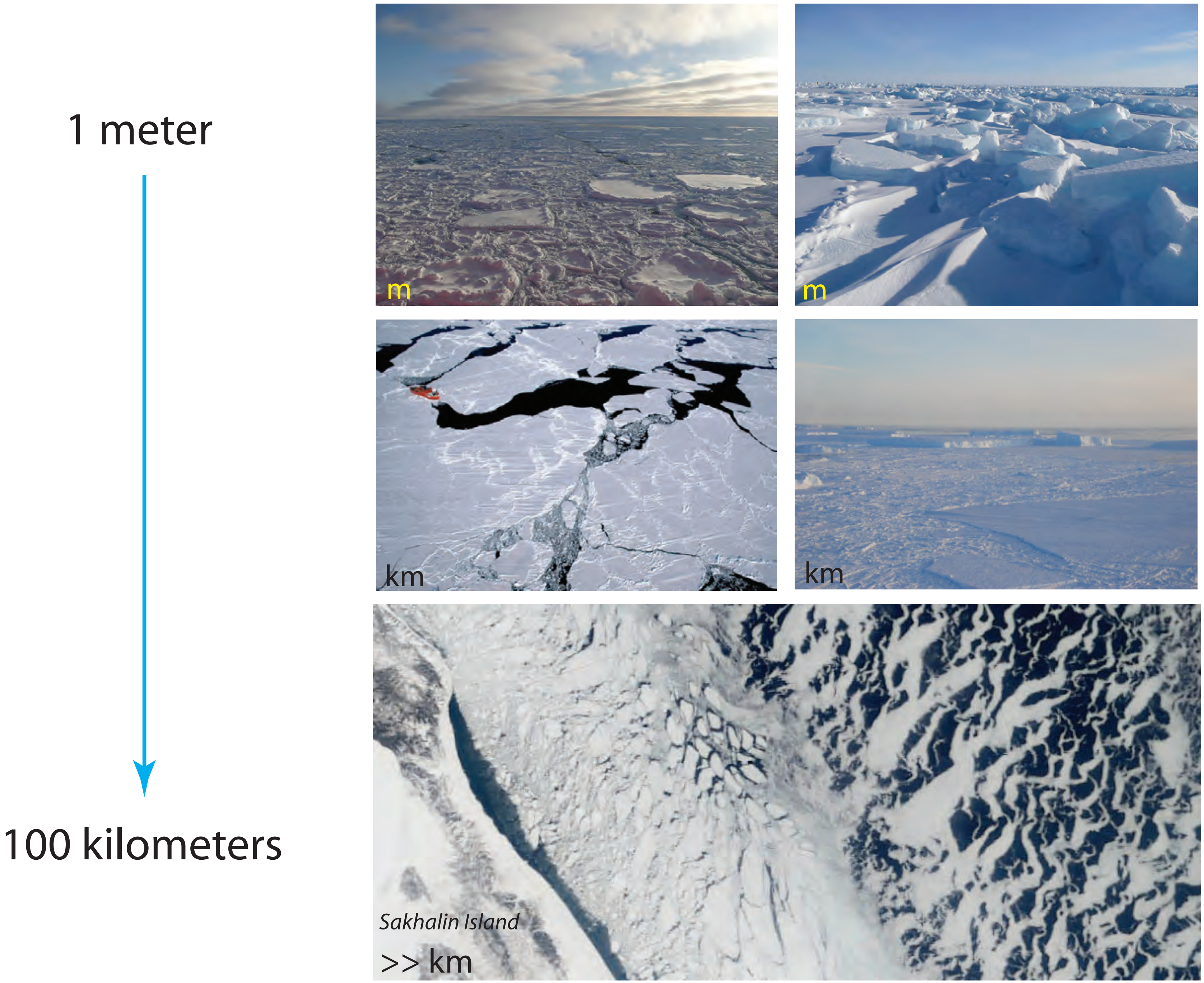
sea ice — ***composite material***



pure ice with ***brine***, air, and salt inclusions

sea ice displays *multiscale* structure over 10 orders of magnitude




1 meter
100 kilometers
m
m
km
km
Sakhalin Island
>> km

What is this talk about?

1. **fluid transport through porous sea ice**
   mediates processes important to polar climate and biology.

2. How do we mathematically model the thermal evolution
   of the fluid permability of sea ice and its critical behavior ?
   bounds, percolation theory, network models

3. **electrical transport in sea ice**
   related to the fluid transport properties, remote sensing -- thickness (EMI)

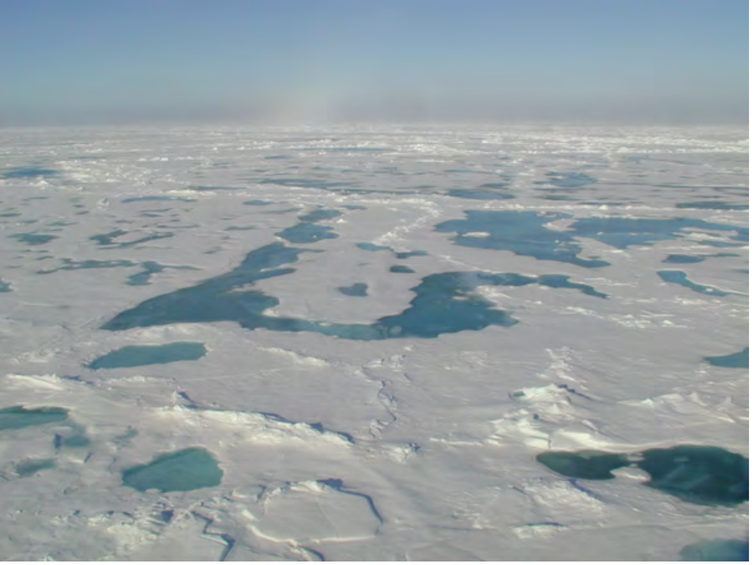
4. SIPEX in Antarctica 2007 - measured fluid and electrical properties
   ***** direct measurements of vertical conductivity***
   ***** find percolation threshold and universal critical behavior (in Arctic also)***
   ***** fluid permeability measurements (higher threshold for granular ice)***

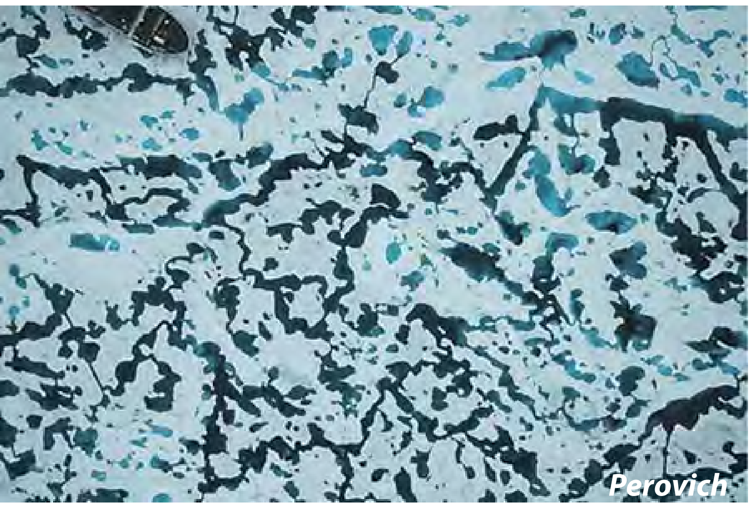
linkage of scales   --   critical behavior

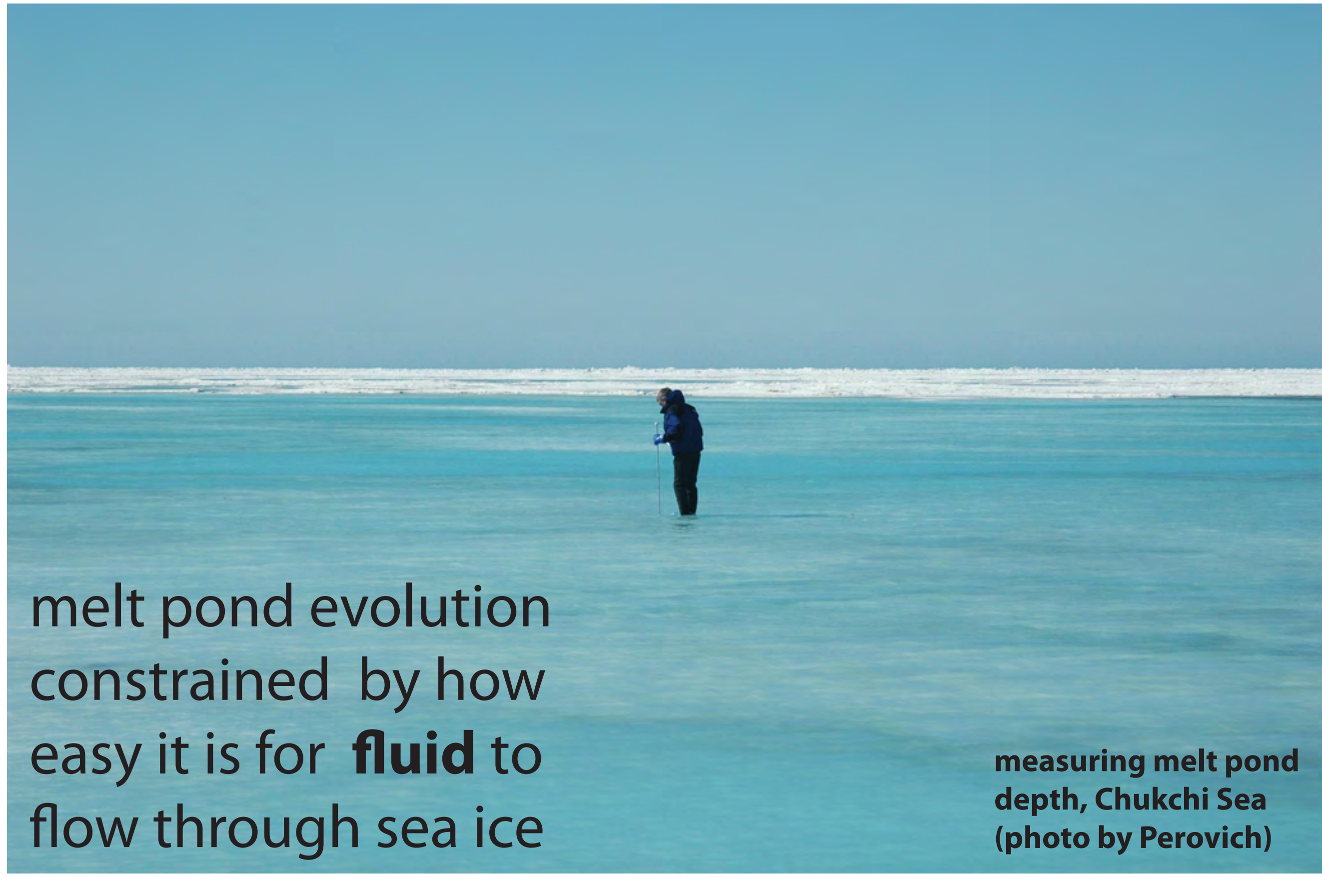
Ice albedo feedback

Perovich

sea ice albedo determined by melt ponds







melt pond evolution constrained by how easy it is for **fluid** to flow through sea ice

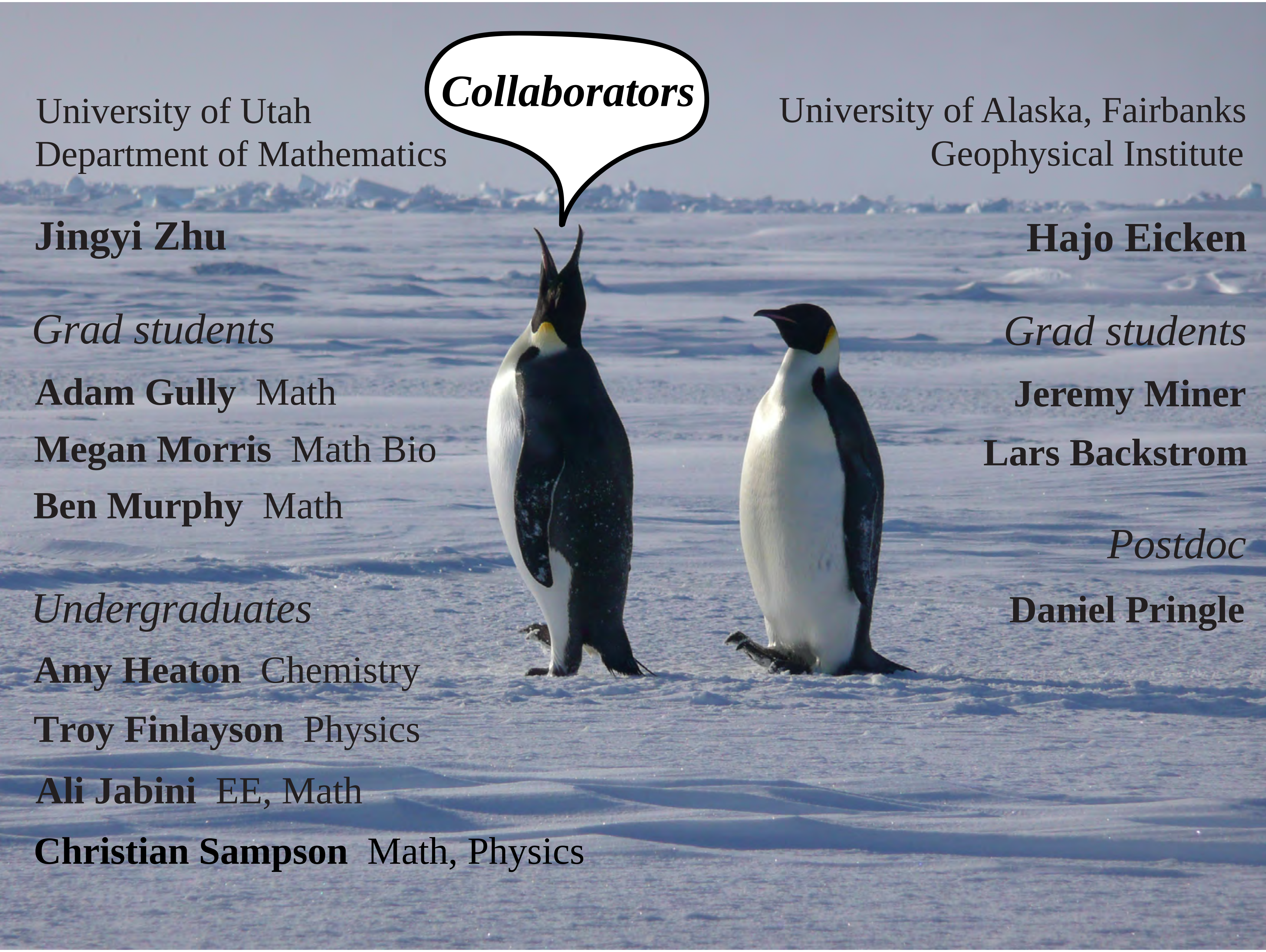
measuring melt pond depth, Chukchi Sea (photo by Perovich)

fluid permeability

depends on microstructure

sea ice microphysics

fluid transport



Collaborators

University of Utah
Department of Mathematics

Jingyi Zhu

Grad students

Adam Gully Math

Megan Morris Math Bio

Ben Murphy Math

Undergraduates

Amy Heaton Chemistry

Troy Finlayson Physics

Ali Jabini EE, Math

Christian Sampson Math, Physics

University of Alaska, Fairbanks
Geophysical Institute

Hajo Eicken

Grad students

Jeremy Miner

Lars Backstrom

Postdoc

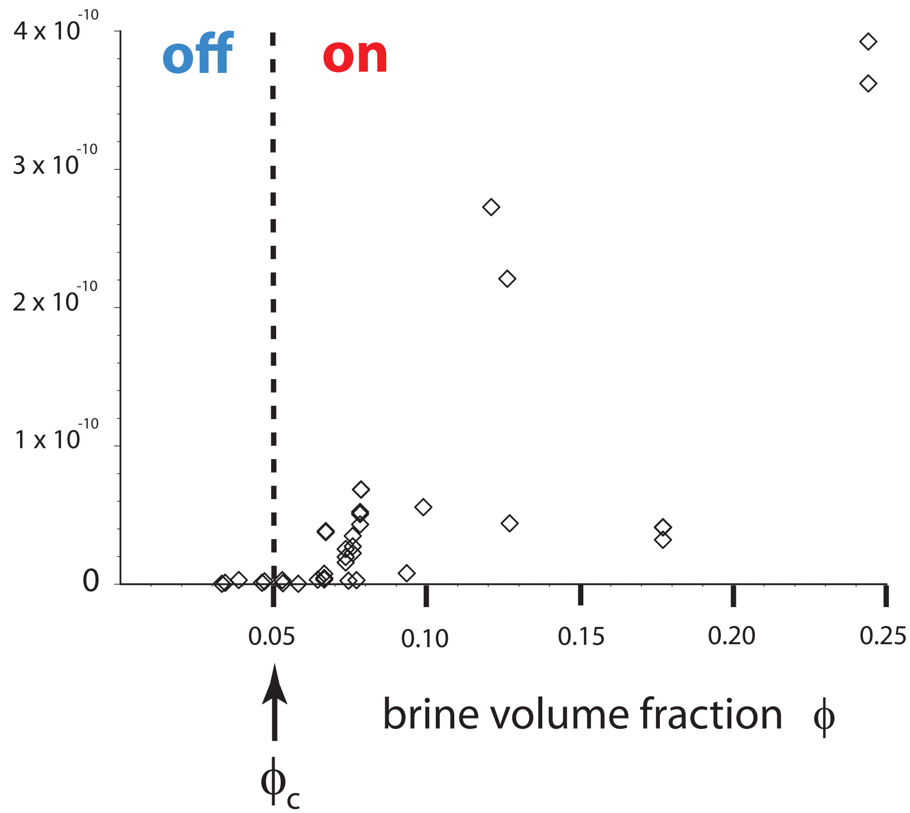
Daniel Pringle

Critical behavior of fluid transport in sea ice

impermeable permeable

Arctic field data

vertical fluid permeability k (m^2)



on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ $\longleftrightarrow$ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

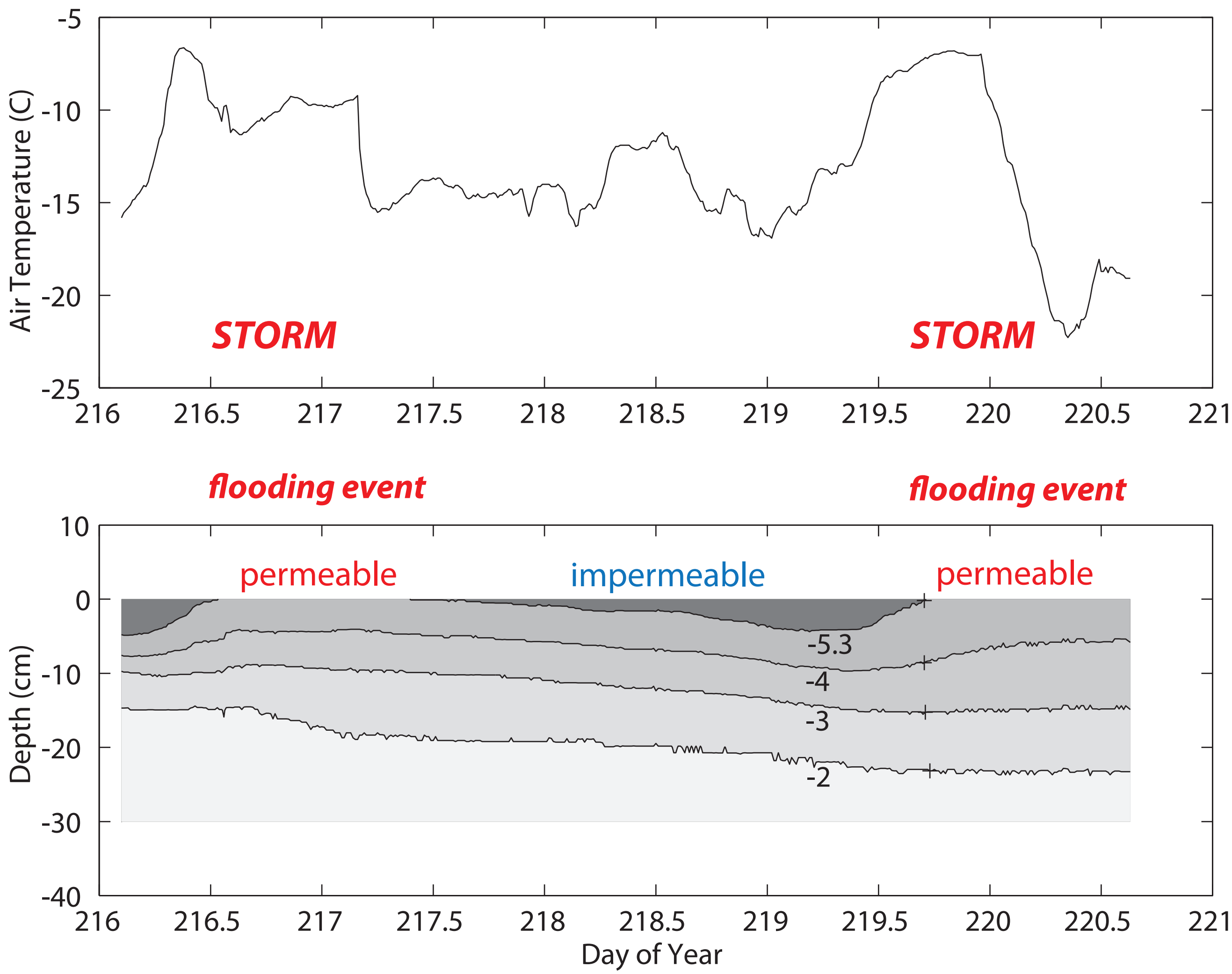
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

on **-** ***off*** switch helps control:

1. melt pond evolution and ice albedo
2. surface flooding and snow-ice formation
3. salinity profile evolution and brine drainage
4. transport of heat and gases (CO_2)
5. microbial activity; nutrient replenishment
6. electromagnetic signatures; remote sensing

Antarctic snow - ice formation



Temperature contours in sea ice and air temperature during Maud Rise drift camp on ANZFLUX cruise.

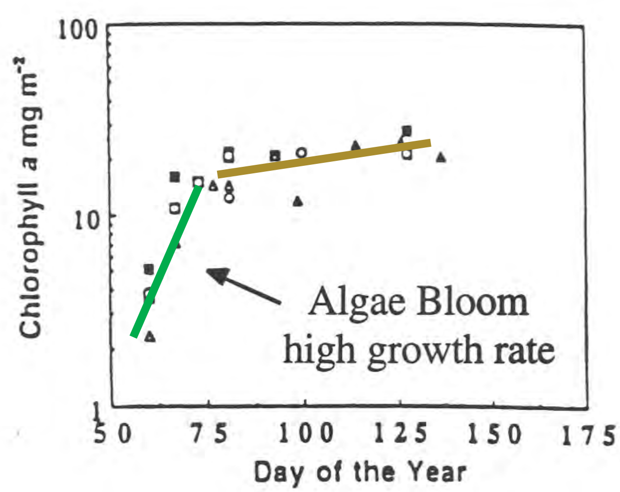
Black, impermeable top layer disappears during storms, allowing percolation of brine to surface.

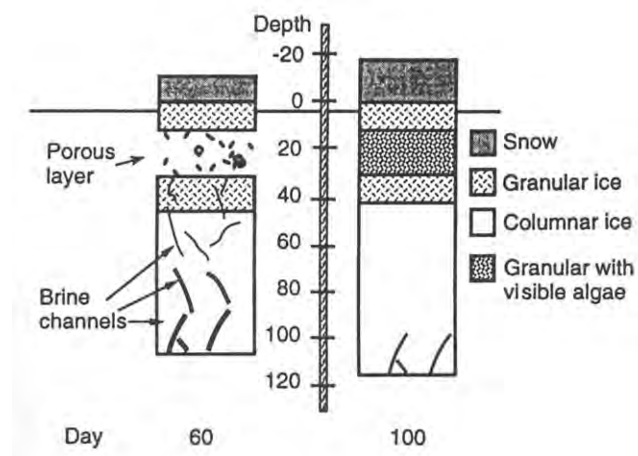
Golden, Ackley, Lytle, Science 1998

critical behavior of microbial activity

Convection-fueled algae bloom
Ice Station Weddell

(Fritsen, Lytle, Ackley, Sullivan, *Science* 1994)





Golden, Ackley, Lytle Science 1998

Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

viscosity

$\mathbf{k}$ = fluid permeability tensor

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

Unified approach to understanding fluid permeability in sea ice:

Thermal evolution of permeability and microstructure in sea ice,
K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and
J. Zhu, *Geophysical Research Letters*, vol. 34, 2007 (+ cover).

1. Homogenization and Darcy's law

2. Rigorous bounds

3. Percolation theory

4. Hierarchical models

5. Network model

+

X-ray CT imaging and pore analysis provide unprecedented look at temperature evolution of brine phase and its connectivity

Validated with lab and Arctic field data.

Arctic Ocean near Point Barrow, June 2007

micro-scale

brine inclusion tomography

controls

macro-scale processes in global climate

Arctic Ocean near Point Barrow, June 2004

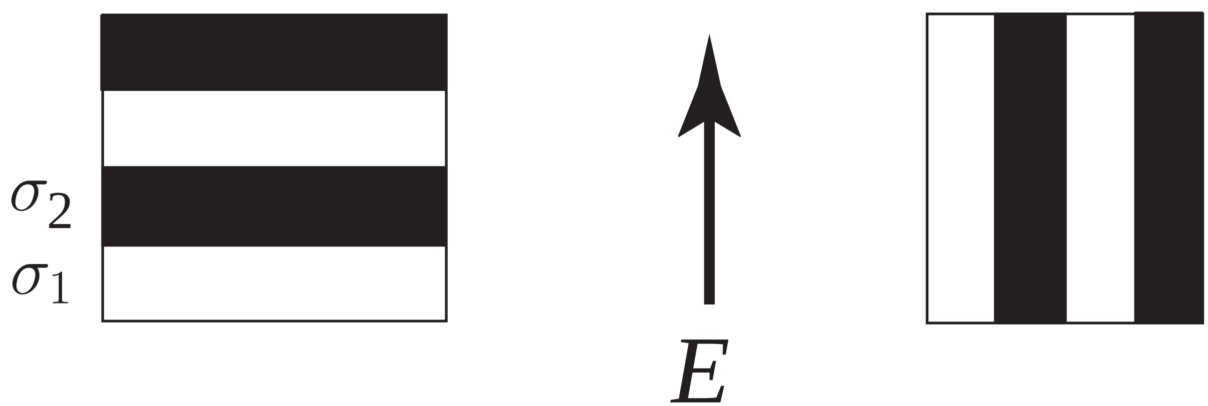
Bounds on the fluid permeability of sea ice

arithmetic and harmonic mean bounds for electrical conductivity σ (dielectric constant ε)

Effective electrical conductivity σ^* for two phase composite of σ_1 and σ_2

optimal bounds on σ^* for known volume fractions p_1 and p_2 :

$$\frac{1}{\frac{p_1}{\sigma_1} + \frac{p_2}{\sigma_2}} \leq \sigma^* \leq p_1\sigma_1 + p_2\sigma_2$$



Wiener, 1912

electrical transport

fluid transport

electrical conductance $g_e = \pi r^2 \sigma$

fluid conductance $g_f = \pi r^4 / 8\eta$

electrical conductivity $\sigma_e = \sigma$

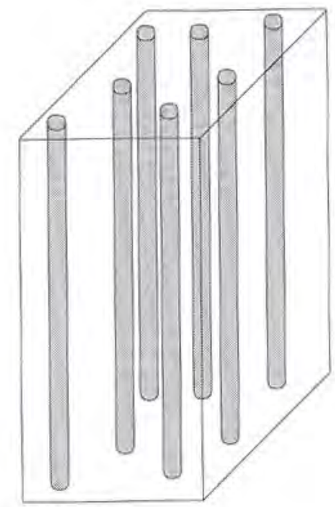
fluid conductivity $\sigma_f = r^2 / 8\eta$

PIPE BOUNDS on vertical fluid permeability k :

vertical pipes

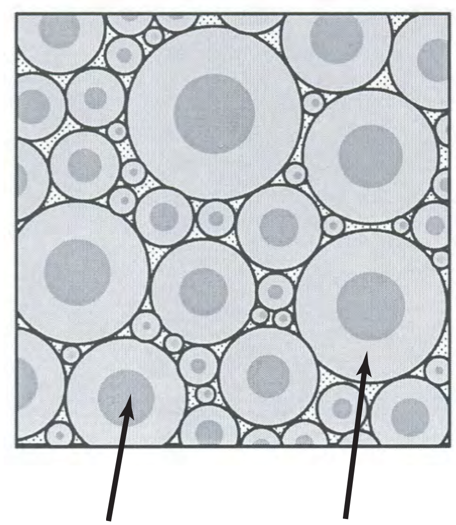
with appropriate radii

maximize k



fluid analog of arithmetic mean upper bound
on effective conductivity (Wiener 1912)

optimal coated
cylinder geometry



special case of optimal VOID BOUNDS

Torquato and Pham, *Phys. Rev. Lett.* 2004

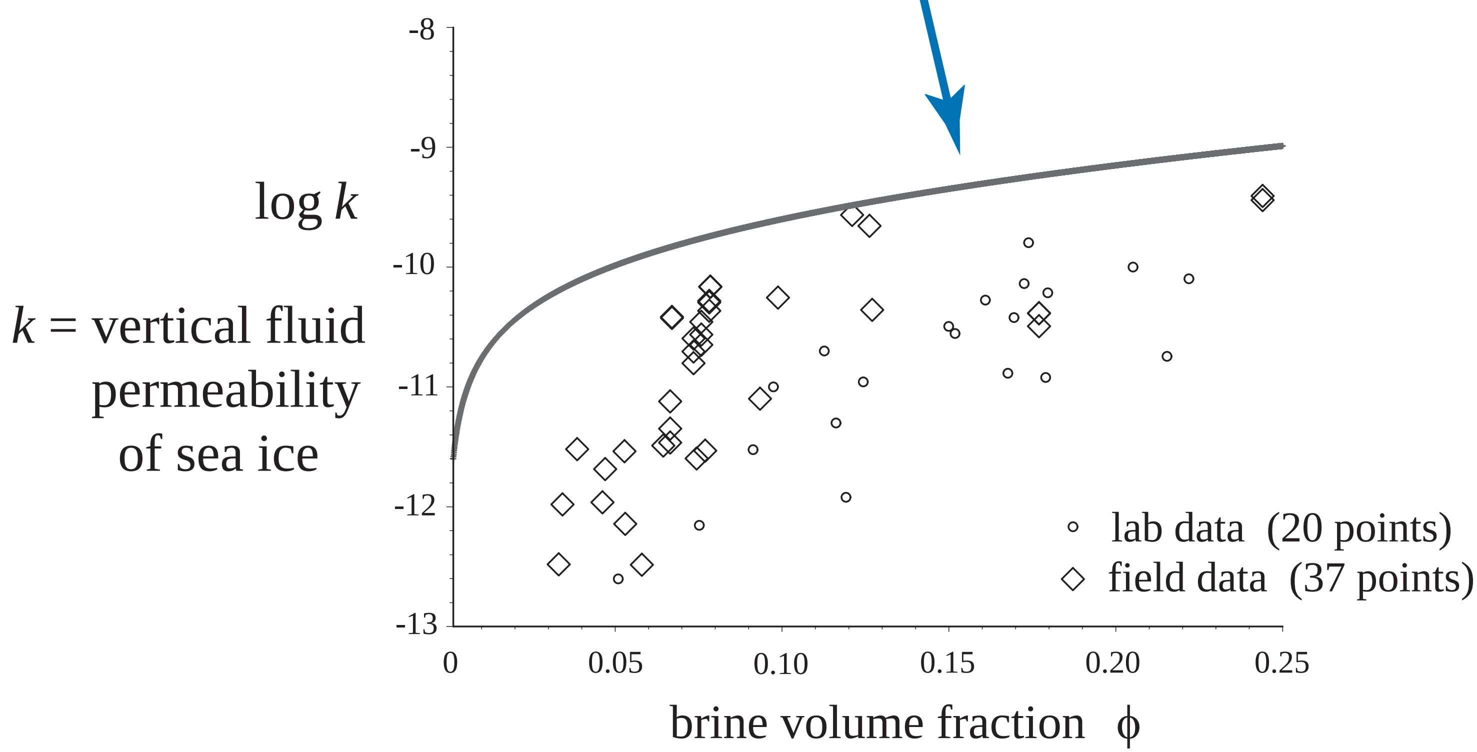
analyze via trapping constant for
diffusion process in pore space

lognormal pipe bound: inclusion cross sectional areas A lognormally distributed

$\ln(A)$ is normally distributed with mean μ and variance σ^2

$$k \le \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

μ increases with T and $\sigma = 1$
(Perovich and Gow, '96, ...)



similar calculations in math-finance
(stock prices tend to be lognormally distributed)

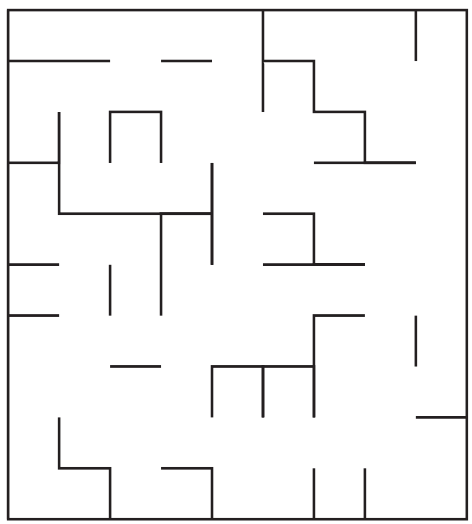
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *GRL* 2007
Golden, Heaton, Eicken, Lytle, *Mech. Materials* 2006

Theoretical predictions of ice permeability using percolation and hierarchical models

percolation theory

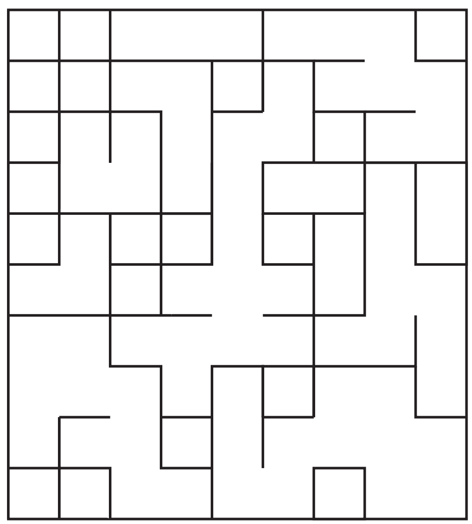
mathematical theory of connectedness

impermeable



$p = 1/3$

permeable



$p = 2/3$

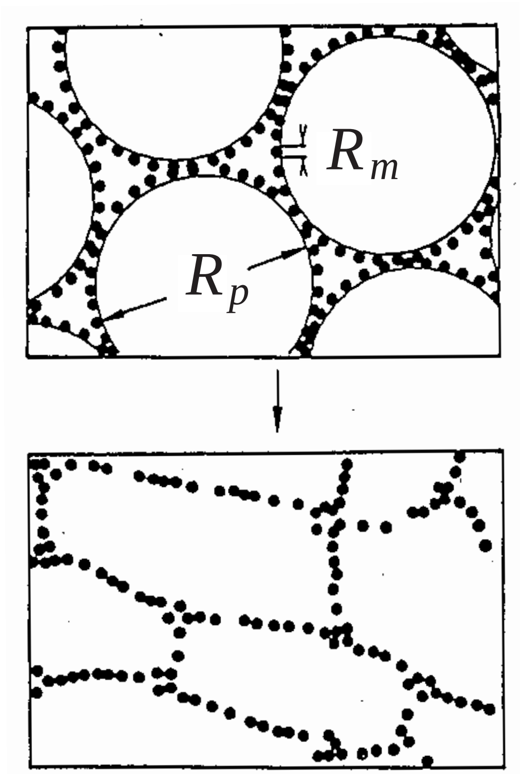
a bond is open with probability p
closed with probability 1-p

percolation threshold
$p_c = 1/2 \quad \text{for} \quad d = 2$

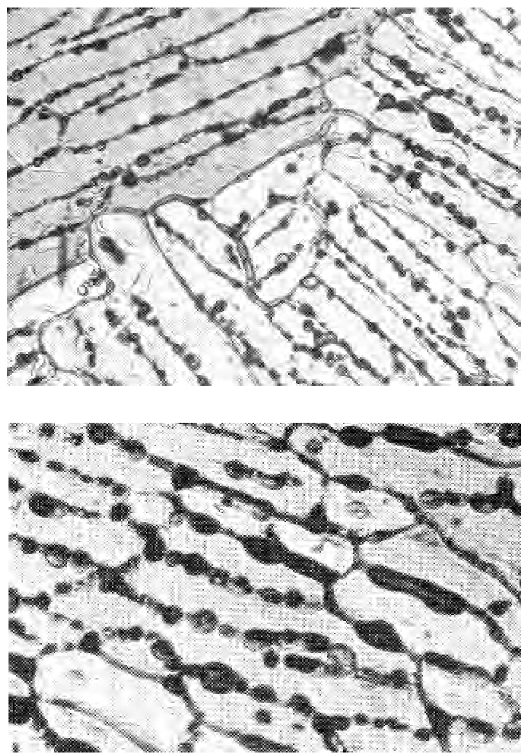
first appearance of infinite cluster

Continuum percolation model for stealthy materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

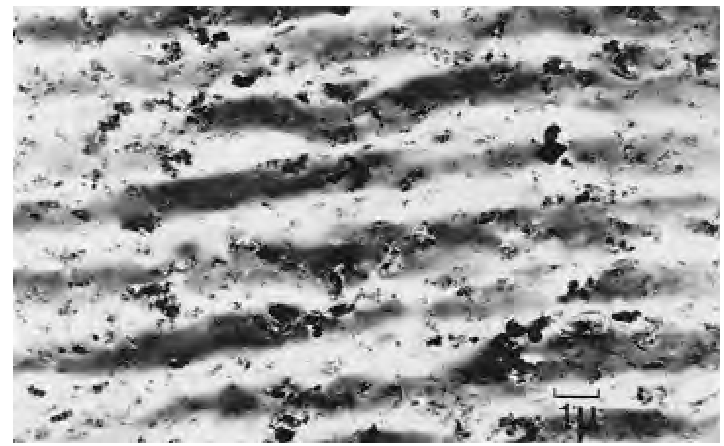
$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science*, 1998



compressed powder



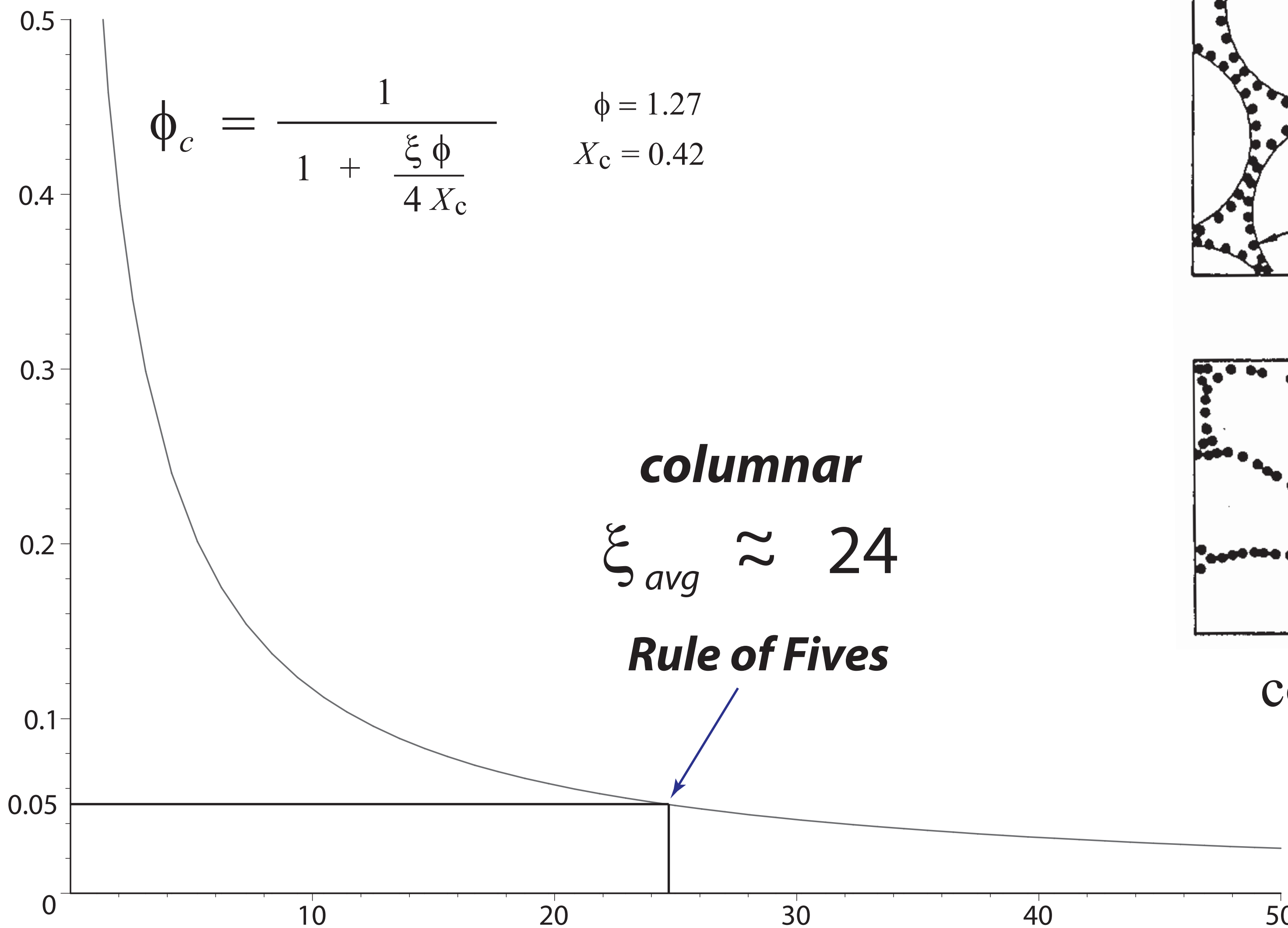
sea ice



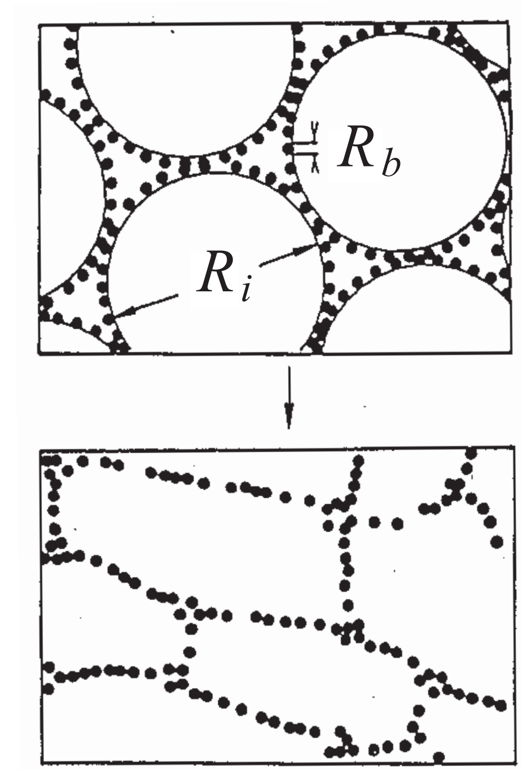
microstructure of radar absorbing composite

percolation threshold calculated from compressed powder model

Kusy and Turner, Nature 1971



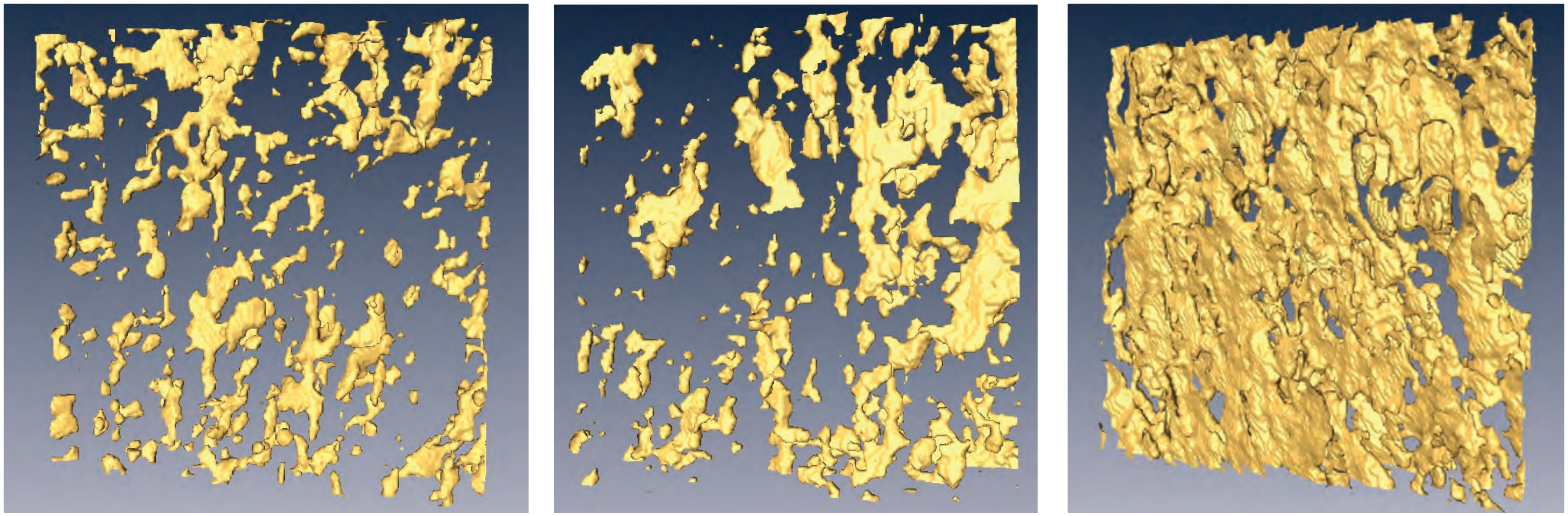
$$\xi = R_i / R_b$$



compressed powder

brine connectivity (over cm scale)

8 x 8 x 2 mm



-15 °C, $\phi = 0.033$ -6 °C, $\phi = 0.075$ -3 °C, $\phi = 0.143$

first direct evidence of conjectured transition in connectivity

3-D images
pores and throats

→

3-D graph
nodes and edges

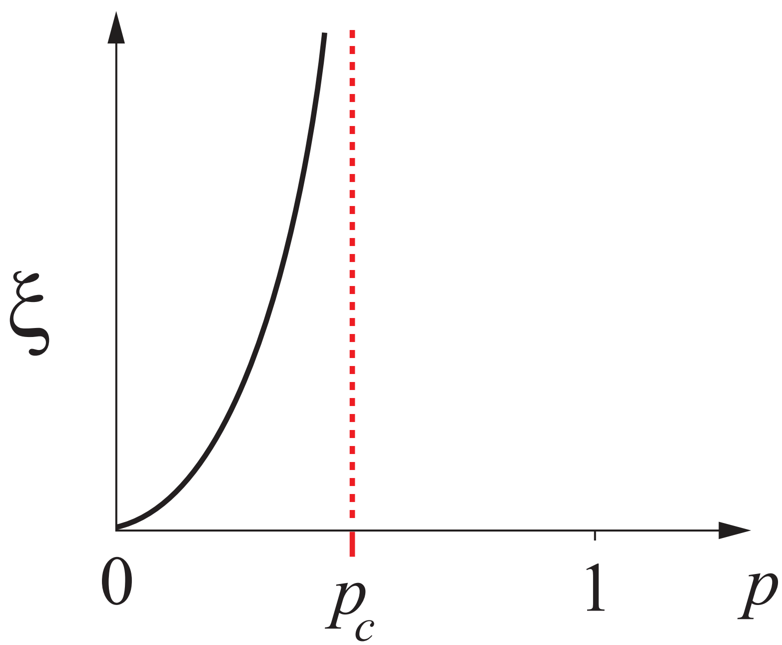
analyze graph connectivity as function of temperature and sample size

use finite size scaling techniques to confirm rule of fives

order parameters in percolation theory

correlation length

characteristic scale
of connectedness

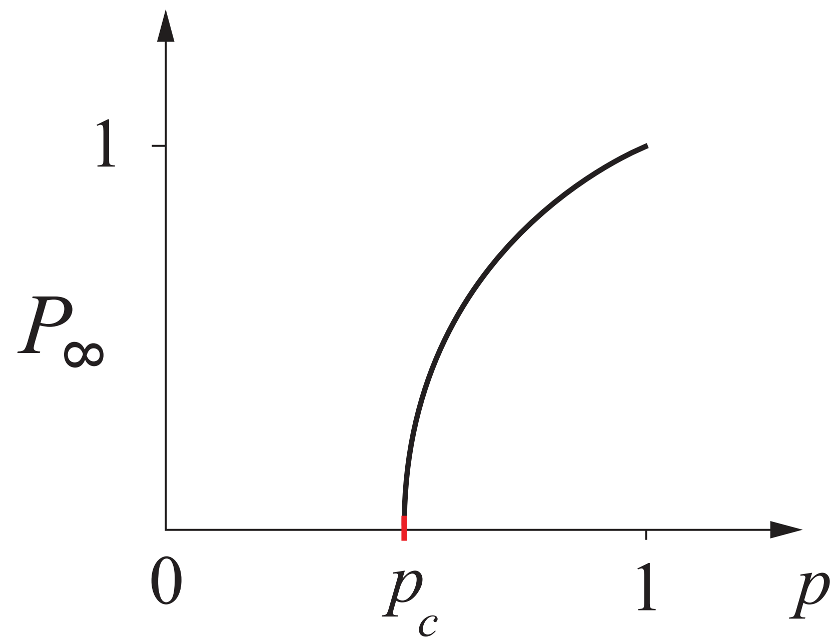


$$\xi(p) \sim |p - p_c|^{-\nu}$$

$$p \to p_c$$

infinite cluster density

probability the origin
belongs to infinte cluster



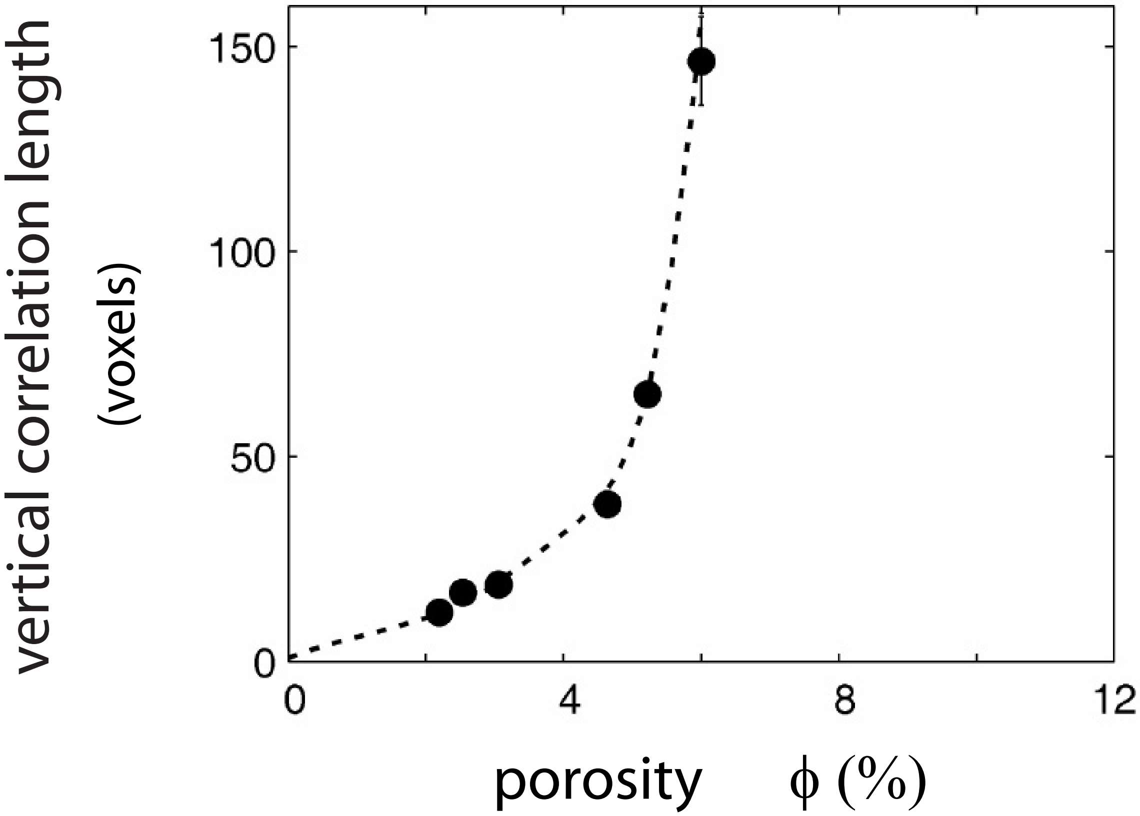
$$P_\infty(p) \sim (p - p_c)^{\beta}$$

$$p \to p_c^{+}$$

The key connectivity functions of percolation theory have been computed extensively for many lattice models, but NOT for natural materials.

We have calculated them for sea ice single crystals -- a first for percolation investigations -- and estimated anisotropic percolation thresholds.

Pringle, Miner, Eicken, Golden, JGR (Oceans) 2009



divergence of correlation length for single crystal data

use finite size scaling

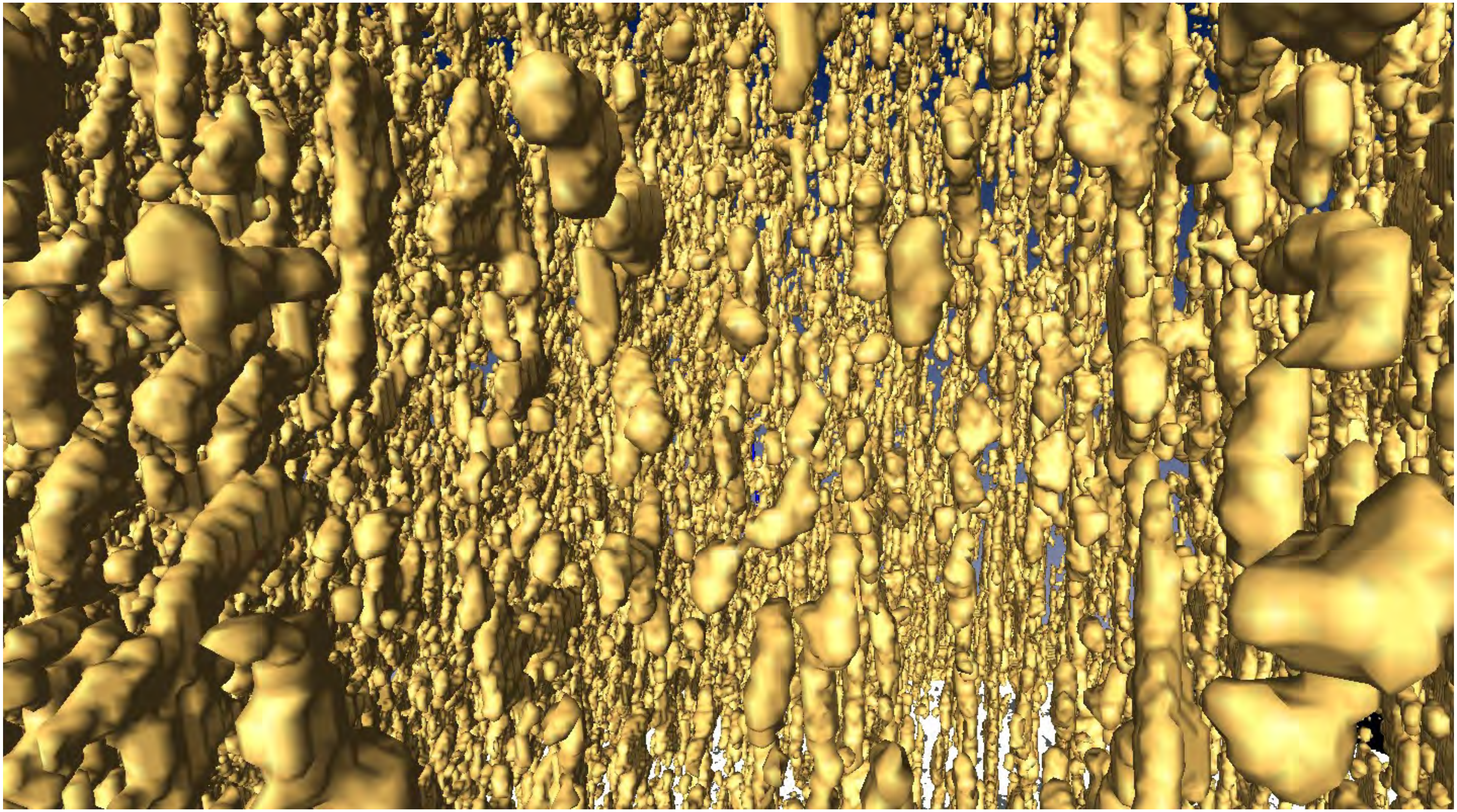
$$P_\infty(p_c, L) \sim L^{-\beta/\nu} \quad (= L^{-0.48})$$

anisotropic percolation threshold estimates

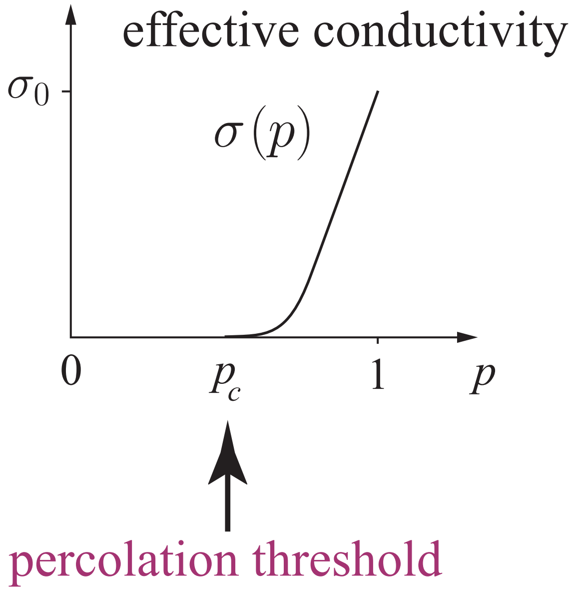
vertical $\phi_c = 4.6 \pm 0.7\,\%$

horizontal $\left\{\begin{array}{ll} \text{parallel to layers} & \phi_c = 9 \pm 2\,\% \\ \text{perpendicular to layers} & \phi_c = 14 \pm 4\,\% \end{array}\right.$

X-ray computed tomography (CT) and medial axis analysis (Prodanovic, et al., 2006) of lab-grown sea-ice single crystals doped with CsCl to improve ice/brine resolution.



$\phi = 5.7\ \%$ $T = -8$



Critical behavior of transport near percolation threshold

conductivity

$$\sigma(p) \sim \sigma_0 \, (p - p_c)^t, \quad p \to p_c^+$$

permeability

$$\kappa(p) \sim \kappa_0 \, (p - p_c)^e, \quad p \to p_c^+$$

conductivity exponent t

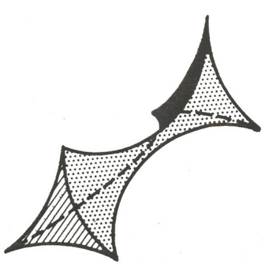
UNIVERSAL for lattices

depends only on dimension, $e = t$

$d = 3$ **numerical** $t \approx 2$

rigorous bound $1 \leq t \leq 2$

Golden PRL 1990, CMP 1992



continuum exponents can be ***non-universal***

SWISS CHEESE

Halperin, Feng, Sen PRL 1985

lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - \phi_c)^2$$

critical
exponent
t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

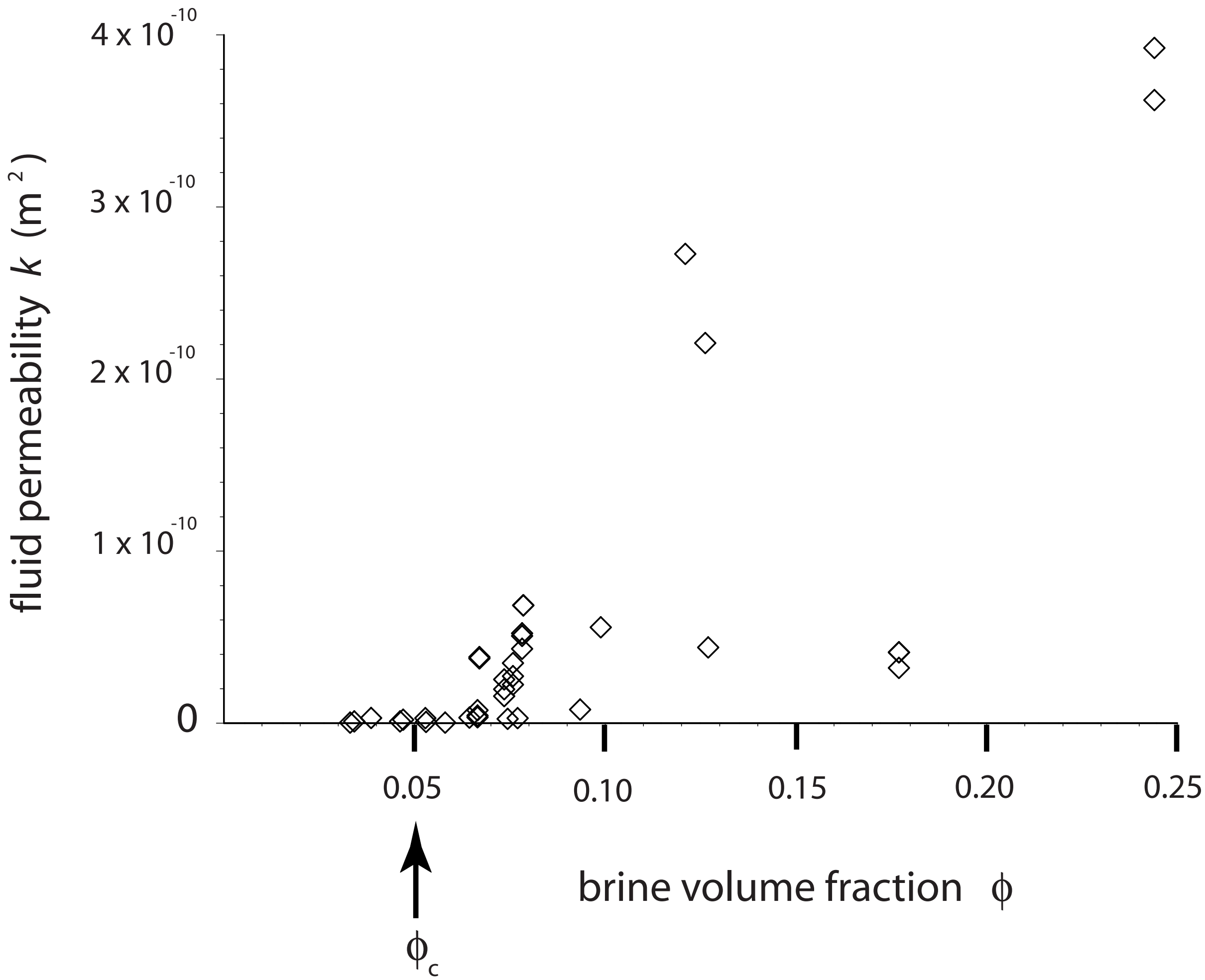
- exponent is UNIVERSAL lattice value $t \approx 2.0$ from general structure of brine inclusion distribution function (-- other saline ice?)

- sedimentary rocks like sandstones also exhibit universality

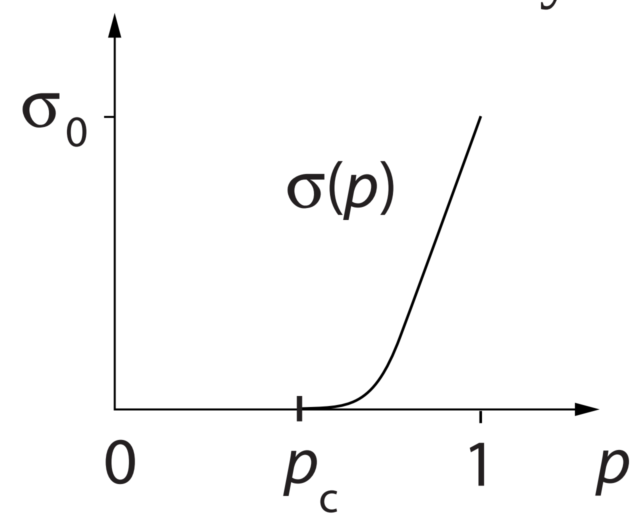
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0

- no free parameters - microstructural input only

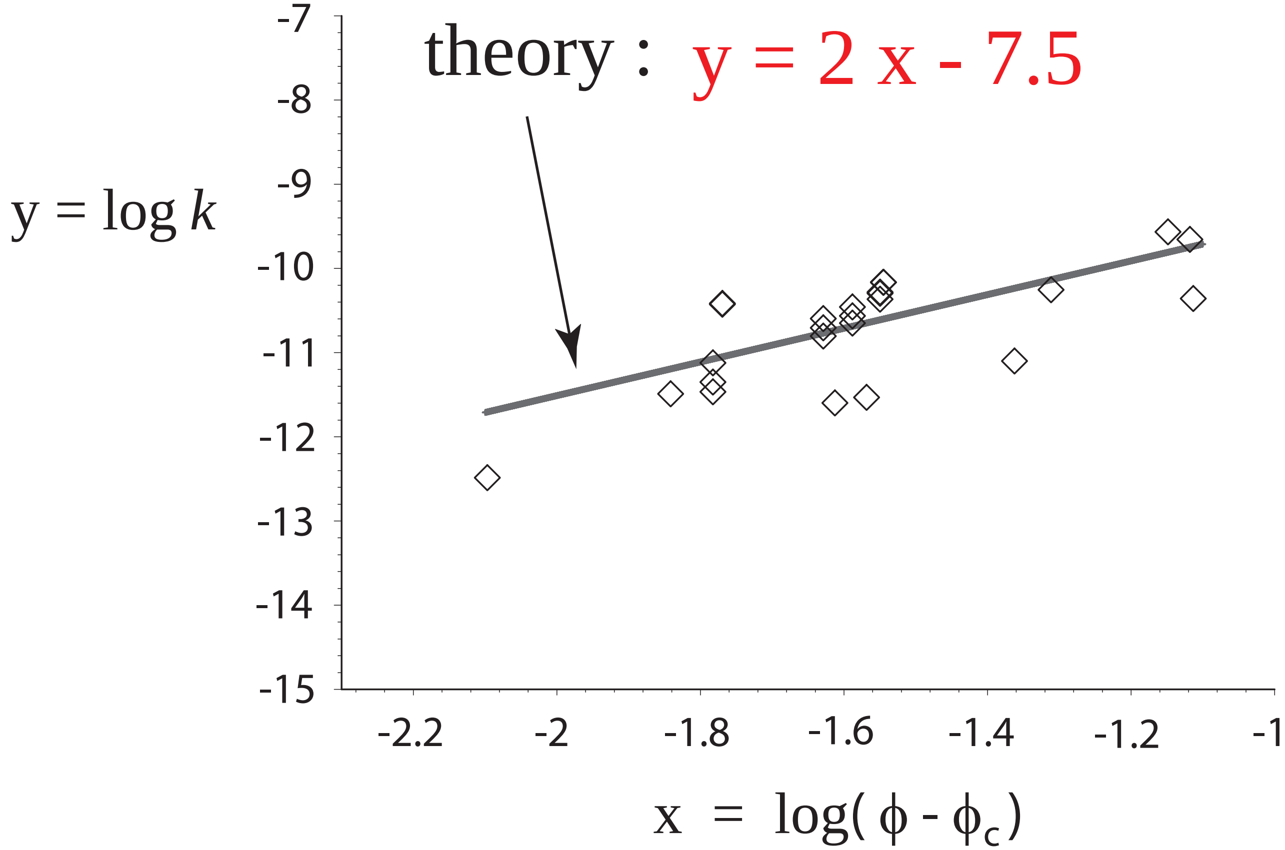
in situ data on vertical fluid permeability of Arctic sea ice



effective conductivity



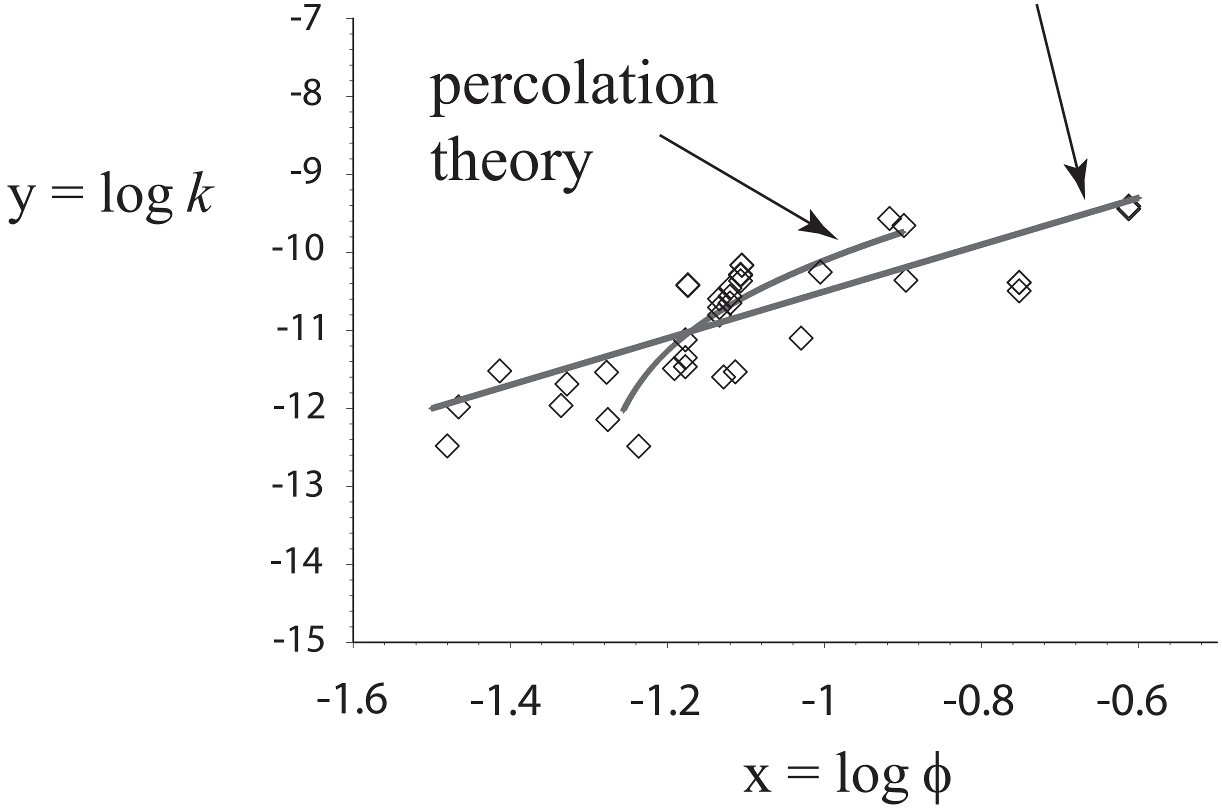
two dimensional lattice percolation

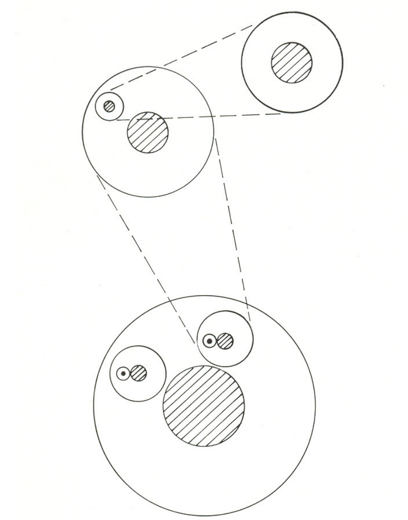


statistical best fit of data:

$$y = 2.07\, x - 7.45$$

hierarchical model : $y = 3x - 7.5$





$$k(\phi) = k_0 \phi^3$$

over full range of porosities

statistical best fit of data: $y = 3.05x - 7.50$

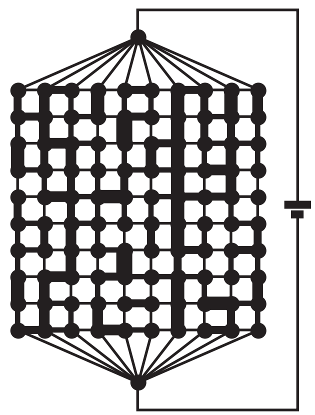
A network model for fluid transport through sea ice

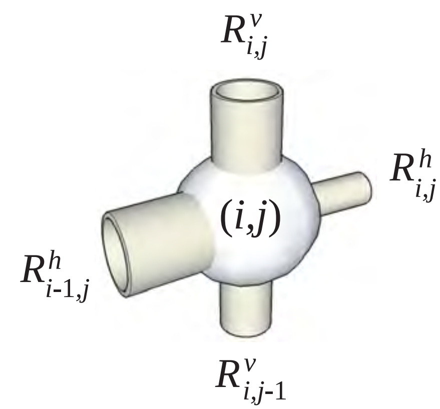
J. Zhu, A. Jabini, K. M. Golden, H. Eicken, M. Morris

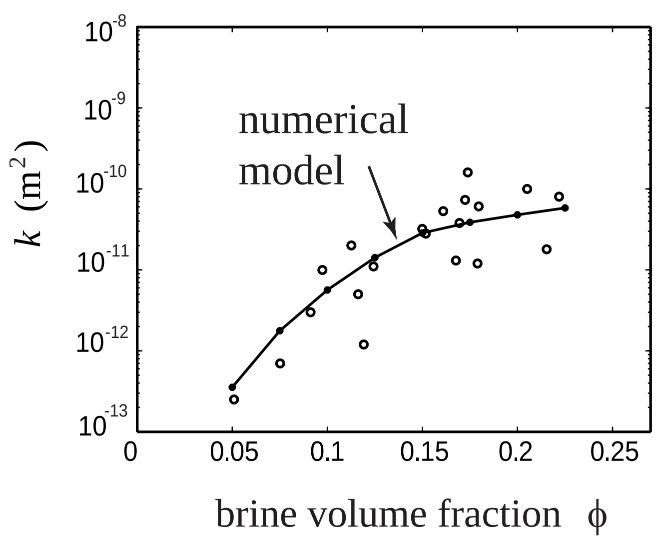
Annals of Glaciology, 2006

- random pipe network with radii chosen from measured inclusion distributions, solved with fast multigrid method

- local velocity field computed, as well as effective permeability





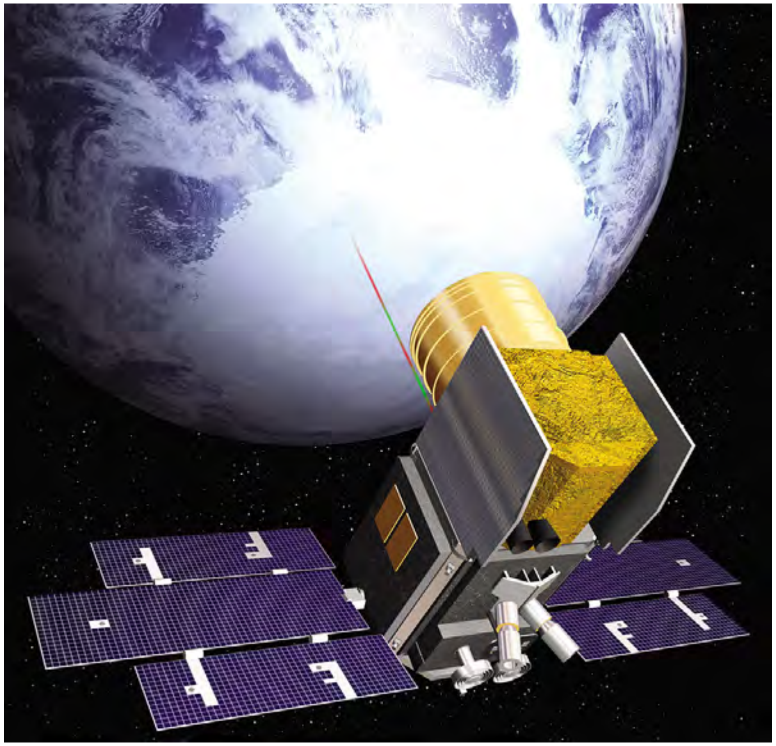


based on connectivity analysis of brine inclusions, introduce *disconnections* into network (Golden et al., GRL 2007)

Remote sensing of sea ice

Recover sea ice properties from electromagnetic (EM) data

INVERSE PROBLEM



NASA's Ice, Cloud and Land Elevation Satellite (ICESat)

The Worbot - a low frequency EM induction instrument for measuring sea ice thickness

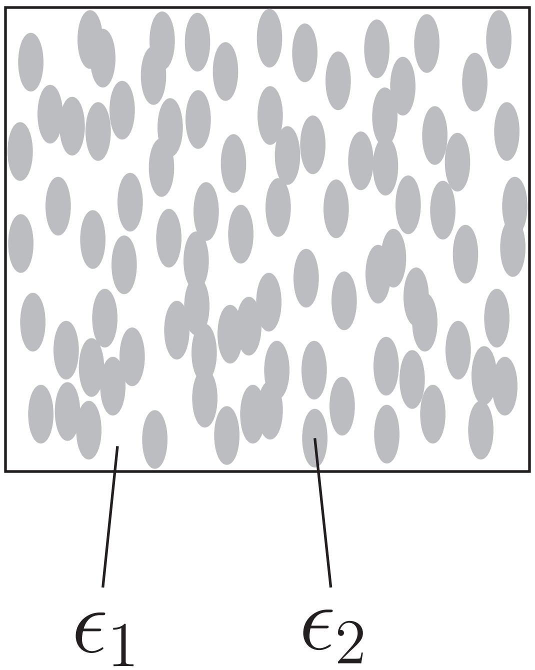
The key parameter in modeling the response of sea ice to an EM field is its

complex *permittivity* or *dielectric constant* ε^*

which depends strongly on the brine microstructure

e.g., interpretation of EM thickness data depends on knowledge of ε^*

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$\Big\}\ \epsilon^*$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \text{ , composite geometry } \right)$$

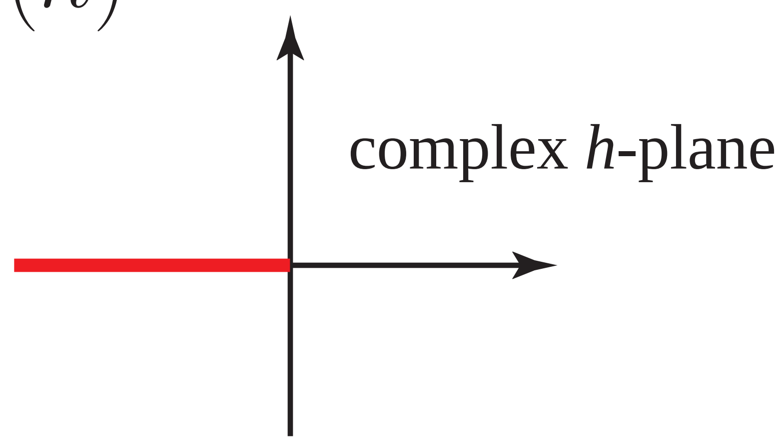
Analytic continuation method for bounding complex ϵ^*

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

$$m(h) = \frac{\epsilon^*}{\epsilon_2}\left(\frac{\epsilon_1}{\epsilon_2}\right) \qquad h = \frac{\epsilon_1}{\epsilon_2}$$

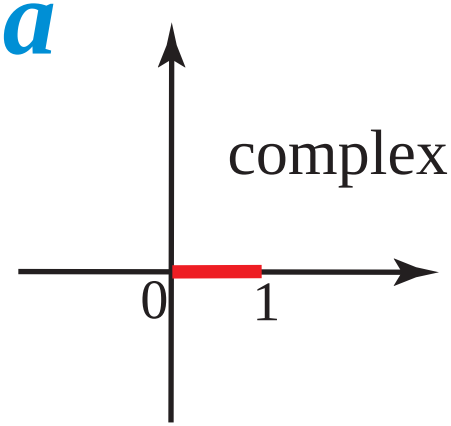
Exploit analytic properties of $m(h)$

- $m(h)$ analytic off negative real axis



- $m(h)$: UHP $\longrightarrow$ UHP

Integral representation formula



$$F(s) = 1 - m(h), \quad s = \frac{1}{1-h}$$

$$F(s) = \int_0^1 \frac{d\mu(z)}{s-z}, \quad \mu$$

- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n-point correlations

representation ***separates***

GEOMETRY μ from

medium parameters in s

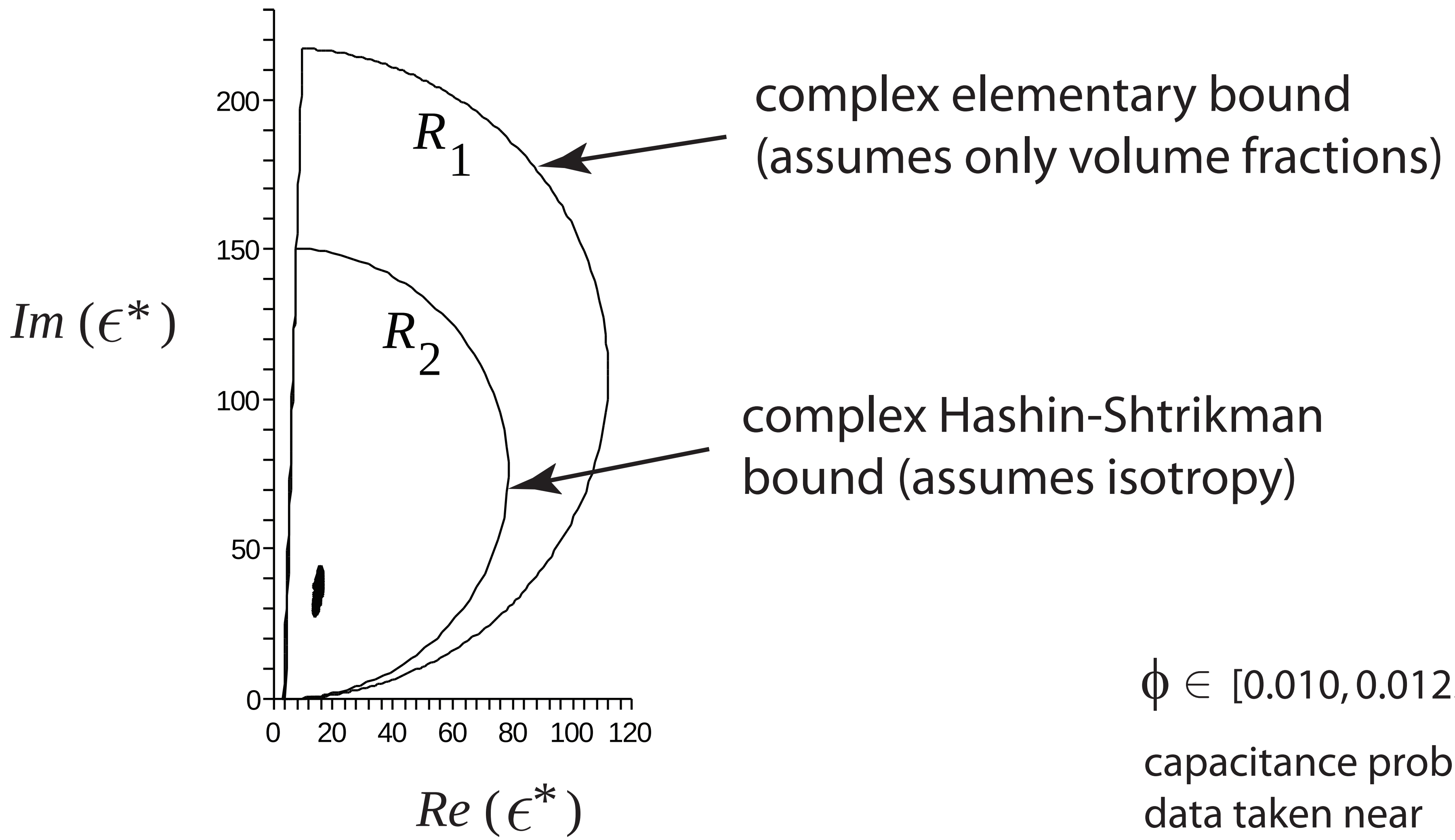
$$E = (s + \Gamma\chi)^{-1} e_k$$

$$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$$

χ = indicator function of medium 1

ice brine

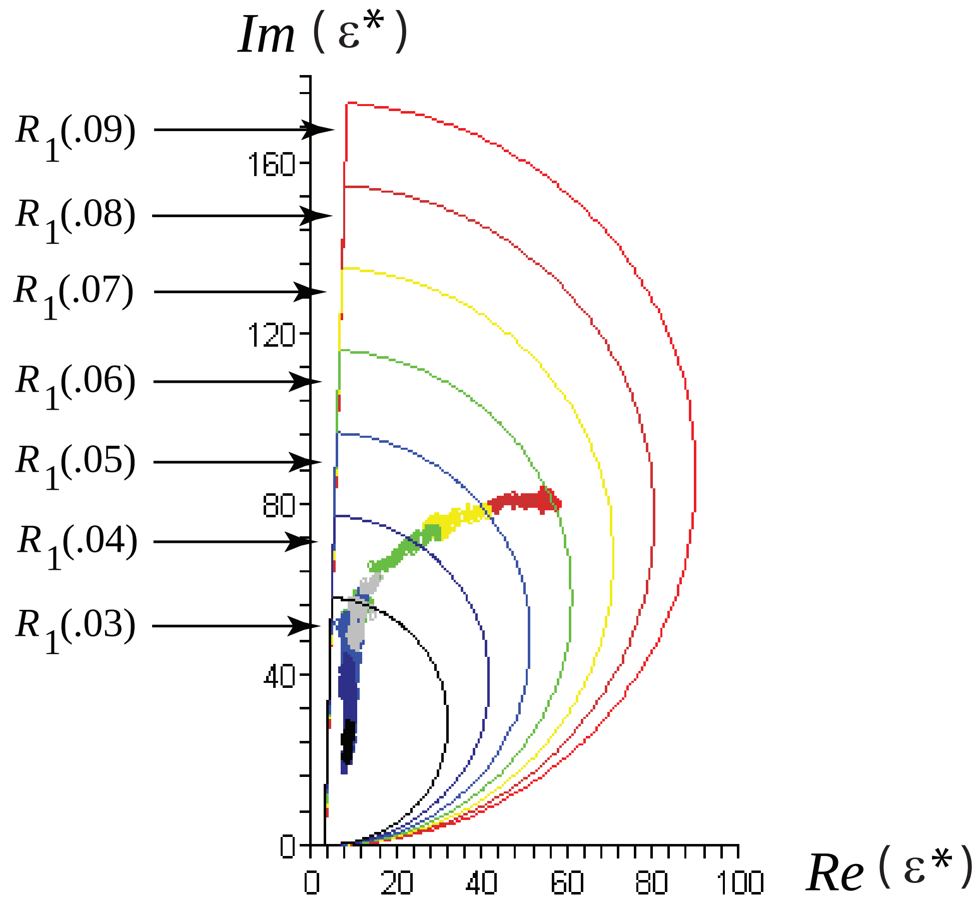
50 Mhz $\epsilon_1 = 3.06$ $\epsilon_2 = 63.3 + i\,1930$



$\phi \in [0.010, 0.0125]$

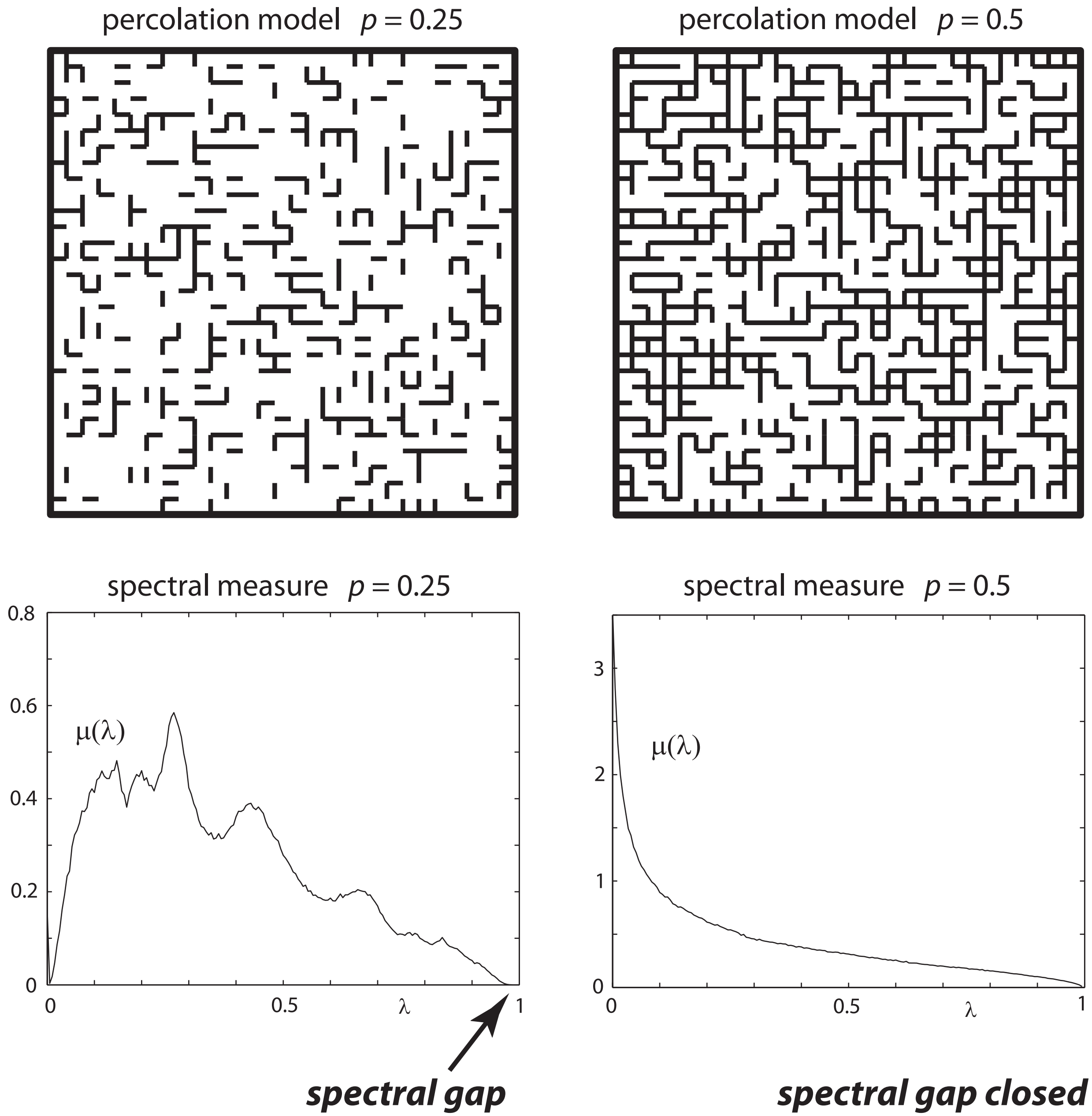
capacitance probe
data taken near
Barrow, AK

Comparison of forward bounds with data



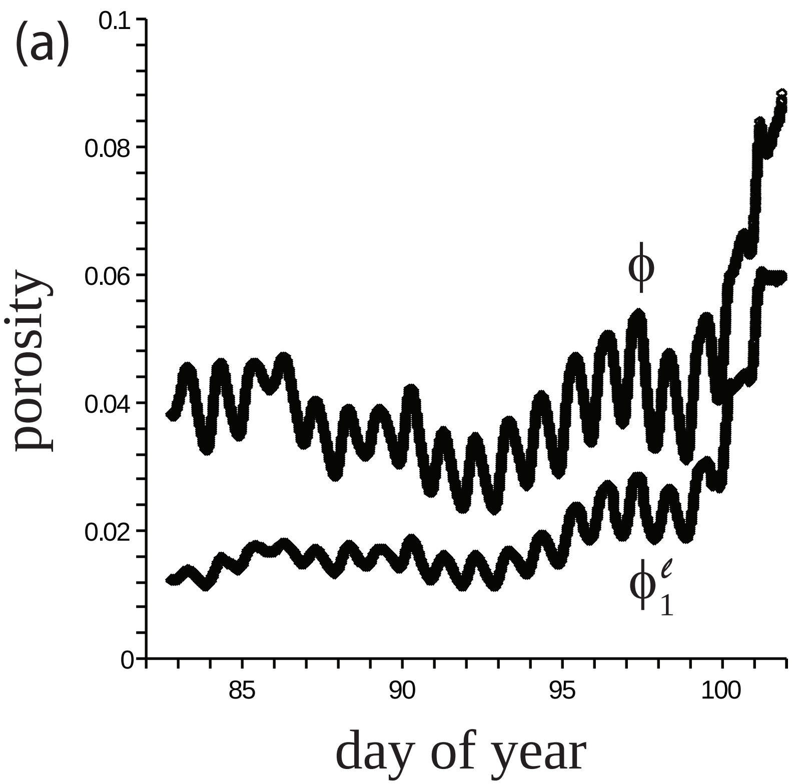
Complex bounds and microstructural recovery from measurements of sea ice permittivity, A. Gully, L. G. E. Backstrom, H. Eicken, and K. M. Golden, *Physica B,* 2007.

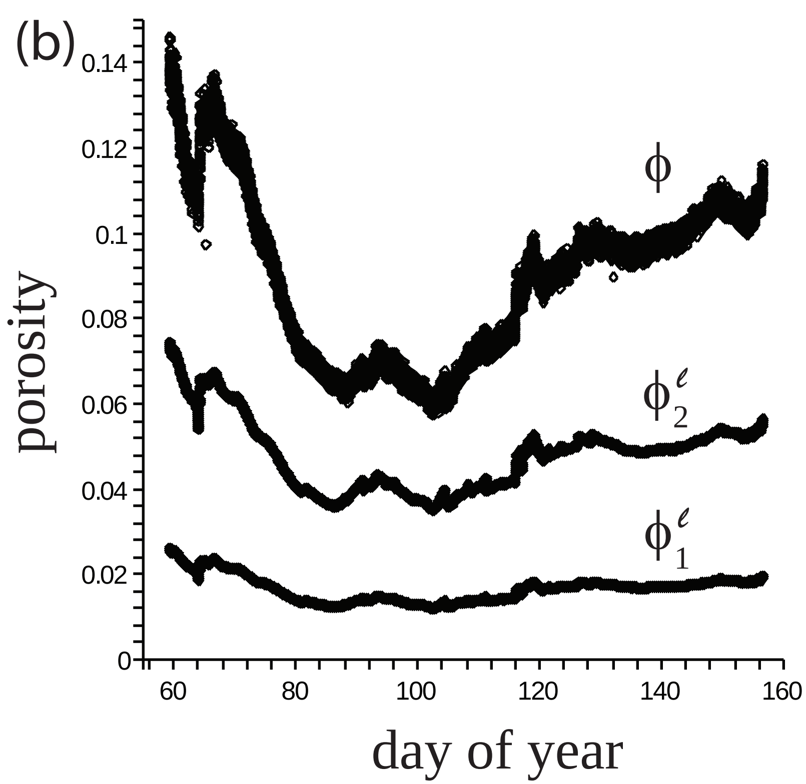
spectral measures for the random resistor network



Golden, Murphy, Cherkaev, J. Biomech. 2010

Inverse bounds and microstructural recovery

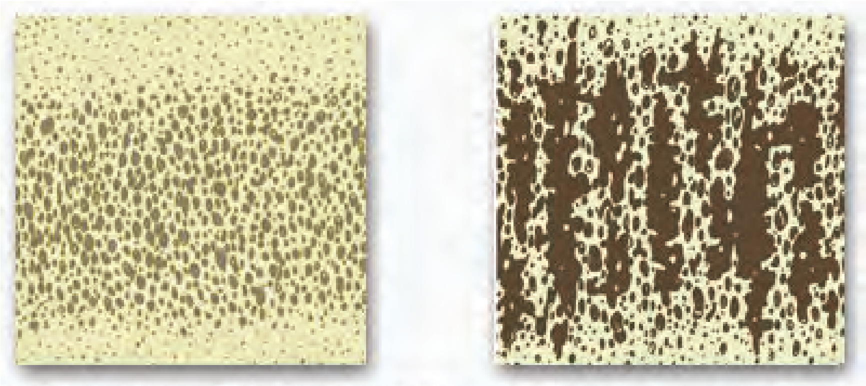




Inverse Homogenization for Evaluation of Bone Structure

Carlos Bonifasi-Lista, Elena Cherkaev, 2006

bone porosity can be estimated from the effective torsional modulus using reconstruction of spectral measure



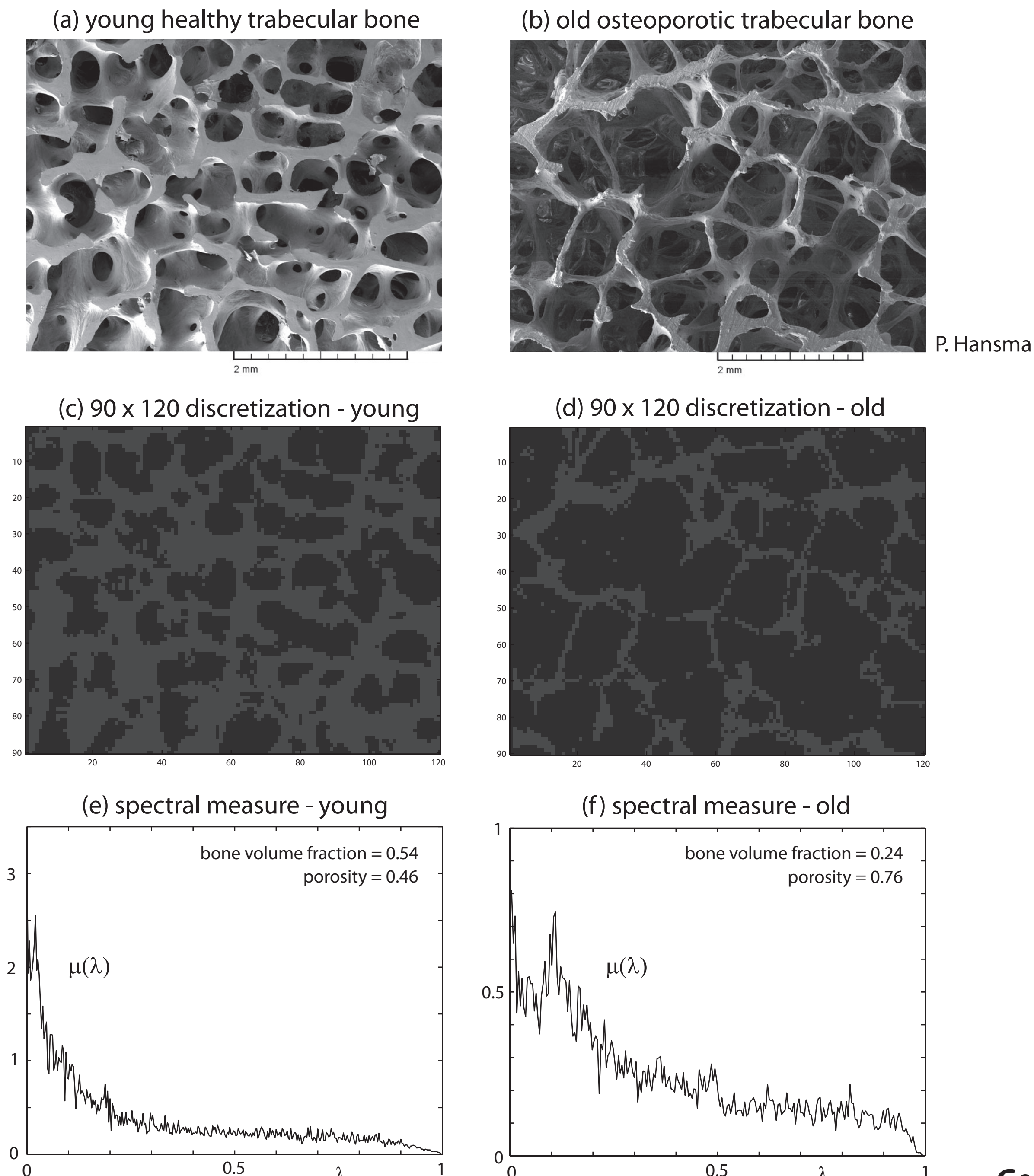
normal osteoporotic

***Application*:**

Monitoring osteoporosis

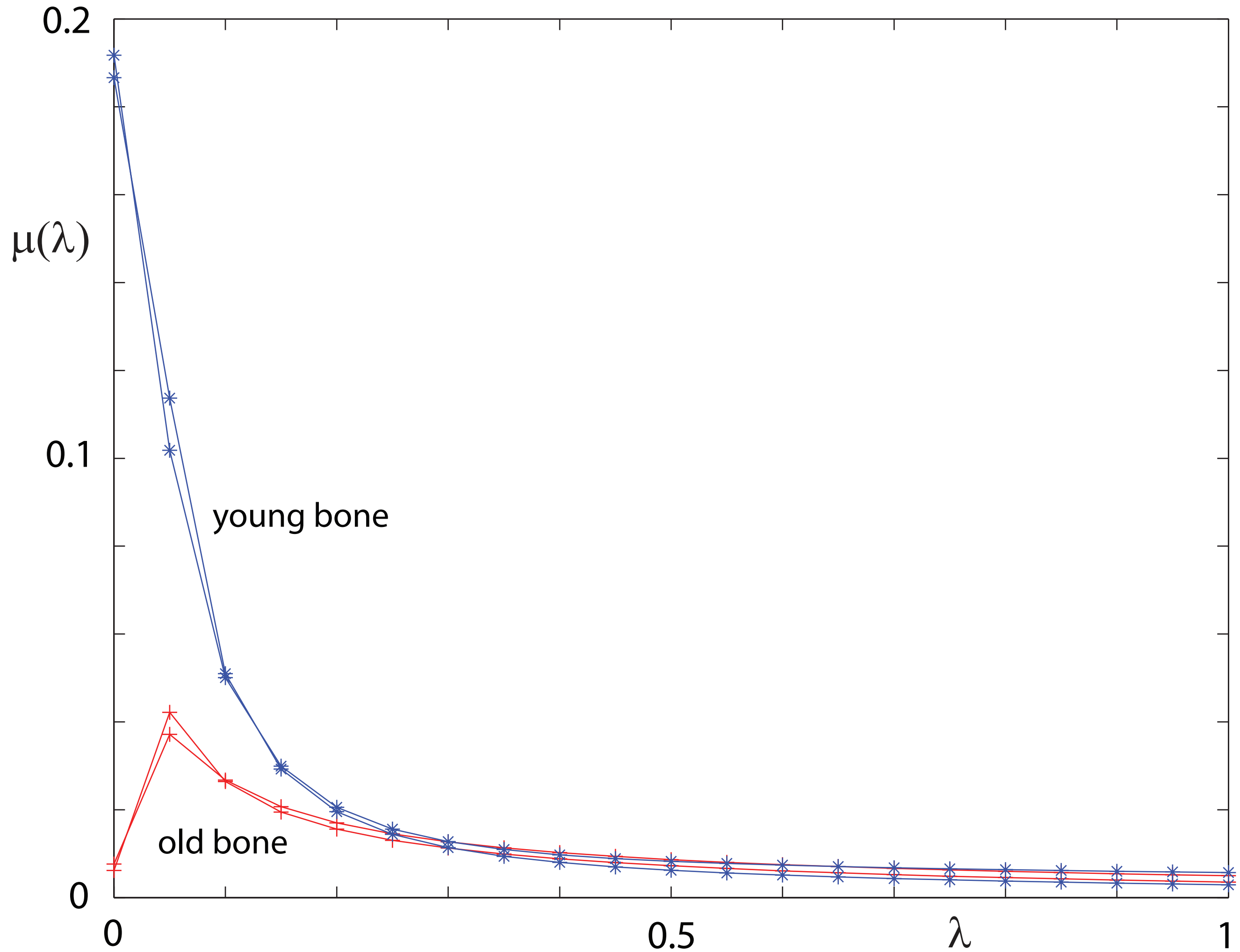
the math doesn't care if it's sea ice or bone

spectral characterization of porous microstructures in bone



Golden, Murphy, Cherkaev
J. Biomechanics 2010

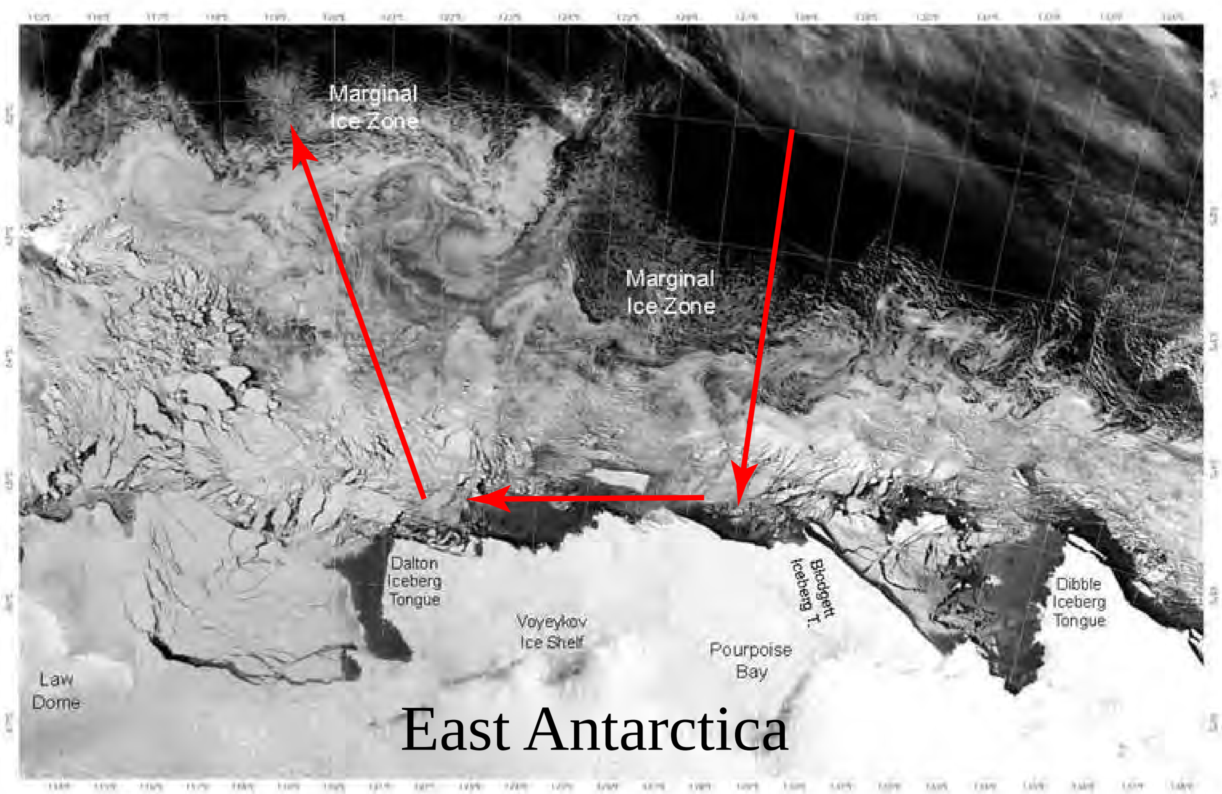
reconstructing the spectral measure from complex permittivity data



Golden, Murphy, Cherkaev
J. Biomechanics 2010

Sea Ice Physics and Ecosystem eXperiment (SIPEX)

4 September - 17 October 2007



Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

International Polar Year (IPY) 2007-2008

ace CRC
ANTARCTIC CLIMATE & ECOSYSTEMS
COOPERATIVE RESEARCH CENTRE

Measuring electrical and fluid transport during SIPEX

with Adam Gully

electrical properties --- help monitor ***fluid*** transport processsses
--- important in ***thickness*** measurements, e.g. EMI

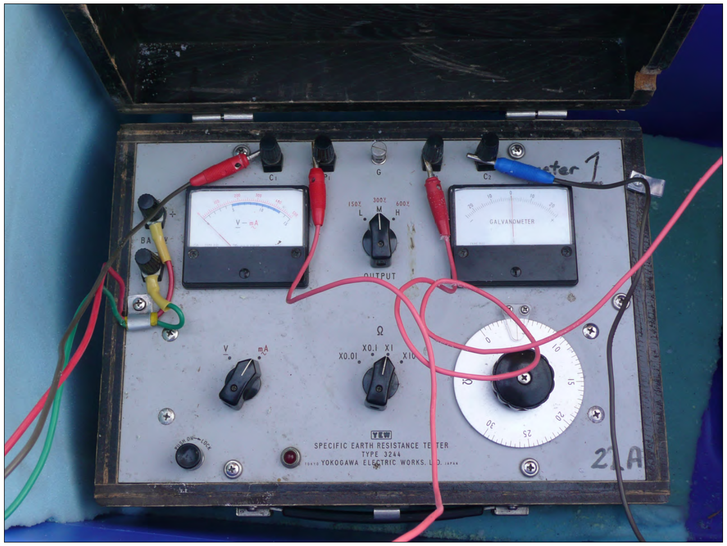
1. Vertical DC conductivity - direct from cores

2. Horizontal DC conductivity - Wenner array
   surface impedance tomography: invert resistance data to recover profile of electrical properties

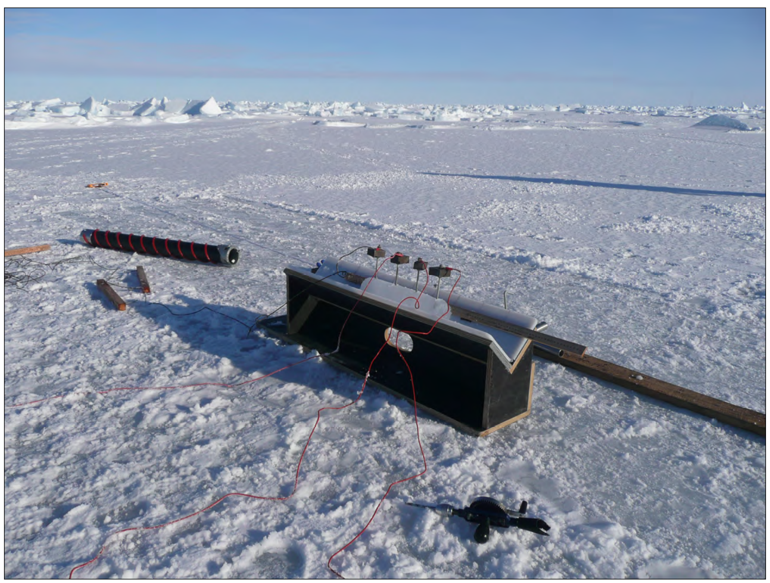
3. Fluid permeability -- first measurements in Antarctic pack ice

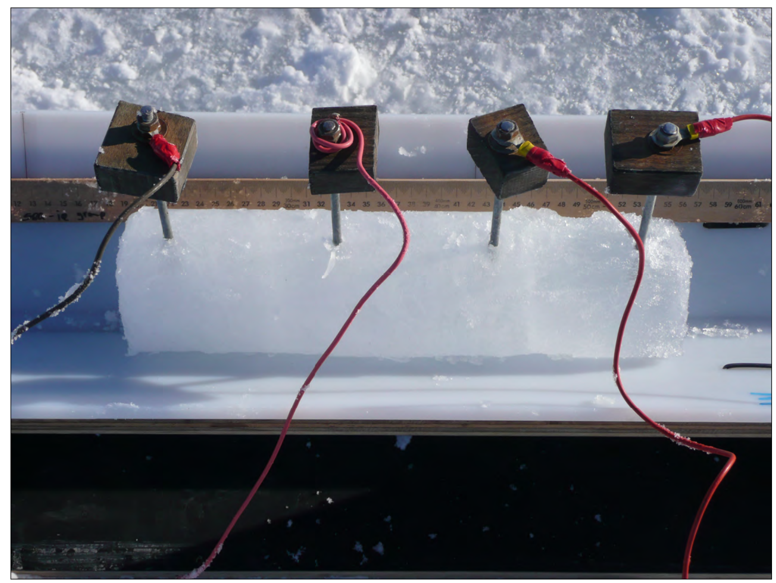
4. Tracer studies

electrical measurements



Wenner array



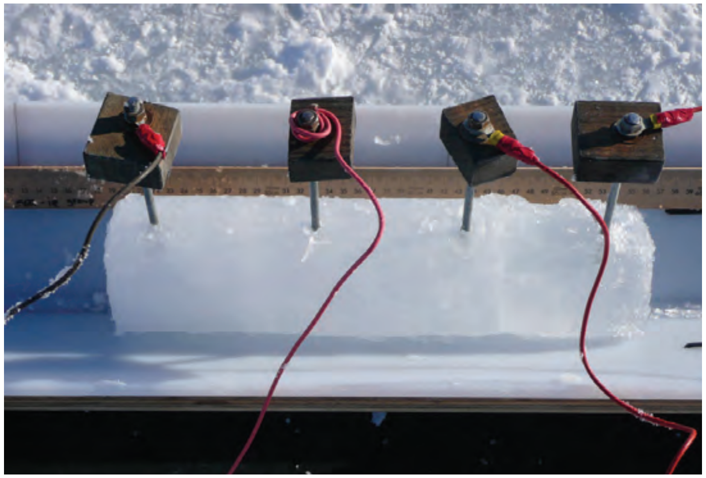


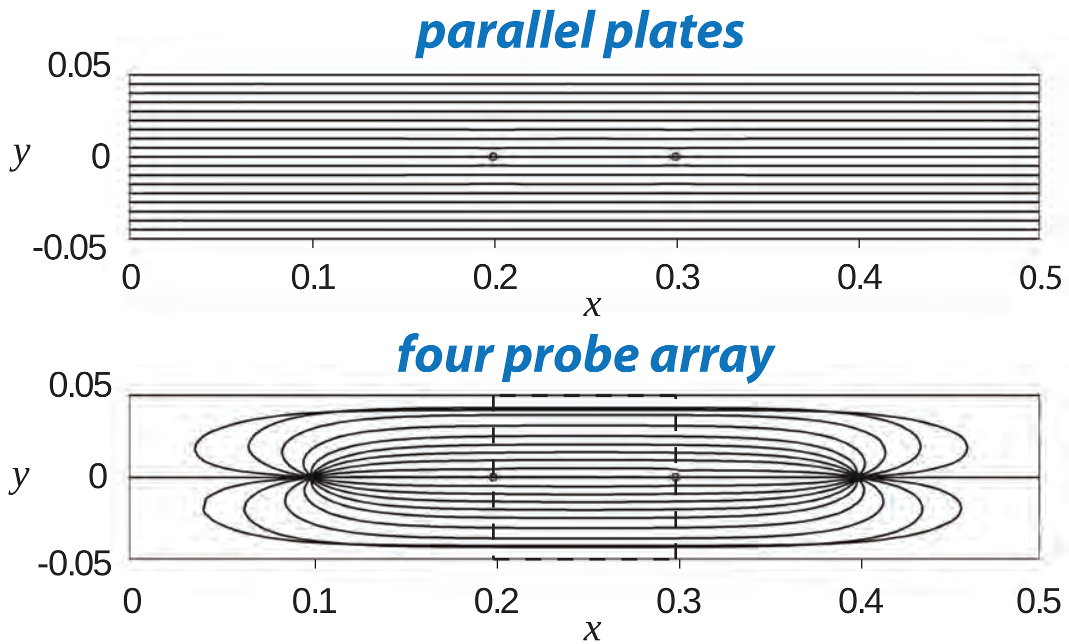
vertical conductivity

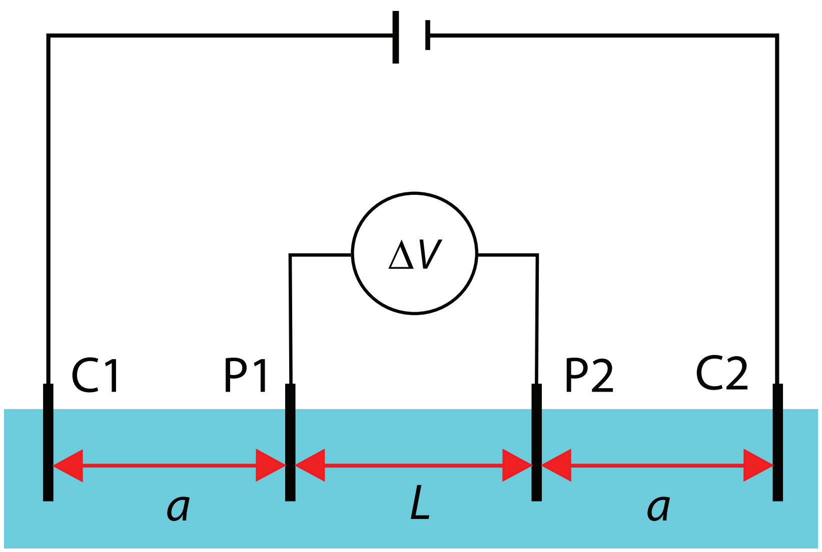
Zhu, Golden, Gully, Sampson ***Physica B*** 2010

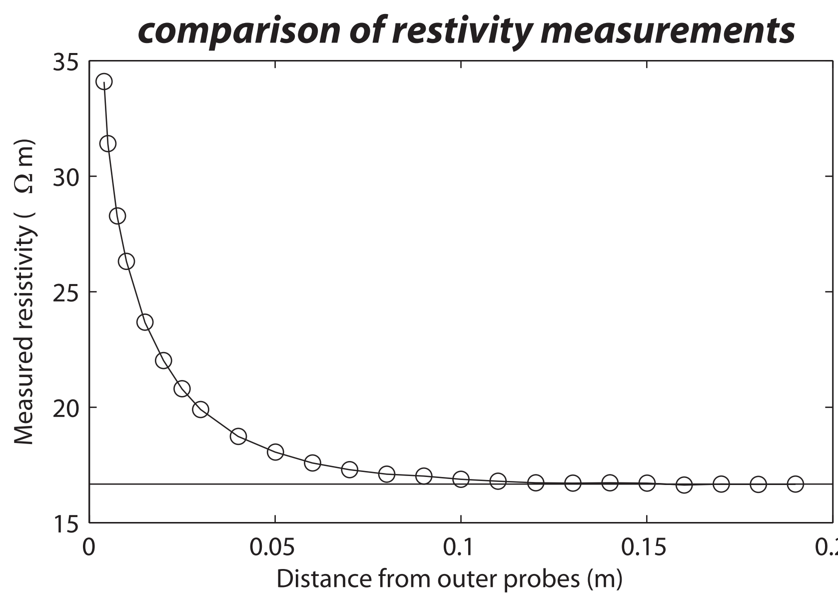
Sampson, Golden, Gully, Worby ***Deep Sea Research*** 2011

comparison of electric current streamlines for parallel plates vs. four probe array

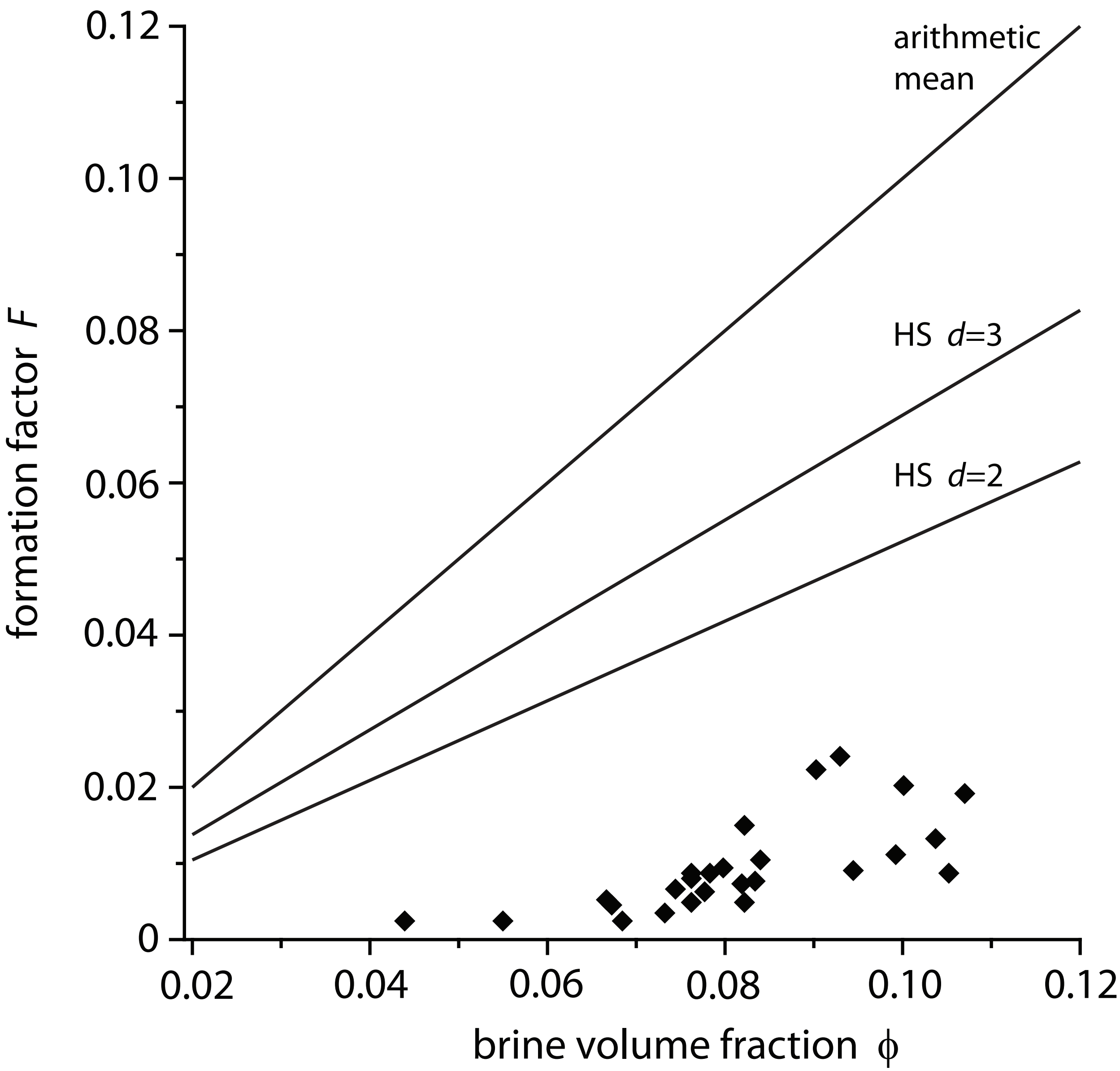




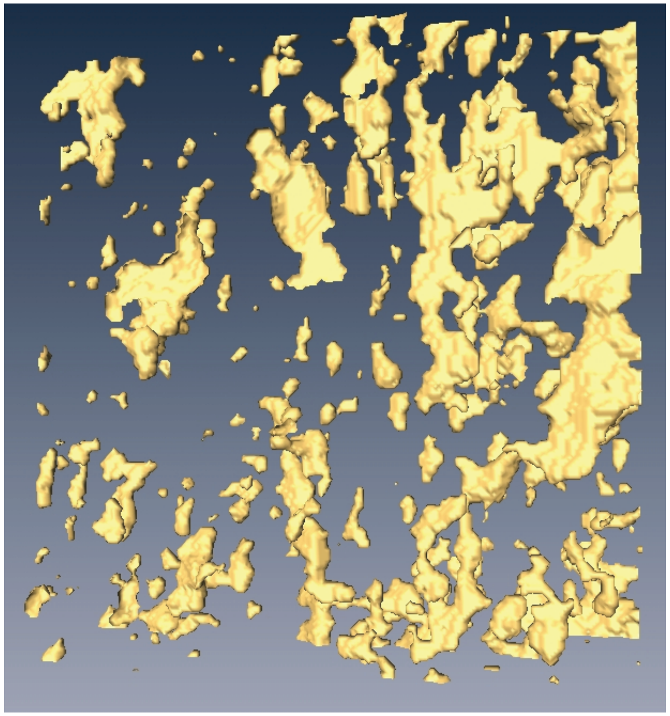




rigorous bounds on the vertical conductivity



coupling fluid permeability and electrical conductivity



$$k^* = \frac{r_{c,e}^2}{8} \frac{\sigma^*}{\sigma_b}$$

Friedman and Seaton 1998

$$r_{c,e} = \textit{critical electrical radius}$$

$$0.1 \text{ mm} \leq r_{c,e} \leq 0.2 \text{ mm}$$

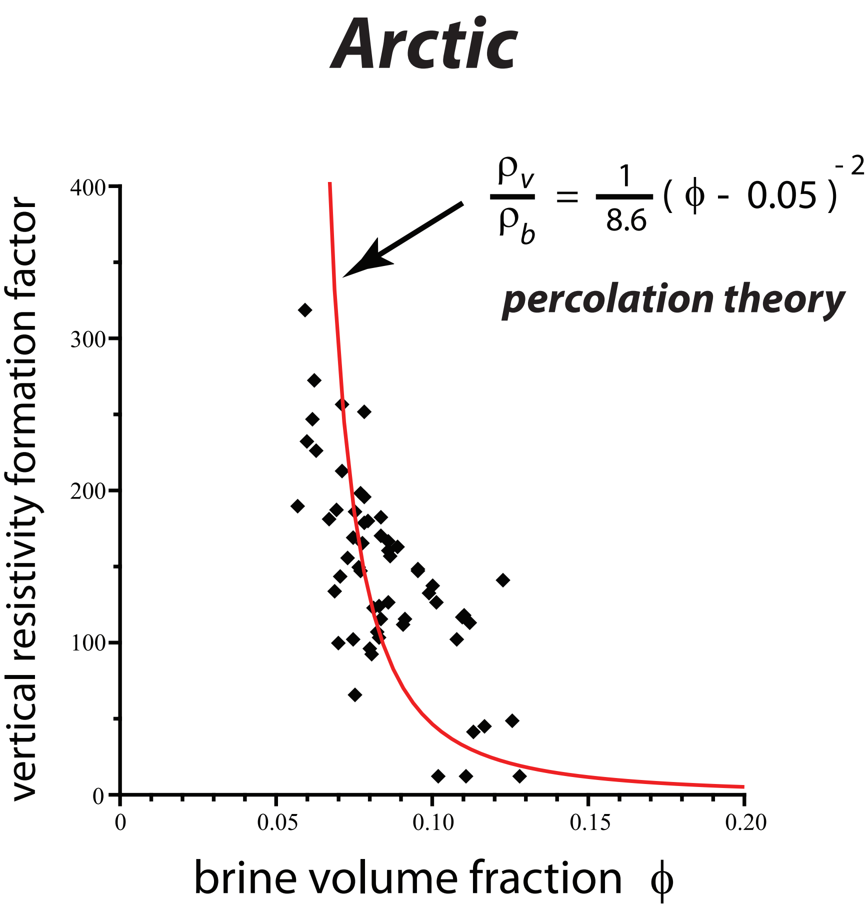
vertical formation factor for conductivity

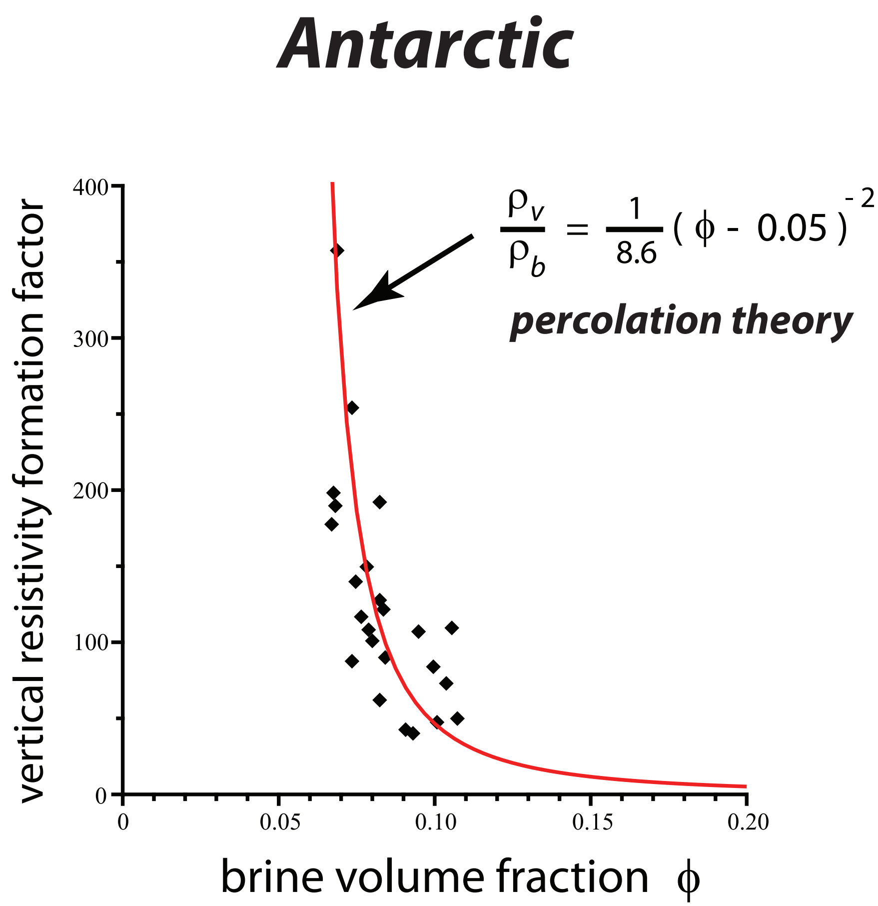
$$F = \sigma_v^* / \sigma_b$$

$$F(\phi) = F_0(\phi - 0.05)^2 \qquad 6 \leq F_0 \leq 24$$

critical behavior of electrical transport in sea ice

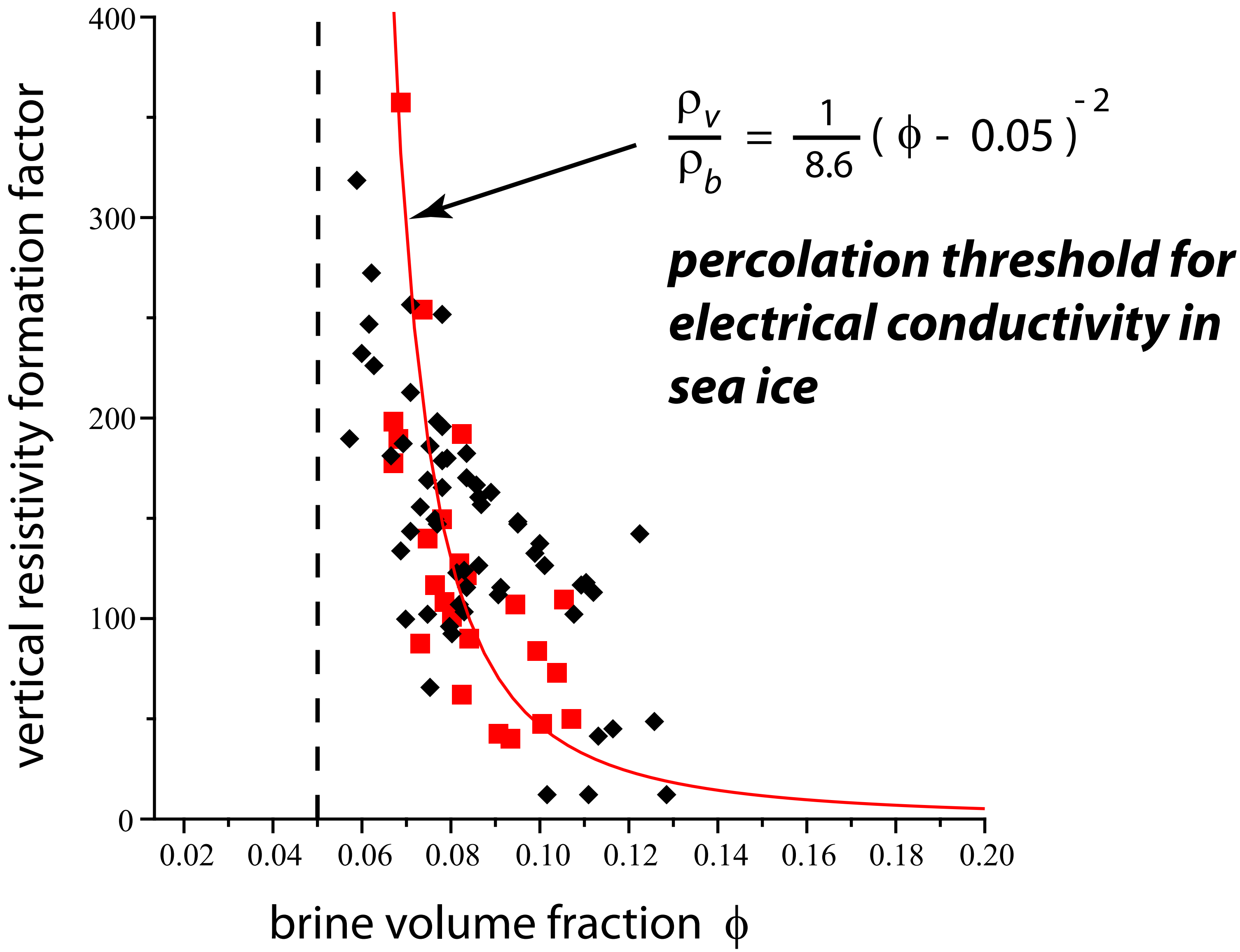
electric signature of the on-off switch for fluid flow



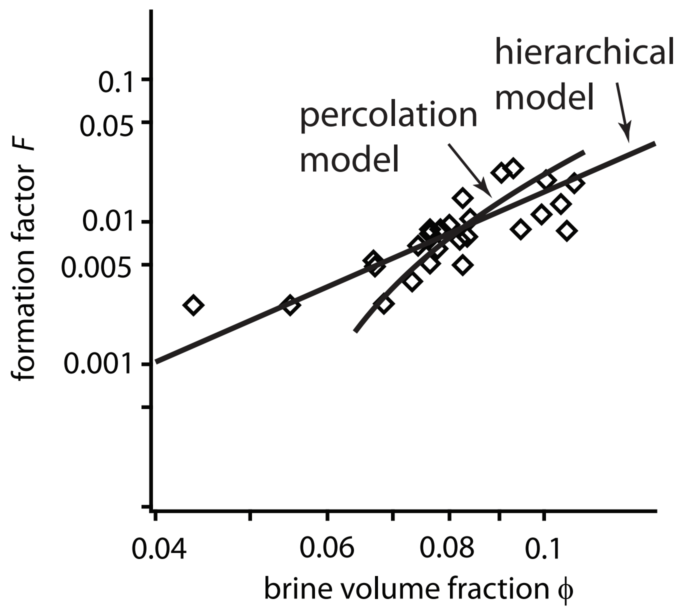


Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2010

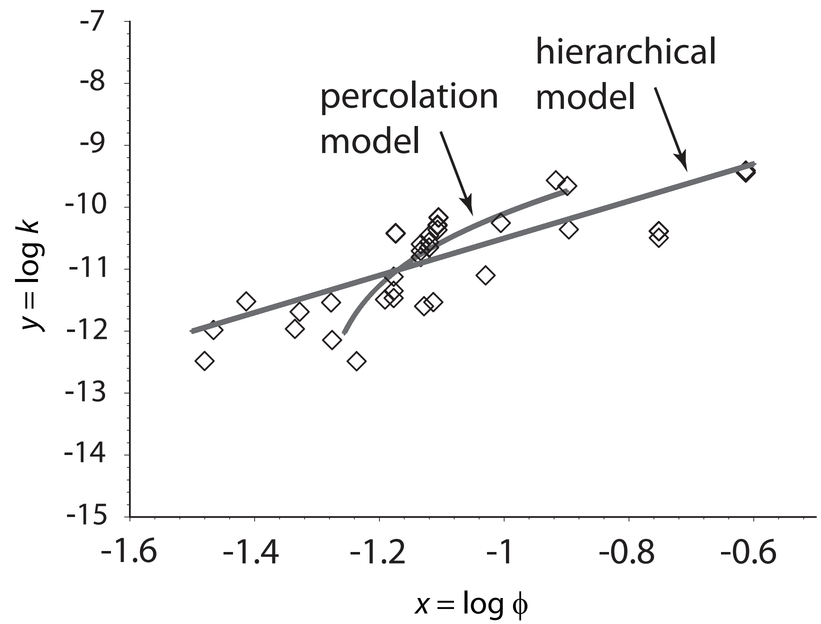
black diamonds -- Malcolm and Keleigh -- Arctic
red squares -- Ken and Adam -- Antarctic



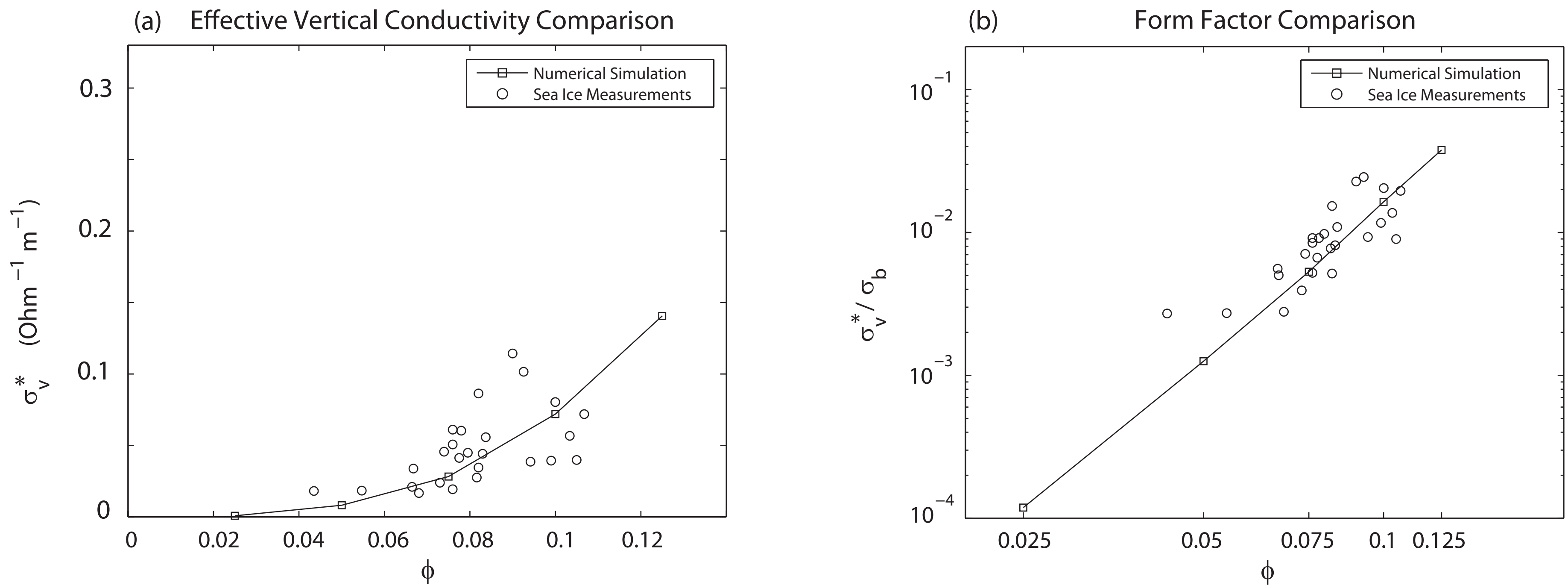
Antarctic electrical conductivity



Arctic fluid permability

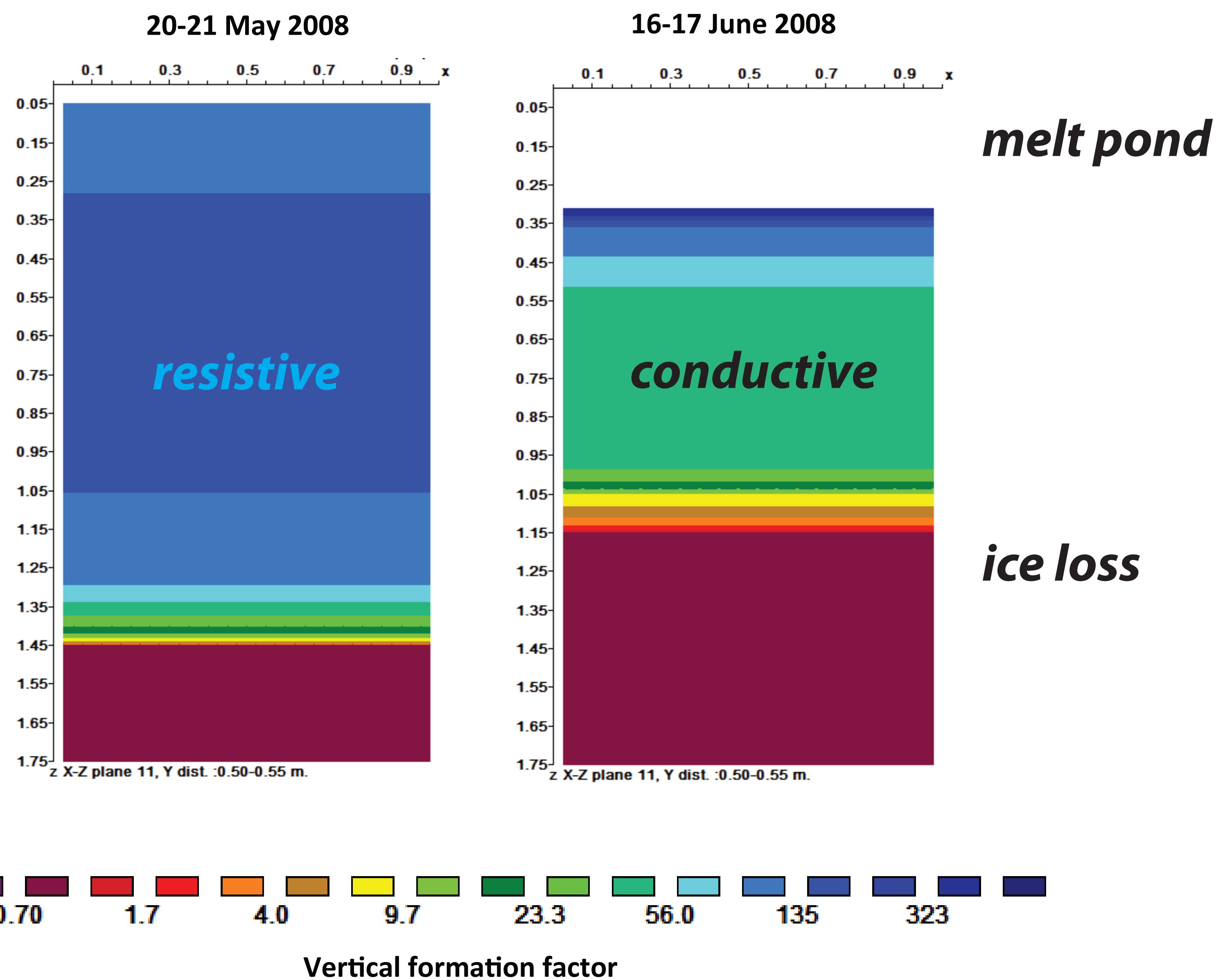


network model vs. vertical conductivity data



Zhu, Golden, Gully, Sampson, Physica B, 2010

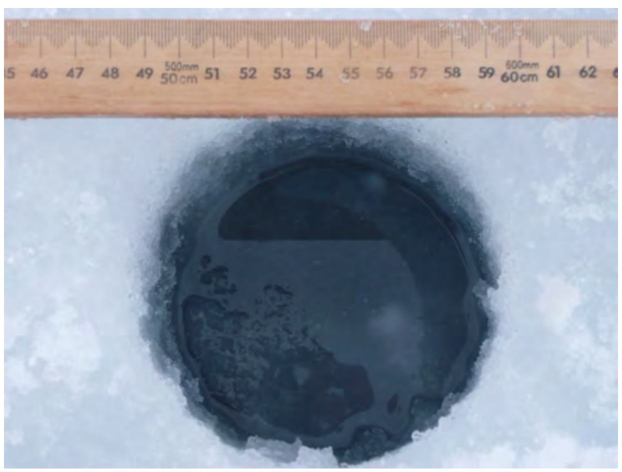
reconstructed resistivity (formation factor) profiles from Arctic sea ice near Barrow

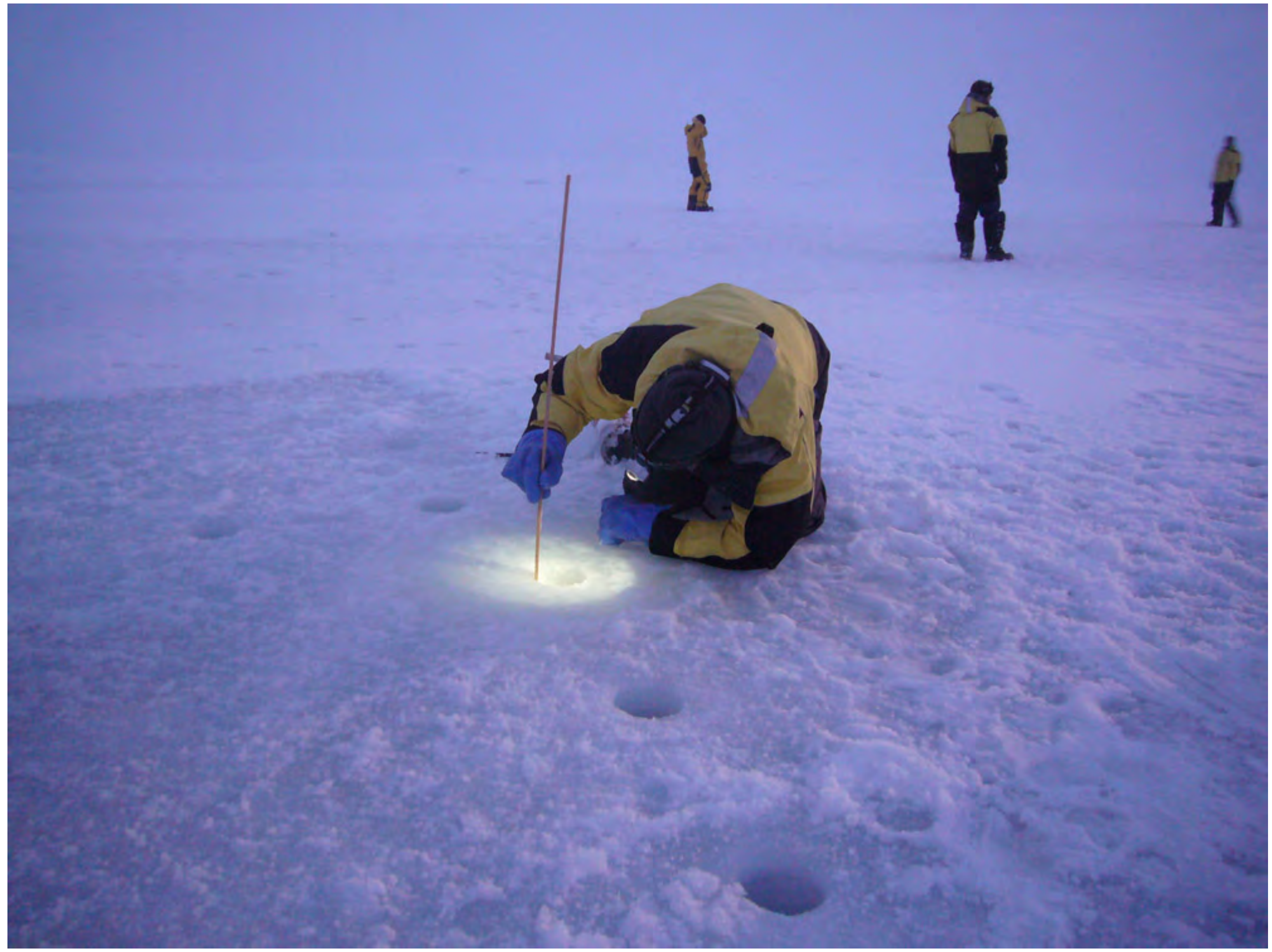


cross borehole tomography

Keleigh Jones, Malcolm Ingham

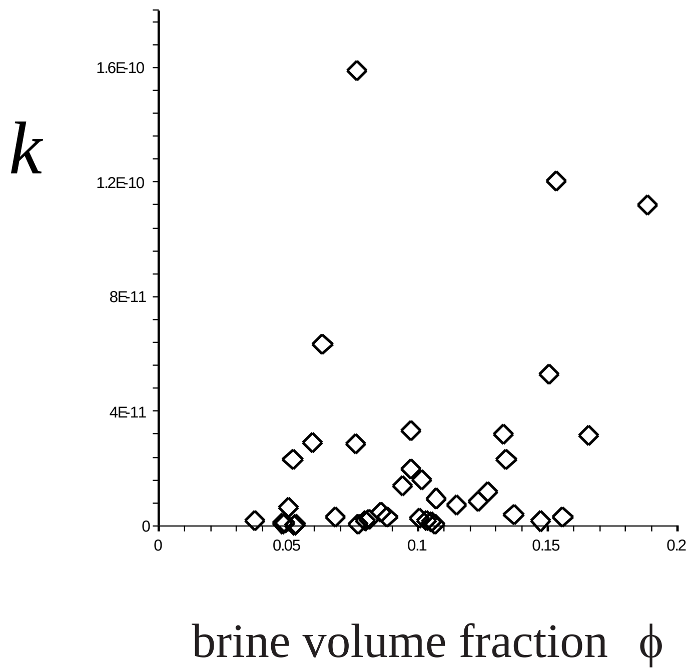
Measurements of fluid permeability

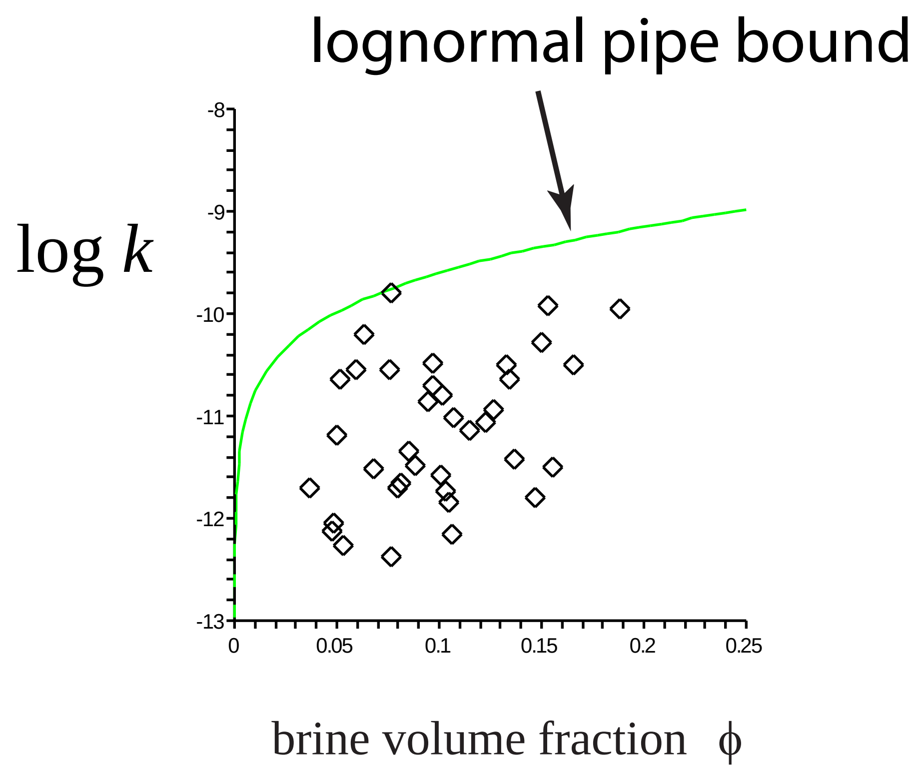




(Photo by Jan Lieser)

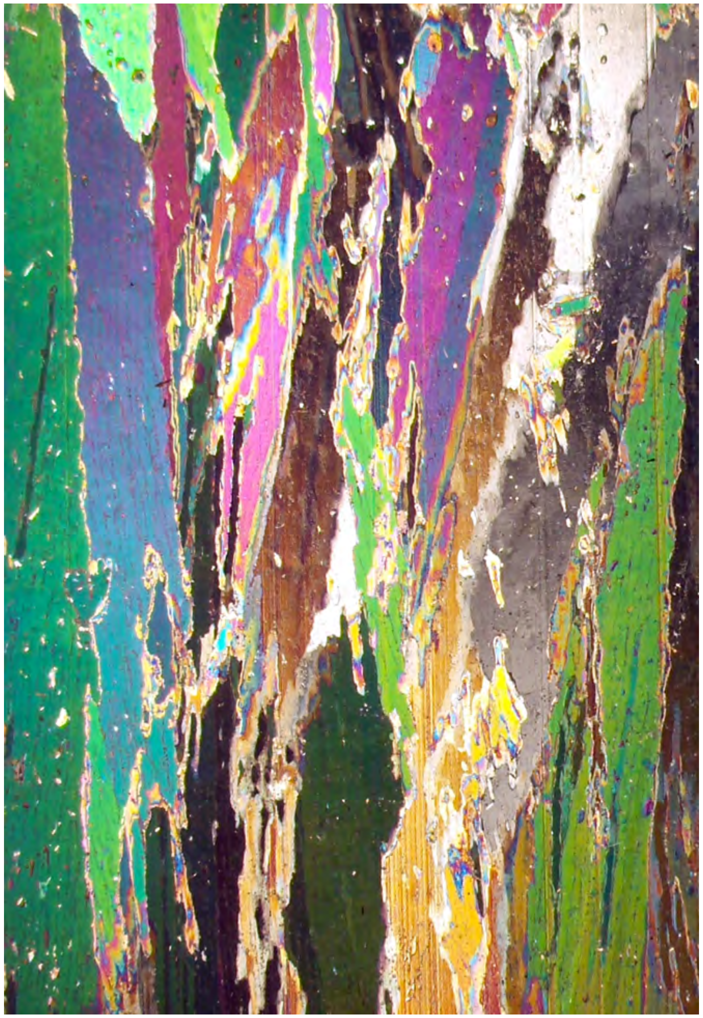
corrected permeability data -- excludes horizontal flow





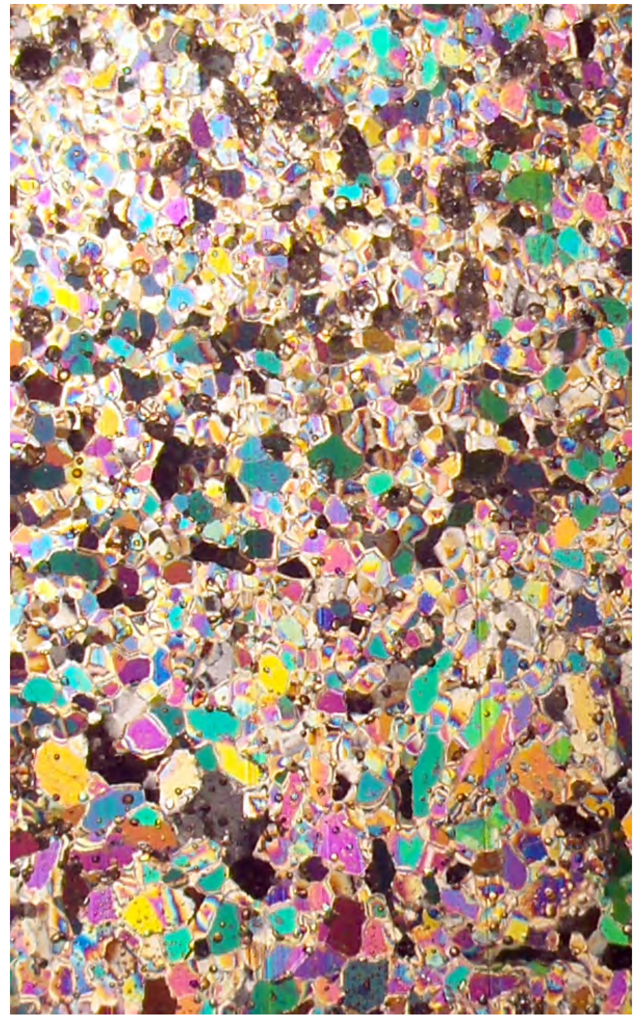
higher threshold for fluid flow in Antarctic granular sea ice

columnar



5%

granular

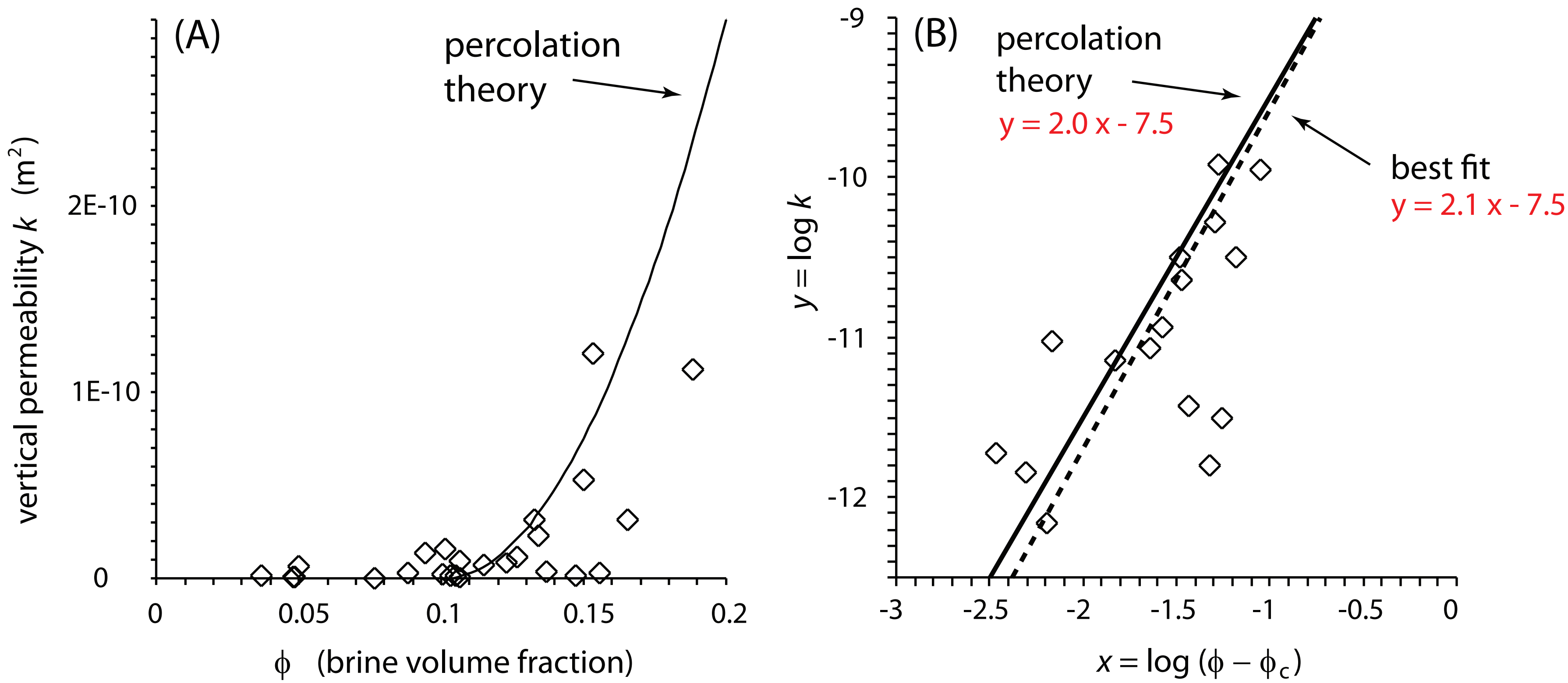


10%

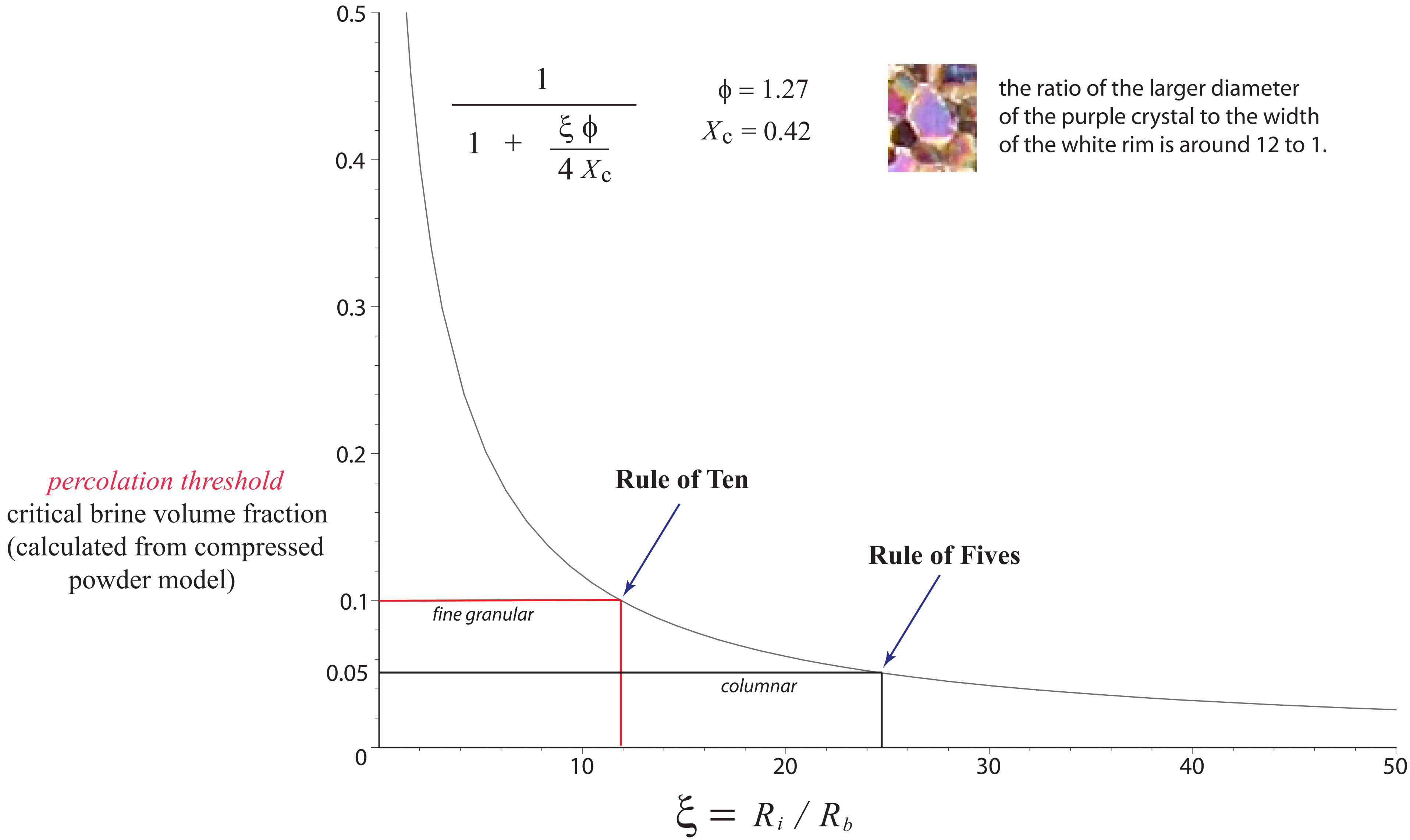
different microstructure -- different threshold

granular ice common in Arctic surface layer

permeability data for granular sea ice

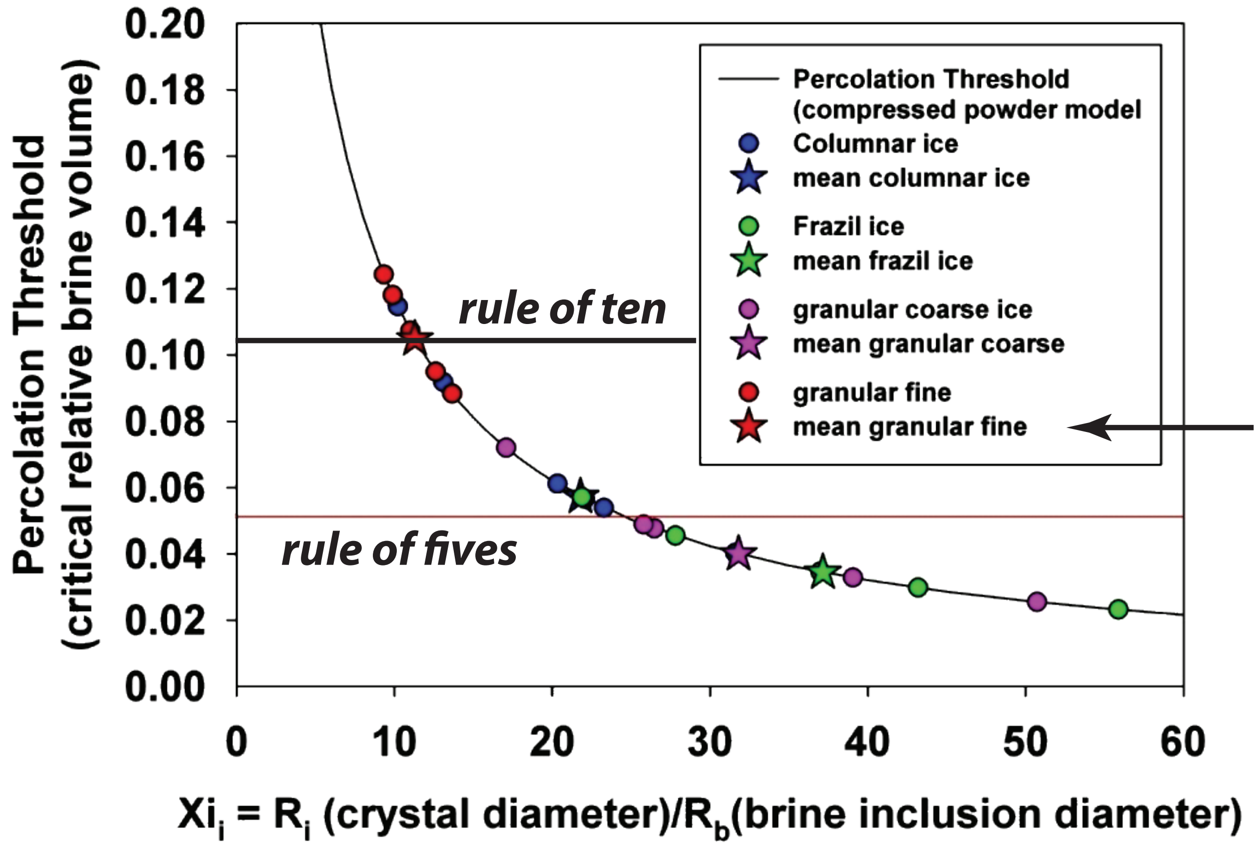


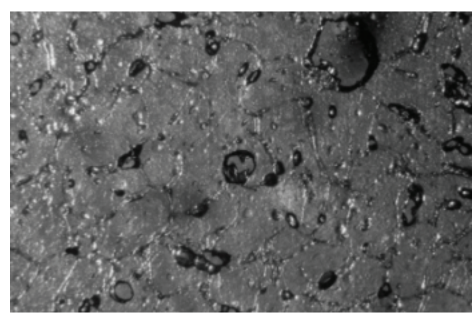
Golden, Gully, Tison, 2010



$\frac{1}{1 + \frac{\xi \phi}{4 X_c}}$
$\phi = 1.27$
$X_c = 0.42$
the ratio of the larger diameter of the purple crystal to the width of the white rim is around 12 to 1.
percolation threshold
critical brine volume fraction (calculated from compressed powder model)
Rule of Ten
Rule of Fives
fine granular
columnar
0.5
0.4
0.3
0.2
0.1
0.05
0
10
20
30
40
50
$\xi = R_i / R_b$

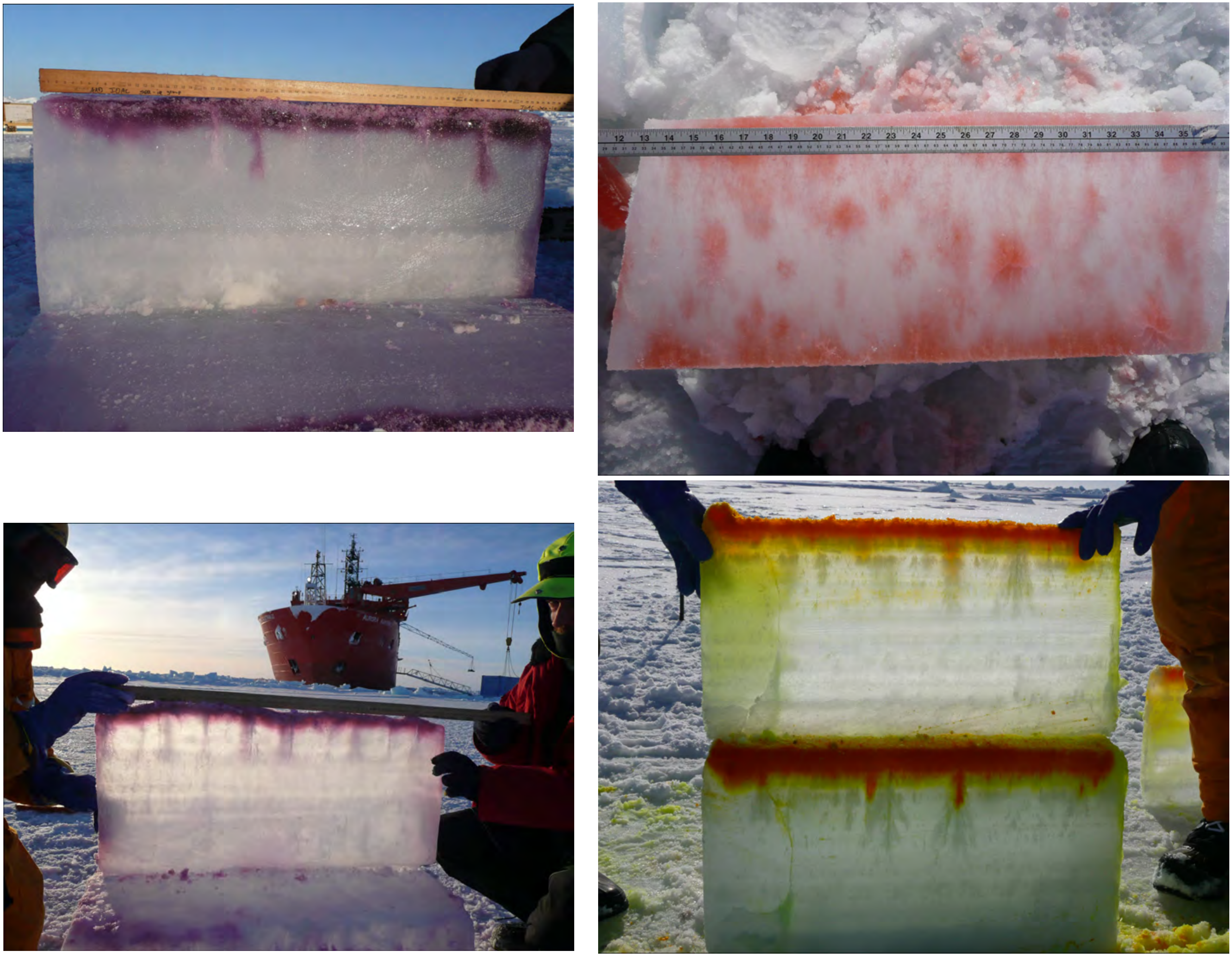
crystal data for compressed powder model





J. L. Tison

tracers flowing through inverted sea ice blocks



Conclusions

1. Polar climate processes exhibit ***critical behavior***.

2. Brine flow through sea ice is a key to geophysics and biology of polar regions, and displays critical behavior.

3. Comprehensive theory of fluid permeability, using mathematical techniques from solid state physics.

4. Measured sea ice transport properties in Antarctica.

5. Sea ice processes such as melt pond evolution, snow-ice formation, nutrient flux can be modeled more realistically.

6. Results can help to predict how climate change may affect sea ice packs and polar ecosystems.

THANKS to:

National Science Foundation

Division of Mathematical Sciences

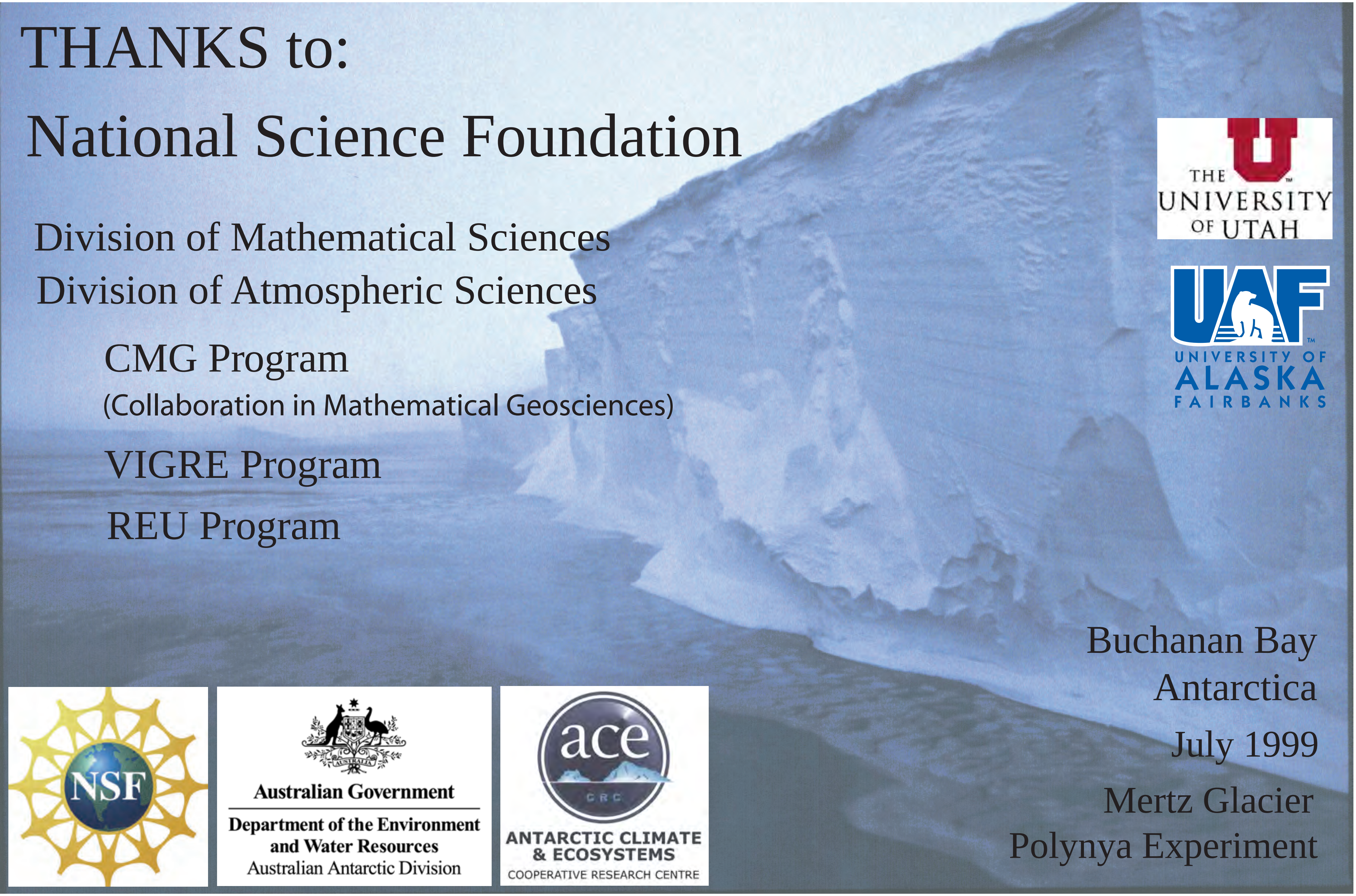
Division of Atmospheric Sciences

CMG Program

(Collaboration in Mathematical Geosciences)

VIGRE Program

REU Program



Buchanan Bay
Antarctica

July 1999

Mertz Glacier
Polynya Experiment

The University of Utah

UAF University of Alaska Fairbanks

NSF

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

ace CRC
Antarctic Climate & Ecosystems
Cooperative Research Centre

ISSN 0002-9920

Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Real analysis in polar coordinates (see page 613)