

Mathematics 1220 PRACTICE EXAM IV Spring 2002

1. Solve the following differential equations.

(a) $xy' + (1+x)y = e^{-x}$, $y(1) = 0$ (b) $y'' + 5y' - 6y = 0$

(c) $y'' - 4y' + 4y = 0$ (d) $y'' + 4y = 0$, $y(0) = 0$ and $y'(0) = 1$

(e) $y'' + y' + y = 0$ (f) $y'' + y' - 6y = 2x^2$ (g) $y'' + 4y' = \cos x$

2. Consider a microwave propagating vertically with electric field in the x -direction at time t given by $E(x, t)$. Assuming the wave is time harmonic with one angular frequency ω , then $E(x, t) = e^{i\omega t}E(x)$. If the wave propagates through a medium (such as a turkey) described by the permittivity, or dielectric constant, $\epsilon(x)$ (related to the index of refraction $n(x)$ by $n = \sqrt{\epsilon}$), then $E(x)$ satisfies the second order non-constant coefficient differential equation $\frac{d^2E}{dx^2} + k_0^2\epsilon(x)E(x) = 0$, where $k_0 = \omega/c$ is the free space wave number, and c is the speed of light. For a homogeneous medium, such as free space where $\epsilon(x) = 1$, E satisfies

$$\frac{d^2E}{dx^2} + k_0^2E = 0. \tag{1}$$

Use the characteristic polynomial of (1) to find its general solution, and then from your general solution find the particular solution satisfying $E(0) = 0$ and $\frac{dE}{dx}(0) = 1$.

3. Consider a pendulum of length ℓ . The angle $\theta(t)$ that the pendulum makes with respect to vertical at time t satisfies the second order nonlinear differential equation $\ell \frac{d^2\theta}{dt^2} = -g \sin \theta$, where g is the acceleration due to gravity. For small oscillations (small θ), $\sin \theta$ can be approximated by θ , which yields the linearized equation

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0, \quad \omega = \sqrt{\frac{g}{\ell}}. \tag{2}$$

Use the characteristic polynomial of (2) to find its general solution, and then from your general solution find the particular solution with initial angle $\theta(0) = \pi/16$ and initial angular velocity $\frac{d\theta}{dt}(0) = 0$.

4. Know how to solve the logistic equation.