Mathematics 1220 PRACTICE EXAM II Spring 2002

1. Calculate the following limits. Be sure to show all of your work.

(a) $\lim_{n\to\infty} n\left(\sqrt[n]{n}-1\right)$ (b) $\lim_{x\to 0}(\cos x)^{\csc x}$ (c) $\lim_{x\to+\infty} x^{25}e^{-x}$ (d) $\lim_{x\to\infty}\dfrac{xe^{-x^2/2}}{e^{-x}}$

(e) $\lim_{x\to-\infty}(e^{-x}-x)$ (f) $\lim_{x\to 1^+}\dfrac{\int_1^x \sin t\,dt}{x-1}$ (g) $\lim_{x\to 0}\dfrac{\int_0^x (e^{t^2}-1)\,dt}{x^3}$

(h) $\lim_{x\to\infty}\dfrac{3x}{\ln(100x+e^x)}$ (i) $\lim_{x\to 0}\dfrac{\cos x-1}{x^2}$

2. Calculate the following integrals. Be sure to show all of your work.

(a) $\int_{-\pi}^{\pi}\sin mx\cos nx\,dx$ (b) $\int_0^{2\pi}\sin mx\sin nx\,dx\ ,\quad m=n$

(c) $\int \tanh x\ln(\cosh x)\,dx$ (d) $\int \dfrac{e^x dx}{e^{2x}+2e^x+5}$ (e) $\int\sqrt{5-4x-x^2}\,dx$

(f) $\int\cos(\ln x)\,dx$ (g) $\int\dfrac{x\,dx}{2x^3+6x^2}\,dx$

3. # 31, p. 381

4. Determine whether the following improper integrals converge or diverge. Be sure to justify your answer completely.

(a) $\int_e^{\infty}\dfrac{dx}{x\sqrt{\ln x}}$ (b) $\int_1^{\infty}\dfrac{\ln x}{\sqrt{x^3+2x+1}}\,dx$ (c) $\int_0^{\infty}e^{-x}\cos x\,dx$

(d) $\int_{-\infty}^{\infty}\dfrac{e^{-x^2}}{x^2}\,dx$ (e) $\int_0^{\infty}x^{16,000}e^{-x}\,dx$ (f) $\int_0^1\dfrac{e^{-x}\sqrt{1+x}}{\sqrt[3]{\sin x\tan x}}\,dx$

5. Determine whether the following infinite series converge or diverge. Be sure to justify your answers completely.

(a) $\sum_{n=1}^{\infty}\left(\dfrac{3}{e}\right)^n$ (b) $\sum_{n=1}^{\infty}\sqrt{1-\cos\left(\dfrac{1}{n}\right)}$ (c) $\sum_{n=1}^{\infty}\sqrt{n}\,\sin\left(\dfrac{1}{n}\right)$

(d) $\sum_{n=1}^{\infty}\left(1-\dfrac{1}{n}\right)^n$ (e) $\sum_{n=2}^{\infty}\dfrac{1}{n(\ln n)^{\pi}}$ (f) $\sum_{k=2}^{\infty}\left(\dfrac{1}{k}-\dfrac{1}{k-1}\right)$

6. A (zero dimensional) bull frog initially jumps a meter. On each successive jump, he can only go $\frac{3}{4}$ of the distance of the previous jump. If he takes infinitely many jumps, how far does he travel?