# Mathematics 1220 PRACTICE EXAM I Spring 2002

1. Calculate the following limits.

   (a) $\displaystyle\lim_{n\to\infty}\left(\frac{n-1}{n}\right)^{2n}$ (b) $\displaystyle\lim_{x\to 0} x^x$ (c) $\displaystyle\lim_{x\to+\infty} x^{25}e^{-x}$

2. Calculate the following.

   (a) $\dfrac{d}{dx}\,(\ln(\tanh x))$, (b) $\displaystyle\int \frac{z}{2z^2+8}\,dz$ , (c) $\displaystyle\int \frac{\tan(\ln x)}{x}\,dx$ , (d) $\displaystyle\int \frac{dx}{x(1-x)}$ ,

   (e) $\dfrac{dy}{dx}$ , $y = \dfrac{(x^2+3)^{2/3}(3x+2)^2}{\sqrt{x+1}}$ (use log. differentiation), (f) $\displaystyle\int \frac{e^x}{1+e^{2x}}$

3. Experiments show that the rate of change of the atmospheric pressure $P(x)$ with altitude $x$ is proportional to the pressure. Write down the resulting differential equation for $P(x)$, and solve it, assuming that the pressure at 6000 meters is half its value $P_0$ at sea level.

4. p. 345, # 14

5. p. 335, # 32, 38

6. p. 350, # 2

7. Know how to solve the logistic equation.

8. Stewart wants to become a millionaire after 10 years by buying \$5,000 worth of a company's stock, which he wants to choose carefully. What must the sustained, annualized growth rate of the stock be in order to achieve his goal? Is Stewart being realistic?

9. Newton's law of cooling states that the rate at which an object cools is proportional to the difference between the temperature $\theta(t)$ of the object and the constant ambient temperature $T$,

   $$\frac{d\theta}{dt} = -k(\theta - T),$$

   where $k > 0$ is a constant depending on the object. A corpse is discovered at 2 pm, and its temperature is found to be 85°F, with the ambient air temperature being 68°F. Assuming $k = 0.5$ hr$^{-1}$, find the time of death.