MODELING *the* MELT:

What math tells us about the shrinking polar ice caps

Kenneth M. Golden
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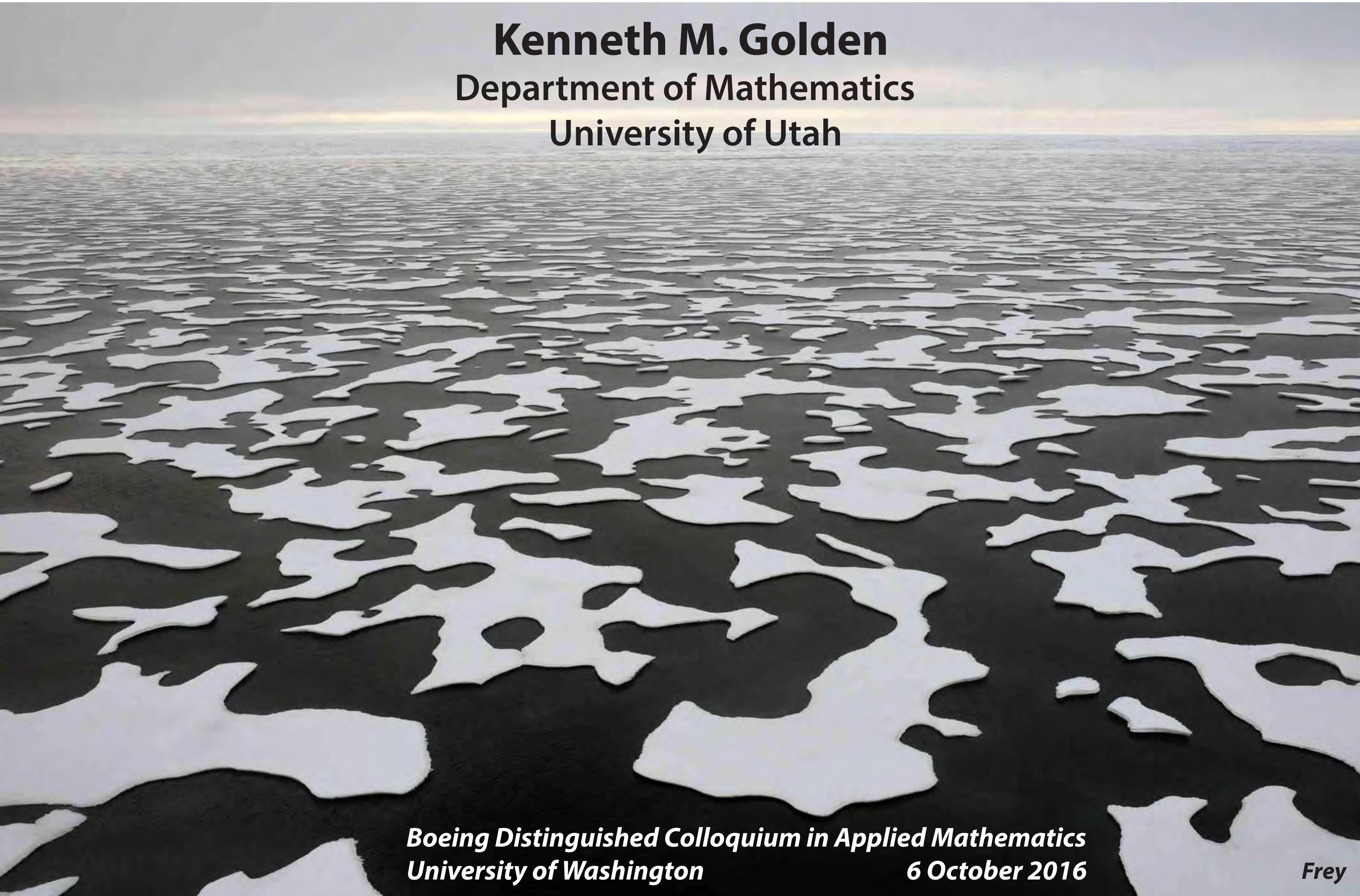
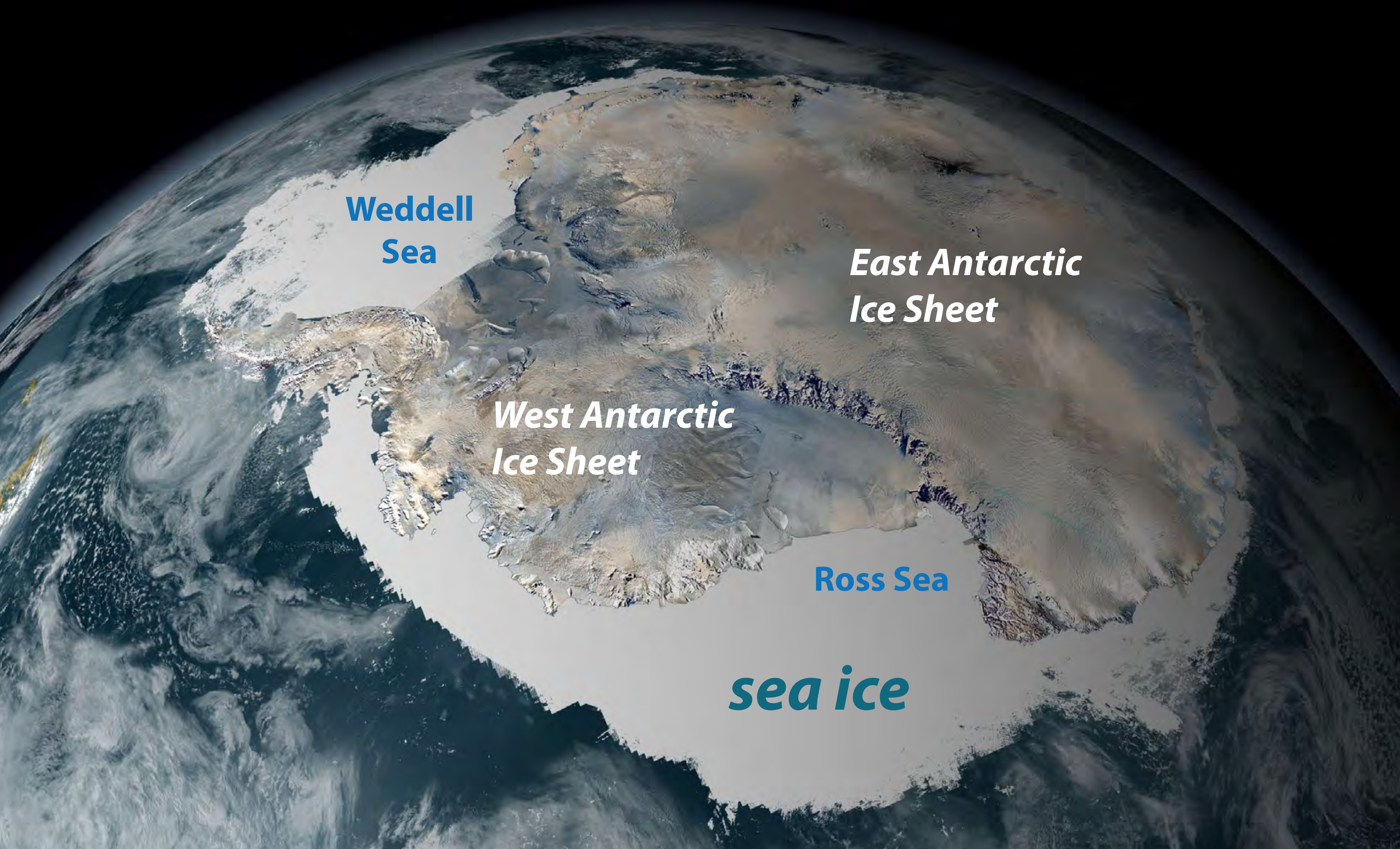


Boeing Distinguished Colloquium in Applied Mathematics
University of Washington *6 October 2016*

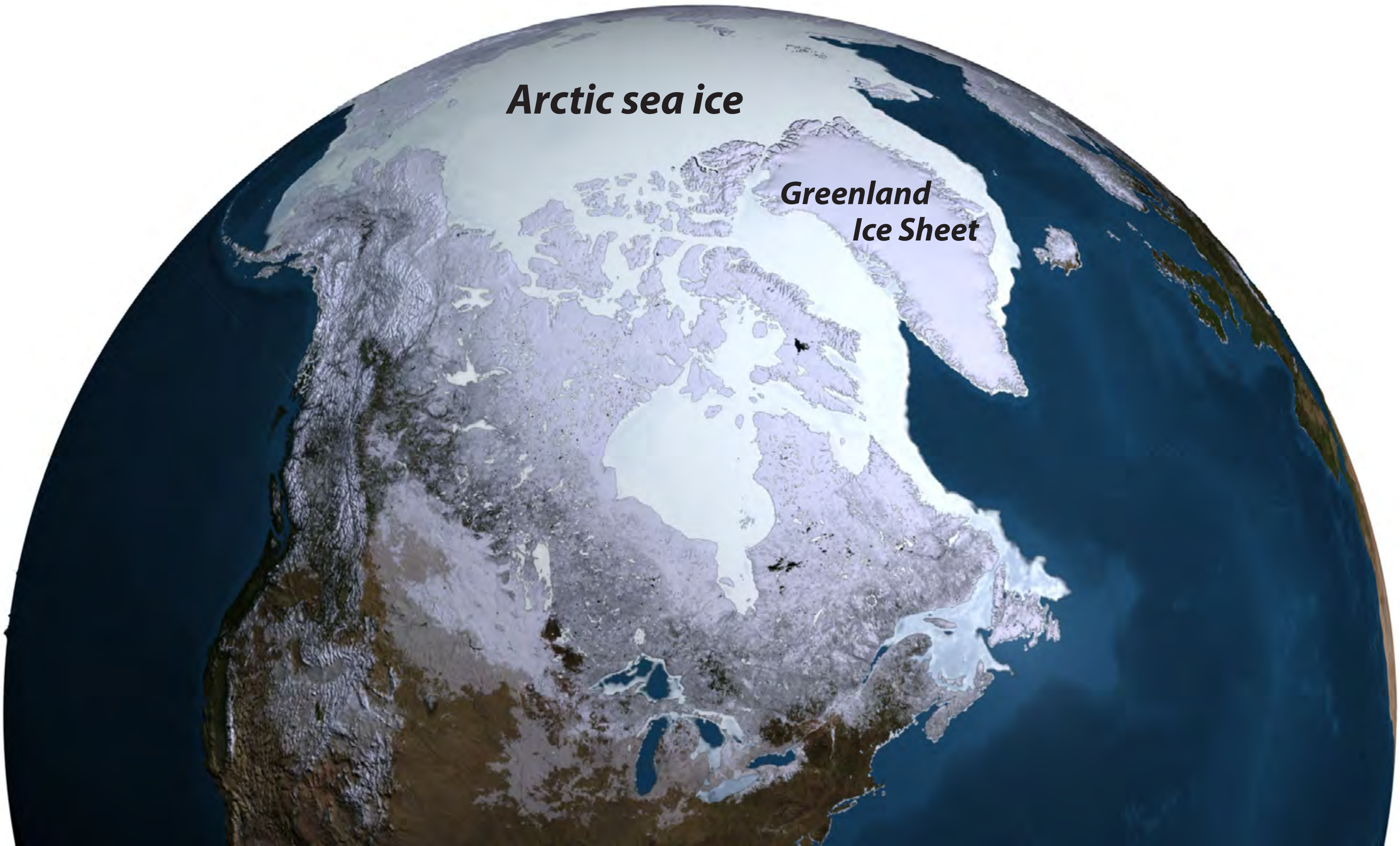
Frey

ANTARCTICA

southern cryosphere



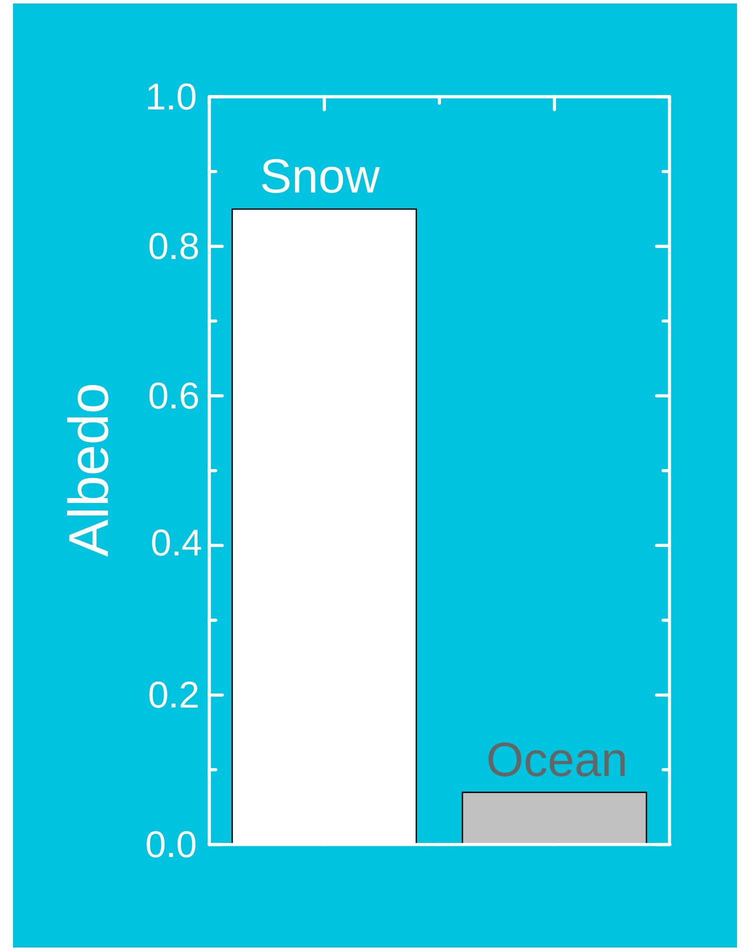
northern cryosphere




SEA ICE covers 7 - 10% of earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

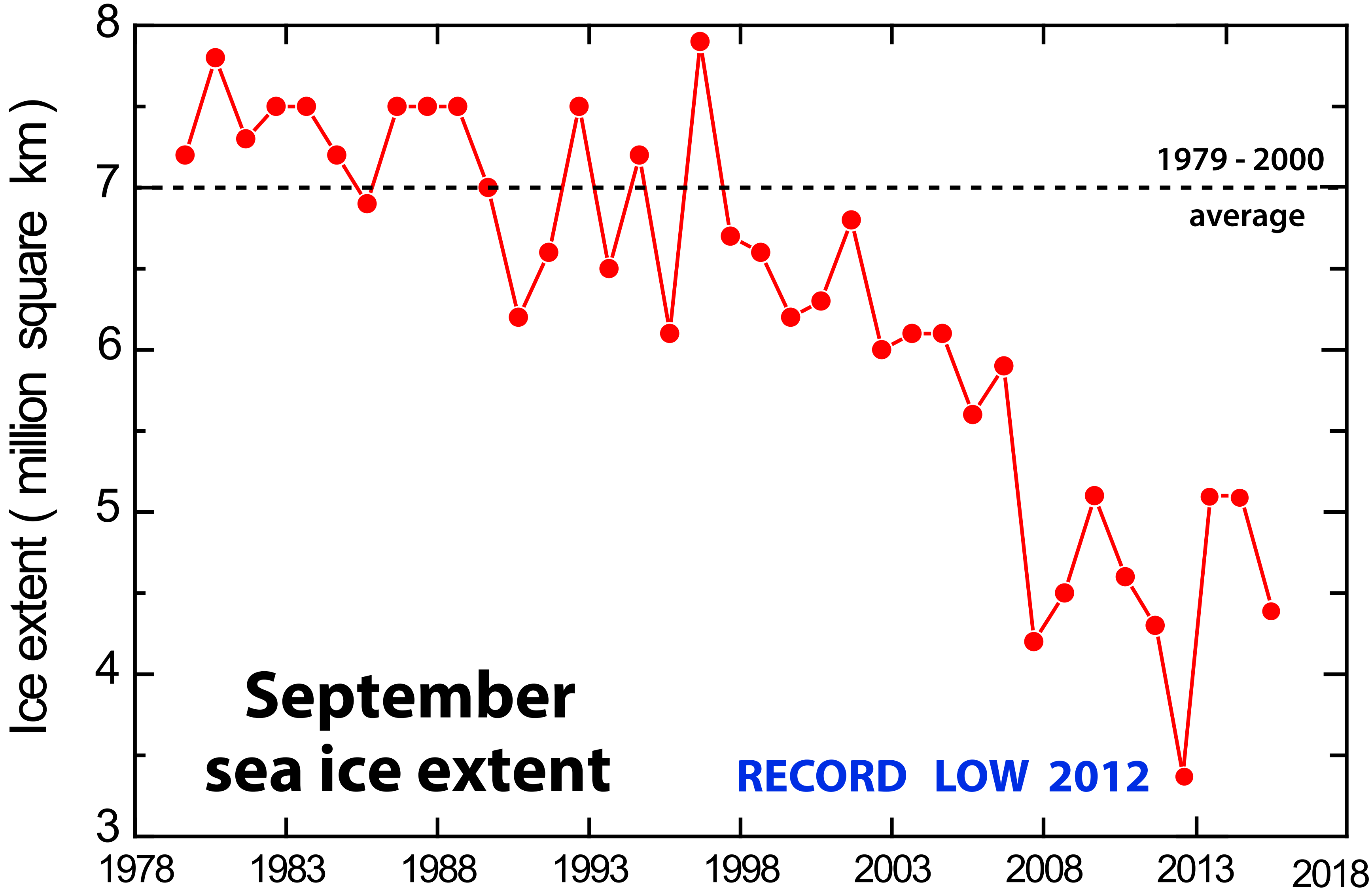
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

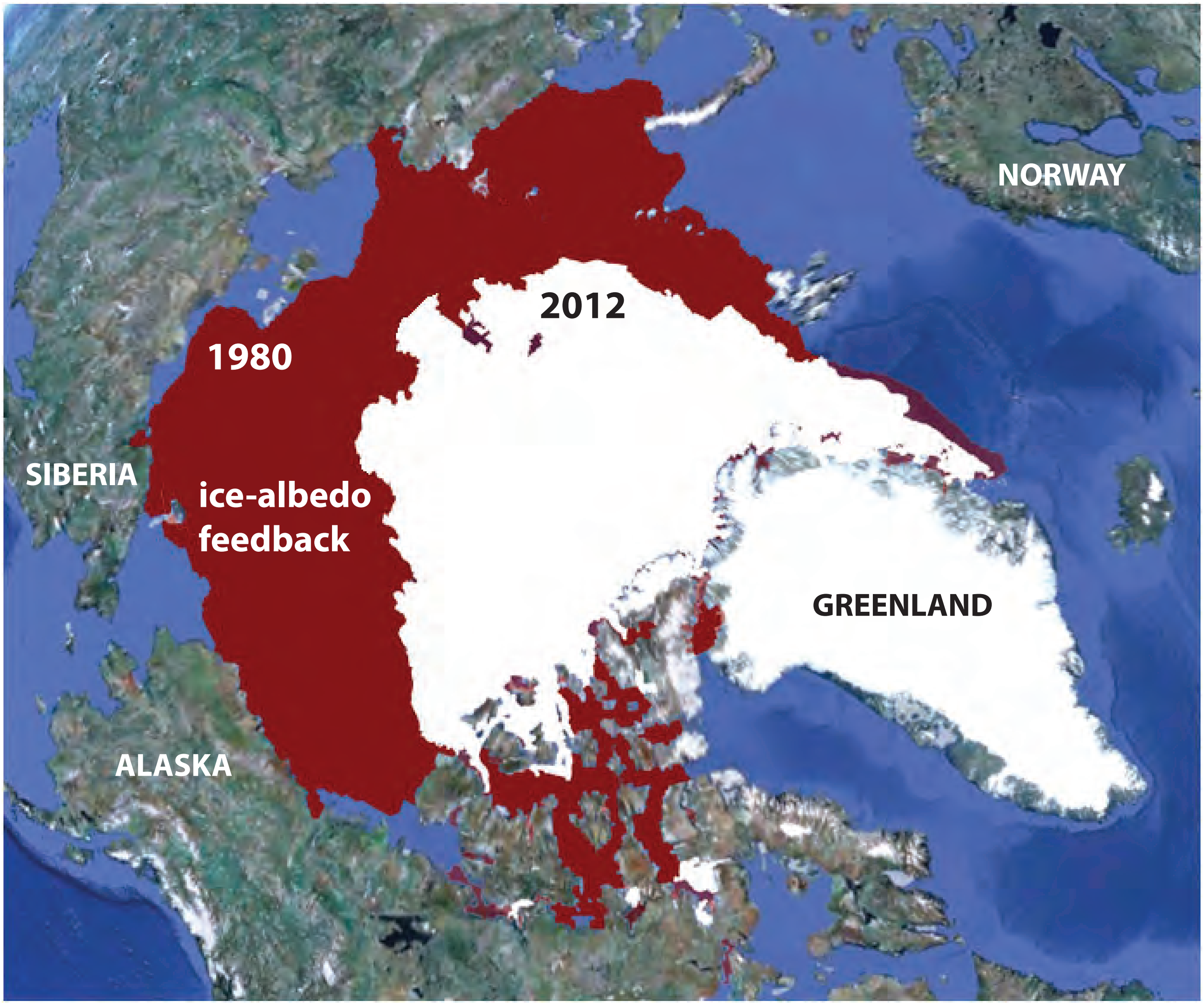
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers

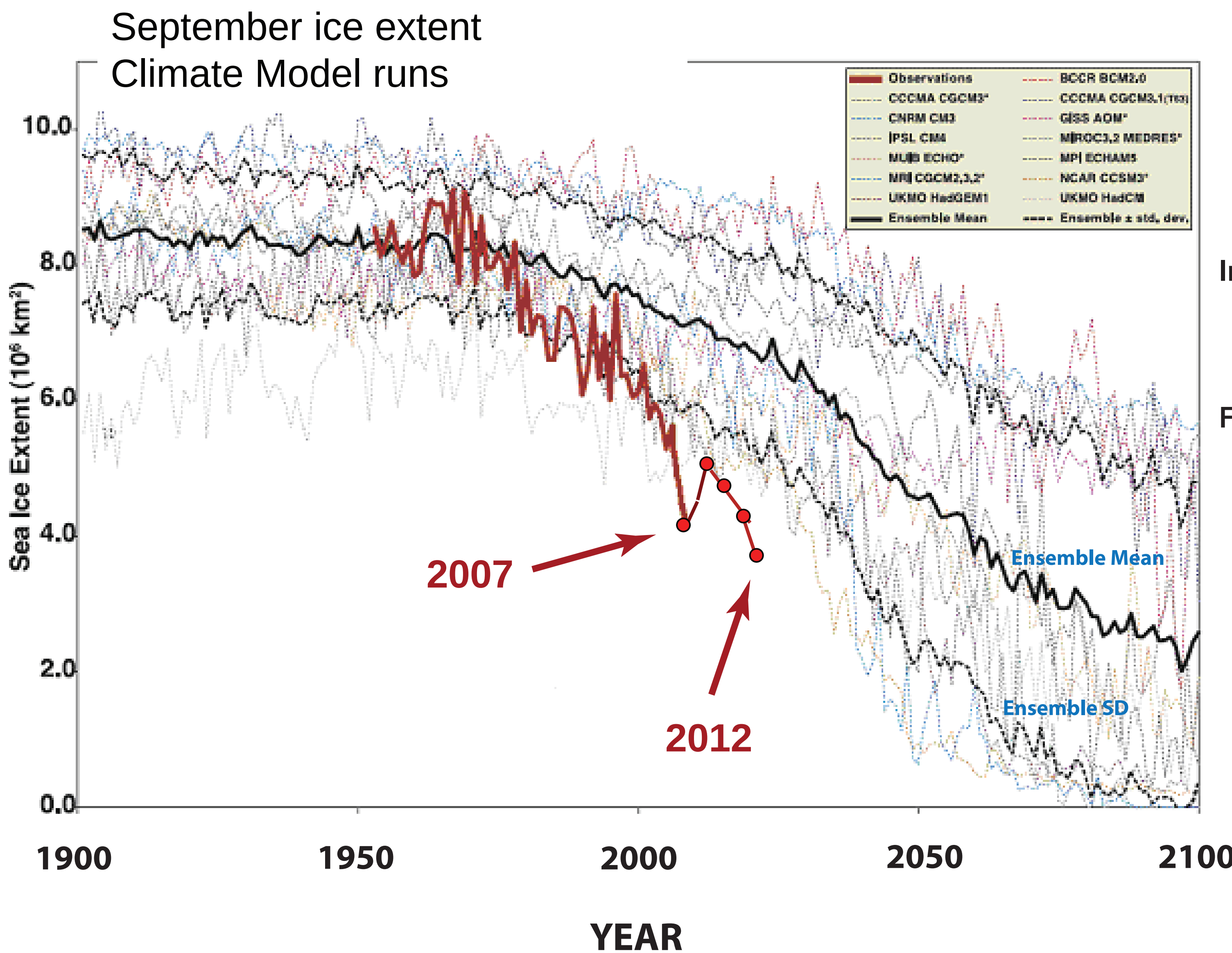


Perovich

recent losses in comparison to the United States

Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

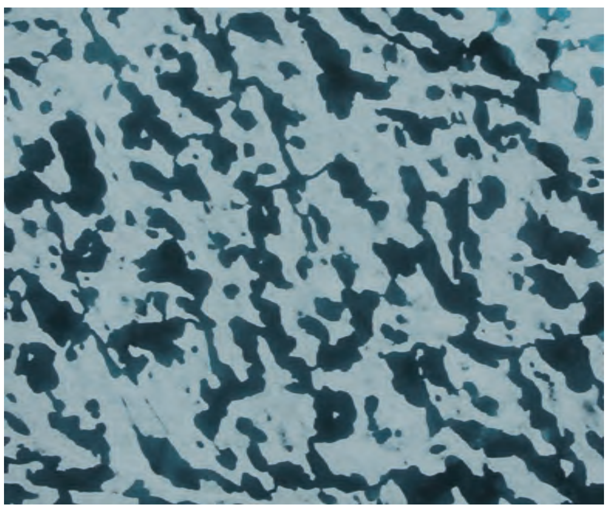
Fourth Assessment AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes

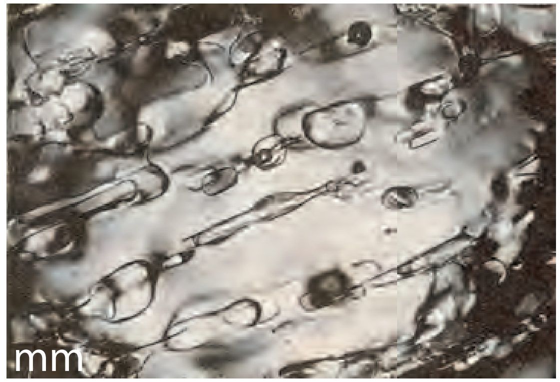
linkage of scales

sea ice is a multiscale composite

displaying structure over 10 orders of magnitude

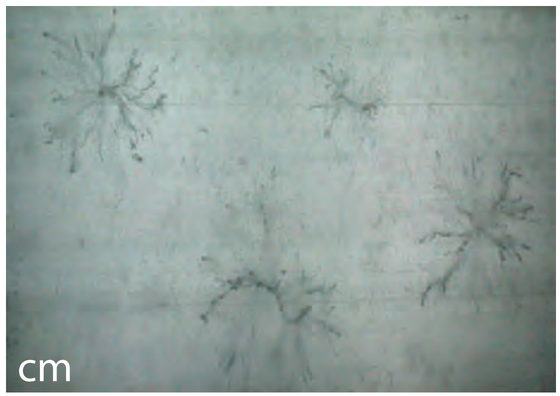
0.1 millimeter

1 meter

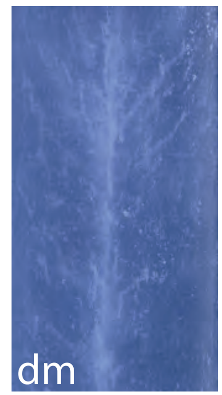


brine inclusions

polycrystals

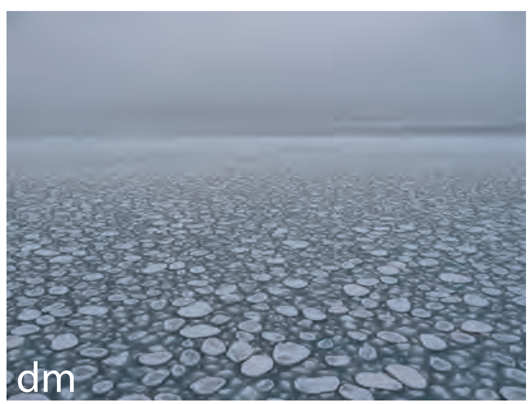


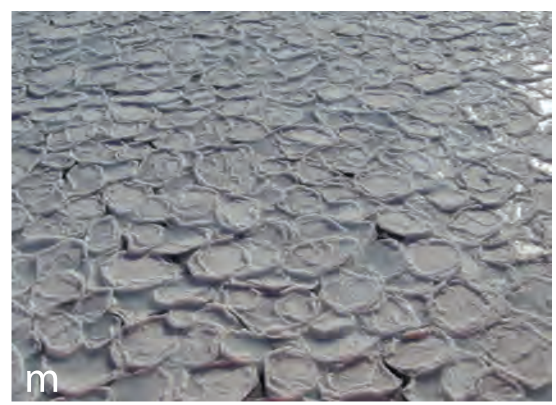
horizontal



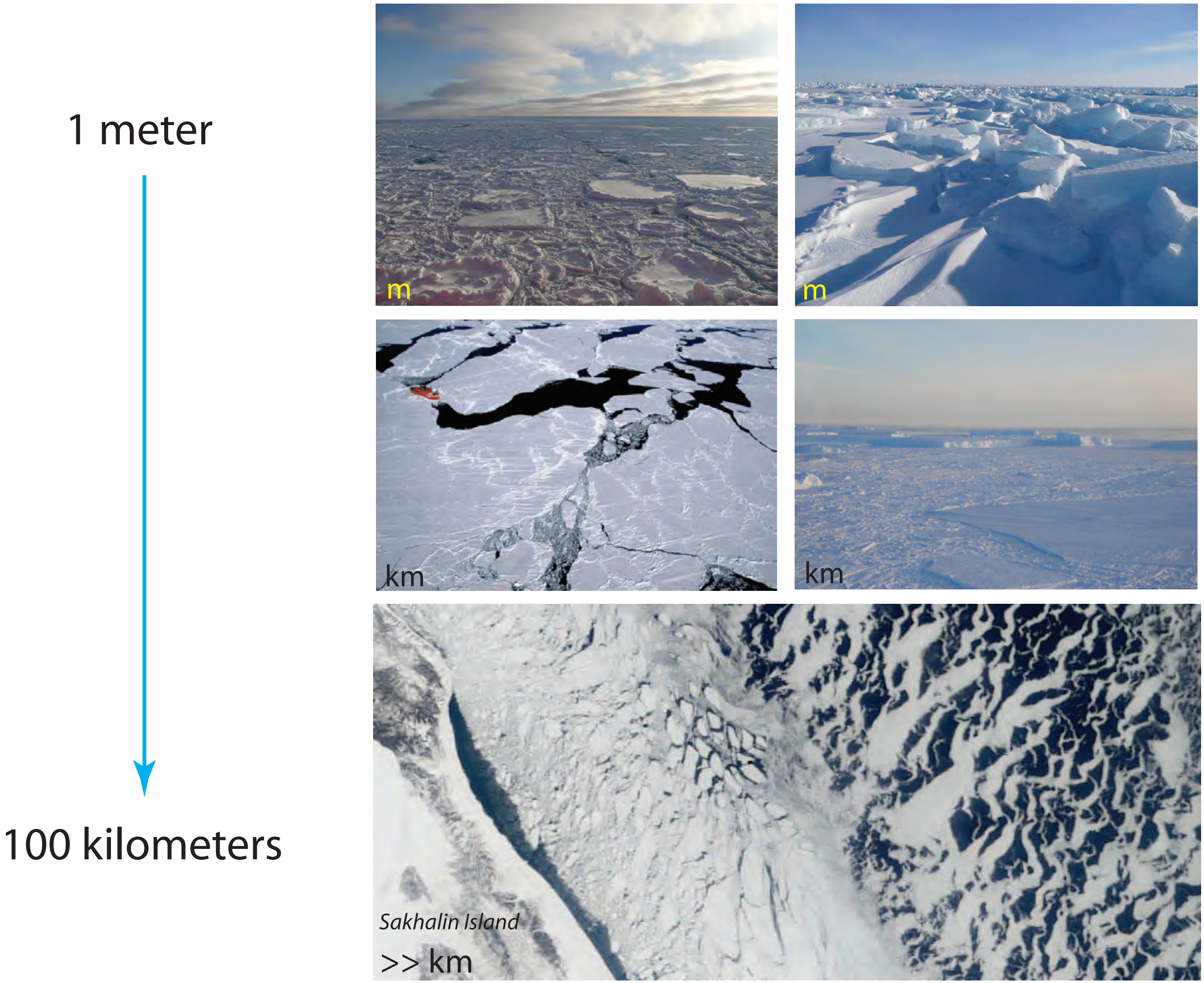
vertical

brine channels





pancake ice

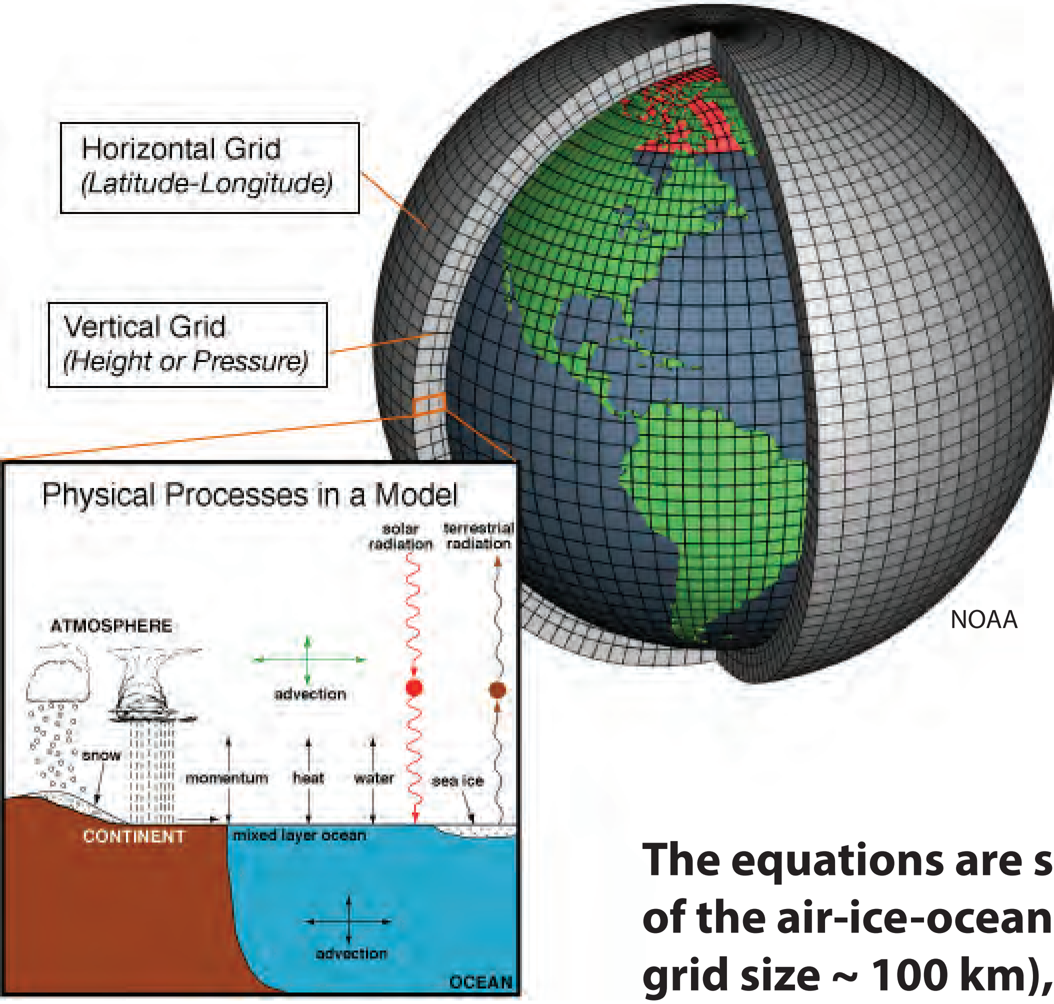

1 meter
100 kilometers
m
m
km
km
Sakhalin Island
>> km

What is this talk about?

Using the mathematics of composite materials and statistical physics to study sea ice structures and processes ... to improve projections of climate change.

1. ***Opposite poles of climate modeling***
2. ***Fluid flow through sea ice, percolation***
3. ***EM monitoring of sea ice, remote sensing***
4. ***Evolution of Arctic melt ponds, fractal geometry***

critical behavior ***cross-pollination***



Global Climate Models

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

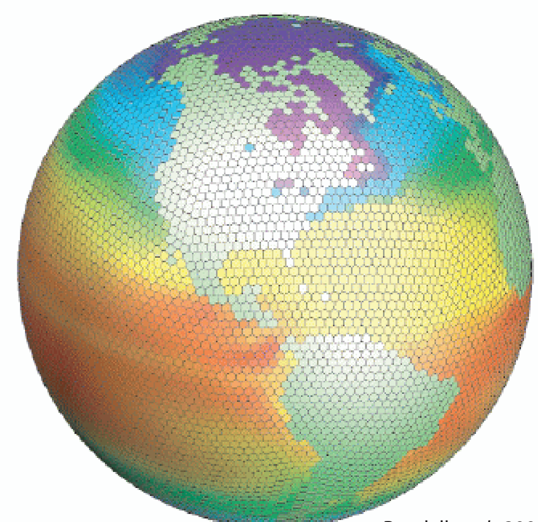
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

key challenge :

incorporating sub - grid scale processes

linkage of scales



Randall *et al.*, 2002

sea ice components of GCM's

What are the key ingredients -- or ***governing equations***
that need to be solved on grids using powerful computers?

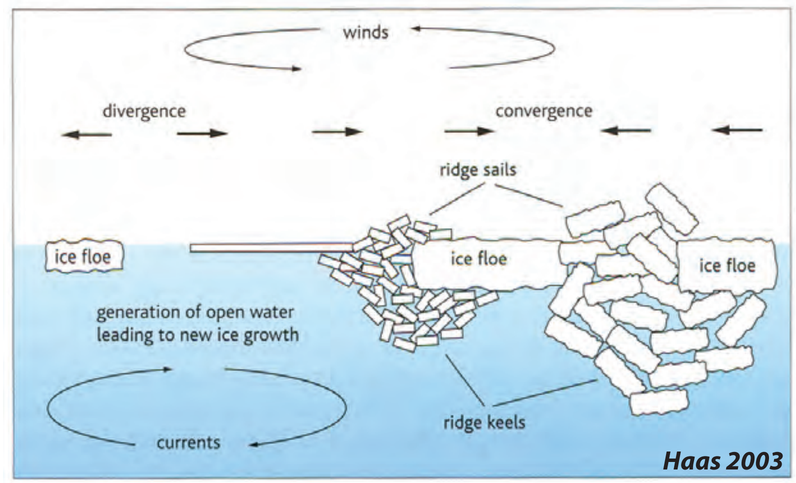
1. Ice thickness distribution $g(x, y, h, t)$ evolution equation

dynamics
+
thermodynamics

(Thorndike *et al.* 1975)

$$\frac{Dg}{Dt} = -g\nabla \cdot \mathbf{u} + \Psi(g) - \frac{\partial}{\partial h}(fg) + \mathcal{L}$$

nonlinear PDE incorporating ice velocity field
ice growth and melting
mechanical redistribution
- ridging and opening



2. Conservation of momentum, stress *vs.* strain relation (Hibler 1979)

$$m\frac{D\mathbf{u}}{Dt} = -mf\mathbf{k} \times \mathbf{u} + \boldsymbol{\tau}_a + \boldsymbol{\tau}_o - mg\nabla H + \mathbf{F}$$

F = ma for sea ice *dynamics*

3. Heat equation of sea ice and snow

(Maykut and Untersteiner 1971)

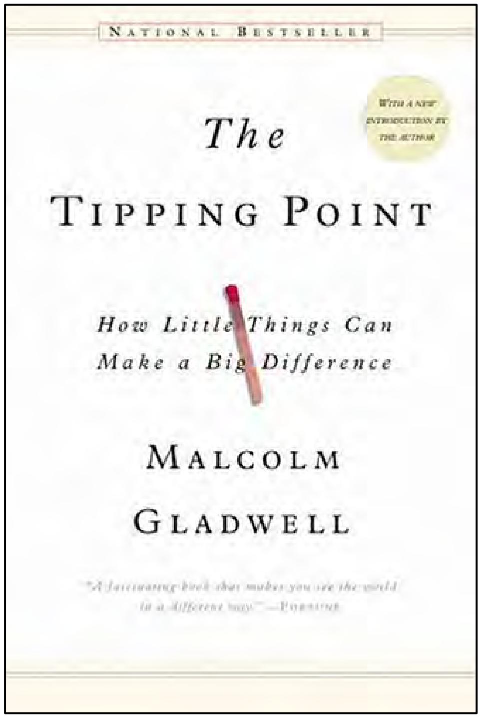
thermodynamics

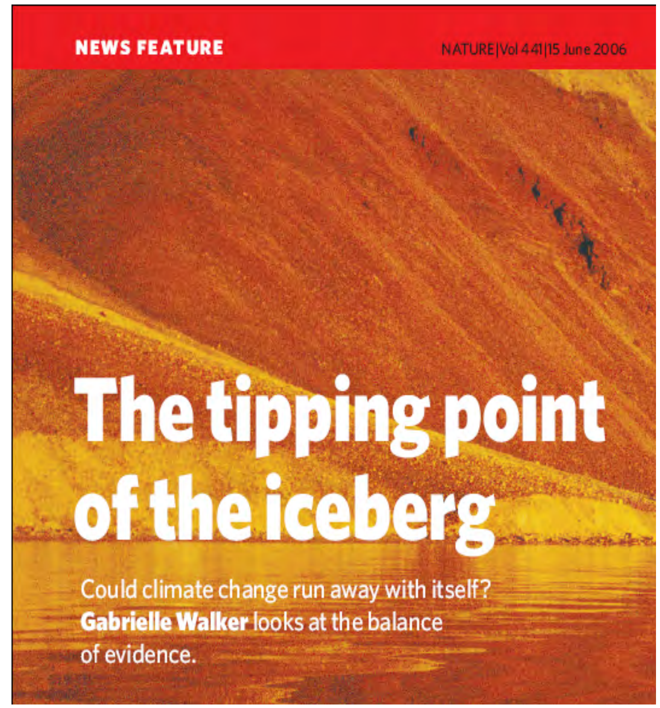
$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot k(T)\,\nabla T$$

+ *balance of radiative and thermal fluxes on interfaces*

tipping points in the mainstream

climate tipping points – September Arctic sea ice cover



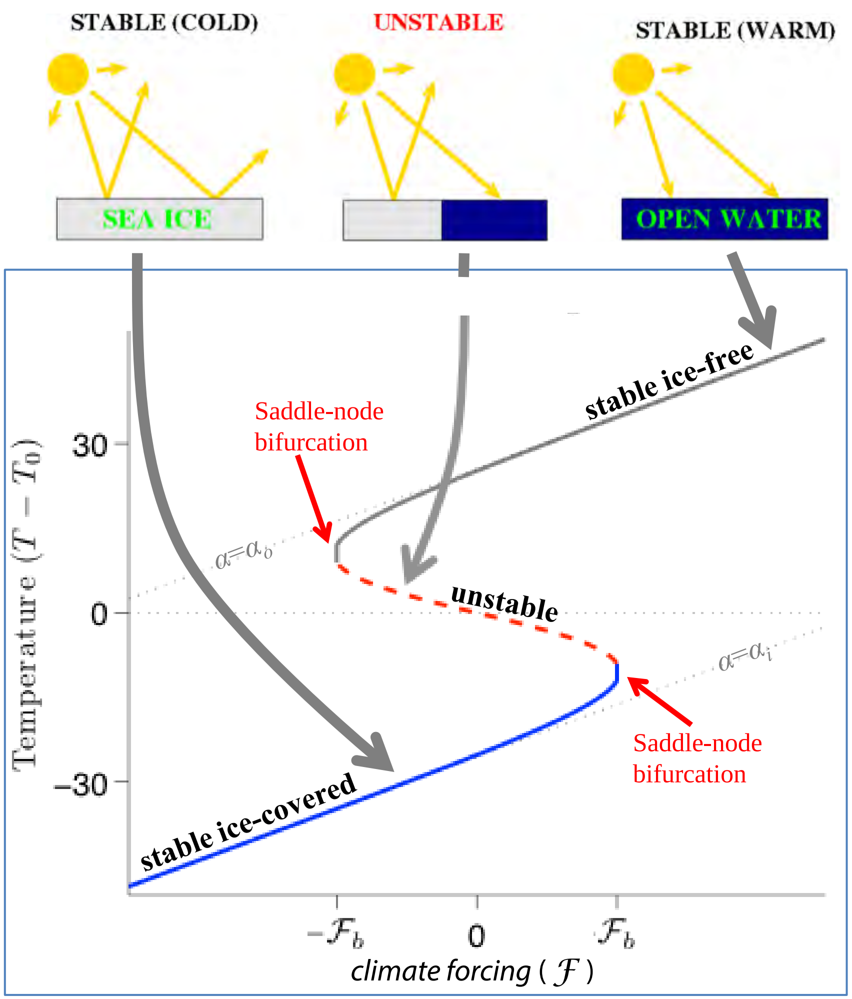


Melting of the Greenland ice sheet
Melting of the West Antarctic ice sheet
Permafrost and tundra loss, leading to the release of methane
Shutoff of N. Atlantic thermohaline conveyor (Gulf Stream) •••

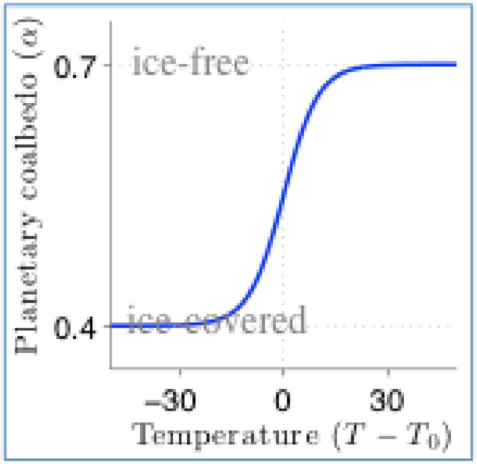
Lenton, et al., PNAS 2008

What would a tipping point in sea ice cover look like?

Bifurcation Diagram



nonlinear ice-albedo feedback

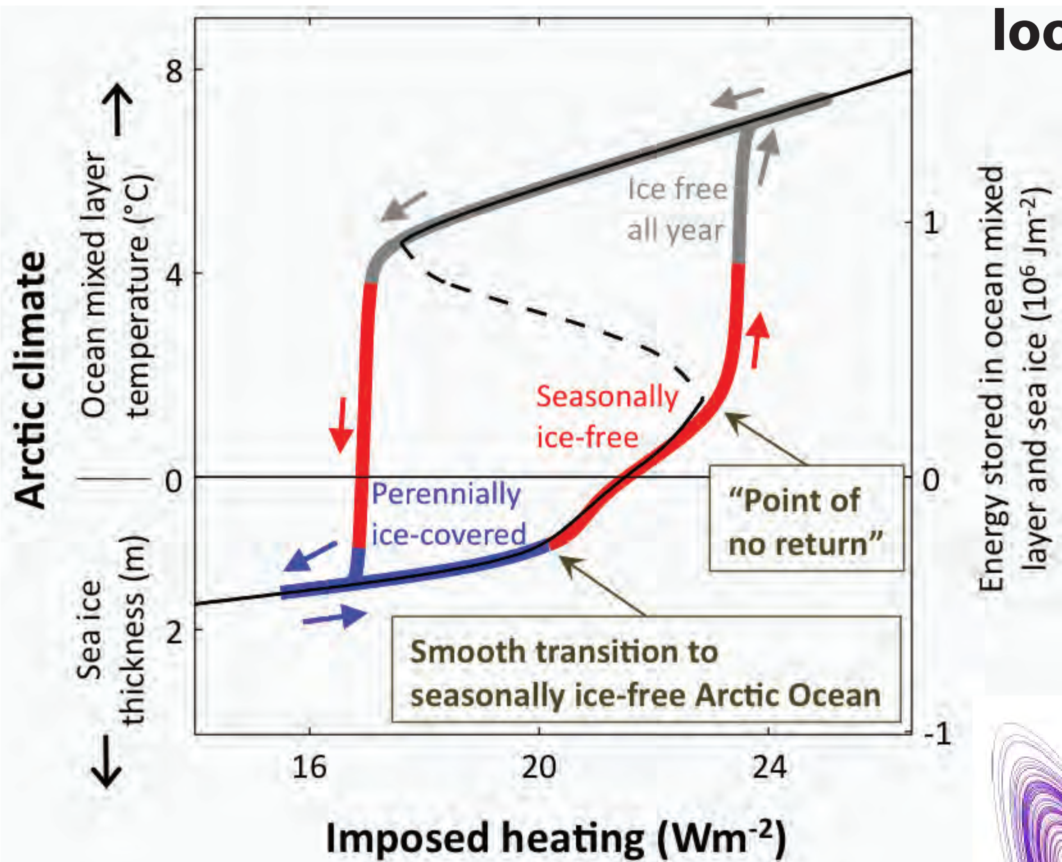


Eisenman

Has Arctic sea ice loss passed through a "tipping point"?

opposite pole from GCM's: low order (toy) models of climate change

Eisenman, Wettlaufer, PNAS 2009: nonlinear ODE for energy in upper ocean



look for bifurcations, multiple equilibria

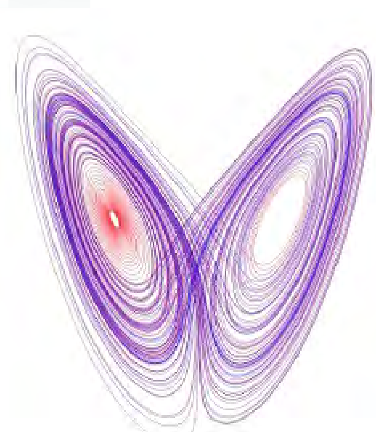
tipping point unlikely in loss of summer ice

Abbot, Silber, Pierrehumbert, JGR 2011

bifurcations with clouds, ice loss

Sudakov, Vakulenko, Golden
Comm. Nonlinear Sci. & Num. Sim., 2014

impact of melt ponds



Lorenz butterfly

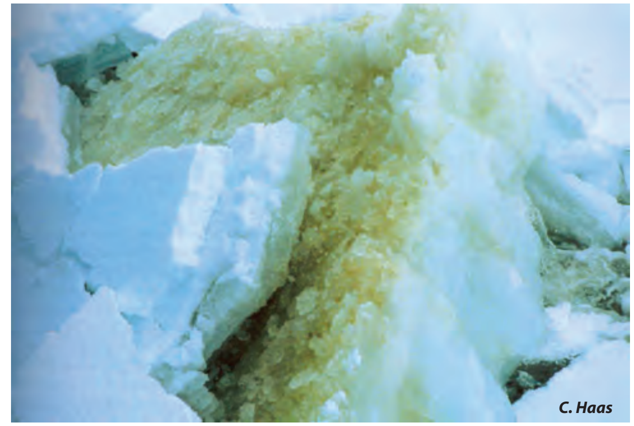
sea ice microphysics

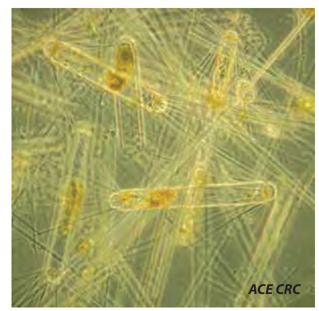
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities





- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO_2*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

linkage of scales

fluid permeability *k* of a porous medium

porous concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Darcy's Law for slow viscous flow in a porous medium

averaged
fluid velocity

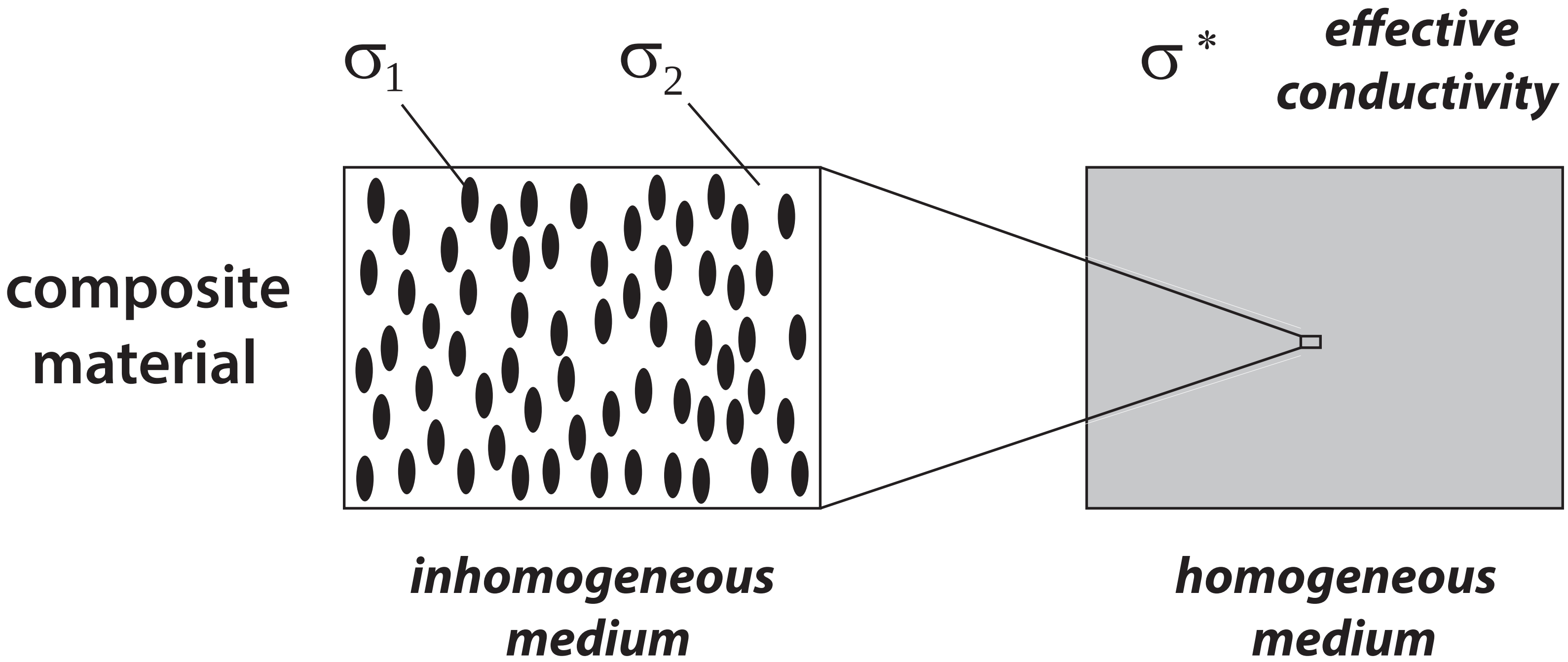
pressure
gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

viscosity

$\mathbf{k}$ = fluid permeability tensor

HOMOGENIZATION

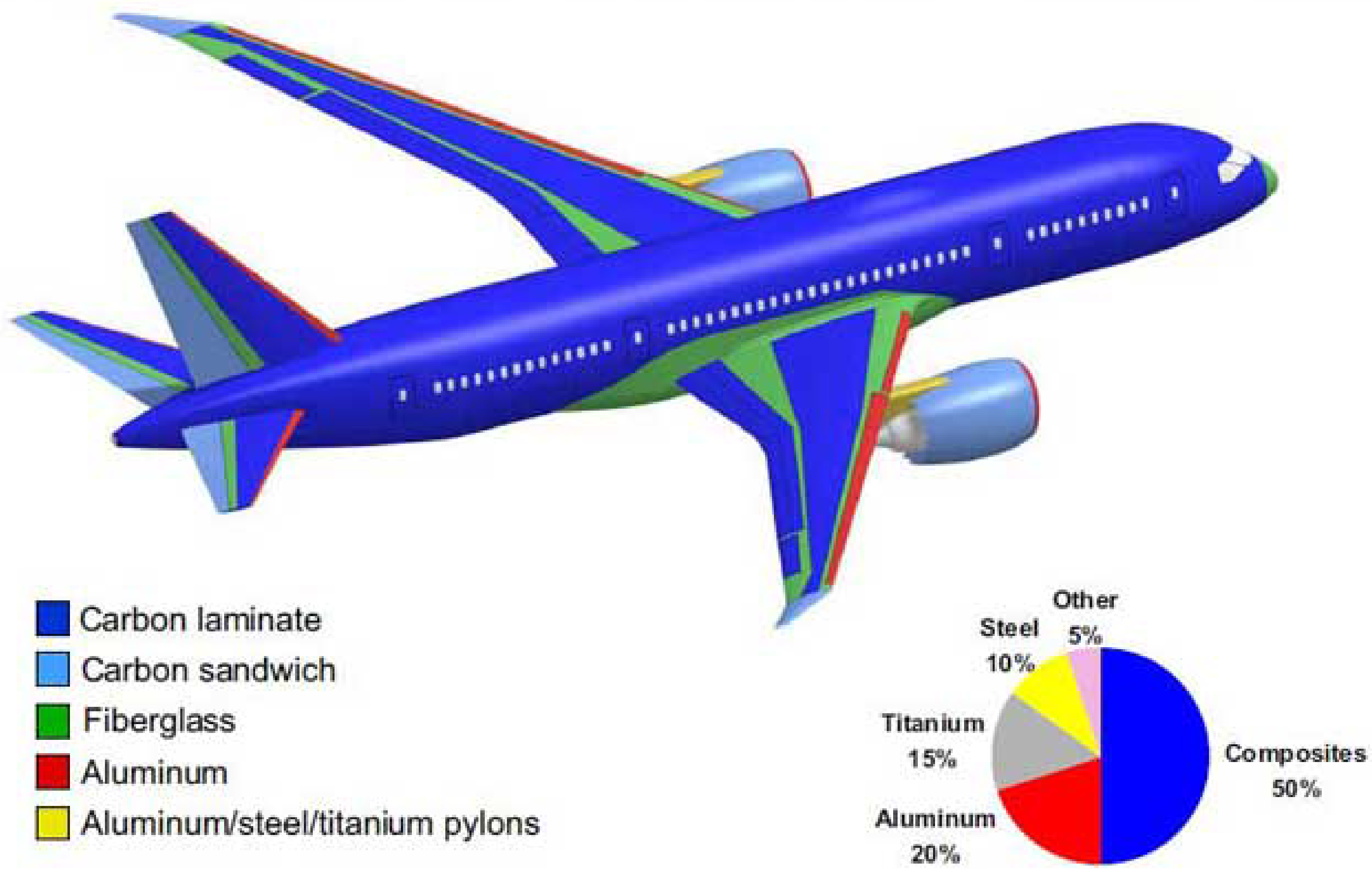


find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

Composite materials in the Boeing 787 Dreamliner

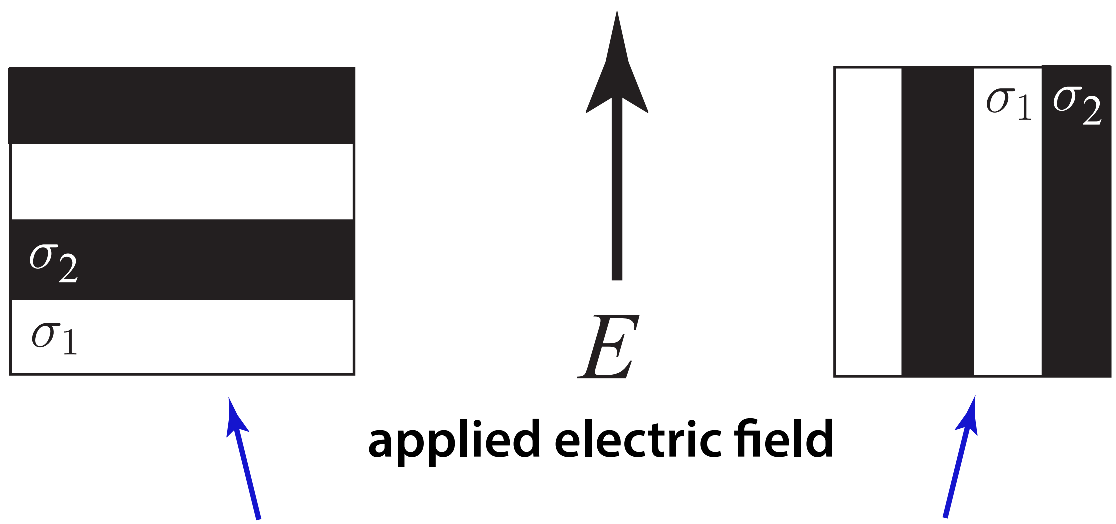


arithmetic and harmonic mean bounds on transport properties

effective electrical conductivity σ^* for two phase composite of σ_1 and σ_2

optimal bounds on σ^* for known volume fractions p_1 and p_2:

$$\frac{1}{\frac{p_1}{\sigma_1}+\frac{p_2}{\sigma_2}} \leq \sigma^* \leq p_1\sigma_1 + p_2\sigma_2$$

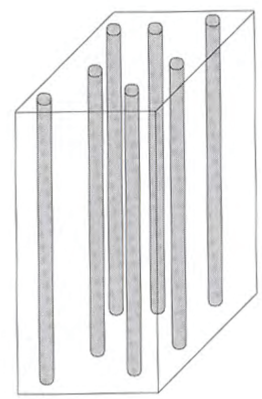


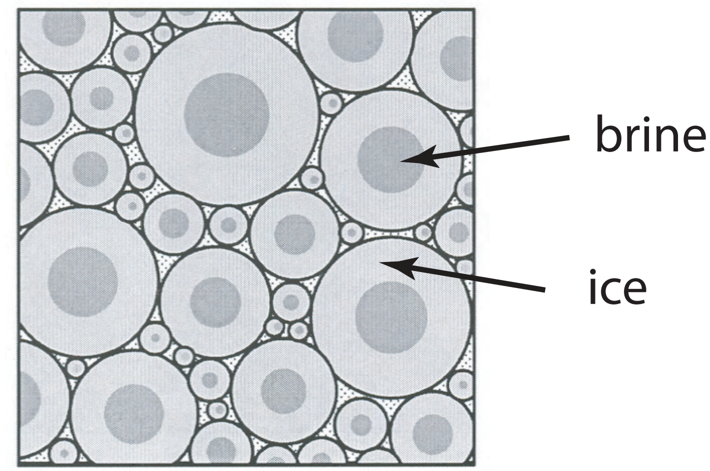
optimal designs are laminates

Wiener 1912, ...

PIPE BOUNDS on vertical fluid permeability k

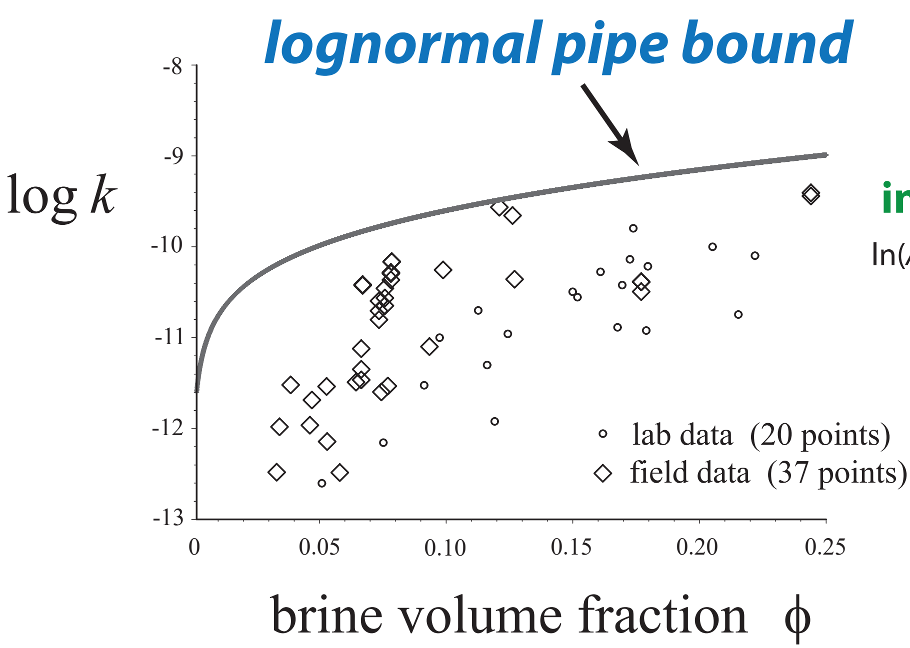
vertical pipes
with appropriate radii
maximize k





fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)

optimal coated cylinder geometry



$$k \;\leq\; \frac{\phi\langle R^4\rangle}{8\langle R^2\rangle} \;=\; \frac{\phi}{8}\langle R^2\rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

ln(A) normally distributed, mean μ (increases with T) variance σ^2 (Gow and Perovich 96)

get bounds through variational analyis of ***trapping constant*** γ for diffusion process in pore space with absorbing BC

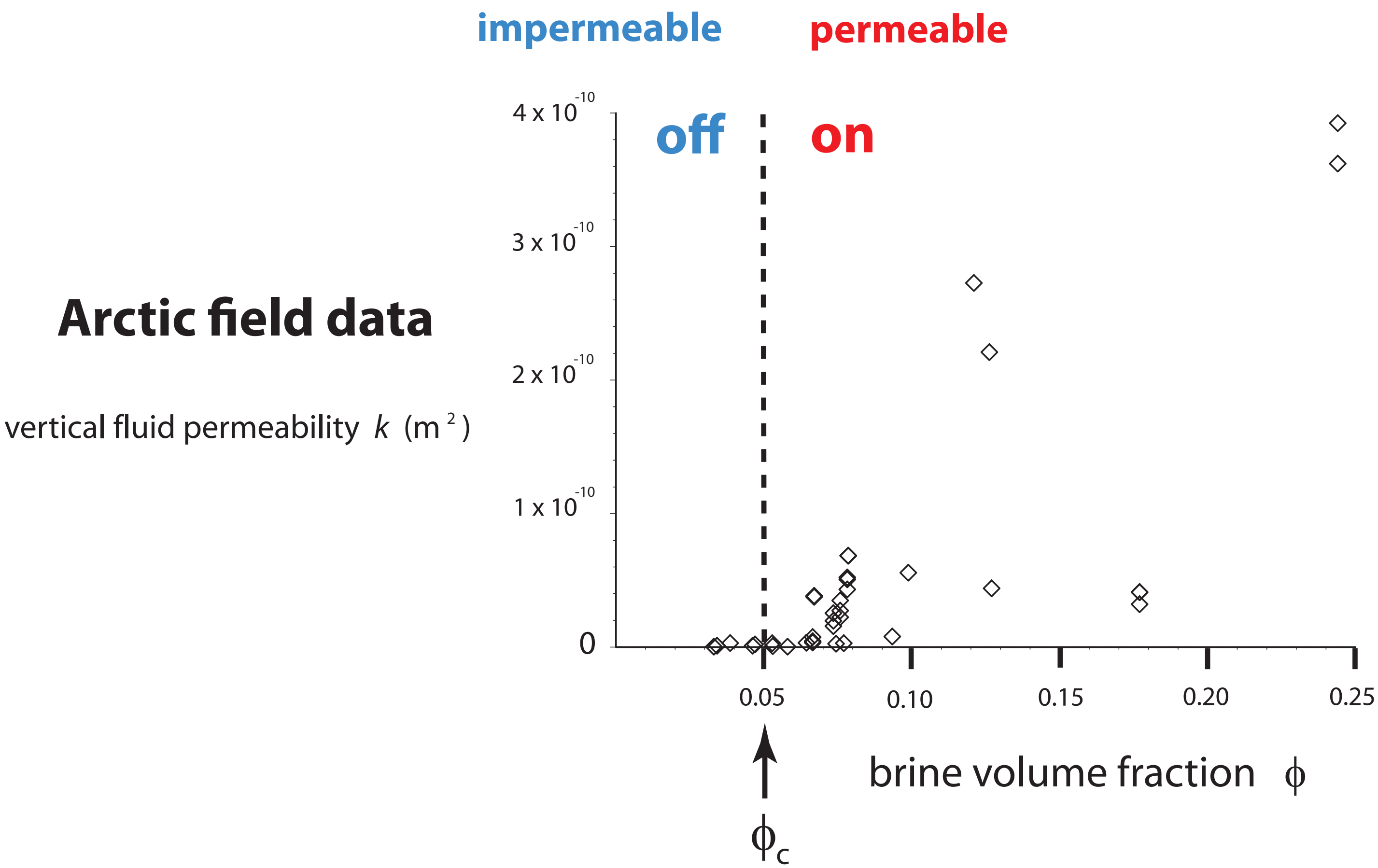
$$\mathbf{k} \;\leq\; \gamma^{-1}\mathbf{I}$$

for any ergodic porous medium (Torquato 2002, 2004)

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007
Golden, Heaton, Eicken, Lytle, Mech. Materials 2006

Critical behavior of fluid transport in sea ice

Arctic field data



on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C, \; S \approx 5$ ppt

RULE OF FIVES

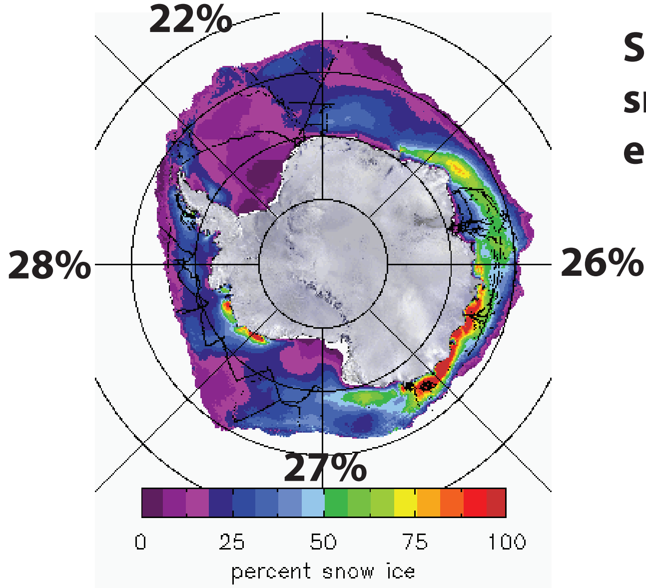
Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

rule of fives constrains:

Antarctic surface flooding and snow-ice formation

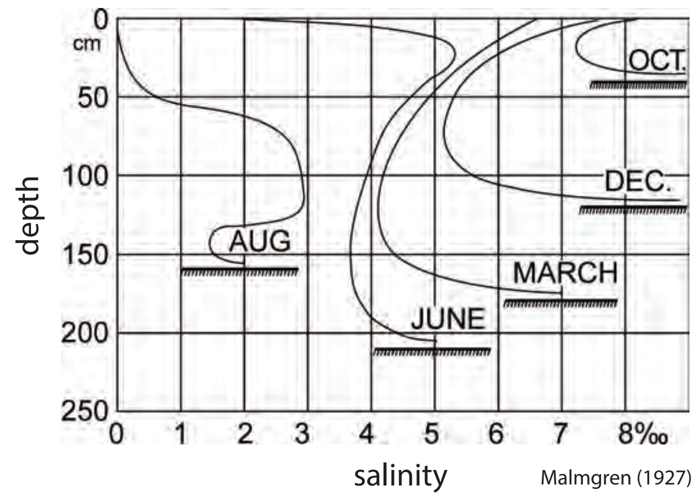


September snow-ice estimates

Antarctic snow-to-ice conversion from passive microwave imagery

T. Maksym and T. Markus, 2008

evolution of salinity profiles

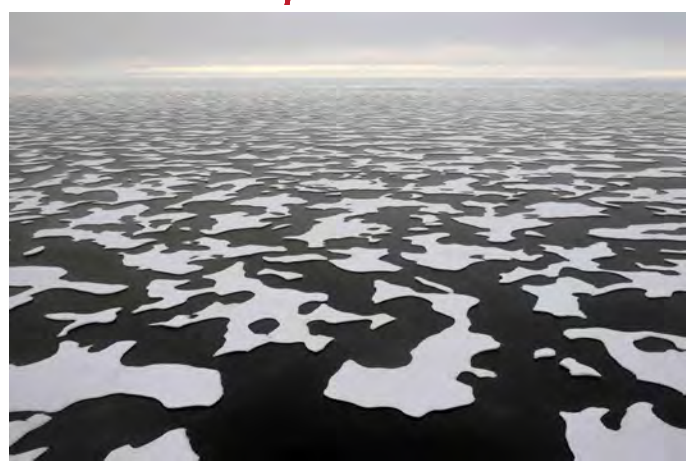


currently assumed constant in climate models

convection - enhanced thermal conductivity

Lytle and Ackley, 1996
Trodahl, et. al., 2000, 2001
Wang, Zhu, Golden, 2016

evolution of melt ponds and sea ice albedo



sea ice algal communities

D. Thomas 2004

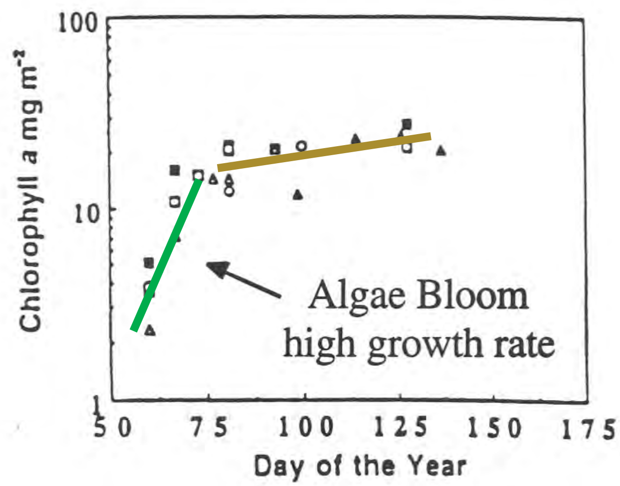
nutrient replenishment
controlled by ice permeability

biological activity turns on or off according to *rule of fives*

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

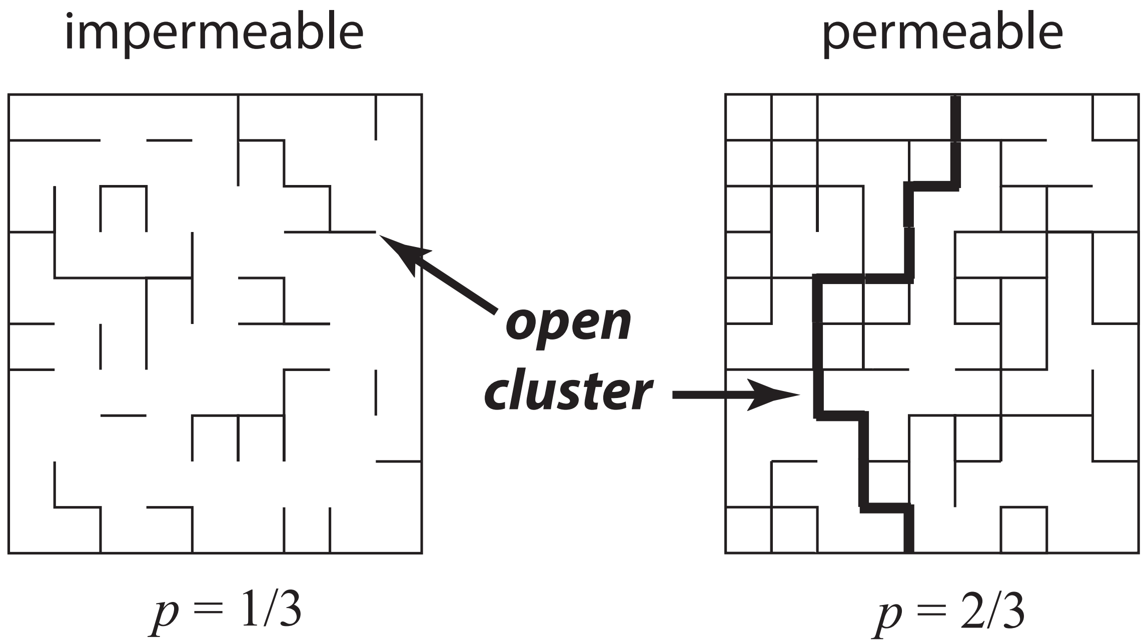
sea ice ecosystem

sea ice algae
support life in the polar oceans

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond → **open** *with probability* p
closed *with probability* $1-p$

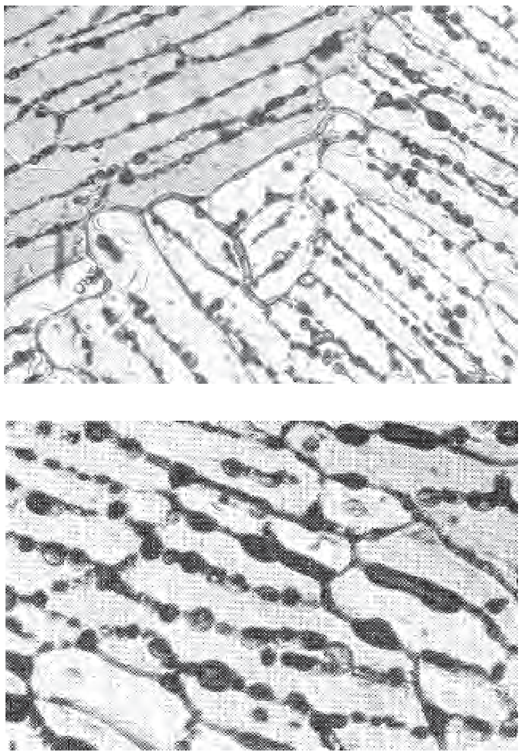
percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

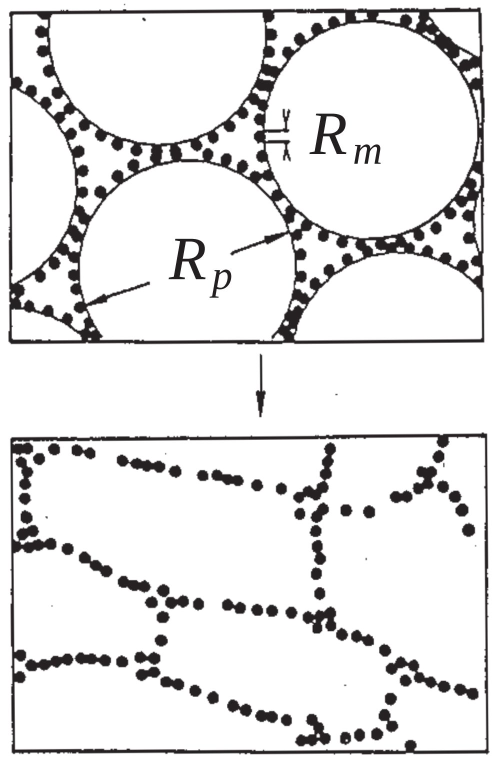
smallest *p* for which there is an infinite open cluster

Continuum percolation model for ***stealthy*** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

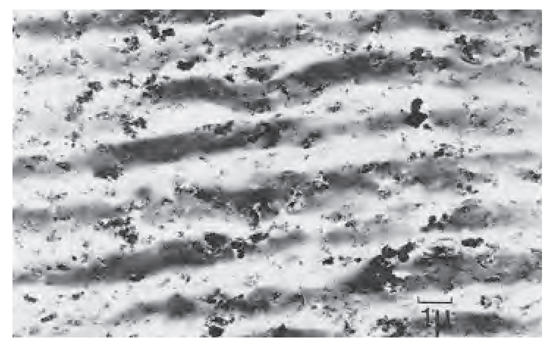
$\phi_c \approx 5\,\%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed powder



radar absorbing composite

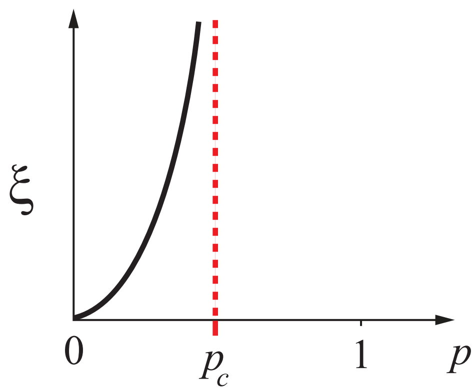
sea ice is radar absorbing

order parameters in percolation theory

geometry

correlation length

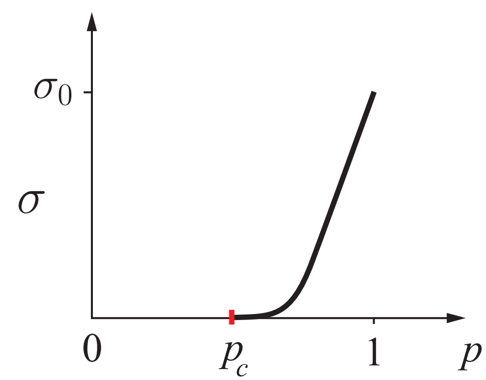
(characteristic scale of connectedness)



$$\xi(p) \sim |p - p_c|^{-\nu}$$

transport

effective conductivity
or fluid permeability



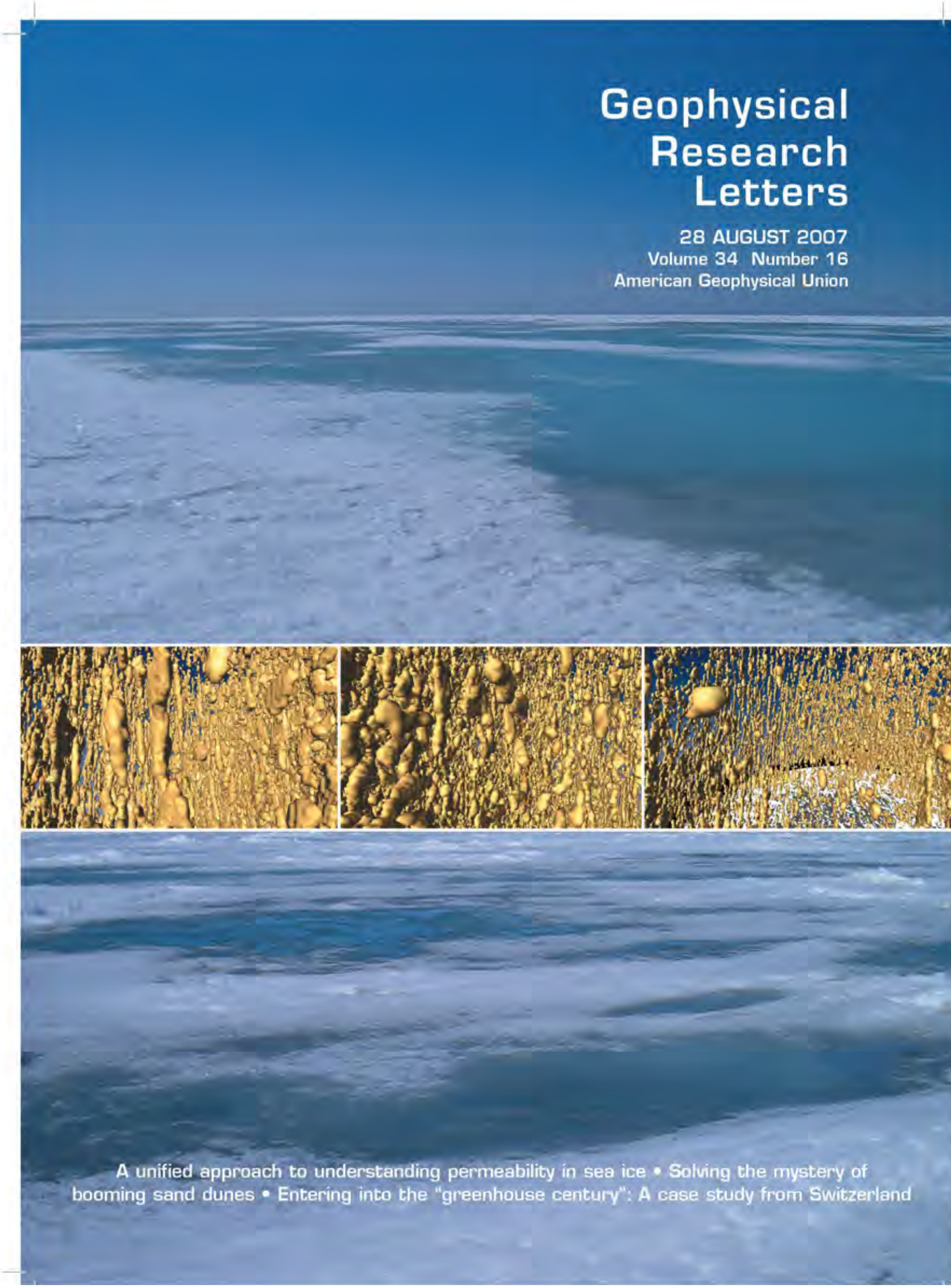
$$\sigma(p) \sim \sigma_0 \, (p - p_c)^t$$

UNIVERSAL critical exponents for lattices -- depend only on dimension

$1 \leq t \leq 2$ (for idealized model) **Golden, *Phys. Rev. Lett.* 1990 ; *Comm. Math. Phys.* 1992**

non-universal behavior in continuum

Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**



rigorous bounds
percolation theory
hierarchical model
network model

field data

micro-scale

controls

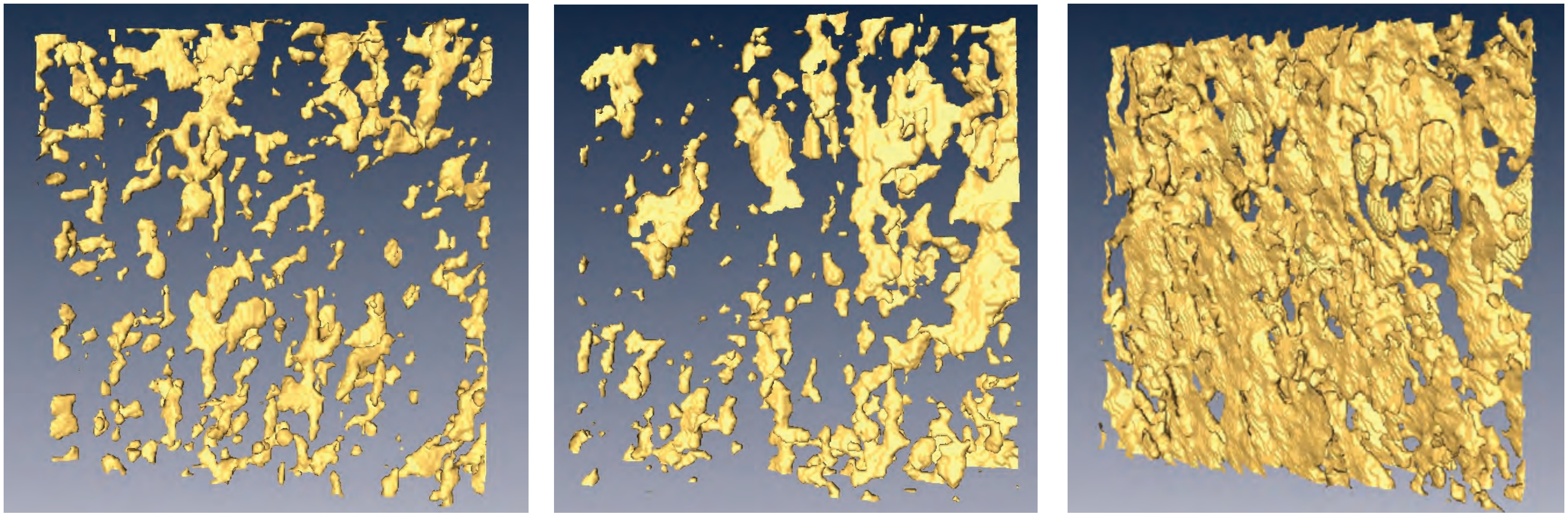
macro-scale processes

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

brine connectivity (over cm scale)

8 x 8 x 2 mm



-15 °C, $\phi = 0.033$ -6 °C, $\phi = 0.075$ -3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats → 3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

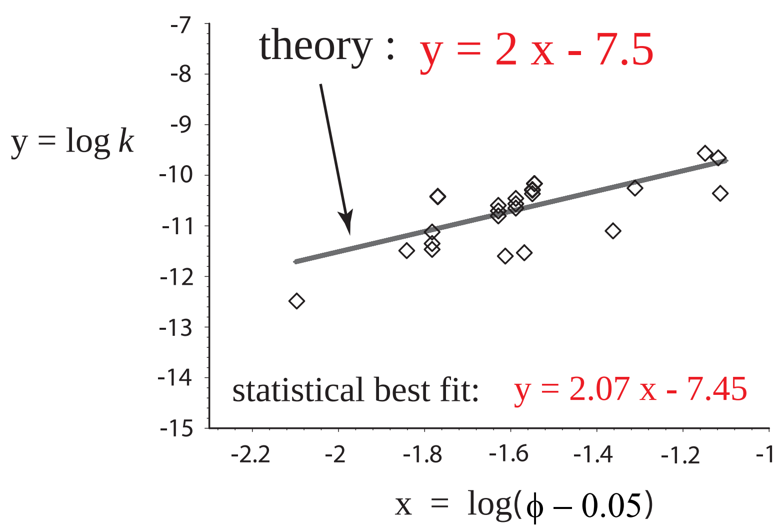
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

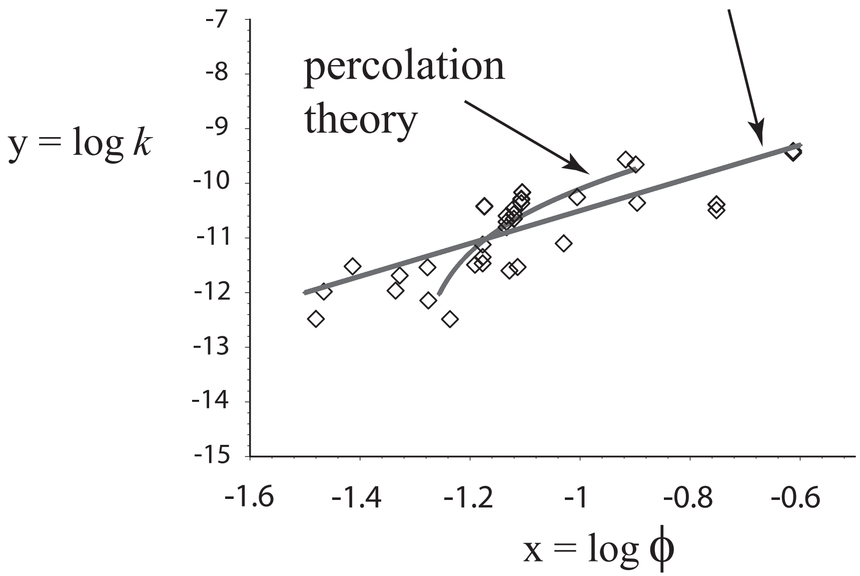
$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0



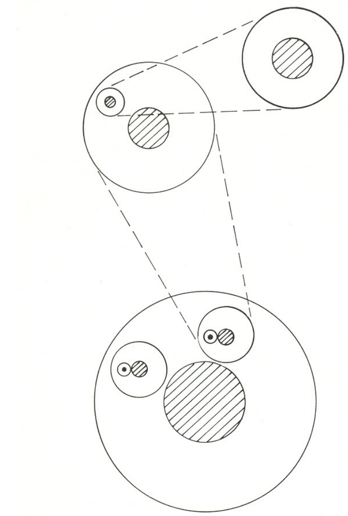
hierarchical and network models

hierarchical model : $y = 3\,x - 7.5$



statistical best fit of data: $y = 3.05\,x - 7.50$

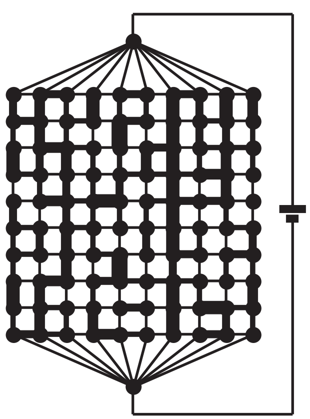
brine-coated spherical ice grains

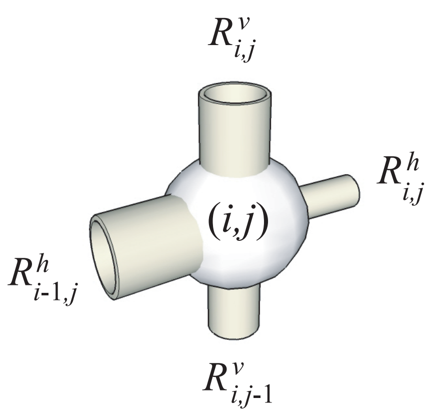


$$k\,(\phi) = k_0\,\phi^3$$

self-similar model used for porous rocks

Sen, Scala, Cohen 1981
Sheng 1990
Wong, Koplick, Tomanic 1984





random pipe network with radii chosen from measured inclusion distributions, solved with fast multigrid method

Zhu, Jabini, Golden, Eicken, Morris, Annals of Glaciology, 2006
Golden et al., Geophysical Research Letters, 2007
Zhu, Golden, Gully and Sampson, Physica B, 2010

diatoms in EPS-filled pores in natural sea ice

protects microorganisms against osmotic shock:

- highly concentrated brine
- fresh water from melt ponds

antifreeze, cryoprotectant

depresses freezing point

physical barrier from ice crystals

Transmitted light with
Alcian Blue stain for EPS

Krembs, Eicken, Deming, PNAS 2011

Extracellular Polymeric Substances (EPS)

gelatinous exopolymers algal mucous

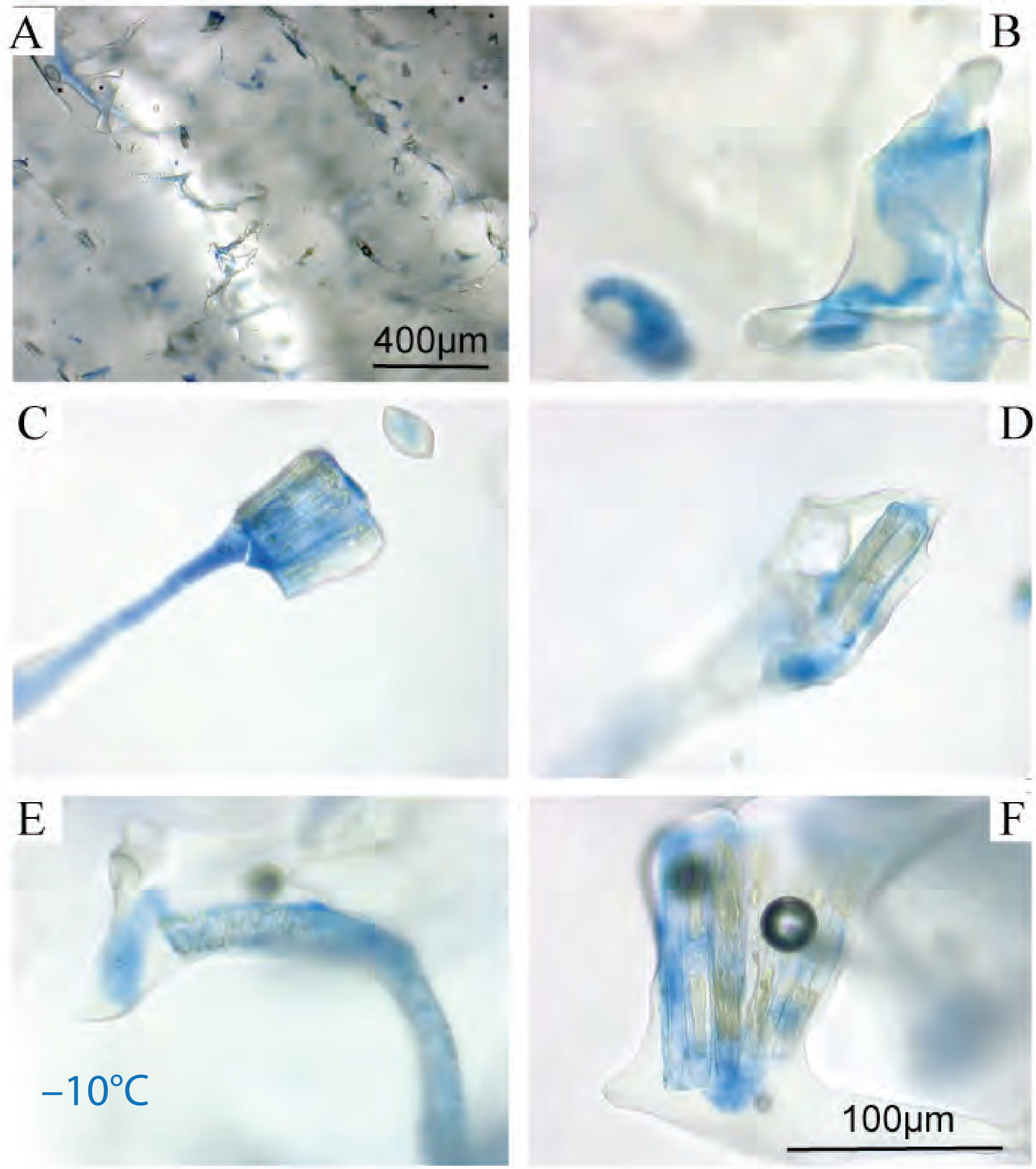
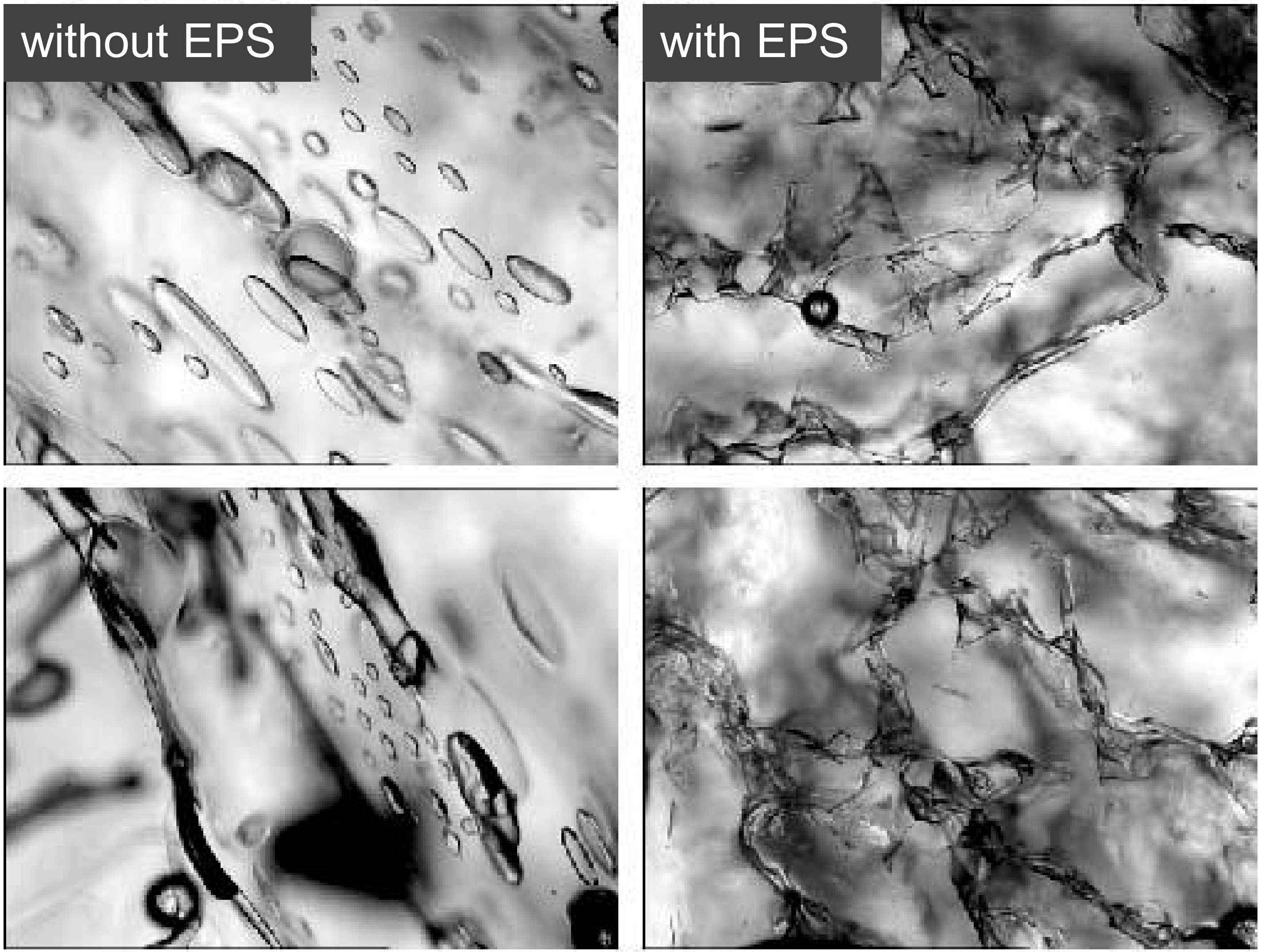


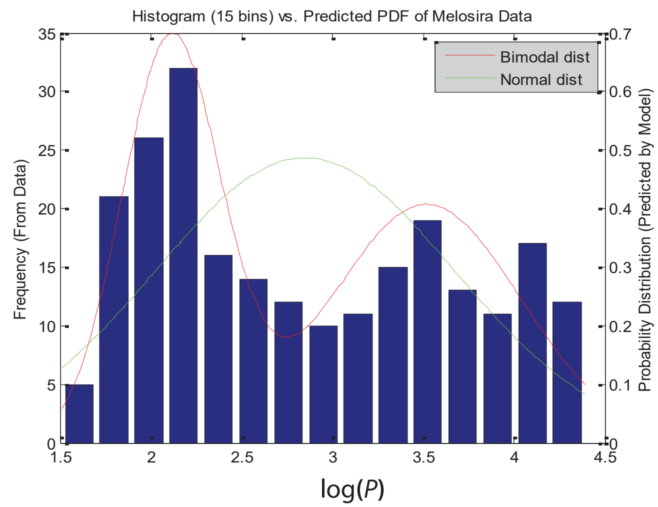
Fig. 1 Krembs et al.

EPS changes the microstructure of sea ice.



Krembs, Eicken, Deming, PNAS 2011

How does EPS affect fluid transport?



- Bimodal lognormal distribution for brine inclusions
- Use random pipe network with bimodal distribution; Develop solver to handle larger variances in sizes.
- Results predict observed drop in fluid permeability *k*.
- Rigorous bound on *k* for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Deming, Golden 2016

Vetter, Deming, Jumars, Krieger-Brockett, *Microb. Ecol.* 1998
A Predictive Model of Bacterial Foraging by Means of Freely Released Extracellular Enzymes

Remote sensing of sea ice

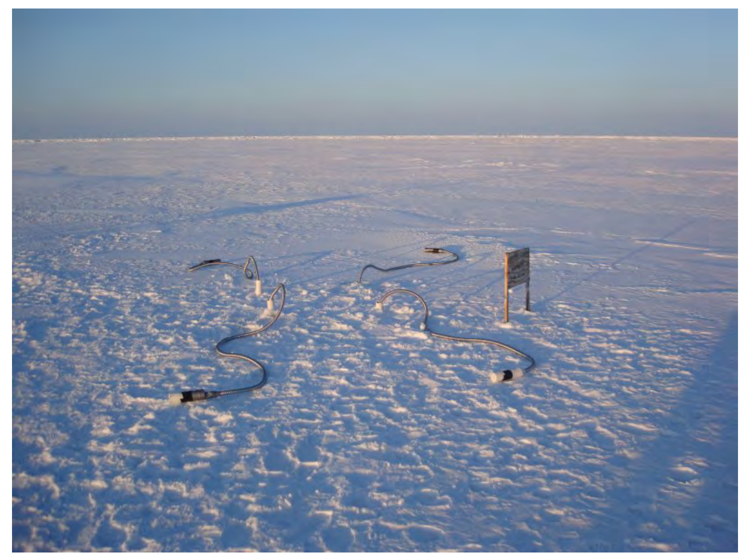
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

$$\varepsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)

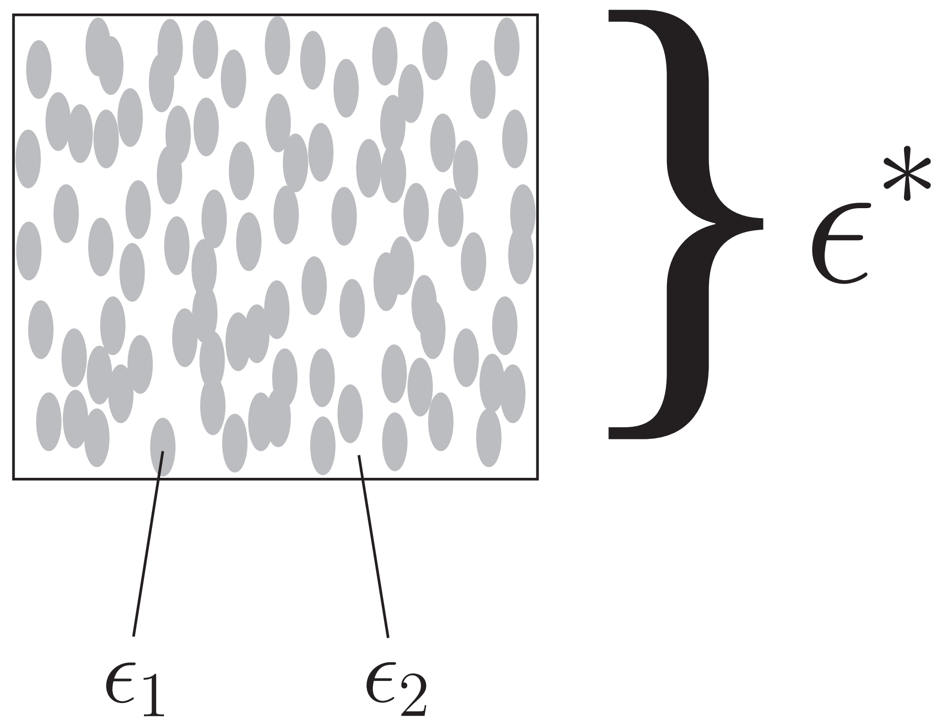


brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

HOMOGENIZATION



p_1 , p_2 = volume fractions of brine and ice

ocean swells propagating through a vast field of pancake ice

HOMOGENIZATION: long wave sees an effective medium, not individual floes

Theory of Effective Electromagnetic Behavior of Composites

analytic continuation method

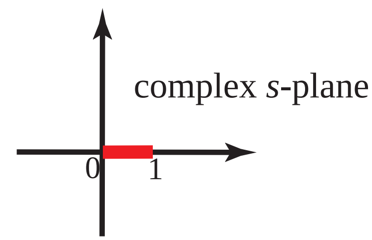
Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

composite geometry
(spectral measure μ) $\longrightarrow$ ε^*

integral representations, rigorous bounds, approximations, etc.

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



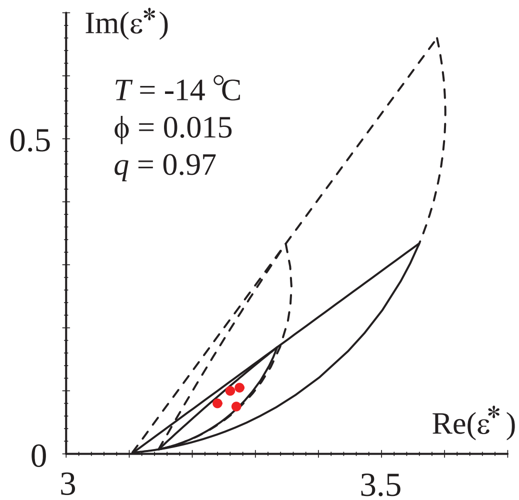
Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001)
(McPhedran, McKenzie, and Milton, 1982)

ε^* $\longrightarrow$ ***composite geometry***
(spectral measure μ)

recover brine volume fraction, connectivity, etc.

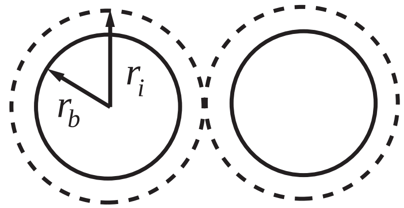
forward and inverse bounds on the complex permittivity of sea ice

forward bounds



4.75 GHz data

matrix particle



$$q = r_b / r_i$$

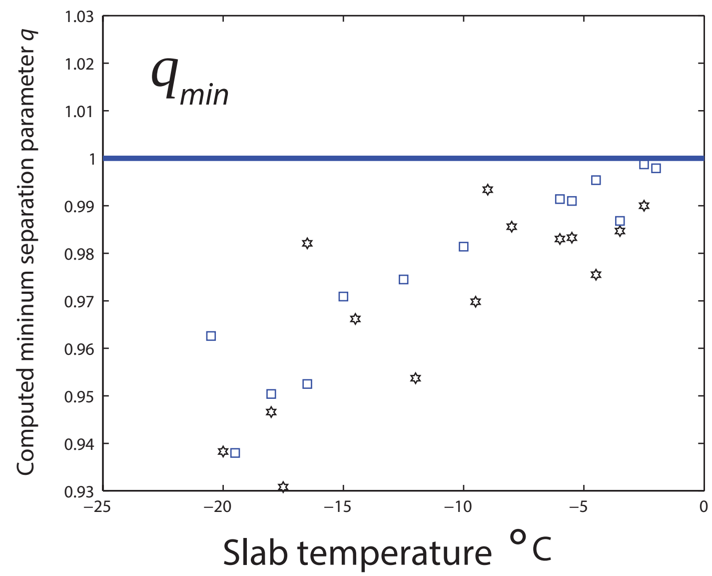
$$0 < q < 1$$

Golden 1995, 1997

inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden Physica B, 2007

inverse bounds



inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden Proc. Roy. Soc. A, 2012

direct calculation of spectral measure

once we have the spectral measure - which depends only on geometry - it can be used in Stieltjes integrals for other transport coefficients:
electrical and thermal conductivity, complex permittivity, magnetic permeability, effective diffusion → *cross-property relations and inversions*

1. Discretization of composite microstructure gives lattice of 1's and 0's (random resistor network).

2. The fundamental operator $\chi\Gamma\chi$ becomes a random matrix depending only on the composite geometry.

3. Compute the eigenvalues λ_i and eigenvectors of $\chi\Gamma\chi$ with inner product weights α_i

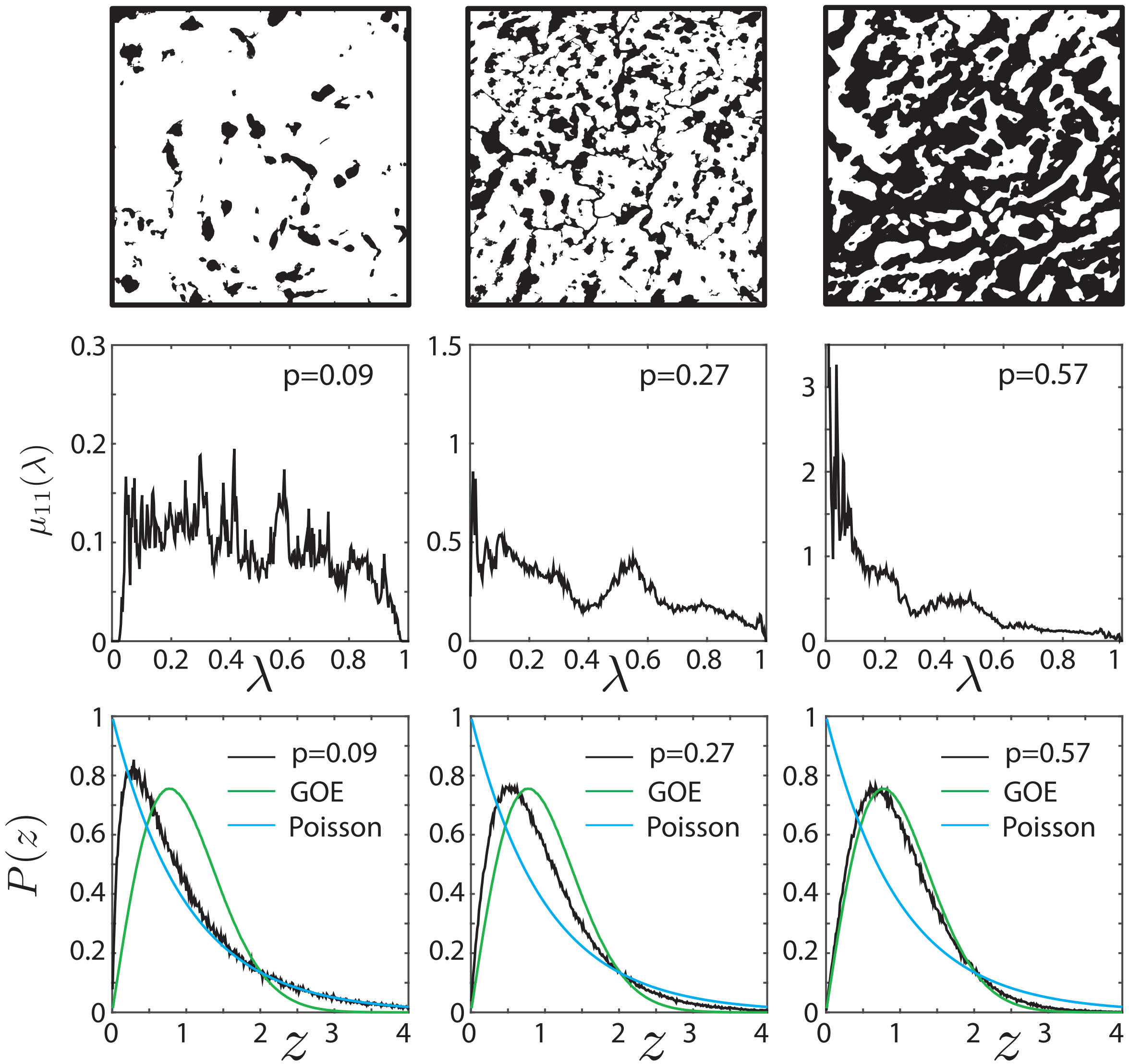
$$\mu(\lambda) = \sum_i \alpha_i \, \delta(\lambda - \lambda_i)$$

Dirac point measure (Dirac delta)

earlier studies of spectral measures

Day and Thorpe 1996
Helsing, McPhedran, Milton 2011

Spectral computations for Arctic melt ponds



spectral measures

eigenvalue spacing distributions

uncorrelated → **level repulsion**

TRANSITION

Ben Murphy
Elena Cherkaev
Ken Golden
2016

eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase percolates

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden 2016

transition to universal eigenvalue statistics
transition to extended states from localized w/ mobility edges

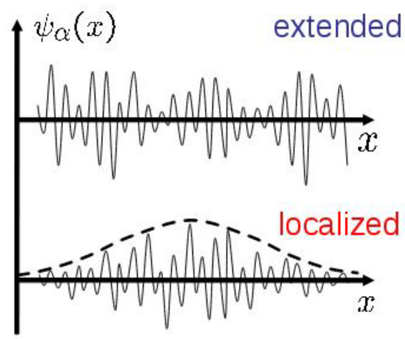
surprising analog

connectedness ~ disorder in potential

Anderson transition in wave physics - quantum, optics, ...

e.g. metal / insulator transition at critical disorder

potential *V*(*x*) wavefunctions energy spacings



low disorder **GOE**

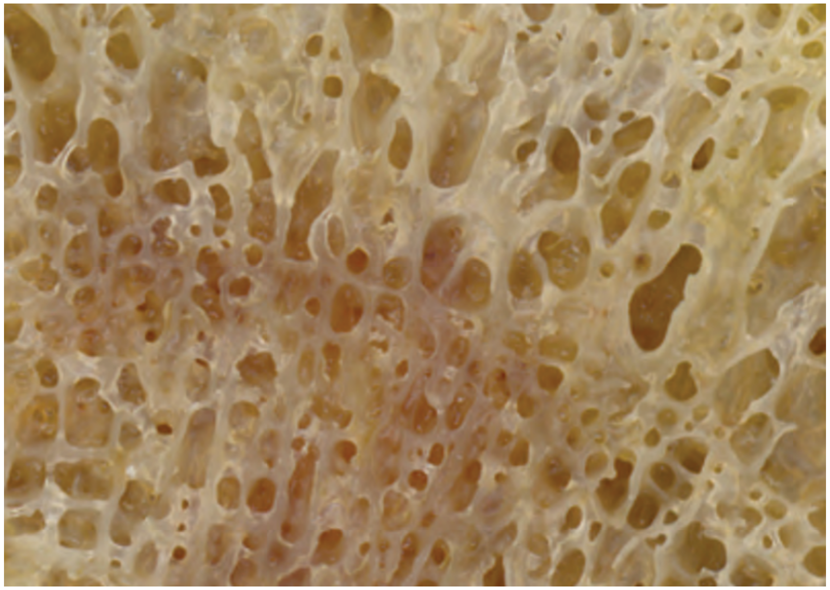
high disorder **Poisson**

Anderson, 1958
Shklovshii et al, 1993
Evangelou, 1992

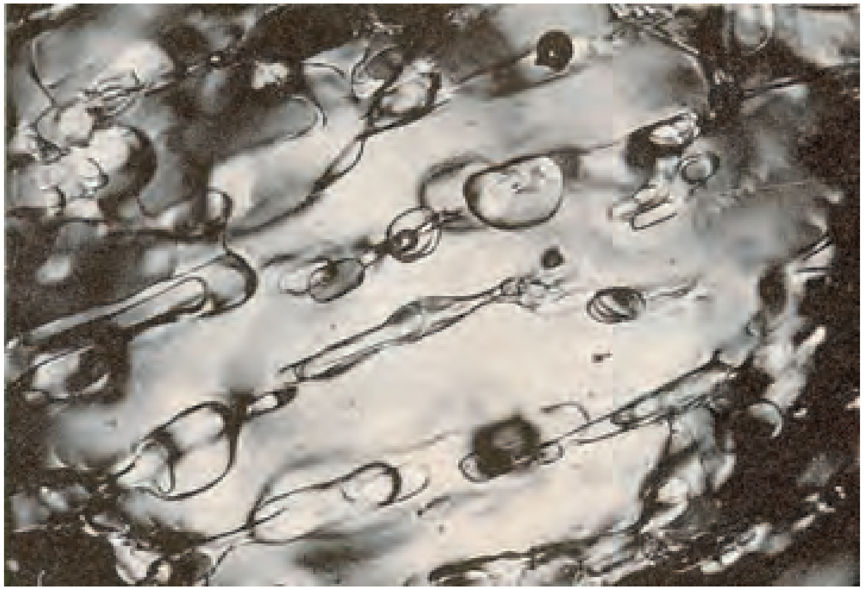
Our framework: $\nabla \cdot (\sigma \nabla \psi) = 0$ $\Big\{$ **NO wave interference or quantum effects !**

the math doesn't care if it's sea ice or bone!

HUMAN BONE



SEA ICE



apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring osteoporosis

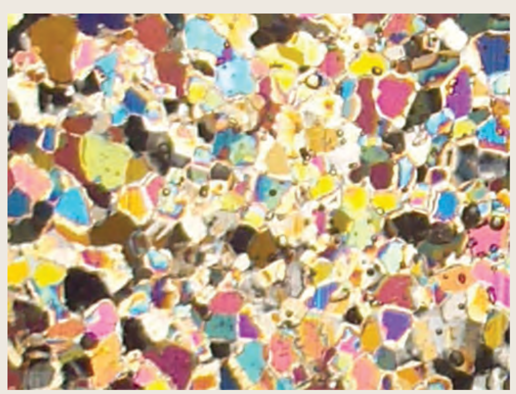
Golden, Murphy, Cherkaev, J. Biomechanics 2011

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

  Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



Proc. Roy. Soc. A 8 Feb 2015

Proc. R. Soc. A | Volume 471 | Issue 2174 | 8 February 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A

350 YEARS OF SCIENTIFIC PUBLISHING

An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques

A computer model to determine how a human should walk so as to expend the least energy

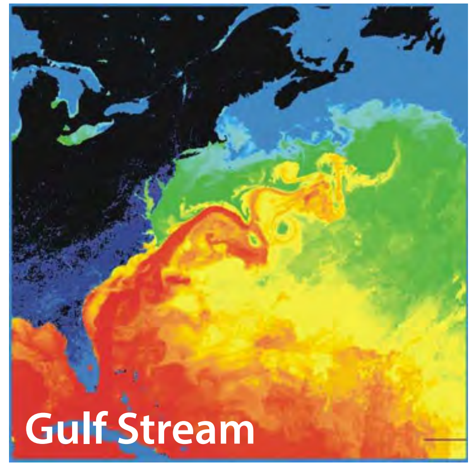
THE ROYAL SOCIETY PUBLISHING

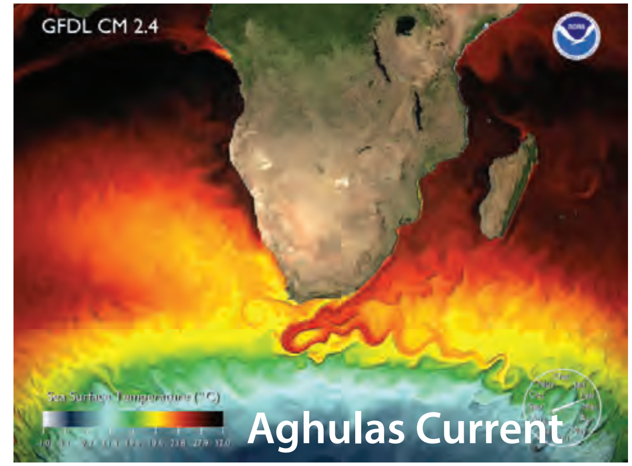
21/01/15 1:47 PM

advection enhanced diffusion

effective diffusivity

tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere
salt and heat transport in ocean





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize

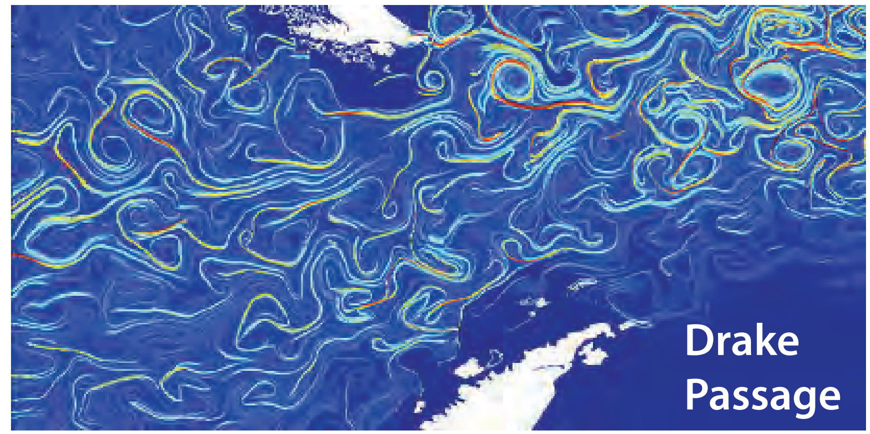
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity***

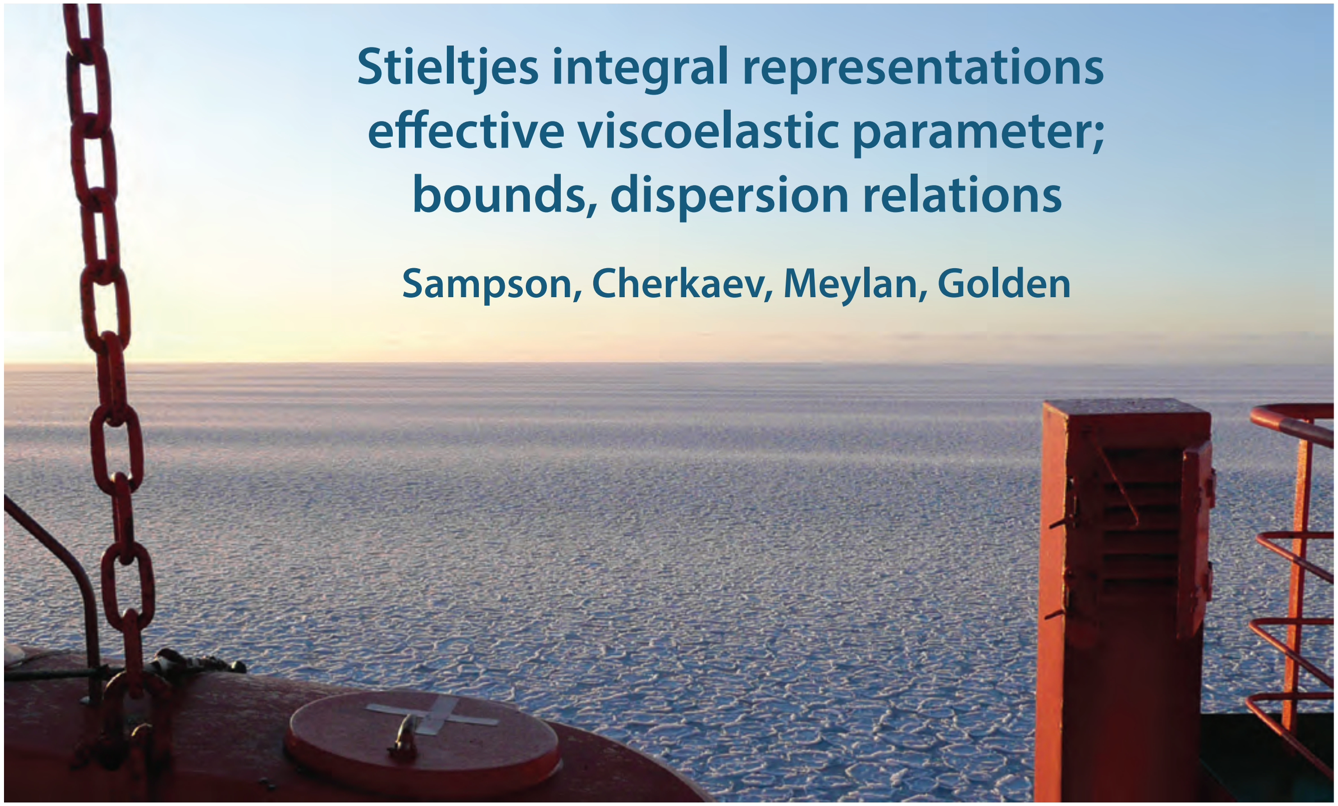
Stieltjes integral for κ^* ***with spectral measure***

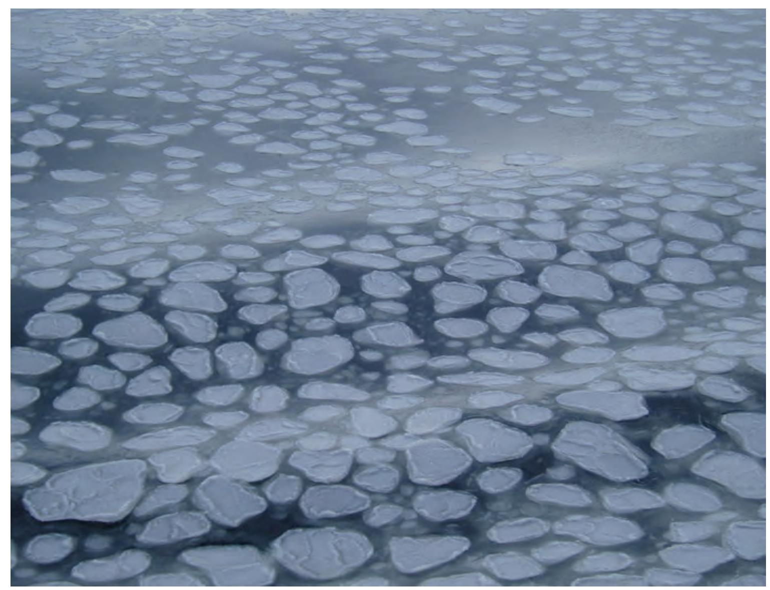
Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Zhu, Xin, Golden 2016



wave propagation in the marginal ice zone

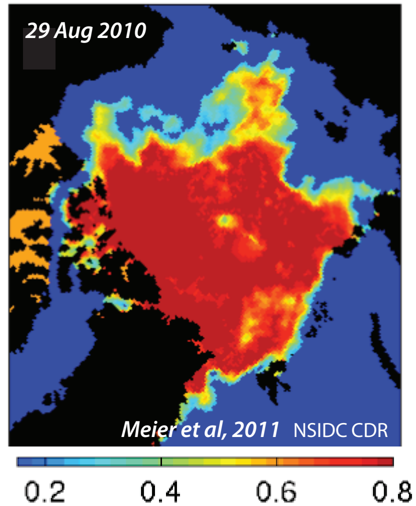




Marginal Ice Zone

MIZ

transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)



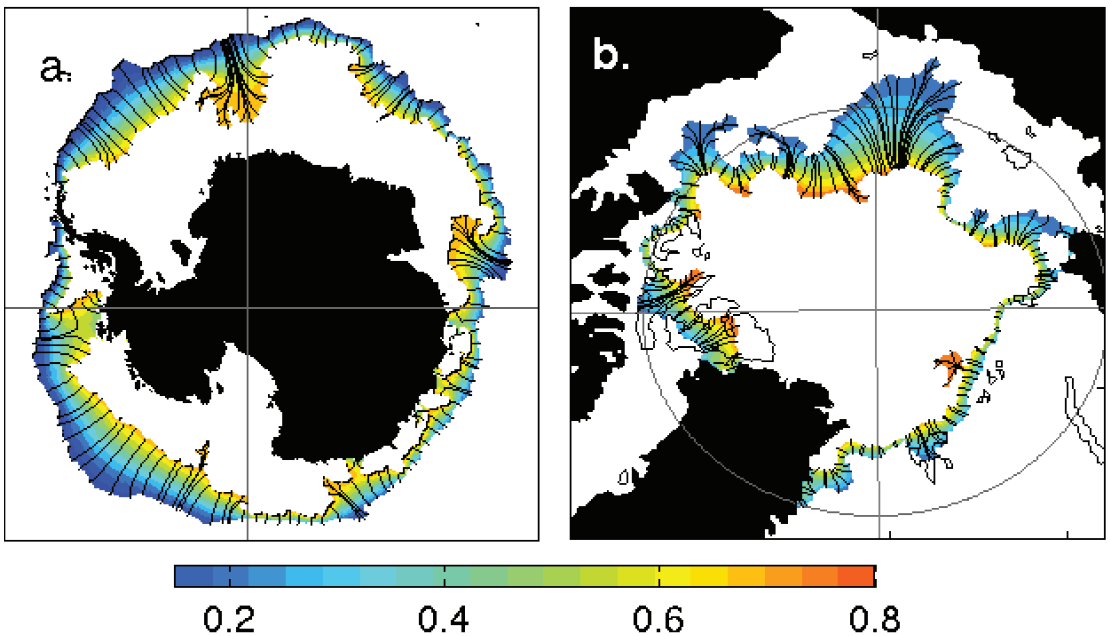
- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions

?? "MIZ WIDTH" ??
fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

To define objective MIZ width
assume idealized sea ice
concentration field ψ satisfying:

$$\nabla^2 \psi = 0 \qquad + \text{BC}$$



MIZ width based on averaging
length of ***electric field lines***

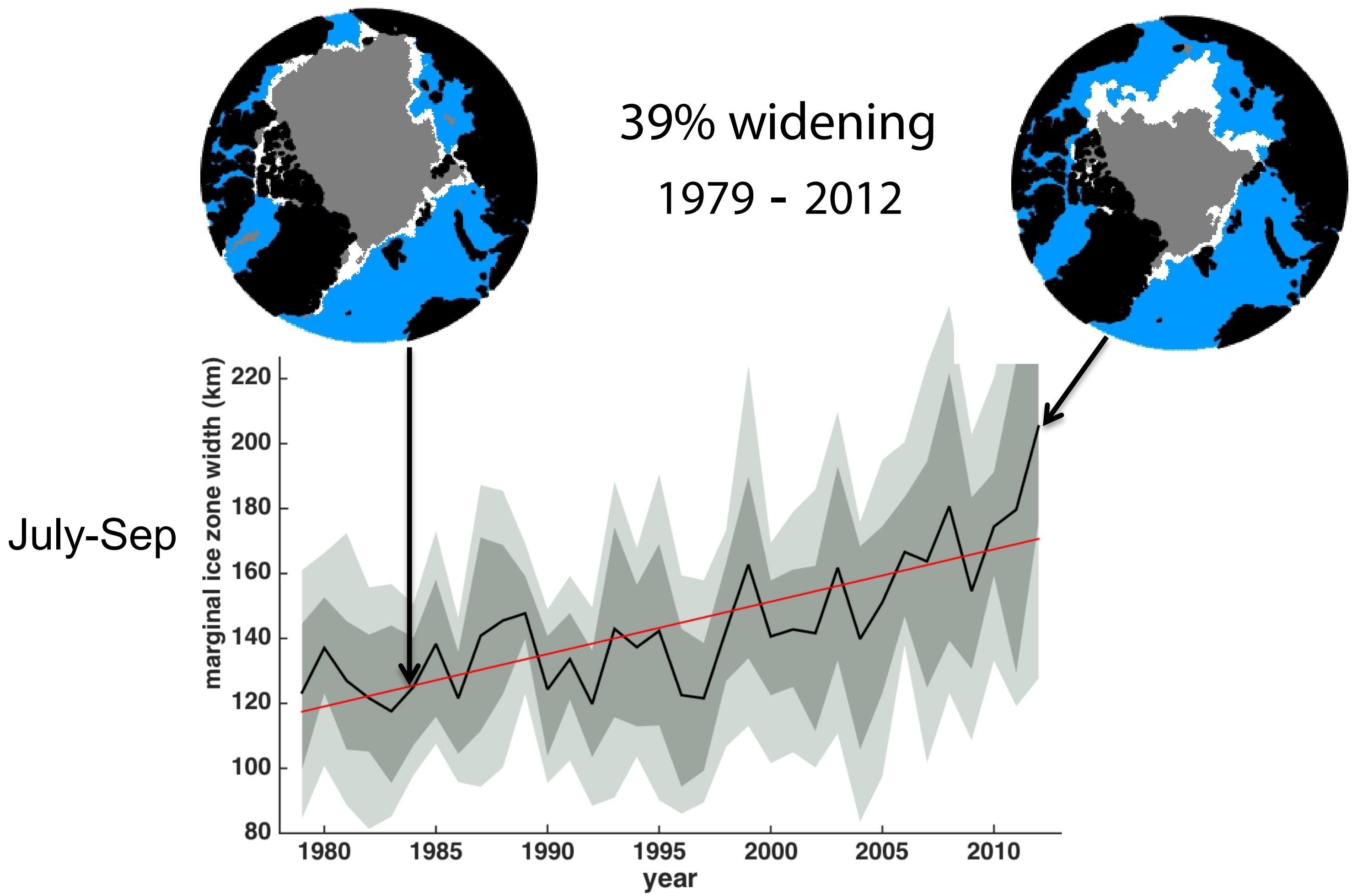
$$\bar{\ell} = \frac{1}{L_\gamma} \int_\gamma \ell(s)\, \mathrm{d}s$$

length of MIZ
perimeter

analysis of different MIZ WIDTH definitions

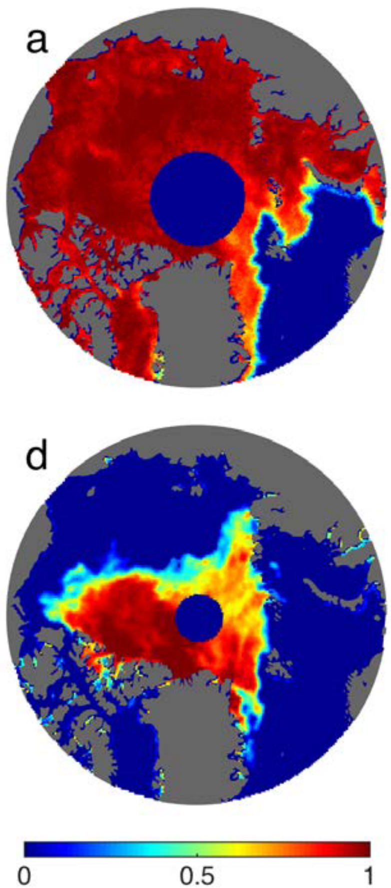
Strong, Foster, Cherkaev, Eisenman, Golden 2016

MIZ width increasing



Strong and Rigor, *Geophysical Research Letters*, 2013

Filling the polar data gap



Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007

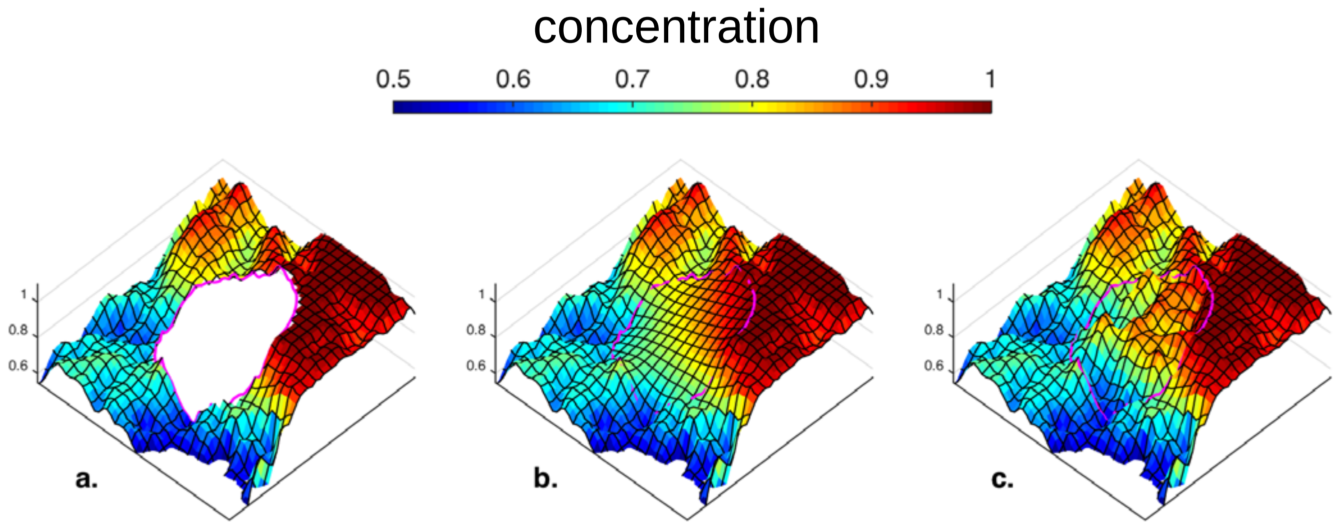
Examples of "polar data gap" where orbiting satellites do not measure sea ice concentration

gap region conventionally assumed ice-covered for sea ice extent calculations

given recent losses this assumption may no longer be valid

Strong and Golden, *Remote Sensing*, 2016.

Filling the polar data gap



polar data gap
30 August 2007

$$\nabla^2\psi = 0$$

$$f(\lambda, \phi) = \psi(\lambda, \phi) + \Omega(\lambda, \phi)$$

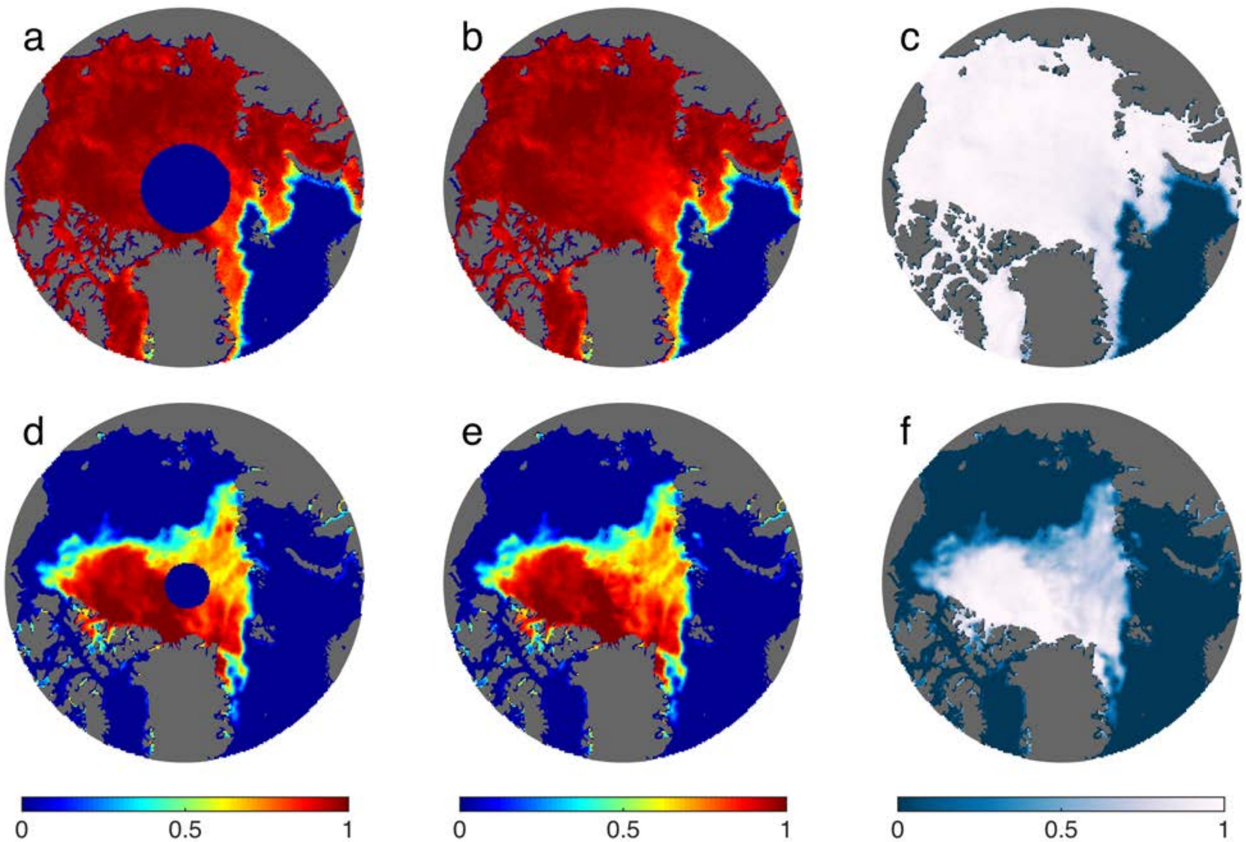
Ω simulates realistically autocorrelated deviations from ψ via convolution of random noise with a Gaussian function

Strong and Golden, *Remote Sensing,* 2016.

Filling the polar data gap

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



Strong and Golden, *Remote Sensing,* 2016.

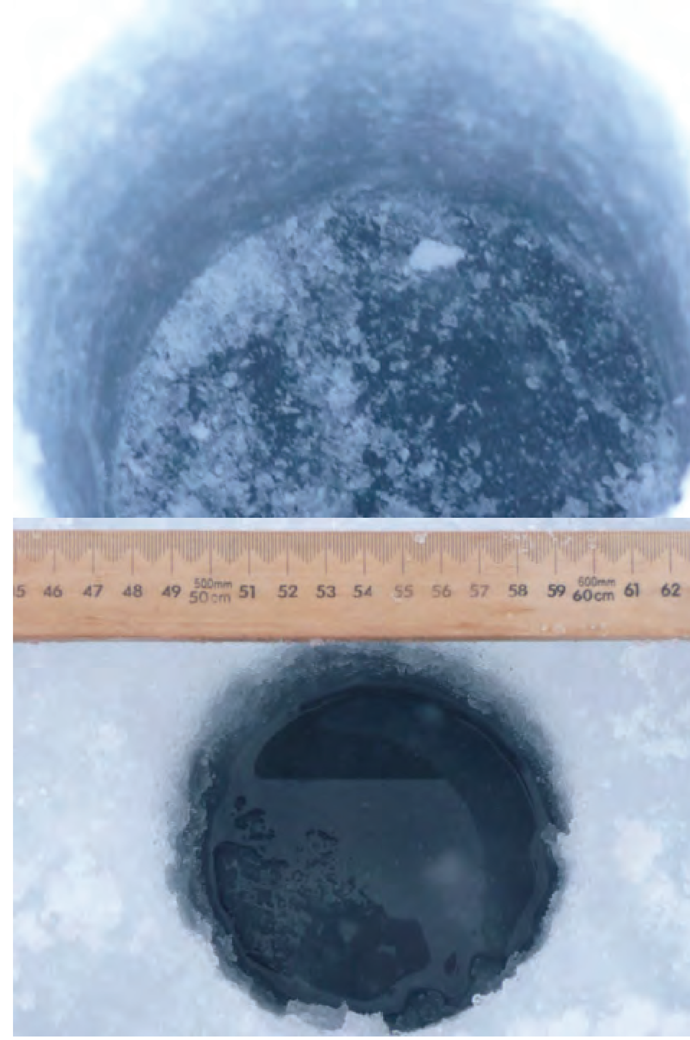
Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*
2014	*Arctic*	*Chukchi Sea*

photo by Jan Lieser

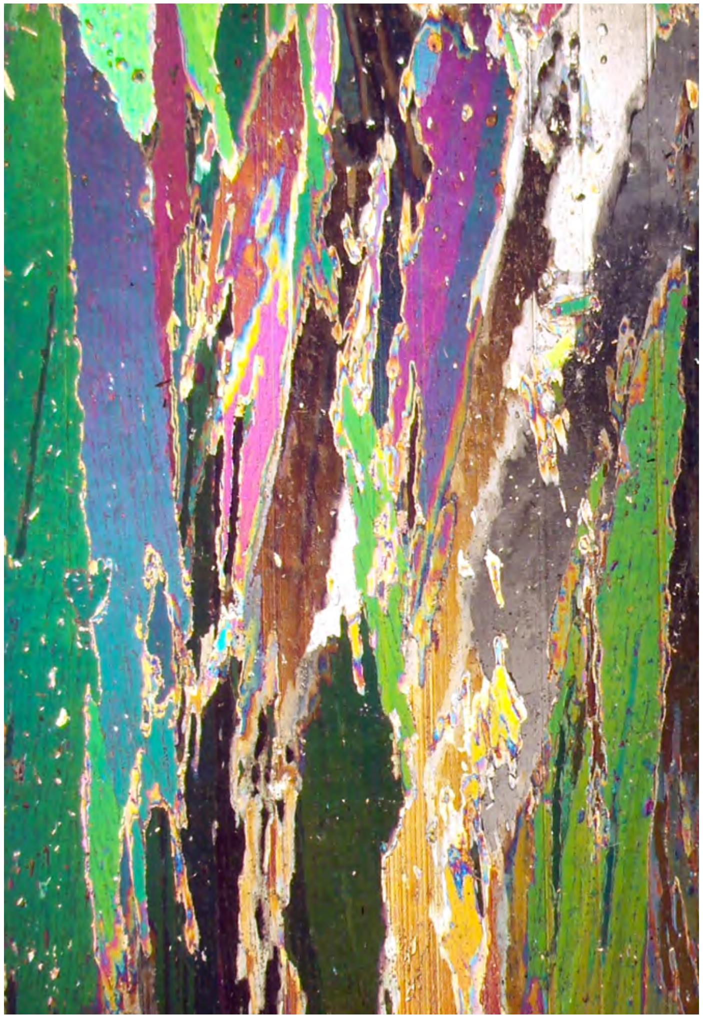


measuring
fluid permeability
of Antarctic sea ice

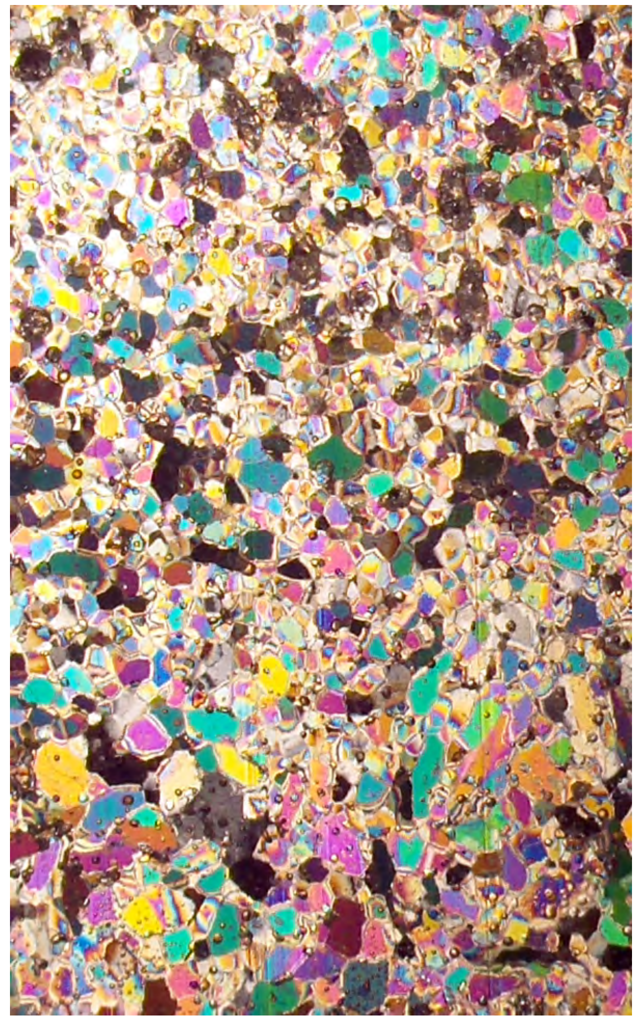
SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar



granular

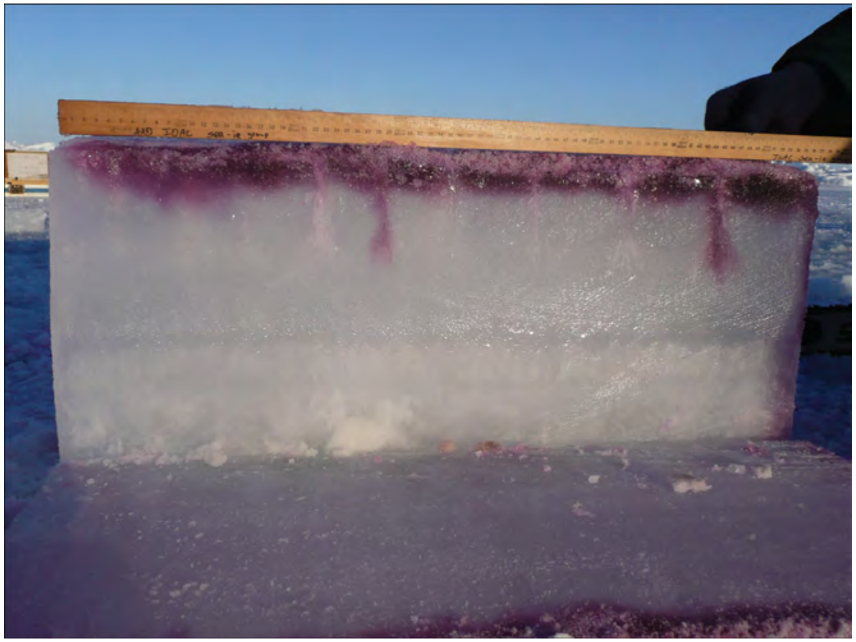


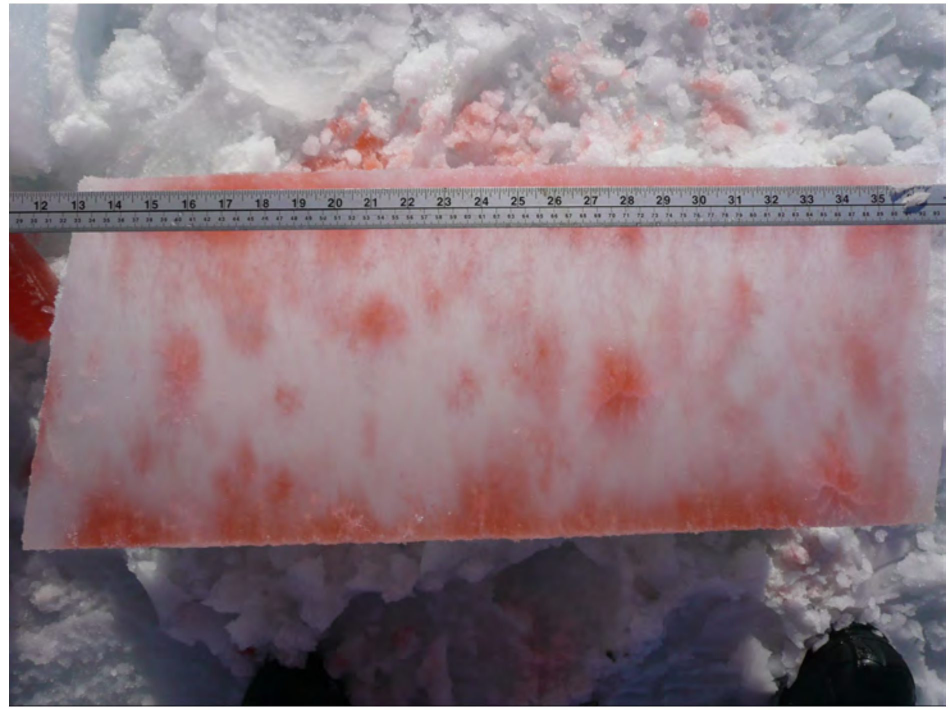
5%

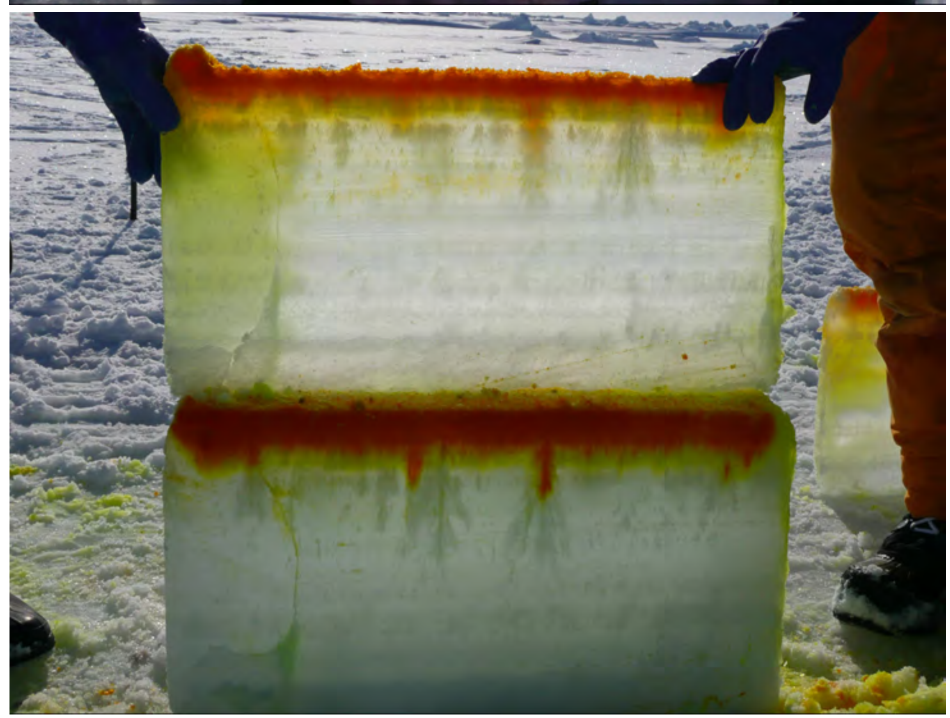
10%

Golden, Sampson, Gully, Lubbers, Tison 2016

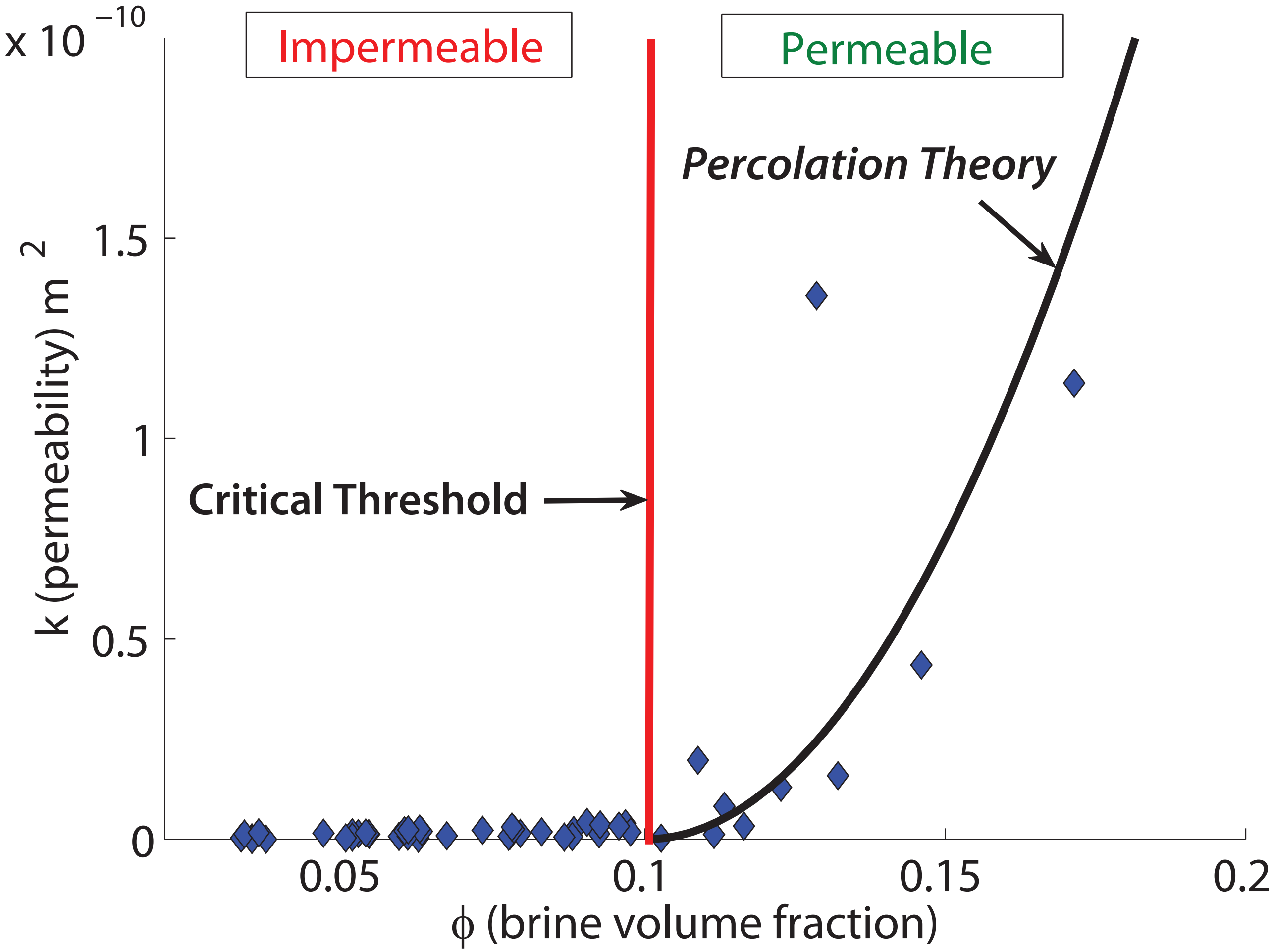
tracers flowing through inverted sea ice blocks





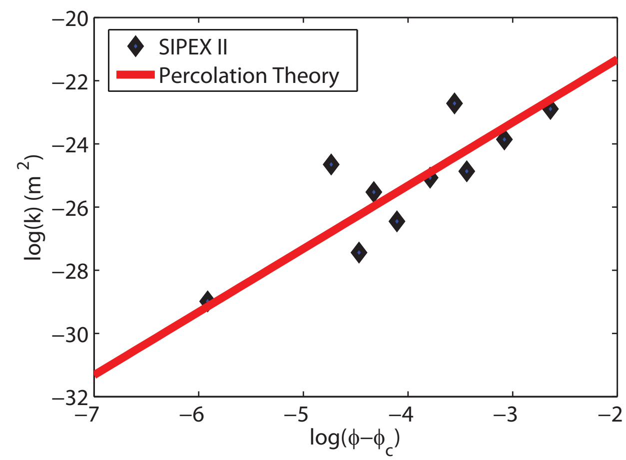


SIPEX II vertical permeability data



same universal critical exponent as lattice models

data above threshold

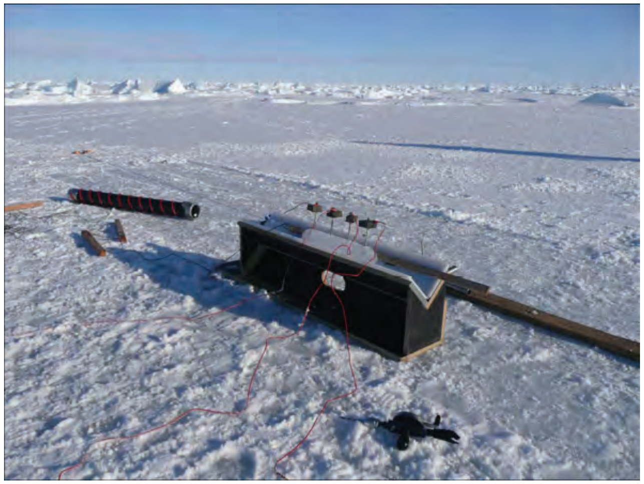


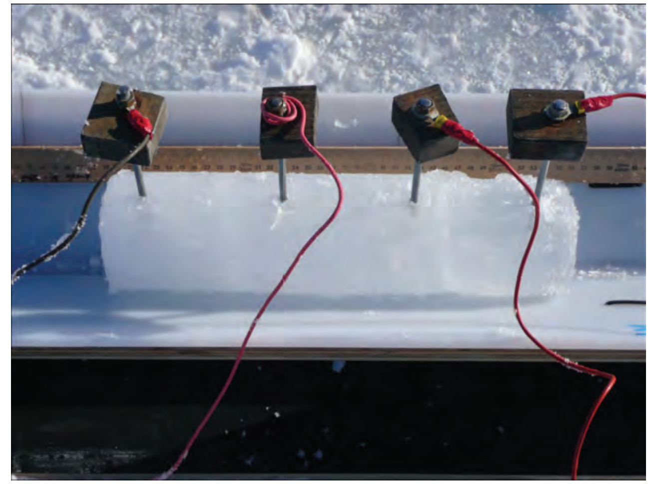
higher threshold in granular ice predicted with percolation theory by Golden, et al. (Science, 1998)

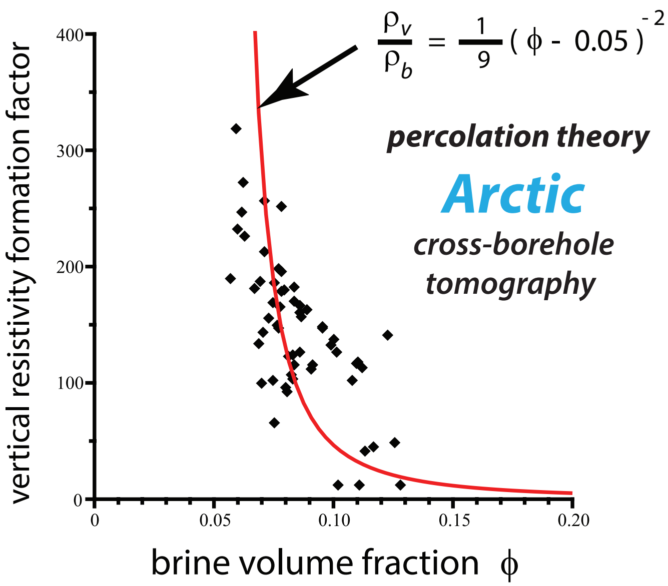
not confirmed experimentally until SIPEX I (2007) and SIPEX II (2012)

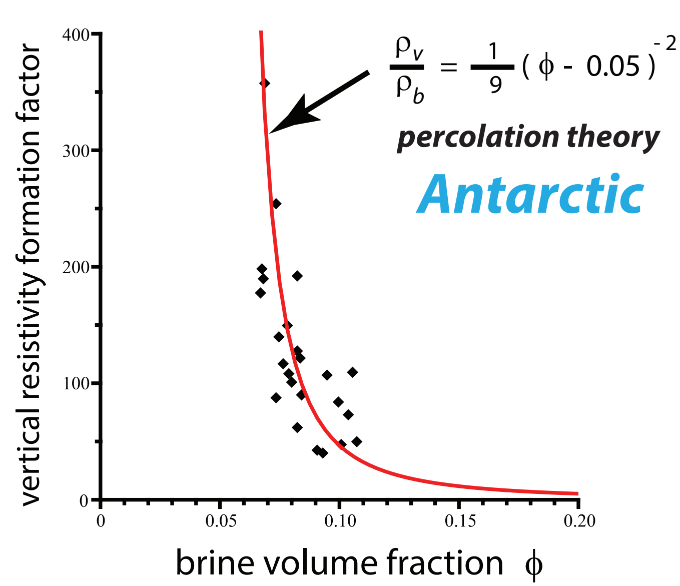
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow







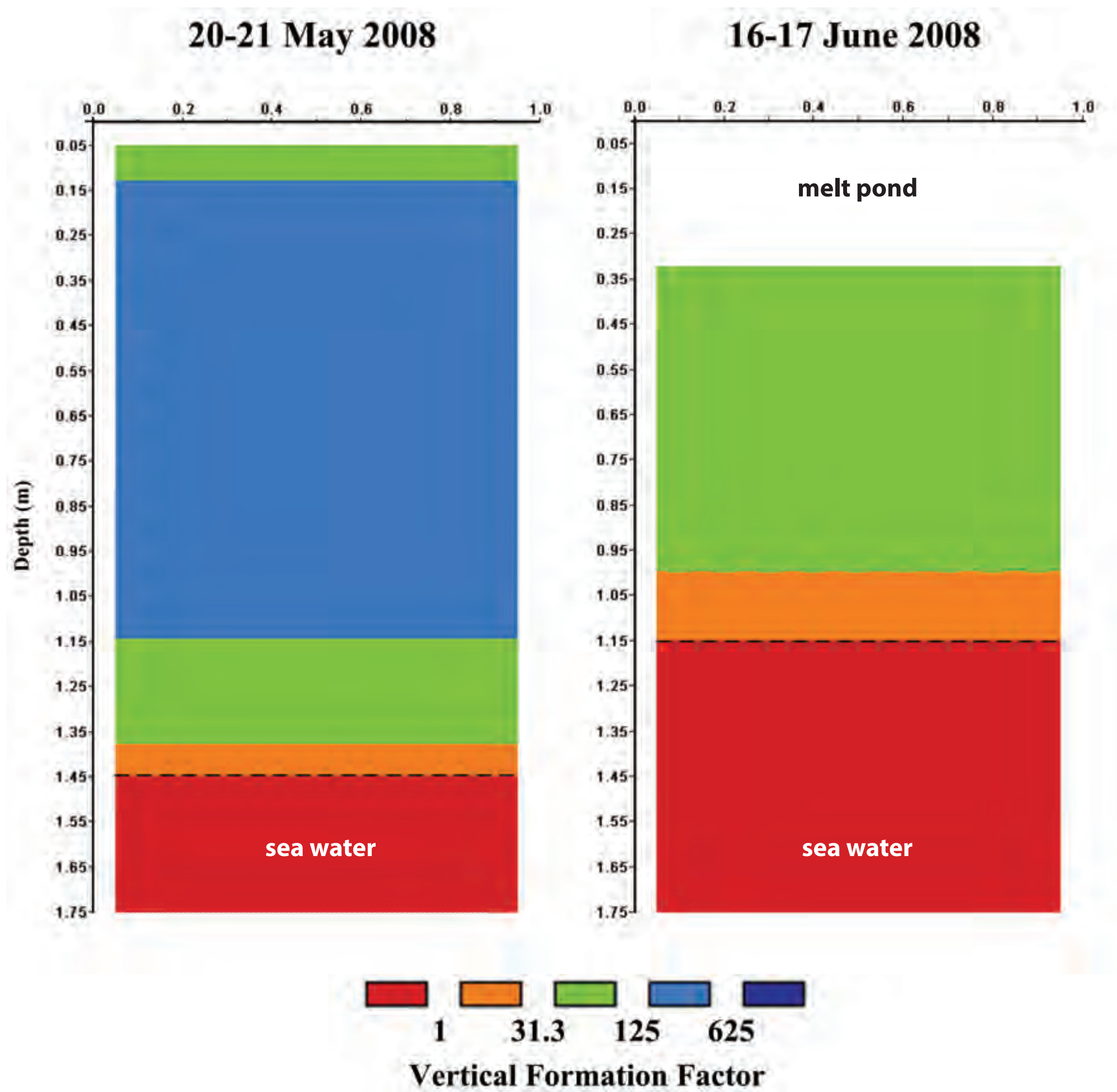


cross-borehole tomography - electrical classification of sea ice layers

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2016

Cross-borehole tomographic reconstructions of sea ice resistivity

before and after melt pond formation



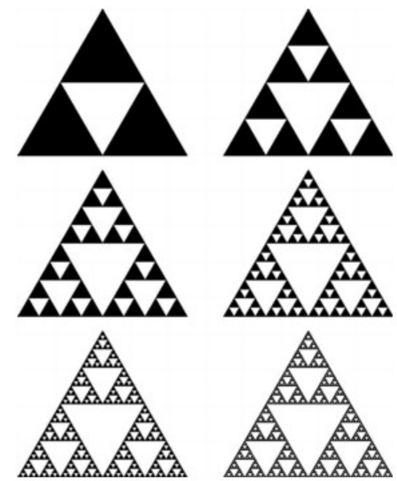
impermeable - ponds can grow

impermeable for CO2 exchange & build-up of nutrients and biomass; supports surface ponding

highly permeable - CO2 pumping, build-up of nutrients and biomass

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, and Worby 2016

fractals and multiscale structure



melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

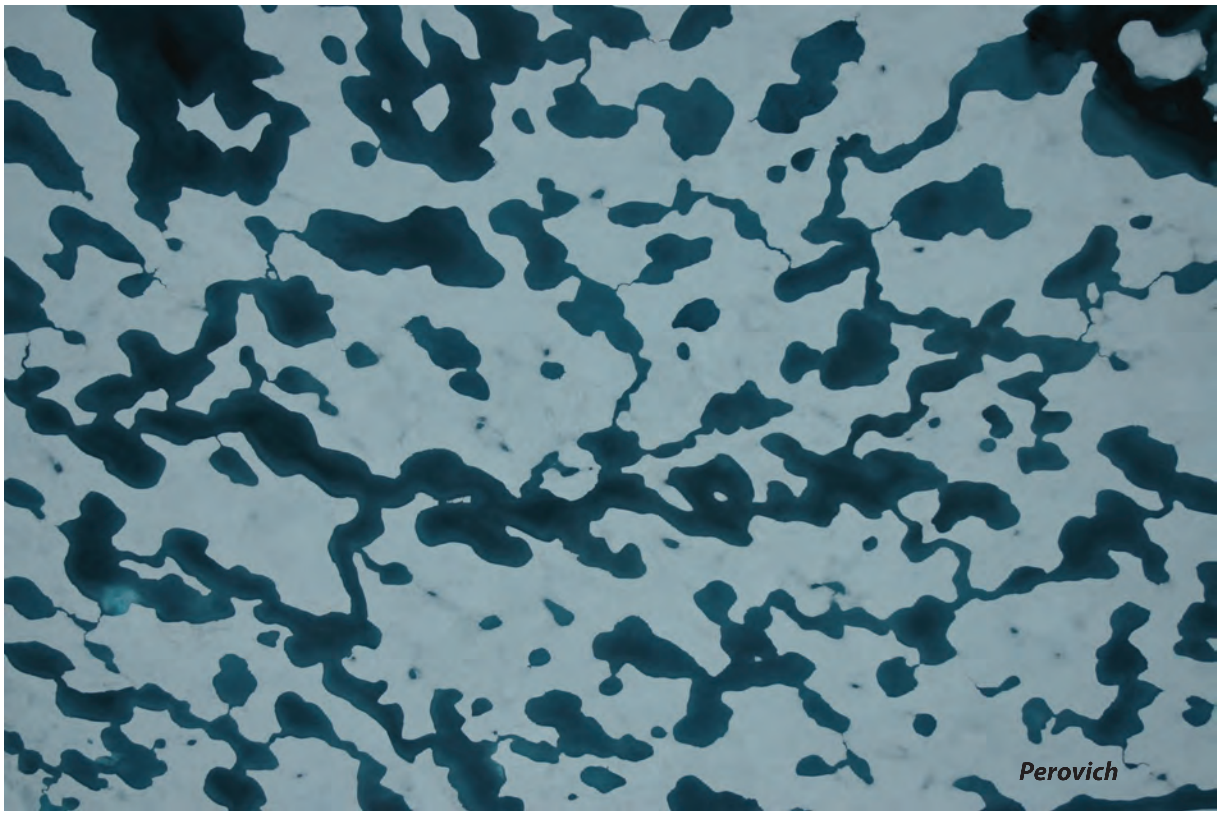
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

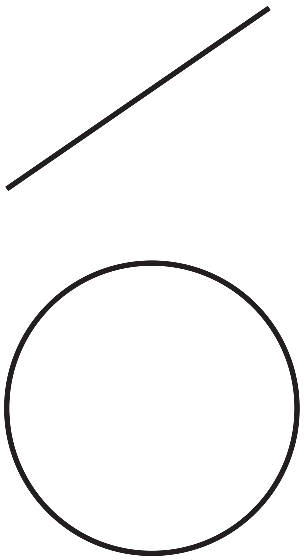
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

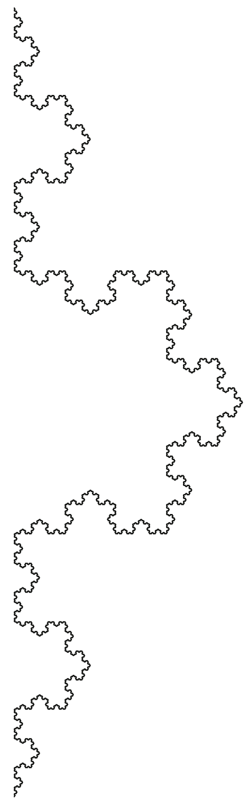
fractal curves in the plane

they wiggle so much that their dimension is >1



simple curves

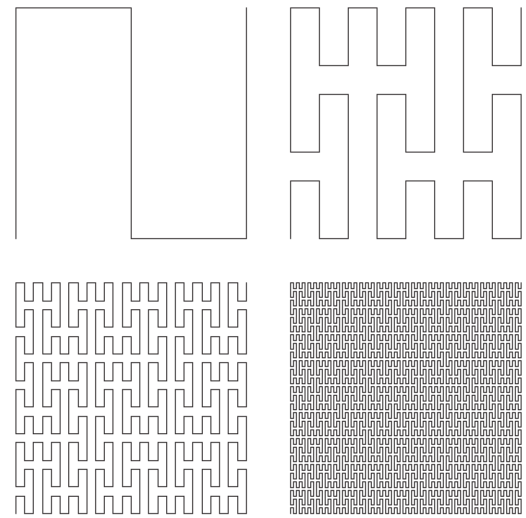
***D* = 1**



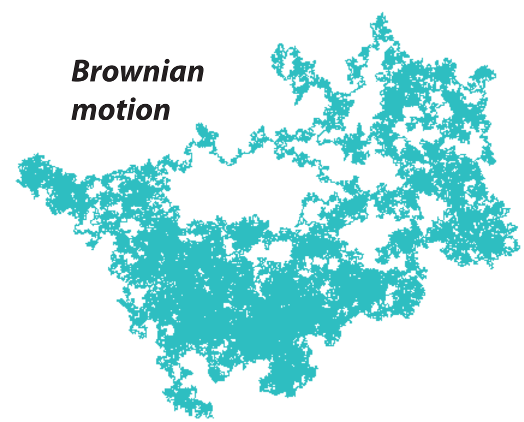
Koch snowflake

***D* = 1.26**

Peano curve



Brownian motion



space filling curves

***D* = 2**

clouds exhibit fractal behavior from 1 to 1000 km

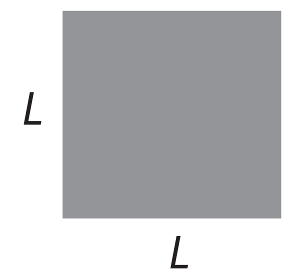
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

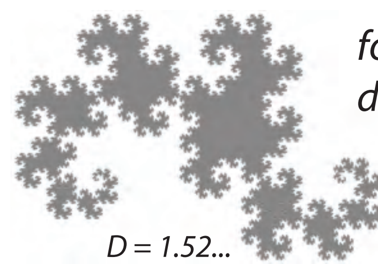
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

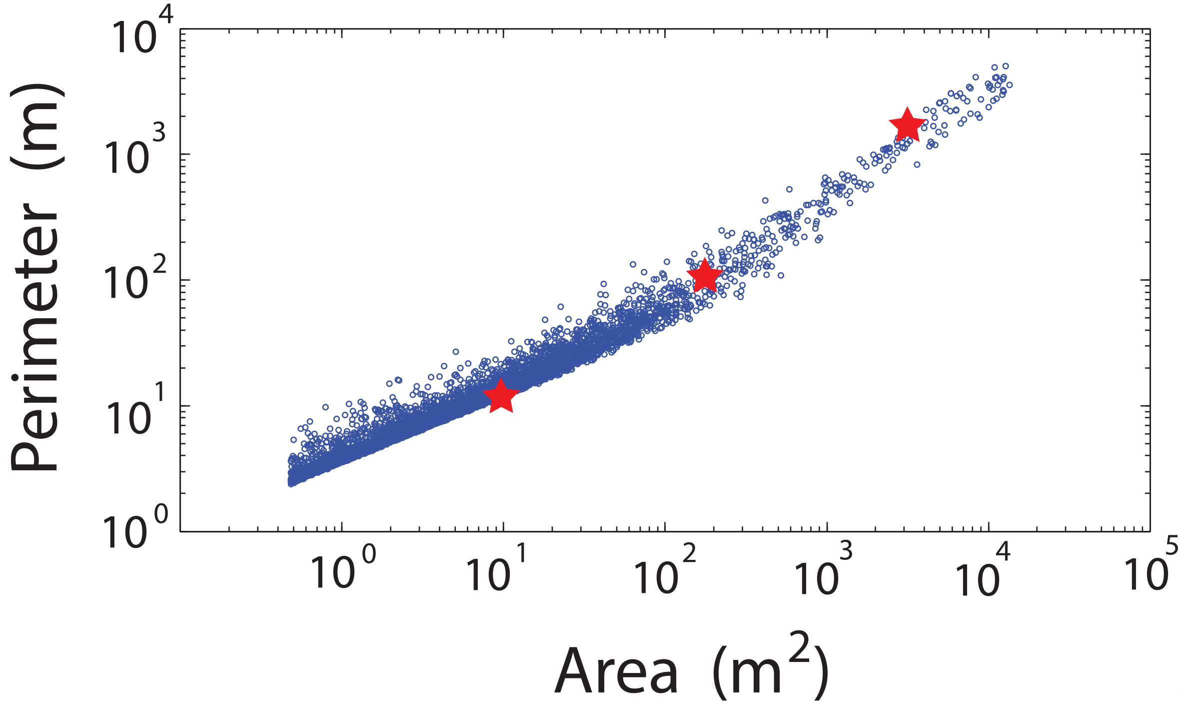
$$P \sim \sqrt{A}^{\,D}$$



for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

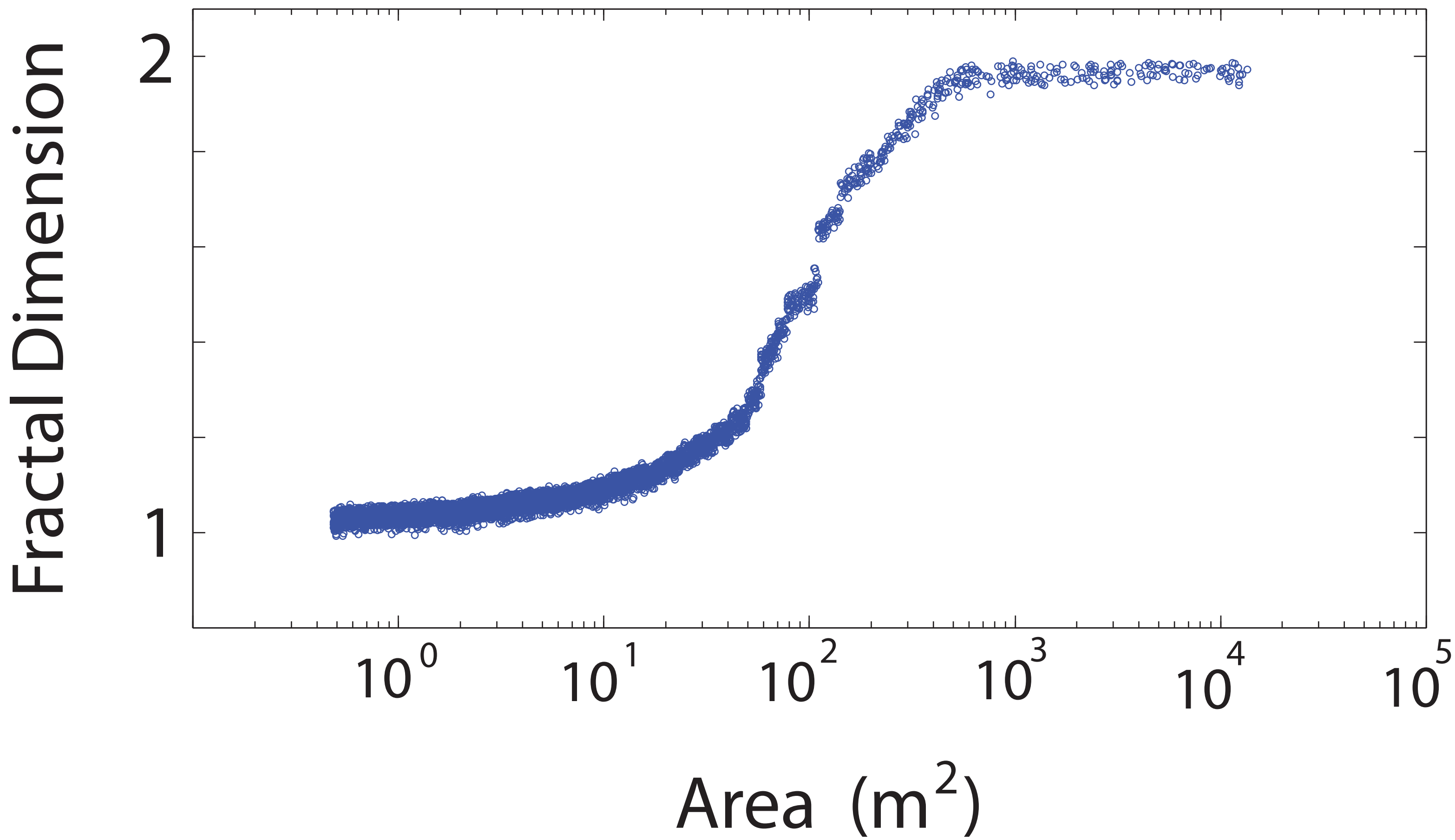


data from 5269 Arctic melt ponds

simple pond ***transitional pond*** ***complex pond***

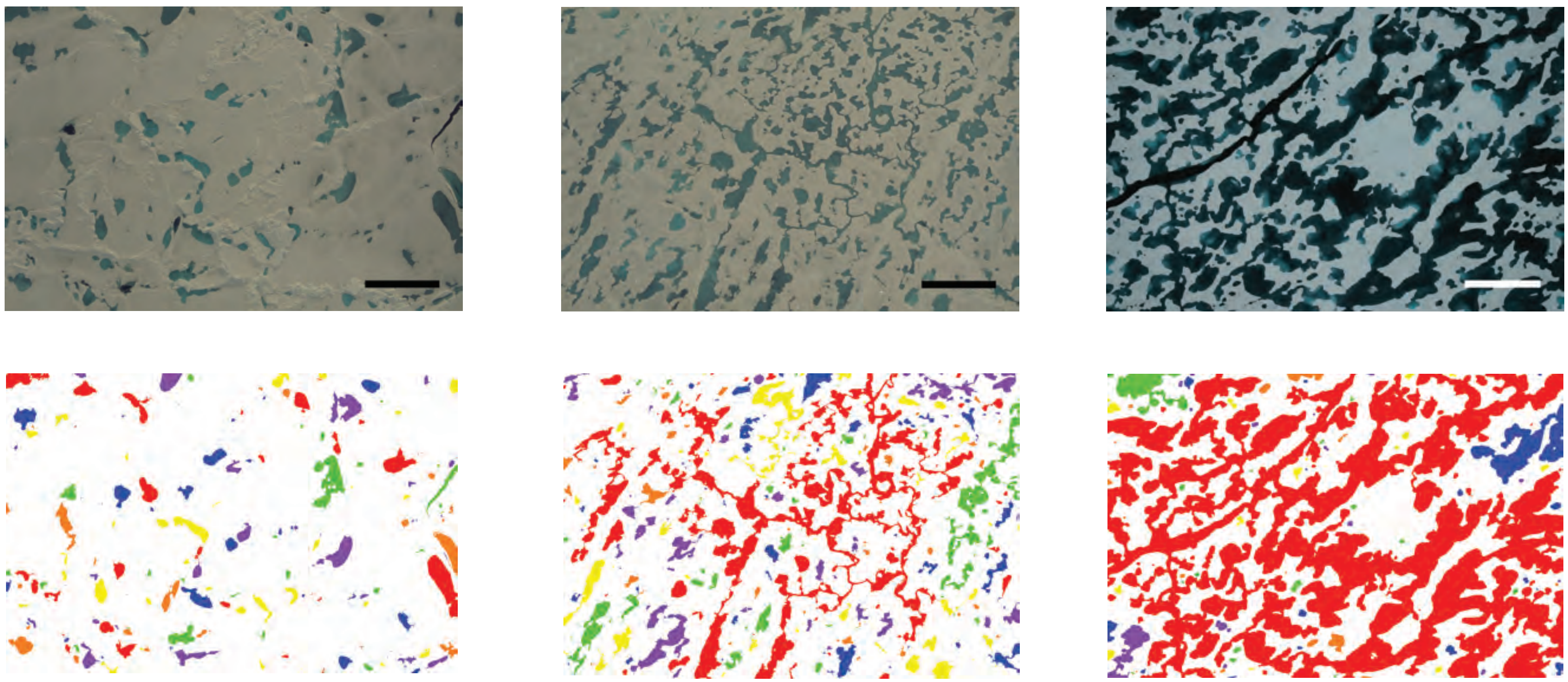
transition in the fractal dimension

complexity grows with length scale



compute "derivative" of area - perimeter data

small simple ponds coalesce to form
large connected structures with complex boundaries



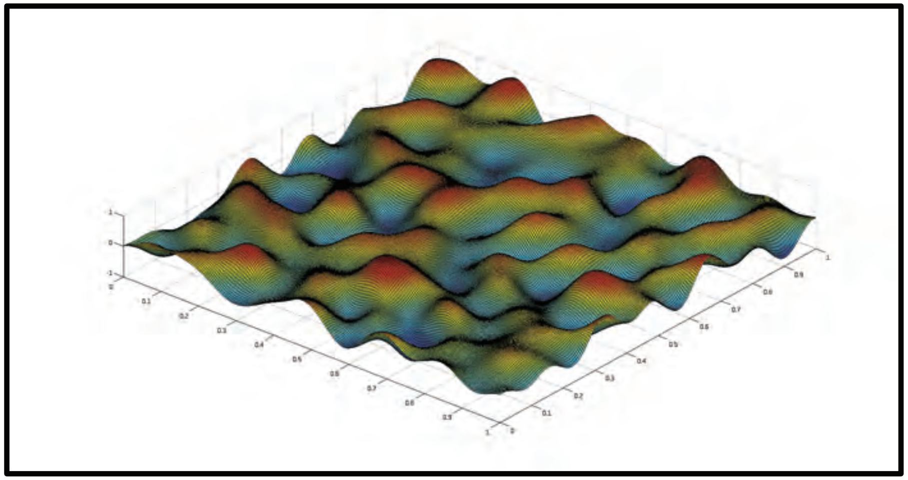
melt pond percolation

results on percolation threshold, cluster behavior

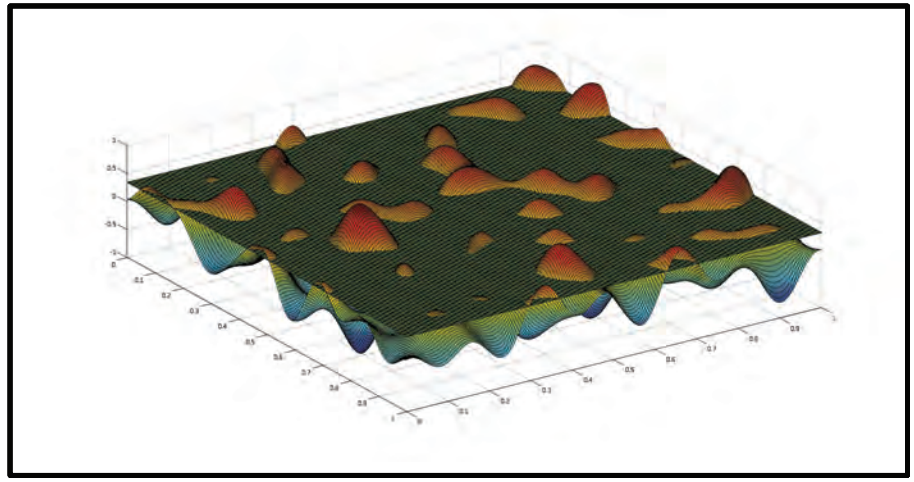
Anthony Cheng (Hillcrest HS), Bacim Alali, Ken Golden

Continuum percolation model for melt pond evolution

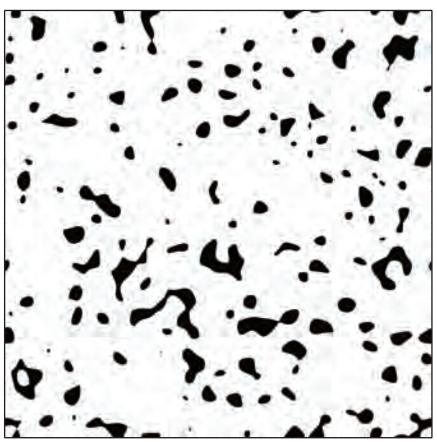
Brady Bowen, Court Strong, Ken Golden, 2016



random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds



electronic transport in disordered media

diffusion in turbulent plasmas

(Isichenko, Rev. Mod. Phys., 1992)

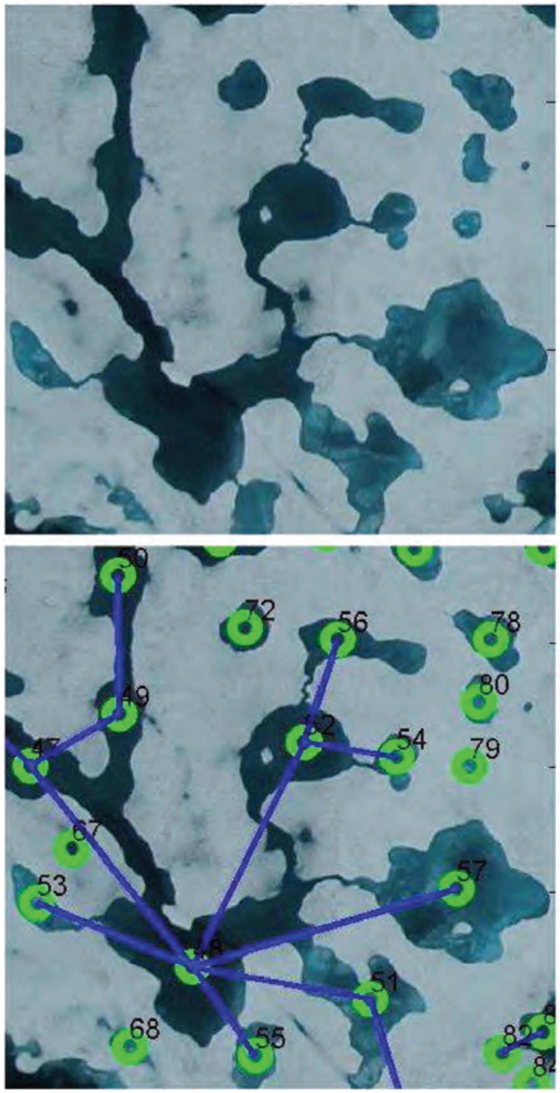
melt pond evolution depends also on large-scale "pores" in ice cover

photos courtesy of C. Polashenski and D. Perovich

Melt pond connectivity enables vast expanses of melt water to drain down seal holes, thaw holes, and leads in the ice

Network modeling of Arctic melt ponds

Barjatia, Tasdizen, Song, Sampson, Golden
Cold Regions Science and Tecnology, 2016



develop algorithms to map images of melt ponds onto

random resistor networks

graphs of nodes and edges with edge conductances

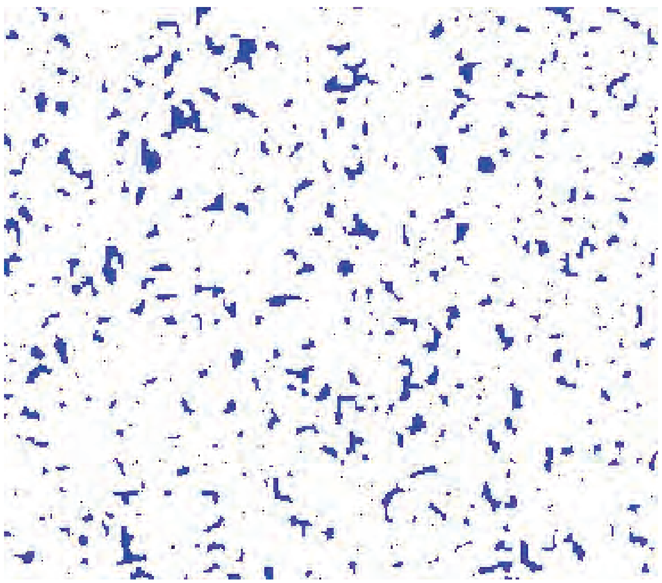
edge conductance ~ neck width

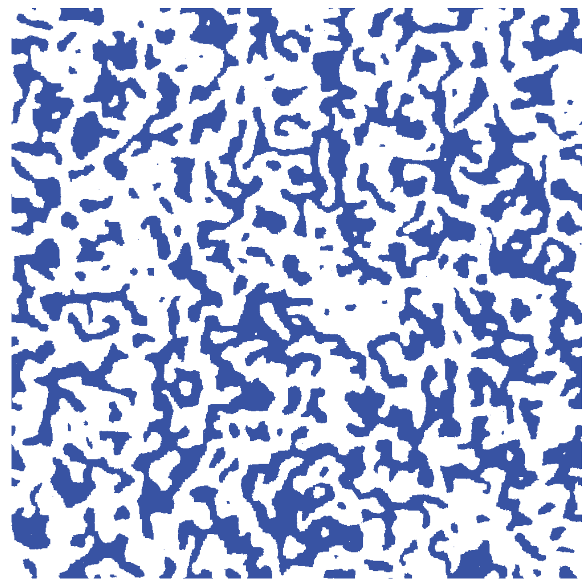
compute effective horizontal fluid conductivity

Ising model for ferromagnets $\longrightarrow$ Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

magnetization $M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$ **pond coverage** $\frac{(M+1)}{2}$

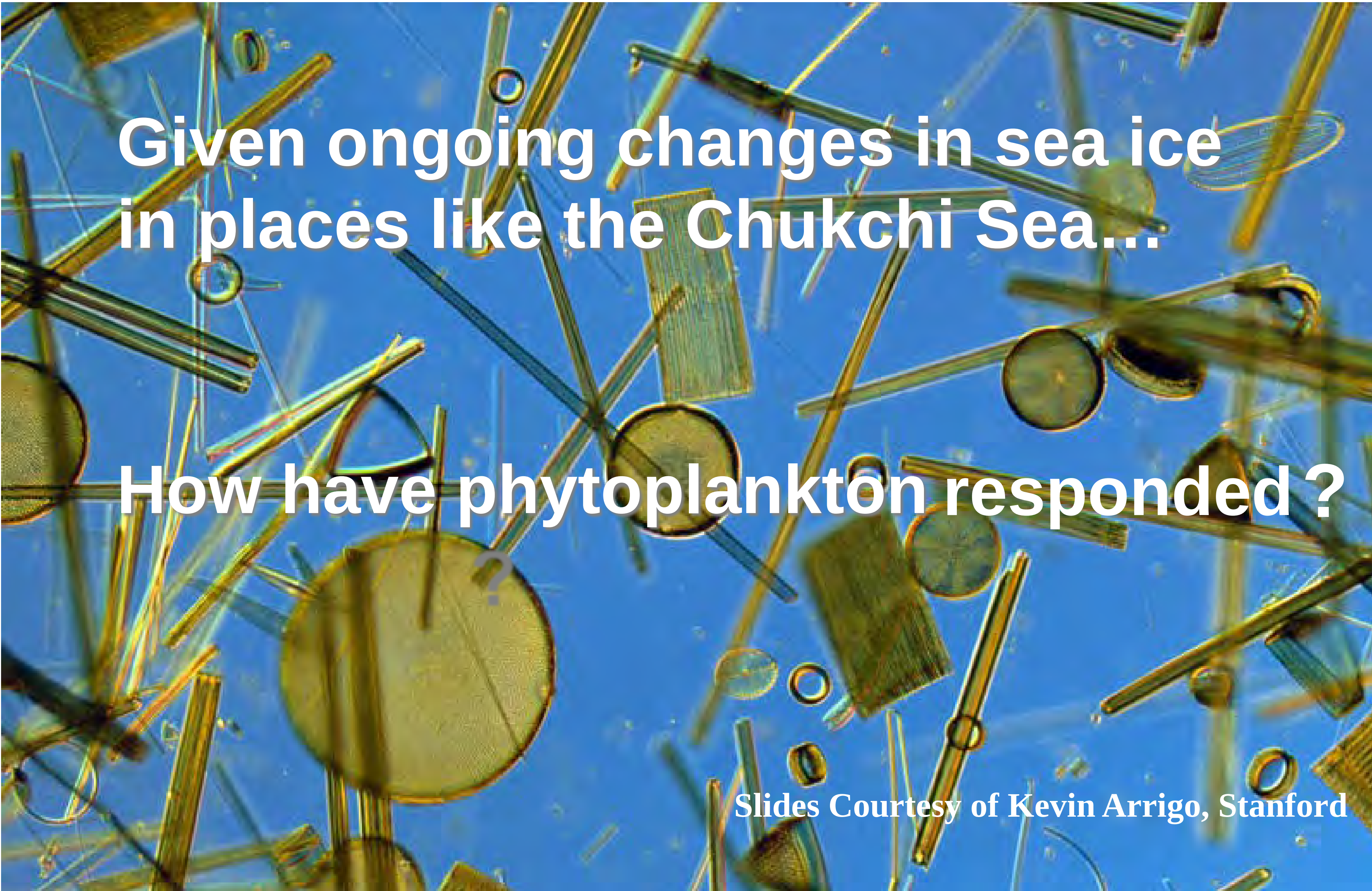




"melt ponds" are clusters of magnetic spins that align with the applied field

predictions of fractal transition, pond size exponent **Ma, Sudakov, Strong, Golden 2016**

Question:



Given ongoing changes in sea ice in places like the Chukchi Sea…

How have phytoplankton responded ?

Slides Courtesy of Kevin Arrigo, Stanford

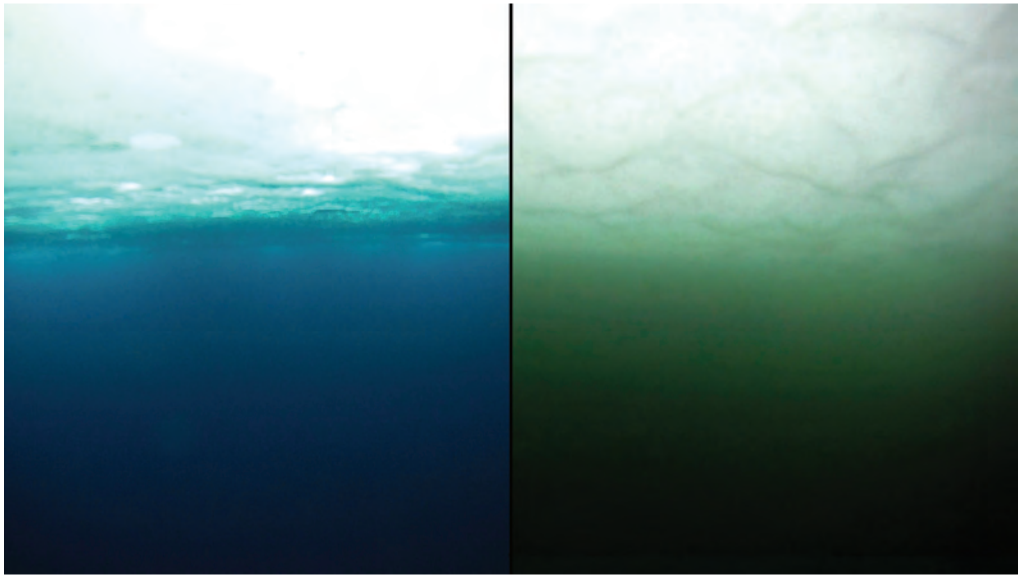
2011 massive under-ice algal bloom

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light through sea ice



no bloom

bloom

Have we crossed into a new ecological regime?

The Melt Pond Conundrum:

How can ponds form on top of sea ice that is highly permeable?

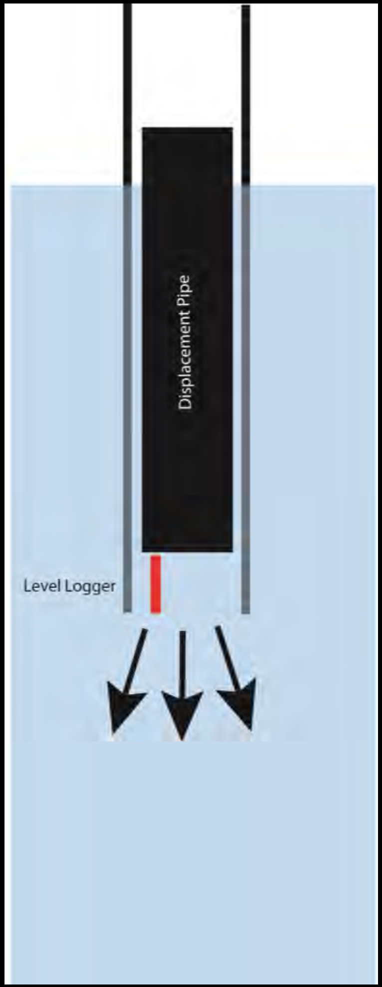
C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: The Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy

Hypothesis – Freshwater re-seals ice

Borehole test with varying salinity



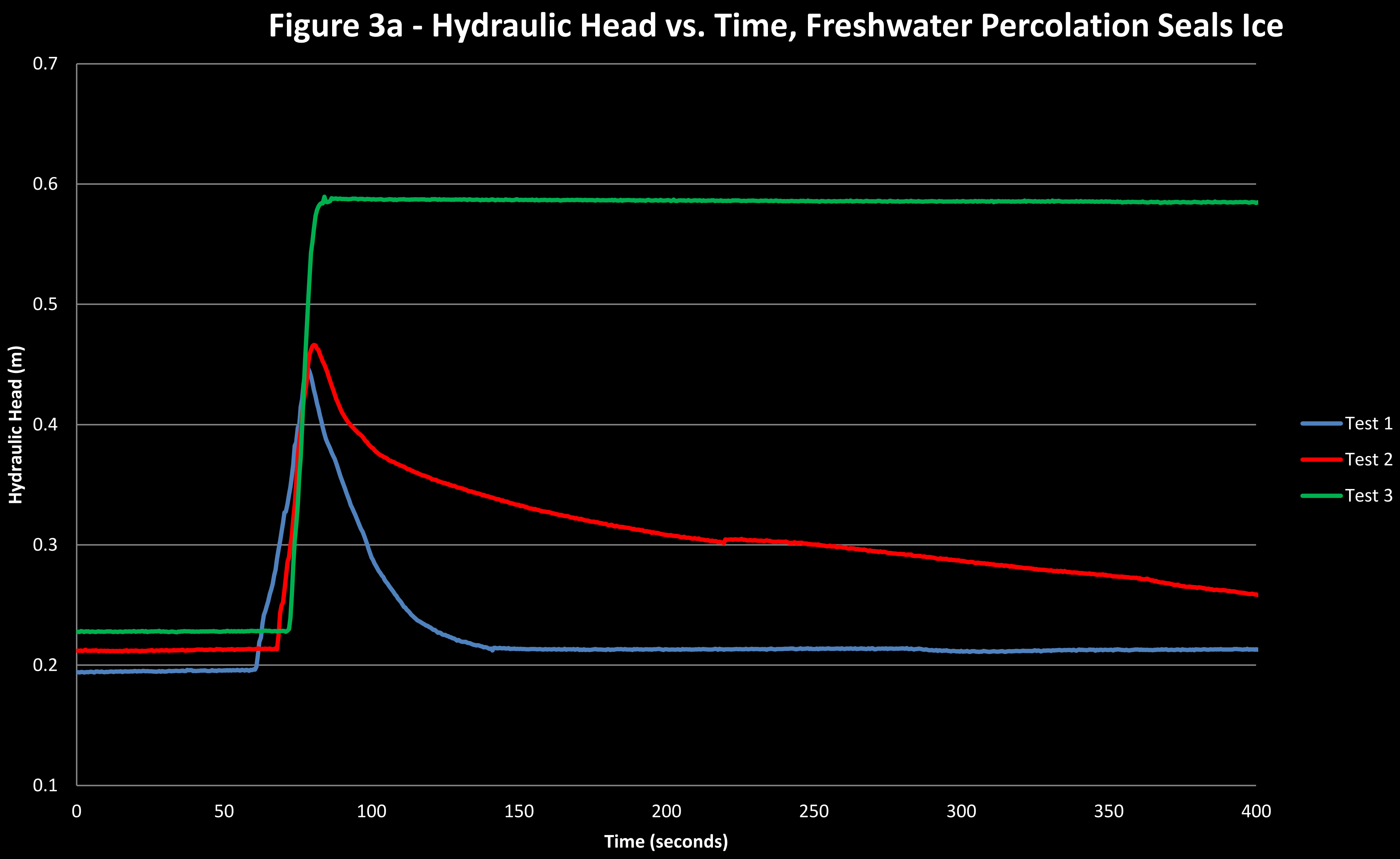


Figure 3a - Hydraulic Head vs. Time, Freshwater Percolation Seals Ice
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Hydraulic Head (m)
0
50
100
150
200
250
300
350
400
Time (seconds)
Test 1
Test 2
Test 3

Conclusions

1. Summer Arctic sea ice is melting rapidly.

2. Fluid flow through sea ice mediates many processes of importance to understanding climate change and the response of polar ecosystems.

3. Mathematical models of composite materials and statistical physics help unravel the complexities of sea ice structure and processes, and provide a path toward rigorous representation of sea ice in climate models .

4. Critical behavior (in many forms) is inherent in the climate system.

5. Field experiments are essential to developing relevant mathematics.

6. Our research will help to improve projections of climate change and the fate of the Earth sea ice packs.

THANK YOU

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Arctic and Global Prediction Program

Applied and Computational Analysis Program

Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

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Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999