

Introduction to Sea Ice

Kenneth M. Golden
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University of Utah



Alison Kohout
September 2012

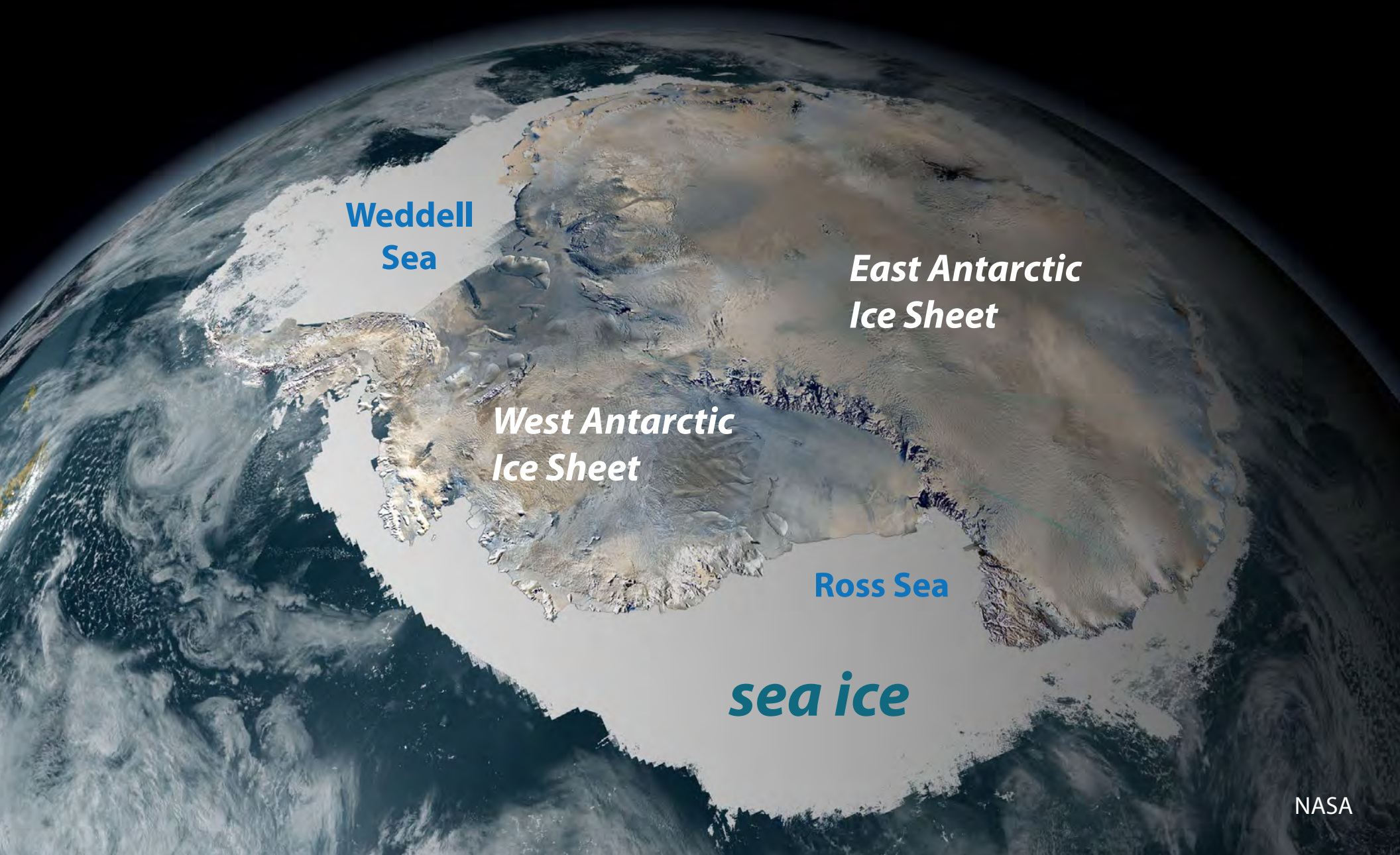
Pattern Formation in Melting Arctic Sea Ice

Kenneth M. Golden
Department of Mathematics
University of Utah

Community Lecture
CNA 2019 Workshop on Mathematical Models for Pattern Formation
Carnegie Mellon University, March 9, 2019

ANTARCTICA

southern cryosphere



**Weddell
Sea**

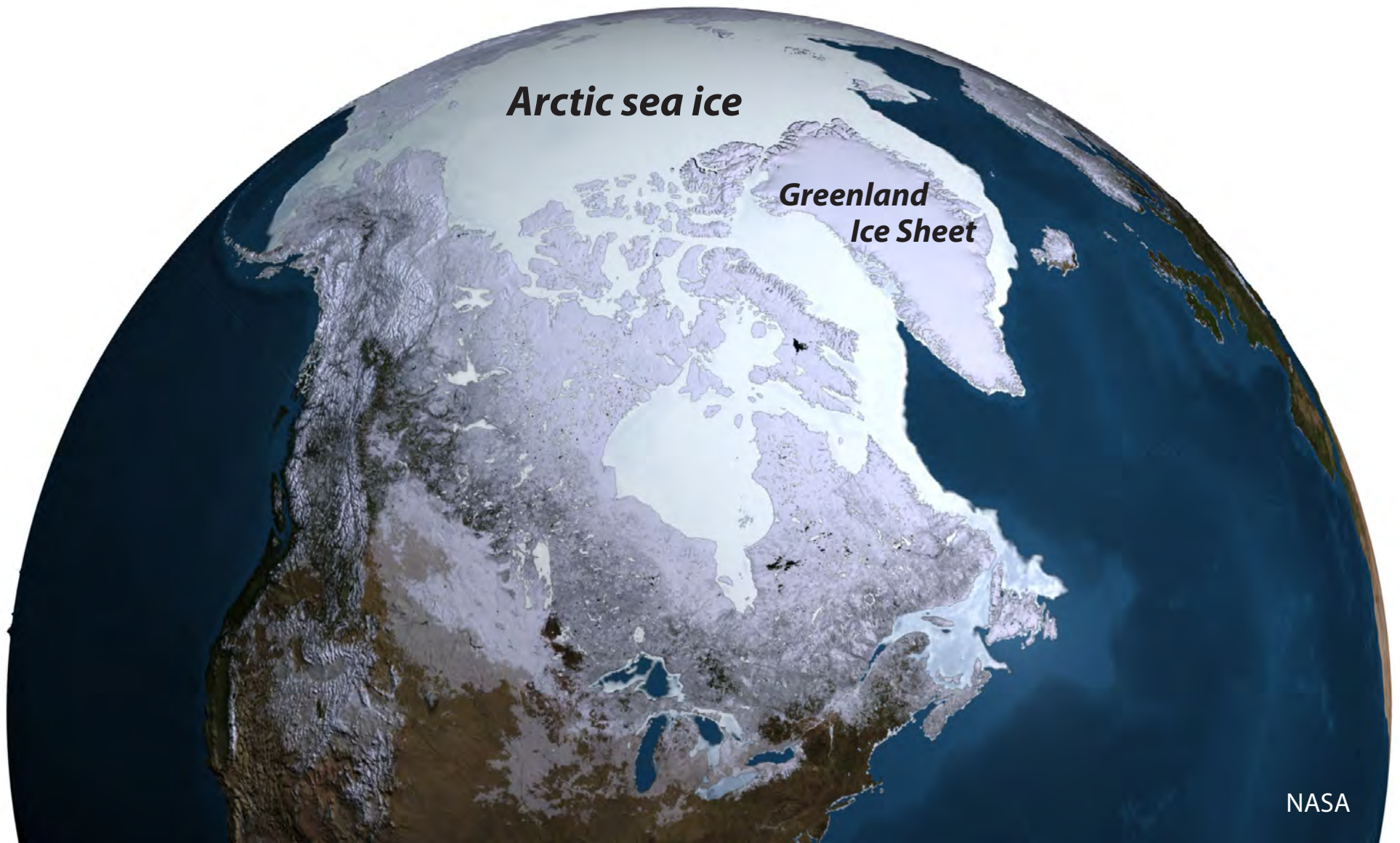
***East Antarctic
Ice Sheet***

***West Antarctic
Ice Sheet***

Ross Sea

sea ice

northern cryosphere



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of **climate change**



polar ice caps critical
to climate in reflecting
sunlight during summer

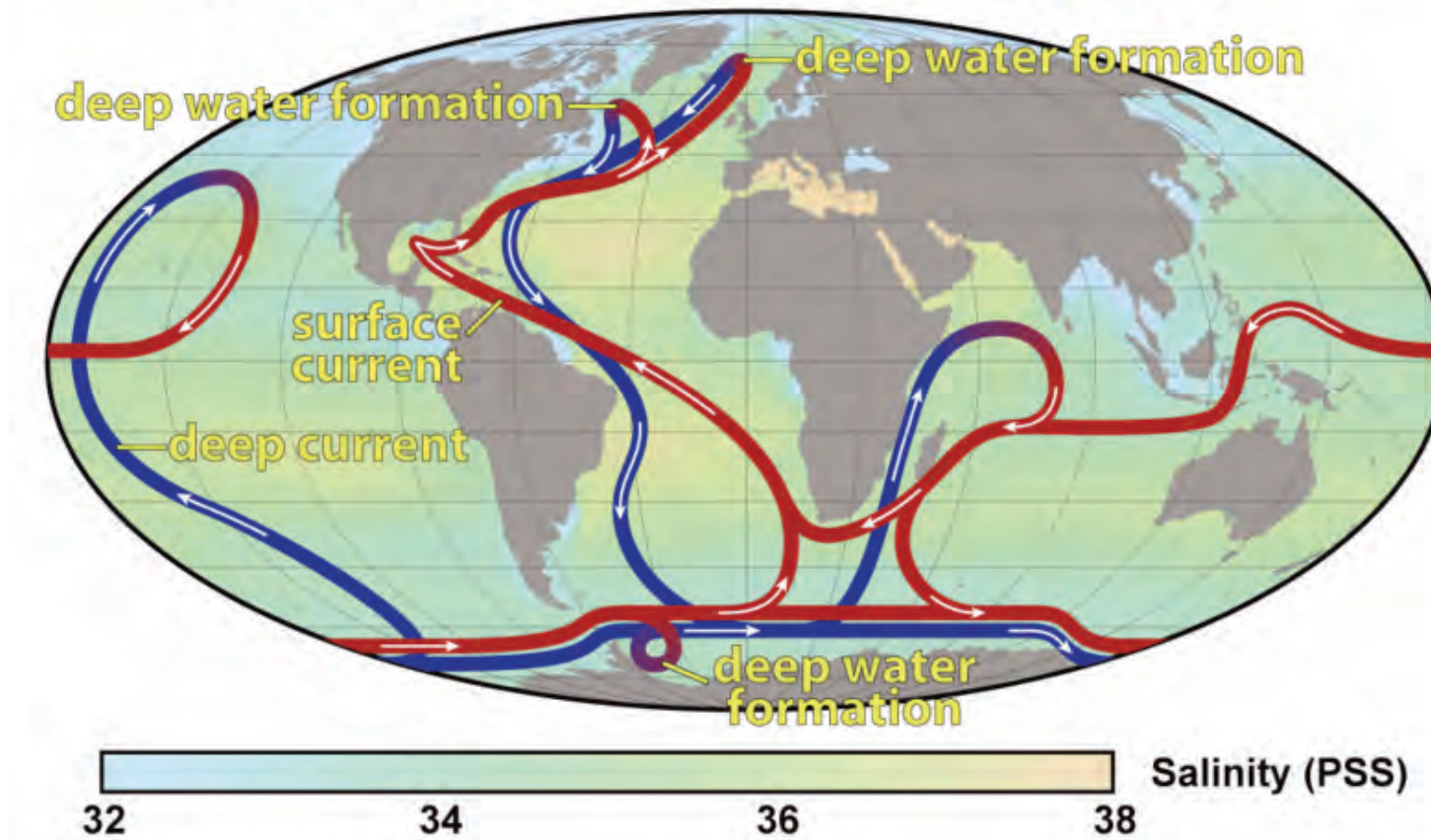
brine expulsion from sea ice formation results in water beneath the ice becoming cooler and saltier



this denser water sinks rapidly to great depths

deep-water formation drives circulation in the world's oceans

Thermohaline Circulation



GLOBAL THERMOHALINE CONVEYOR BELT

polar ice caps critical to global climate in reflecting incoming solar radiation



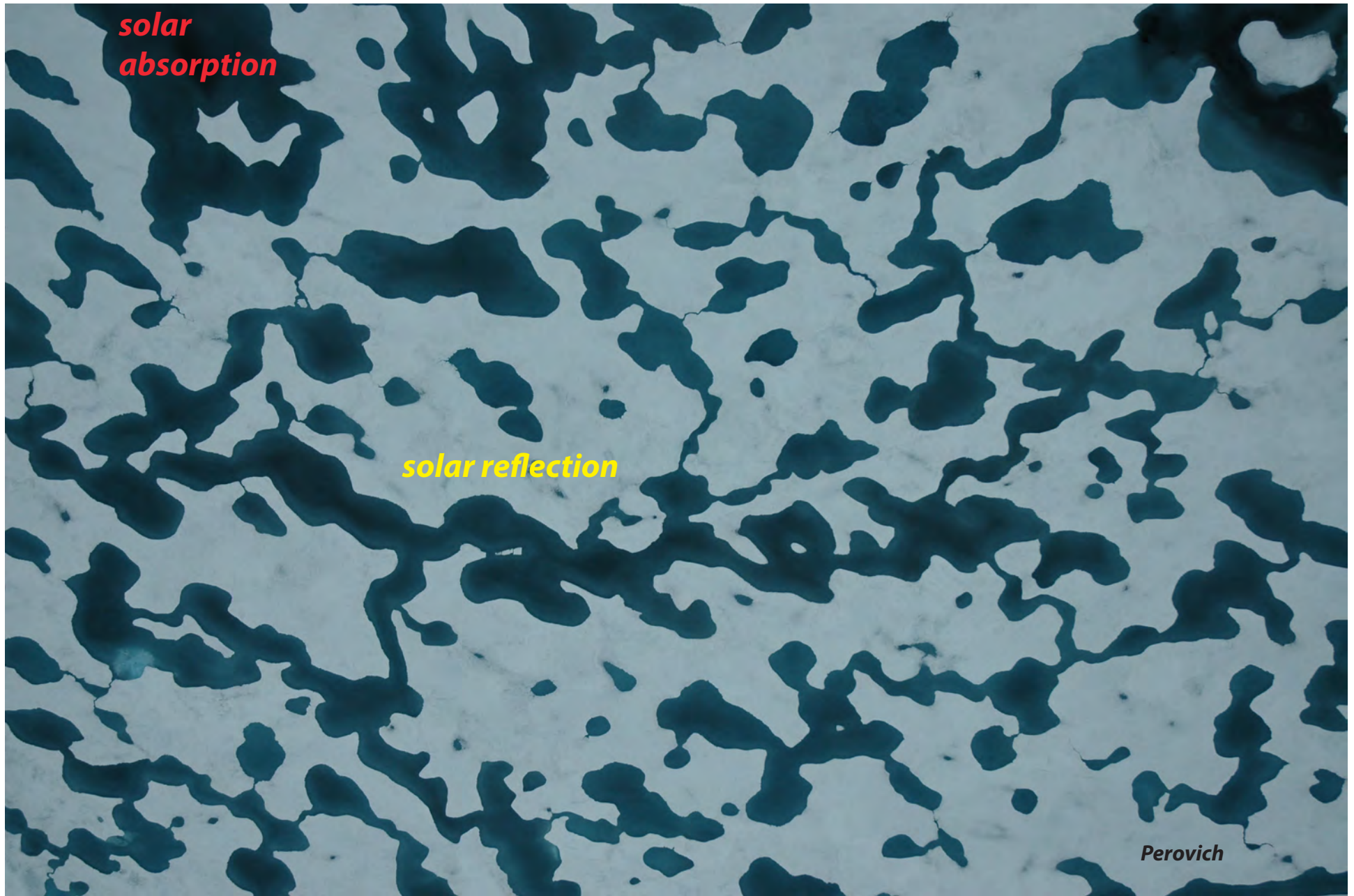
white snow and ice
reflect



dark water and land
absorb

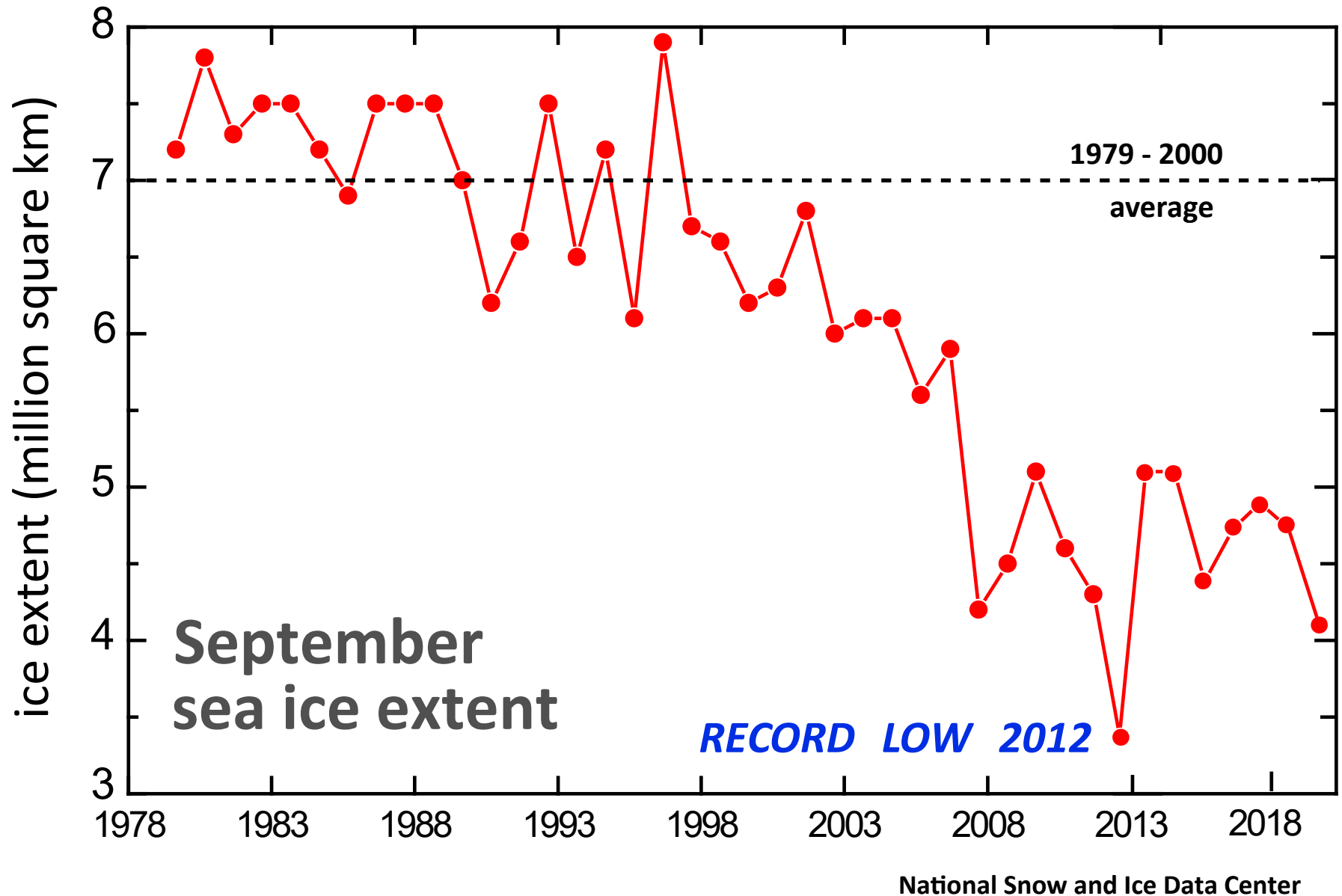
$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Arctic melt ponds



melt pond pattern formation and albedo evolution -- major drivers in polar climate
key challenge for global climate models

the summer Arctic sea ice pack is melting



Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million km²

September 2012 -- 3.4 million km²



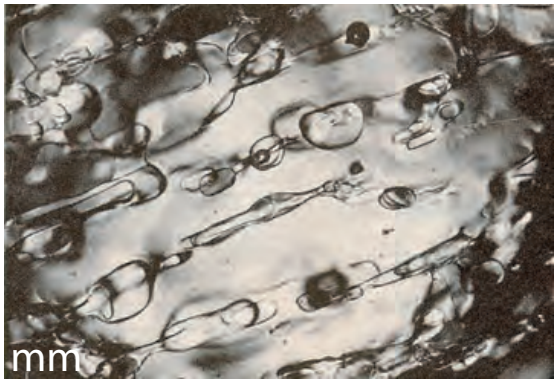
*recent losses
in comparison to
the United States*

Perovich



sea ice displays *multiscale* structure over 10 orders of magnitude

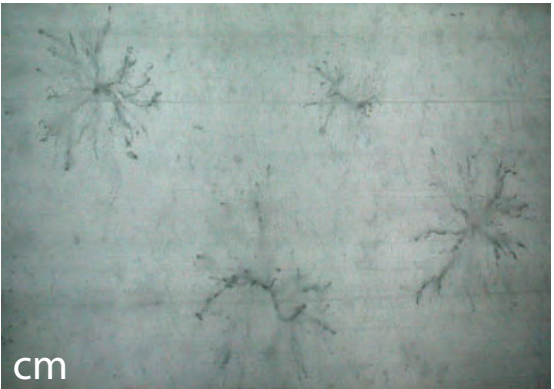
0.1 millimeter



brine inclusions



polycrystals



horizontal

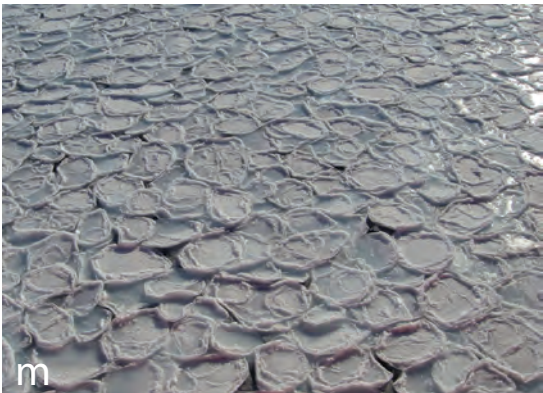


brine channels



vertical

1 meter

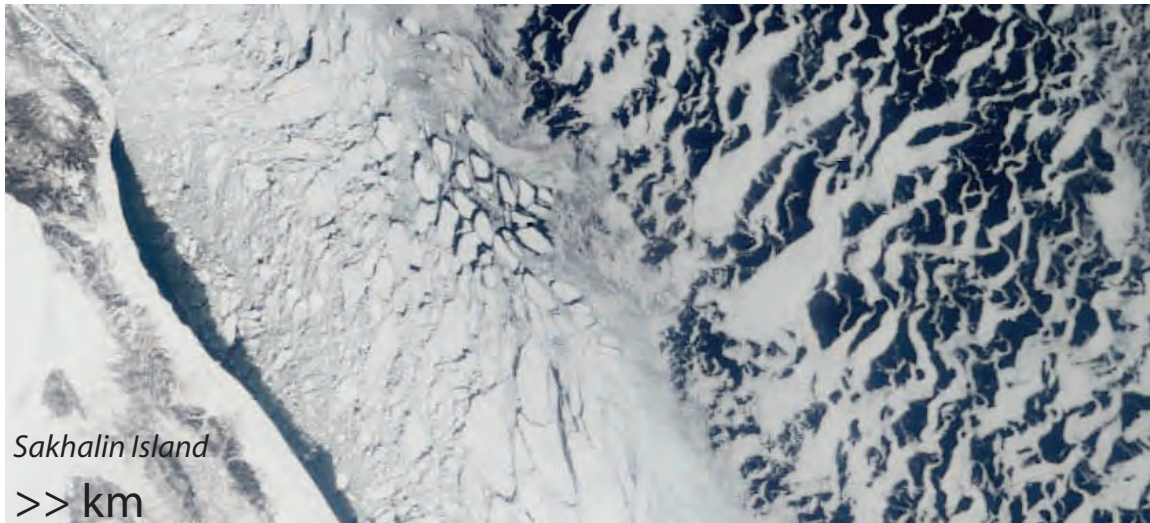


pancake ice

1 meter



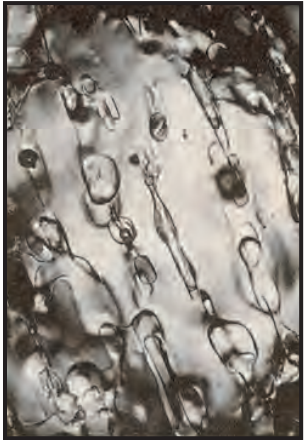
100 kilometers



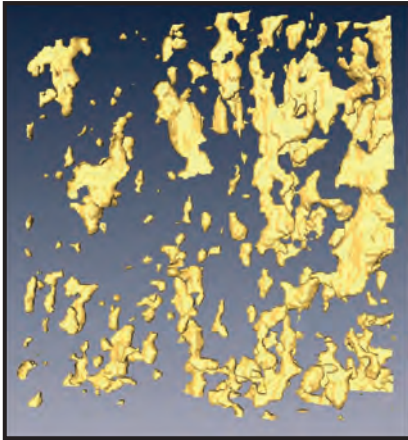
Sea Ice is a Multiscale Composite Material

MICROSCALE

brine inclusions

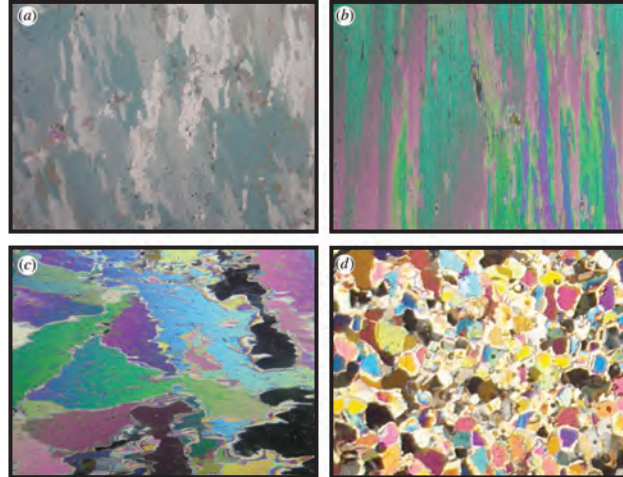


Weeks & Assur 1969



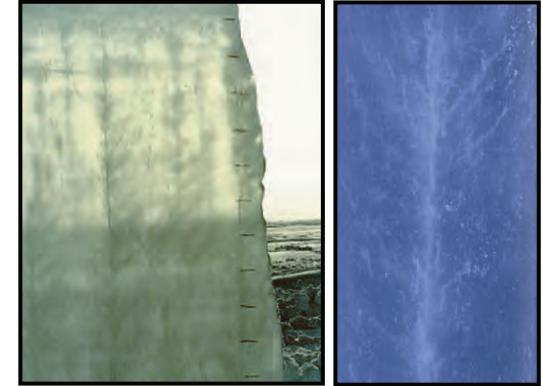
H. Eicken
Golden *et al.* GRL 2007

polycrystals



Gully *et al.* Proc. Roy. Soc. A 2015

brine channels



D. Cole

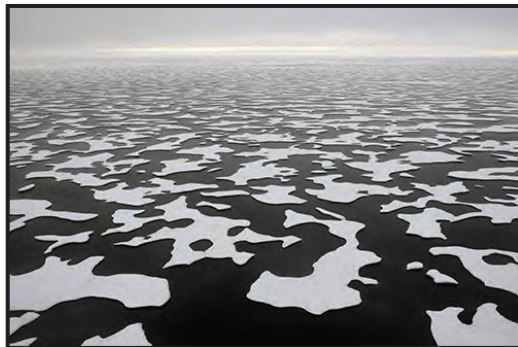
K. Golden

millimeters

centimeters

MESOSCALE

Arctic melt ponds



K. Frey

pressure ridges



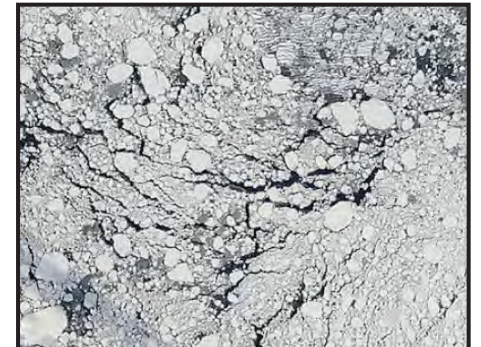
K. Golden

sea ice floes



J. Weller

sea ice pack



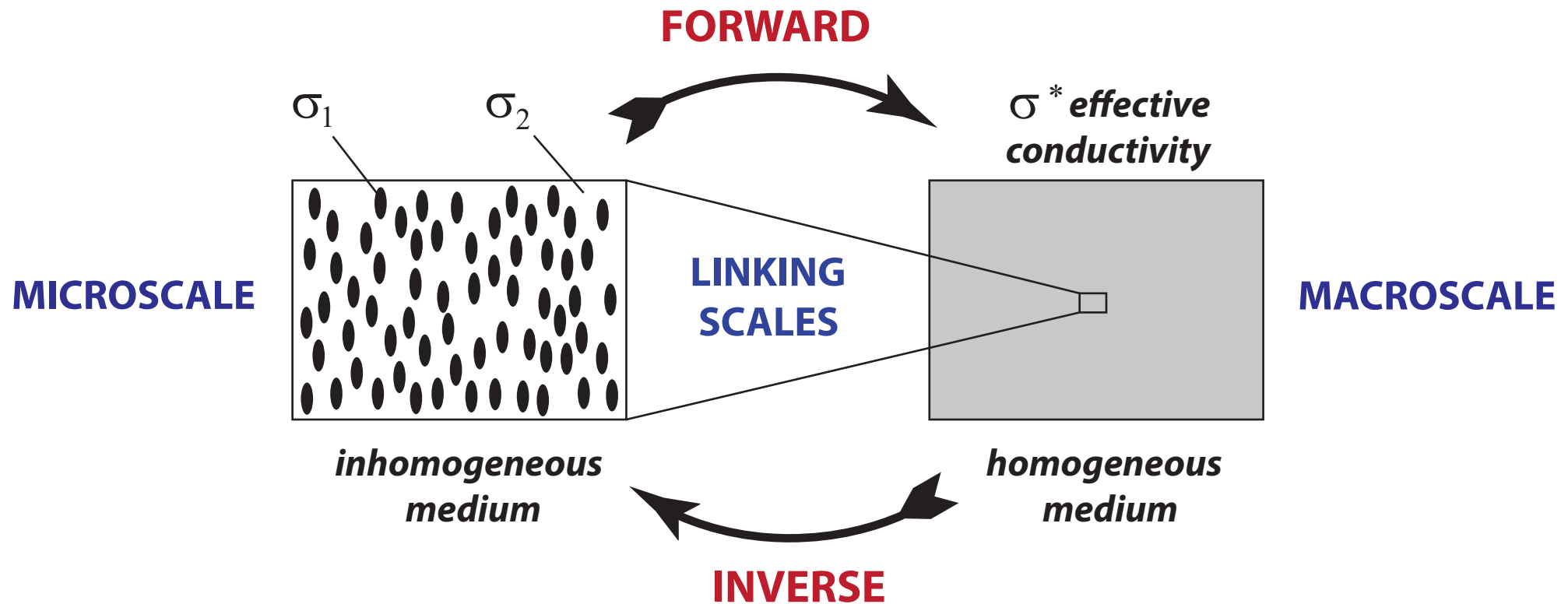
NASA

meters

kilometers

MACROSCALE

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres

Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

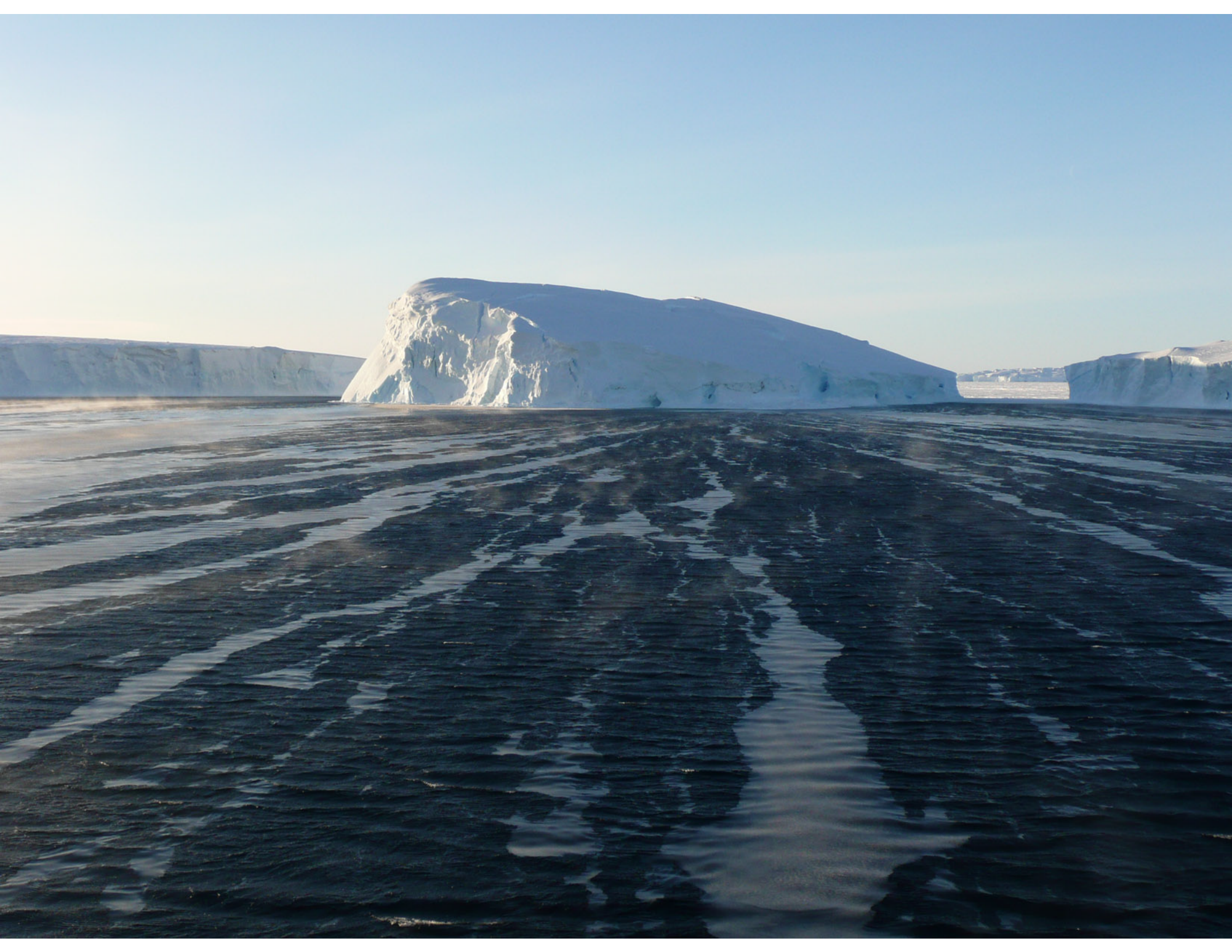
widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

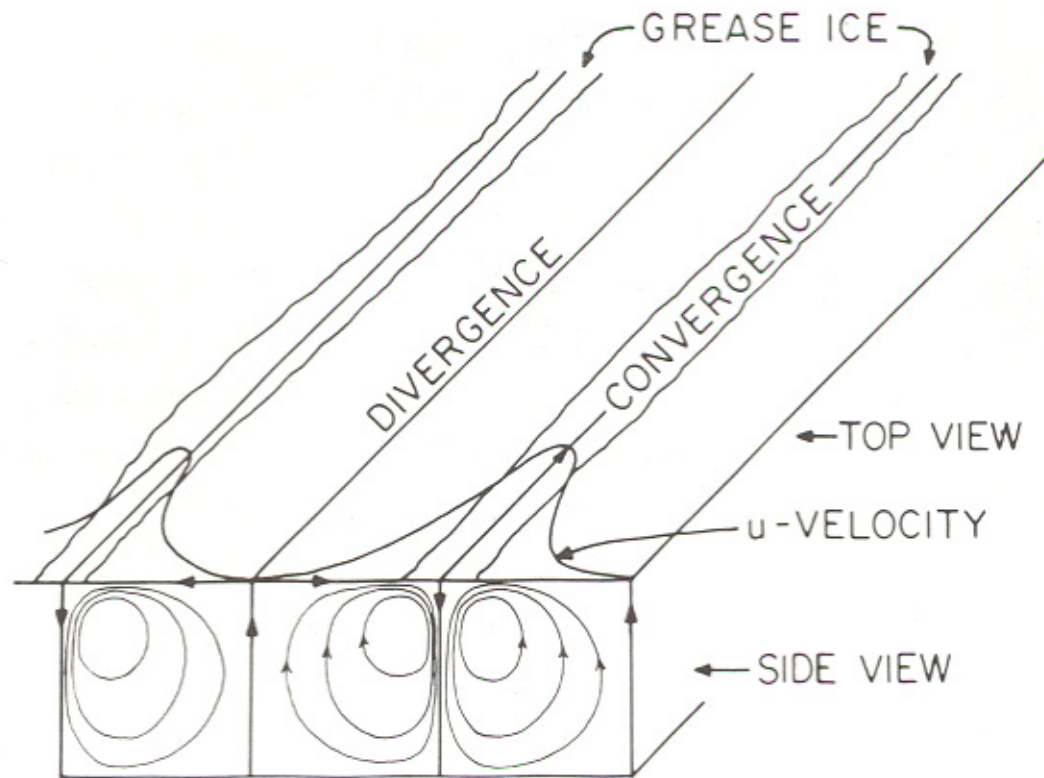
Sea ice structure, properties, and processes



sea ice formation

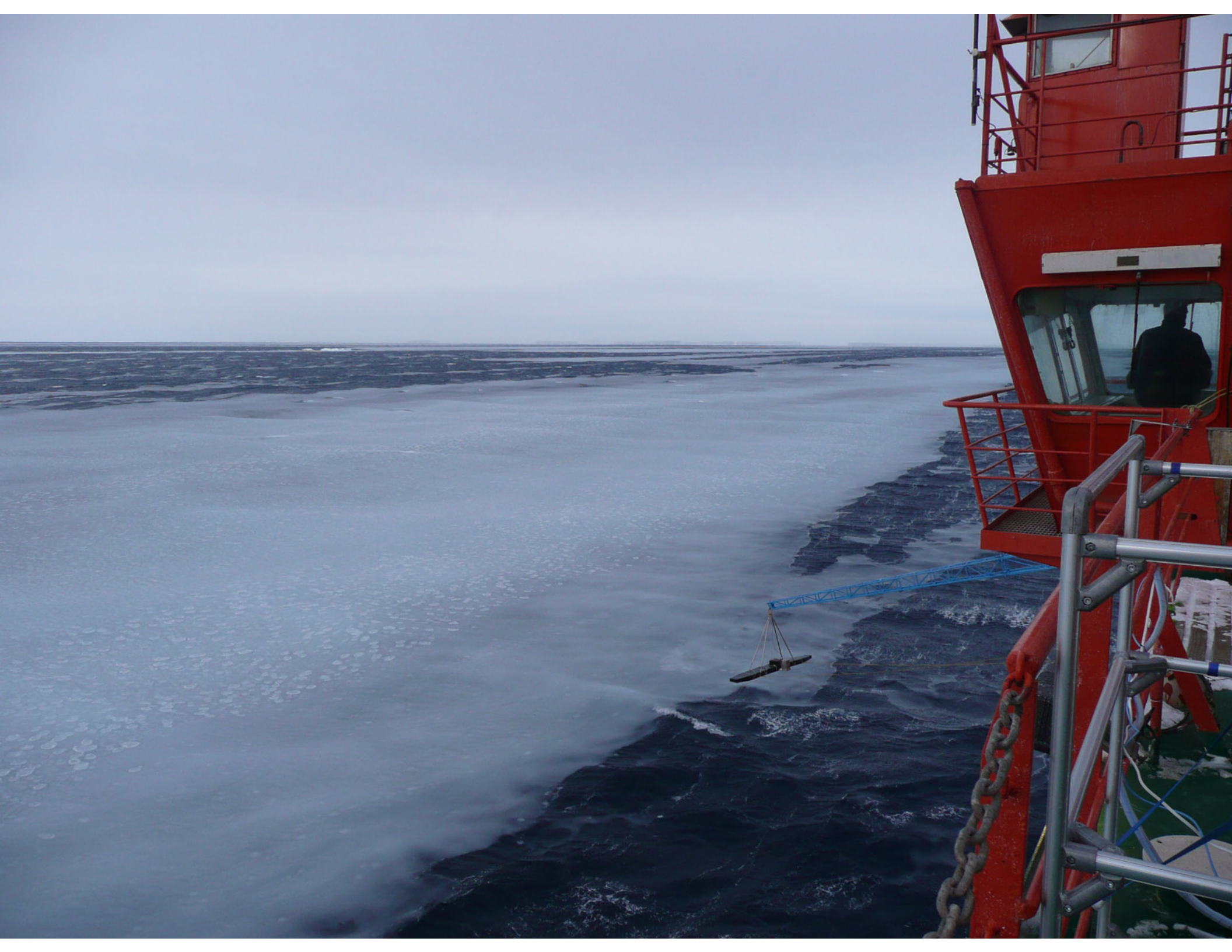






effect of Langmuir circulation
on grease and pancake ice





sea ice formation in a wave field

- turbulence in the wave field maintains the new ice as a dense suspension of frazil, rather than forming nilas
- suspension undergoes cyclic compression
- during the compression phase crystals can freeze together to form small coherent cakes of slush
- they grow larger by accretion from the frazil and more solid through continued freezing between the crystals



pancake ice

- where the wave field is calm, the pancakes begin to freeze together, eventually coalescing to first form large floes, then a continuous sheet of first-year ice known as **consolidated pancake ice**, with frazil as glue

CAN PRODUCE RAPID GROWTH - DYNAMIC THICKENING

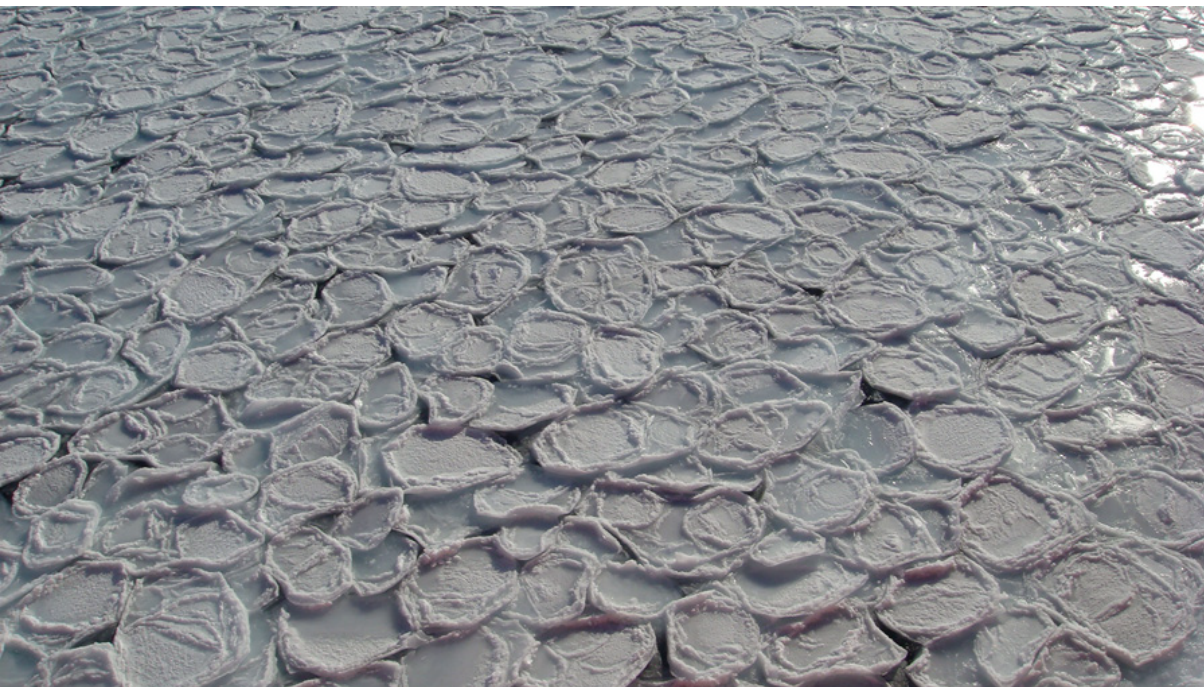
AGGREGATION PROCESS - CONNECTEDNESS TRANSITION

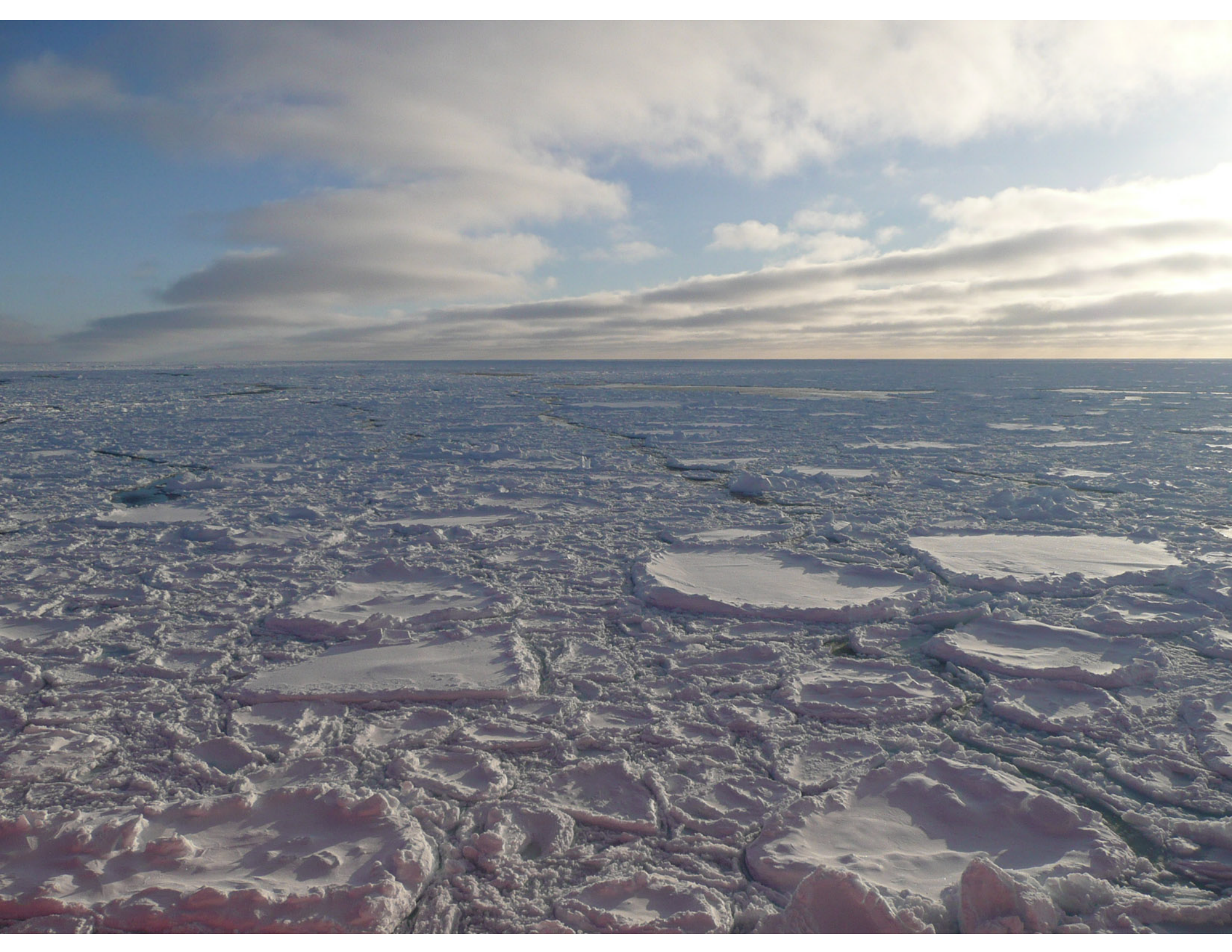
ocean swells propagating through a vast field of pancake ice

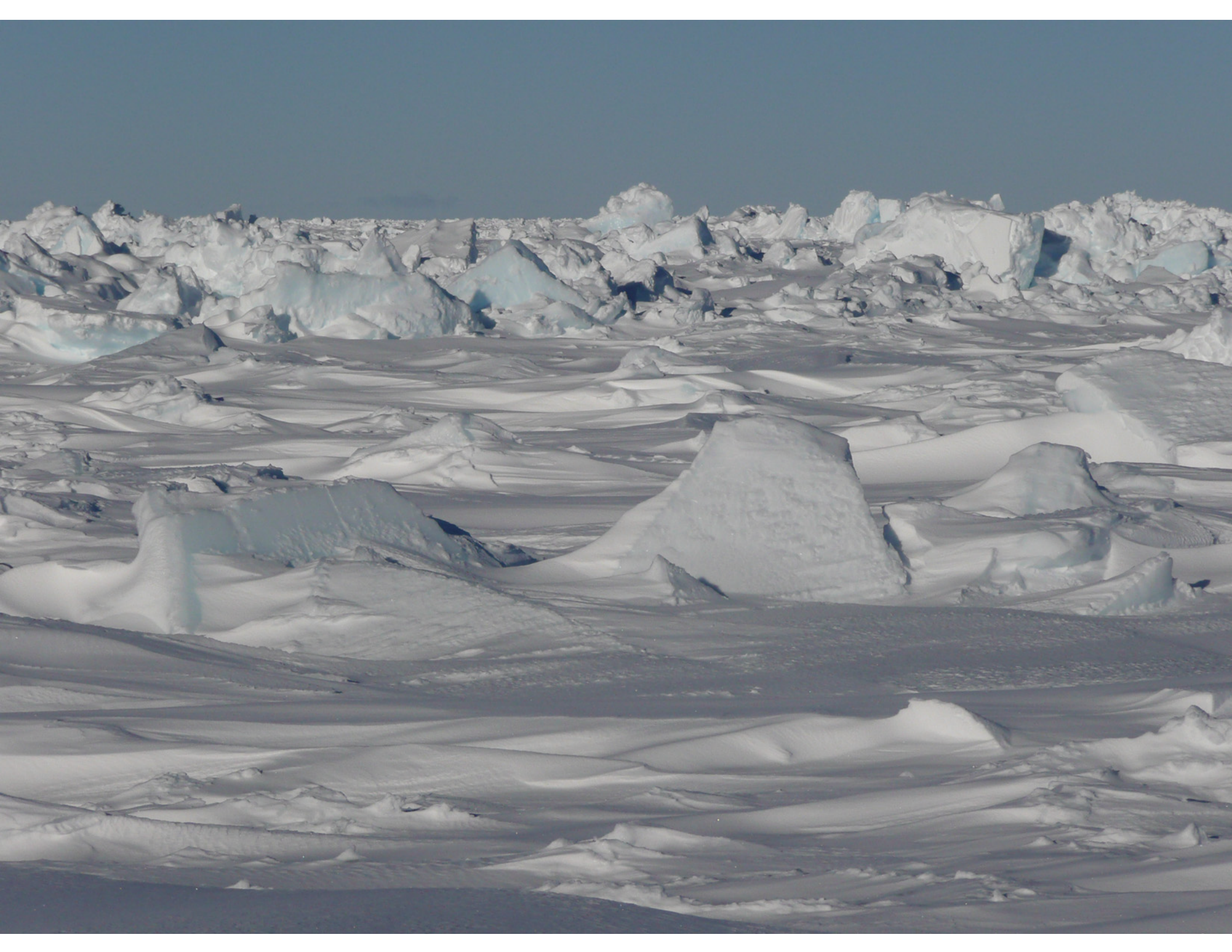
HOMOGENIZATION: long wave sees an effective medium, not individual floes, like long EM wave interacting with brine inclusion microstructure



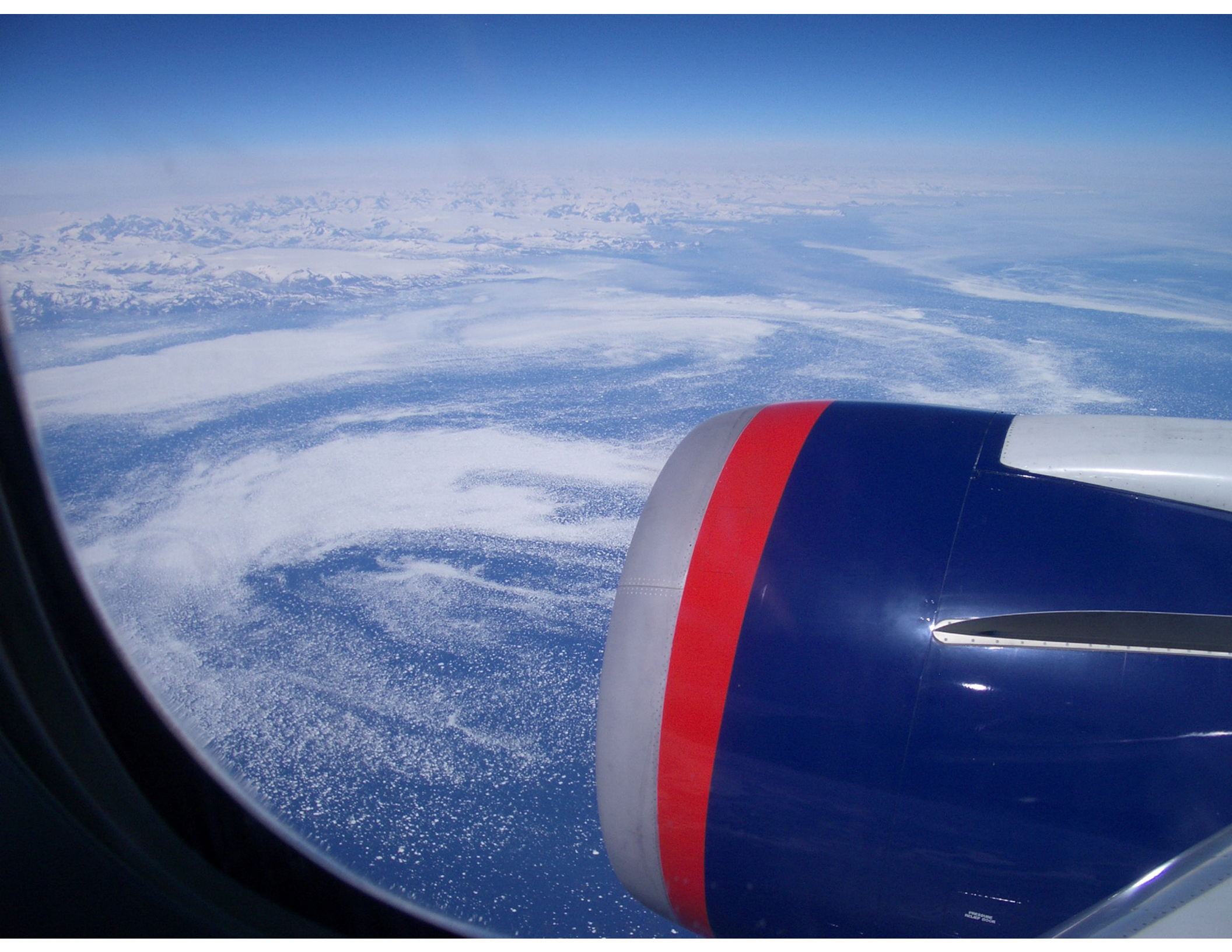
pancake ice

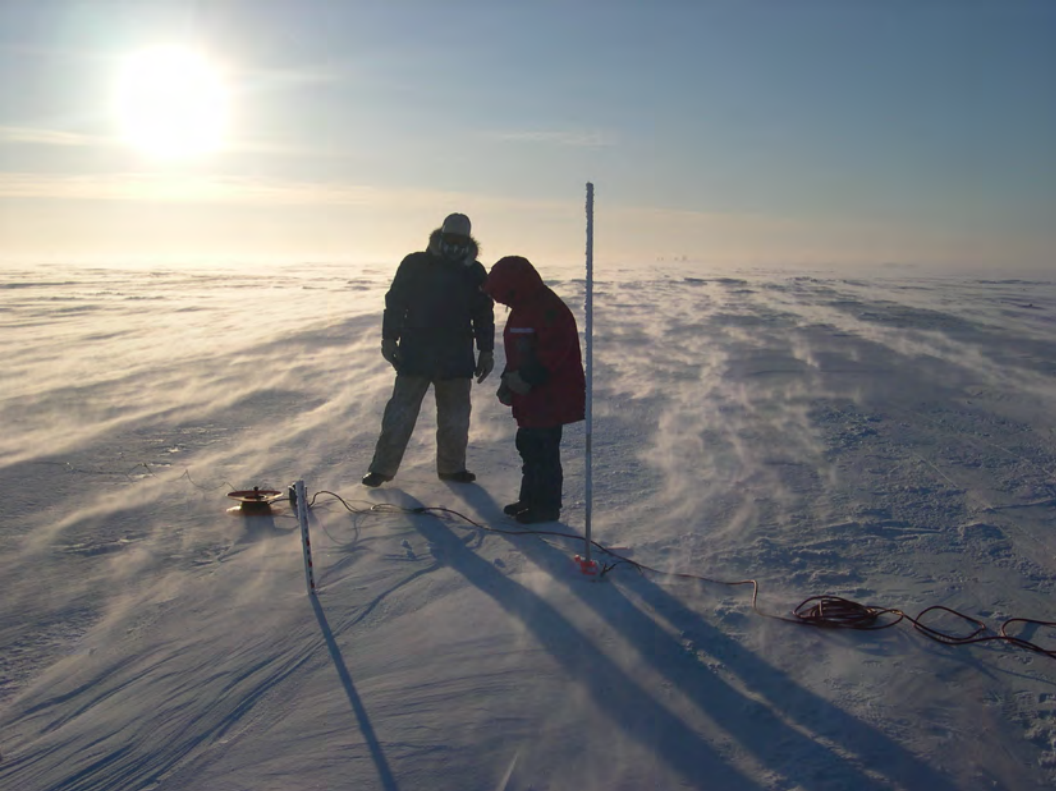




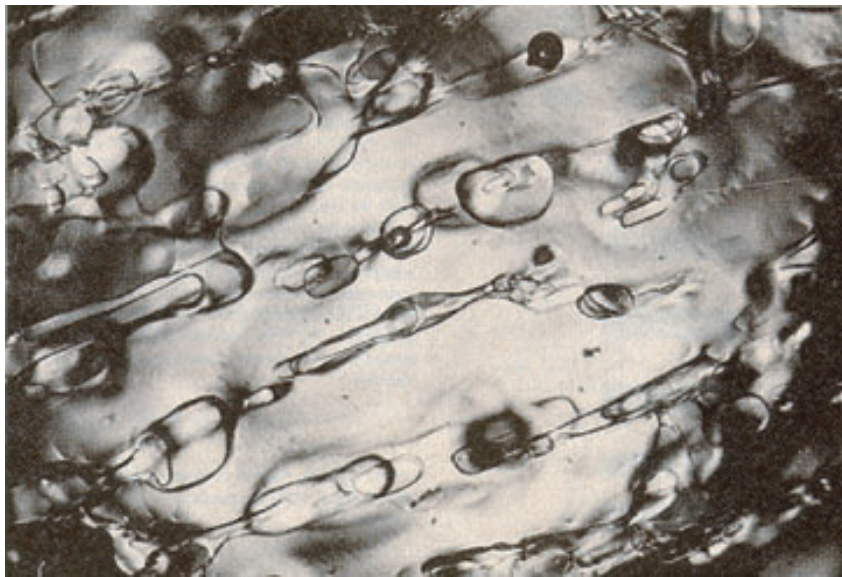




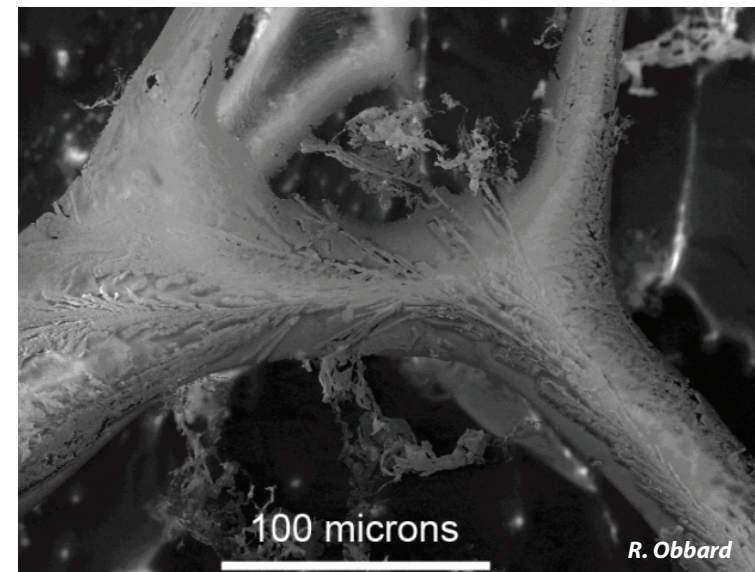




*sea ice may appear to be a
barren, impermeable cap ...*



brine inclusions in sea ice (mm)



micro - brine channel (SEM)

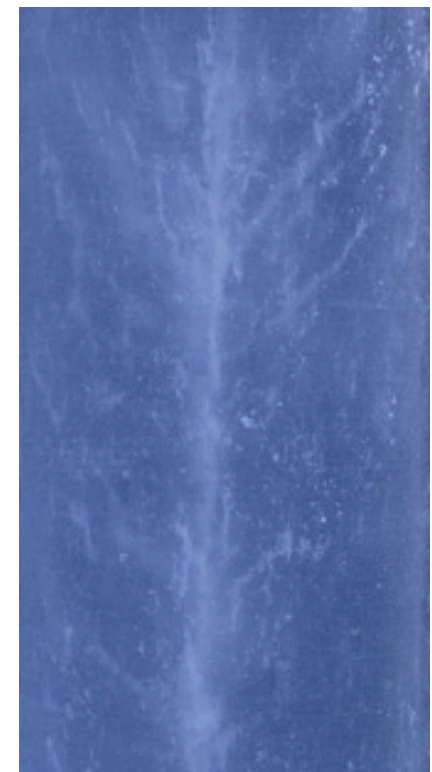
***sea ice is a
porous composite***

pure ice with brine, air, and salt inclusions

brine channels (cm)



horizontal section

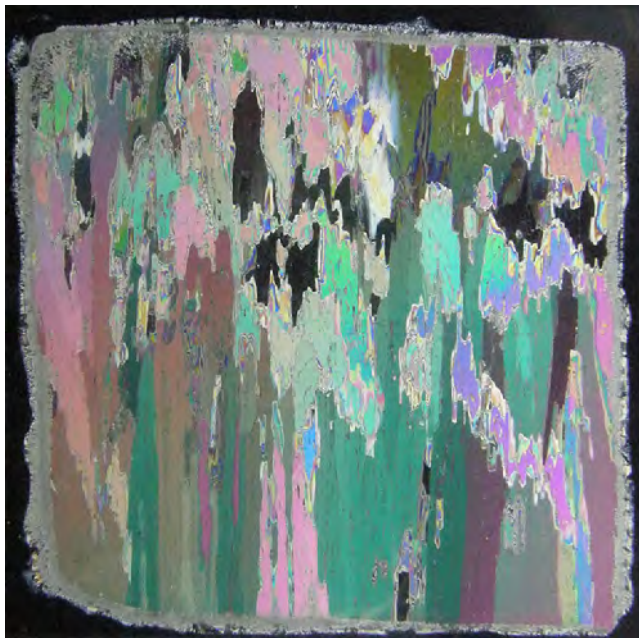


vertical section

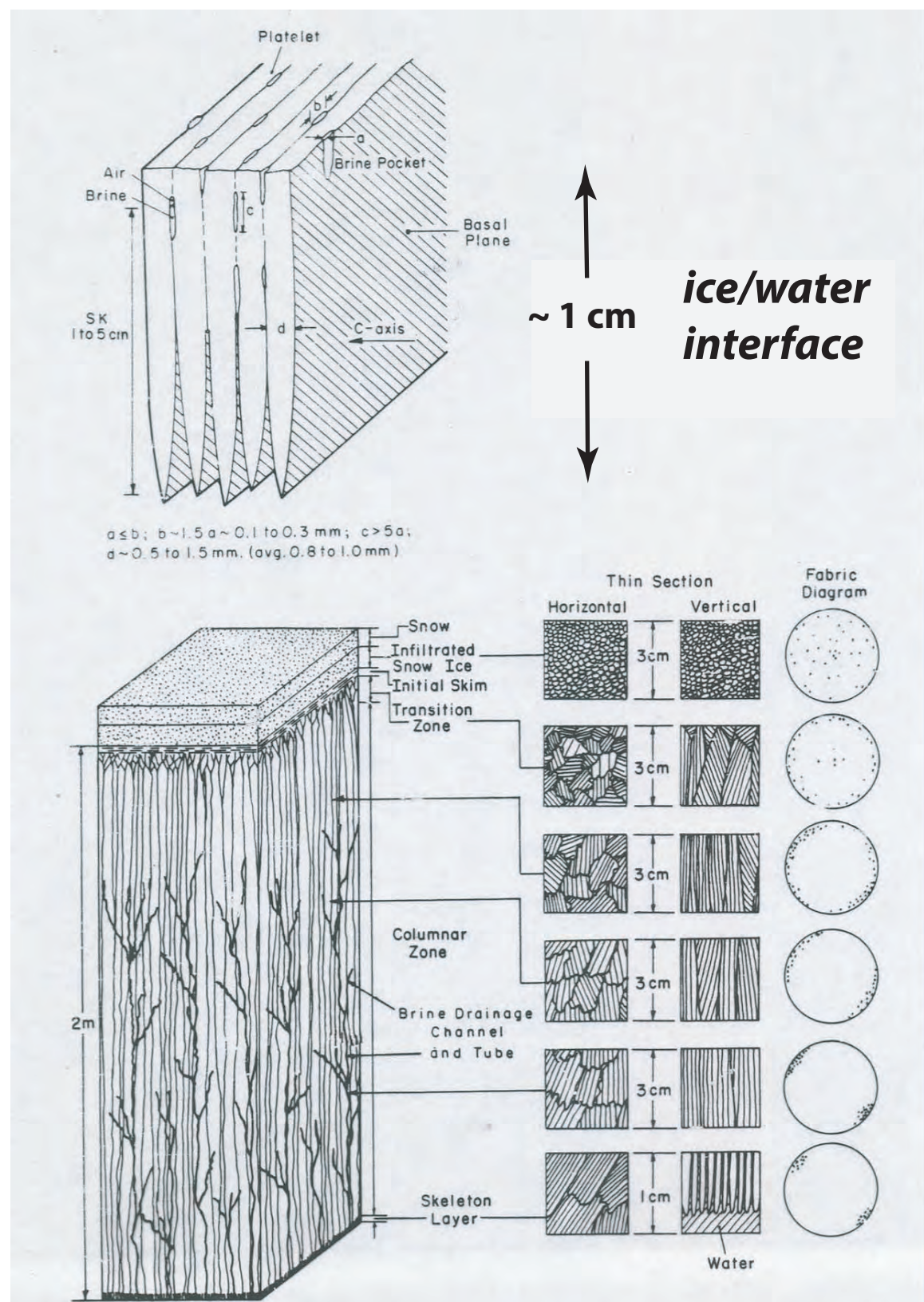
cross-sections of sea ice structure

$$T_{freeze} = -1.8^{\circ}\text{C}$$

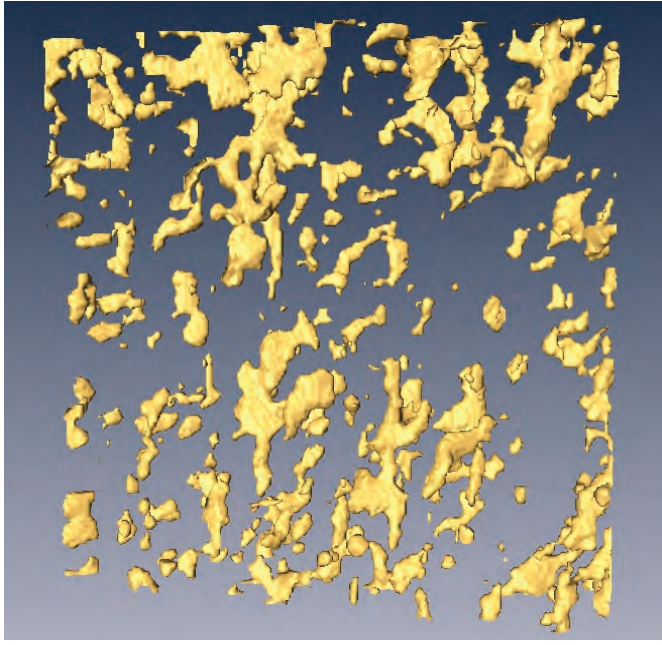
crystallographic texture



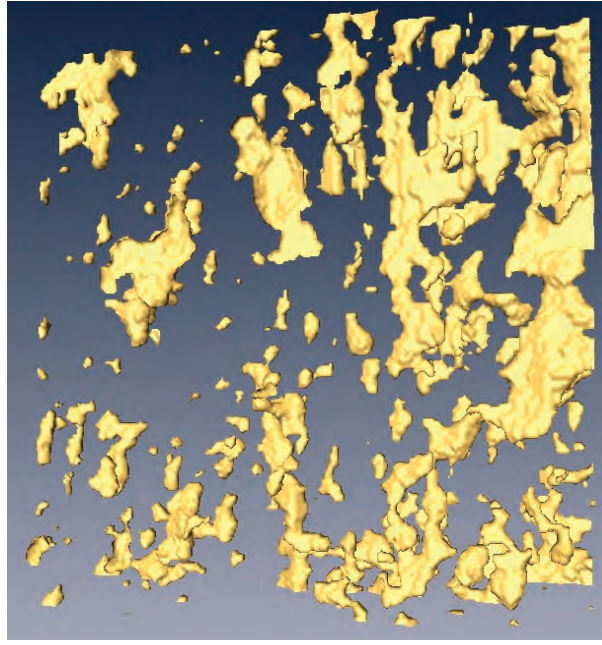
vertical thin section



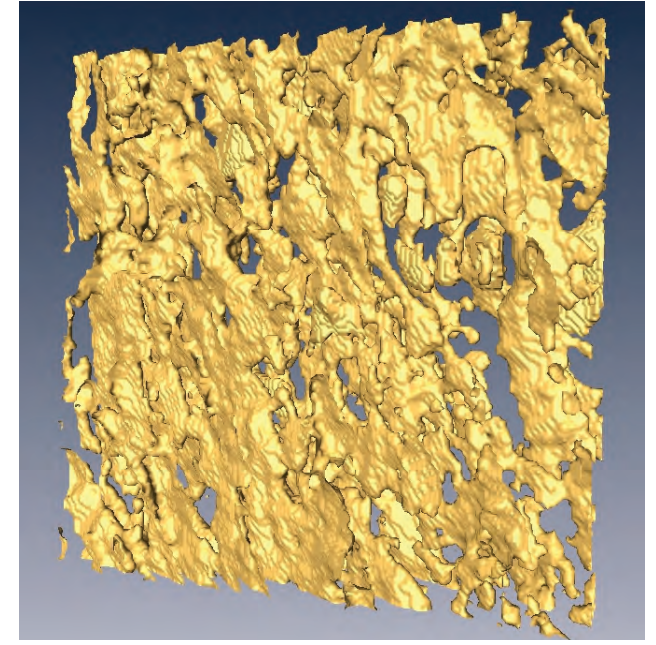
brine volume fraction and **connectivity** increase with temperature



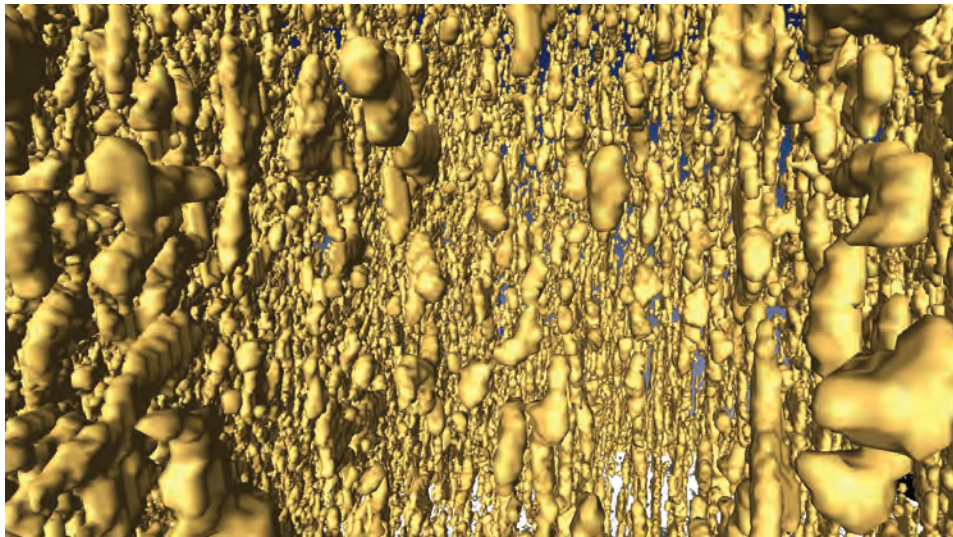
$T = -15\text{ }^{\circ}\text{C}$, $\phi = 0.033$



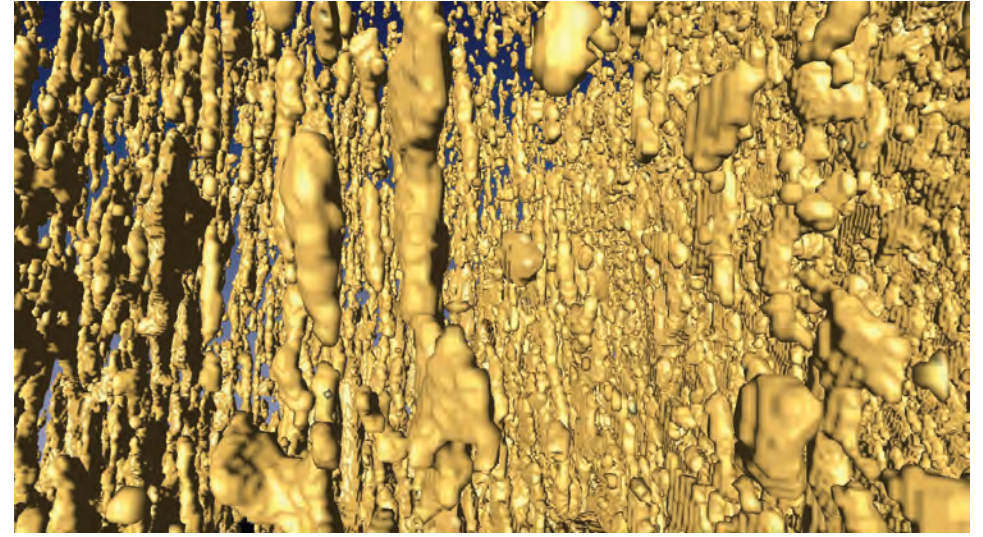
$T = -6\text{ }^{\circ}\text{C}$, $\phi = 0.075$



$T = -3\text{ }^{\circ}\text{C}$, $\phi = 0.143$



$T = -8\text{ }^{\circ}\text{C}$, $\phi = 0.057$



$T = -4\text{ }^{\circ}\text{C}$, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden et al., *Geophysical Research Letters*, 2007

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



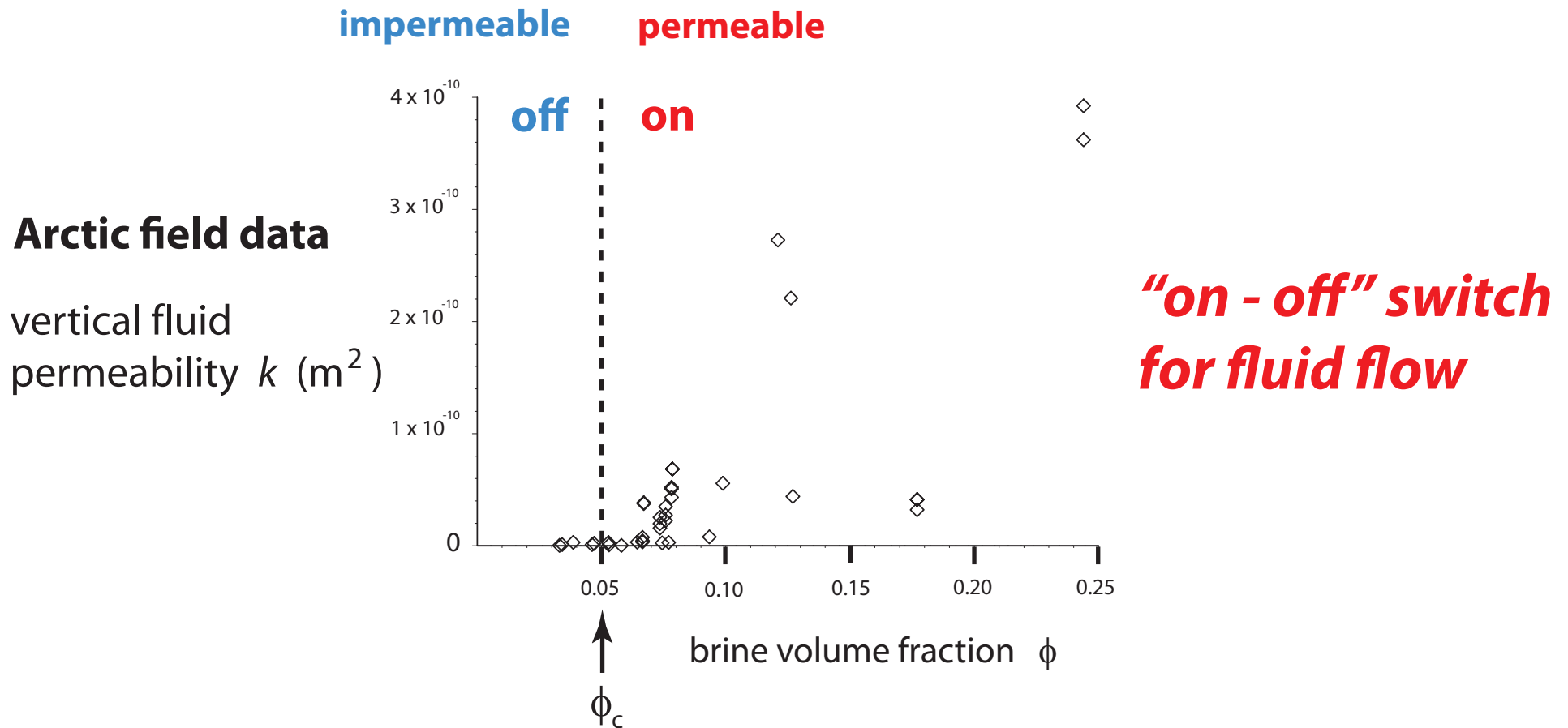
T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

September
snow-ice
estimates

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO₂*

Critical behavior of fluid transport in sea ice



critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ \text{C}, S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu Geophys. Res. Lett. 2007

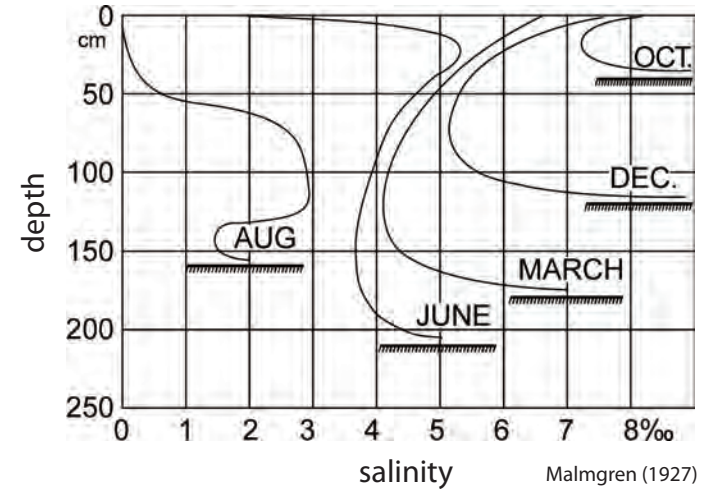
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

rule of fives constrains:

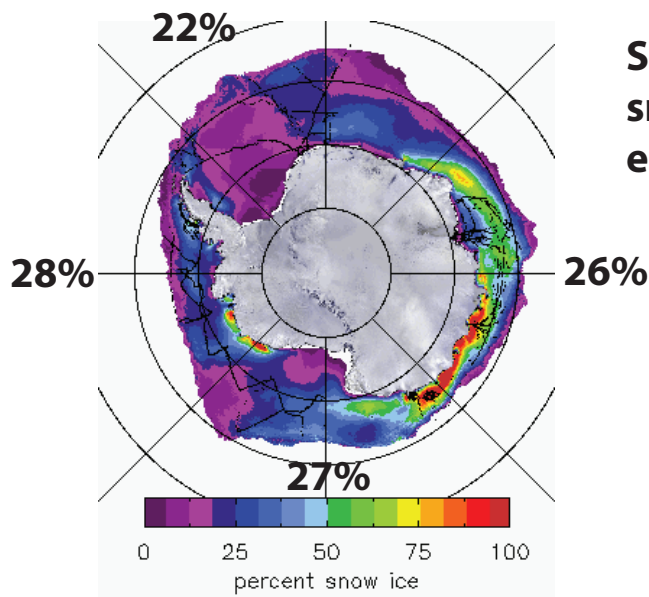
Antarctic surface flooding and snow-ice formation



evolution of salinity profiles



currently assumed constant in climate models



September
snow-ice
estimates

Antarctic snow-to-ice conversion from passive microwave imagery

T. Maksym and T. Markus, 2008

convection - enhanced thermal conductivity

Lytle and Ackley, 1996

Trodahl, et. al., 2000, 2001

Wang, Zhu, Golden, 2012





sea ice algal communities

D. Thomas 2004

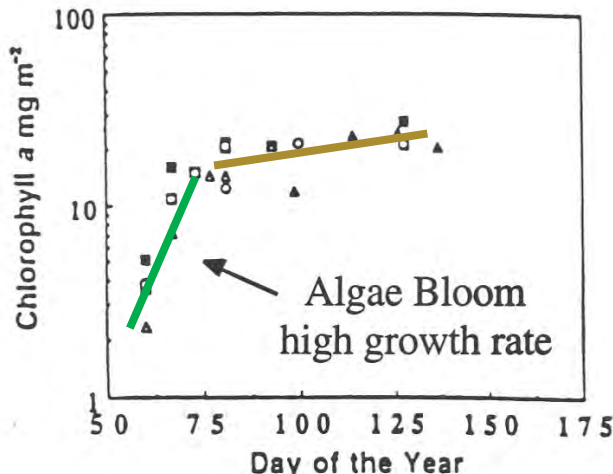
nutrient replenishment
controlled by ice permeability

biological activity turns on
or off according to
rule of fives

Golden, Ackley, Lytle Science 1998

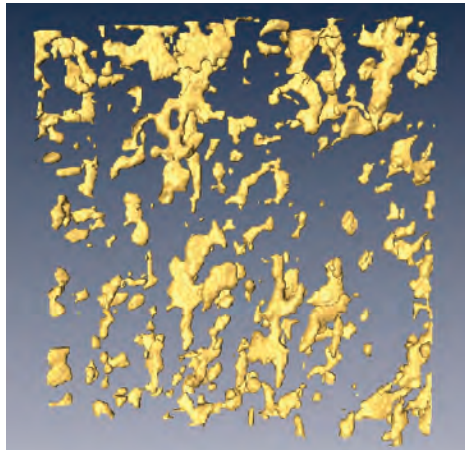
Fritsen, Lytle, Ackley, Sullivan Science 1994

critical behavior of microbial activity

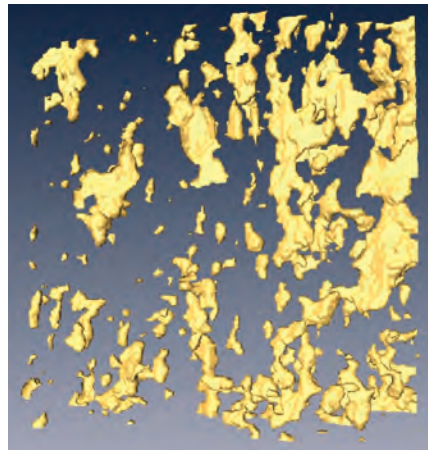


Convection-fueled algae bloom
Ice Station Weddell

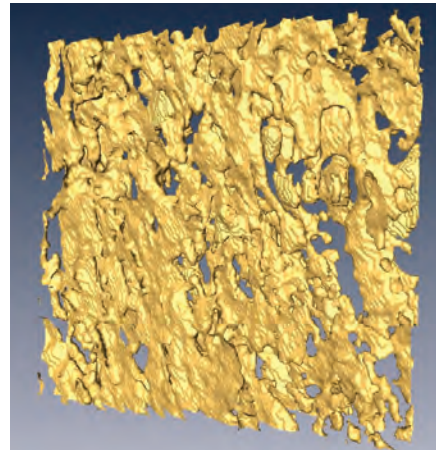
brine volume fraction and **connectivity** increase with temperature



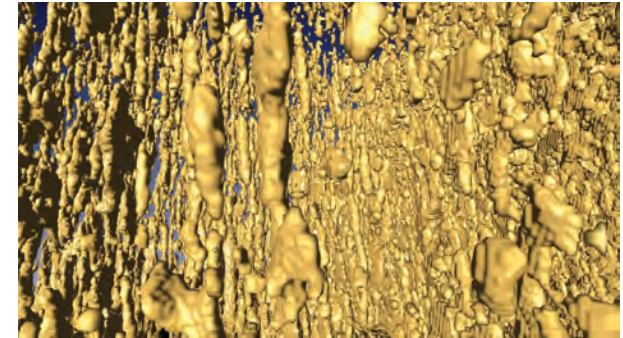
$T = -15\text{ }^{\circ}\text{C}$, $\phi = 0.033$



$T = -6\text{ }^{\circ}\text{C}$, $\phi = 0.075$



$T = -3\text{ }^{\circ}\text{C}$, $\phi = 0.143$



$T = -4\text{ }^{\circ}\text{C}$, $\phi = 0.113$

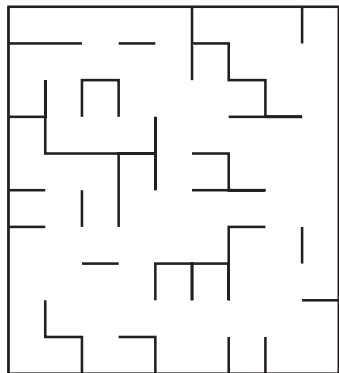
X-ray tomography for brine phase in sea ice

Golden, Eicken, *et al.*, *Geophysical Research Letters* 2007

PERCOLATION THRESHOLD $\phi_c \approx 5\%$

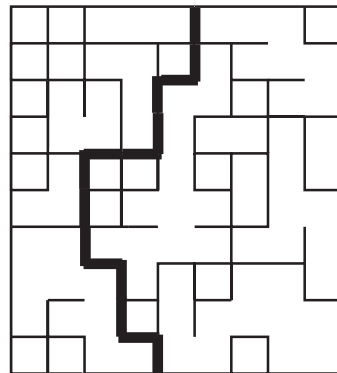
Golden, Ackley, Lytle, *Science* 1998

impermeable



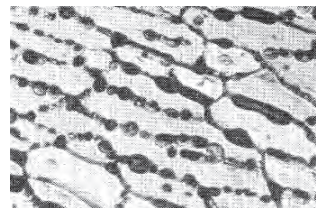
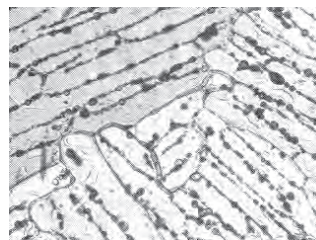
$p = 1/3$

permeable

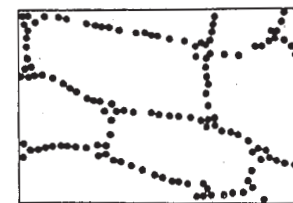
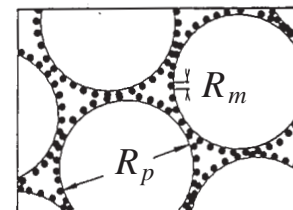


$p = 2/3$

lattice percolation



sea ice



compressed powder

Kusy, Turner
Nature 1971



B-2 Stealth Bomber
F-117 Nighthawk
F-35

stealth

continuum percolation

The Melt Pond Conundrum:

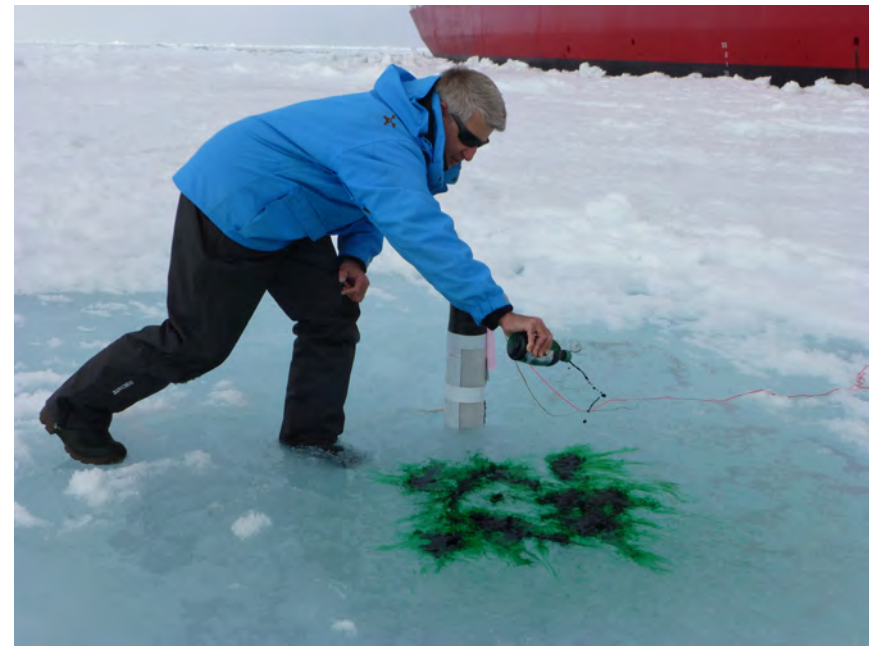
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

*2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy*

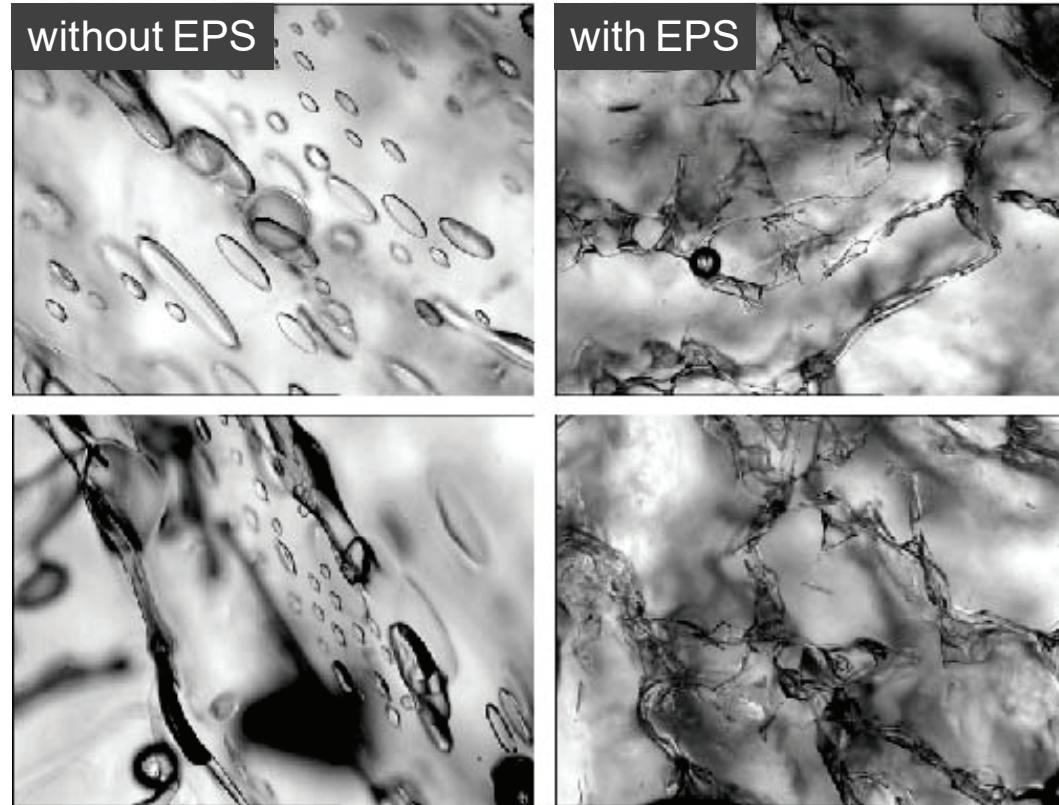


tracers flowing through inverted sea ice blocks



Sea ice algae secrete extracellular polymeric substances (EPS)

EPS changes brine microstructure



ellipsoidal inclusions

fractal inclusions

Krembs, Eicken, Deming *PNAS* 2011

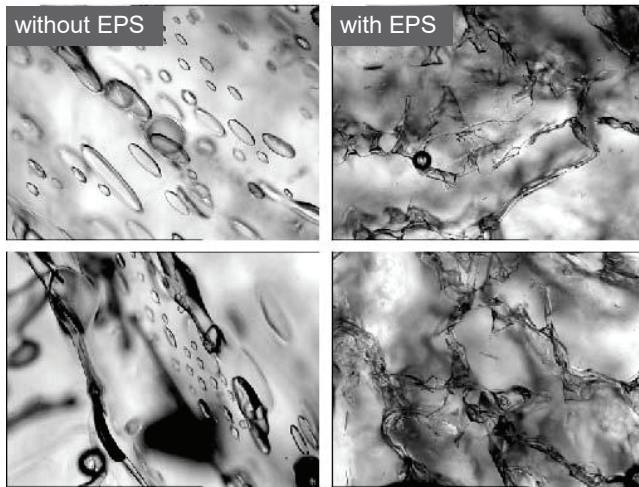
numerical model
bounds on fluid
permeability

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

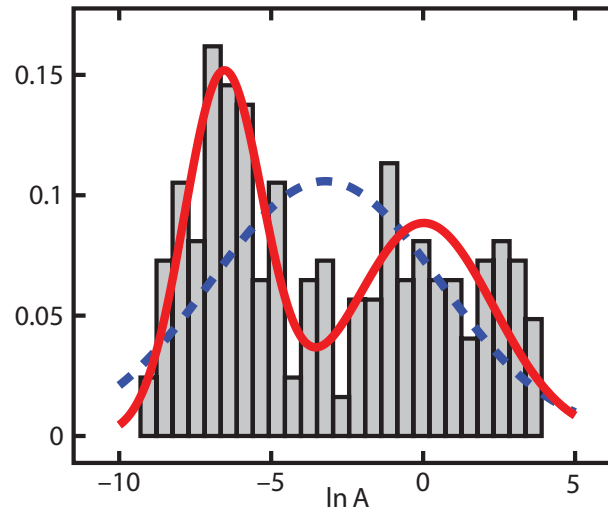
How does the biology affect the physics?

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

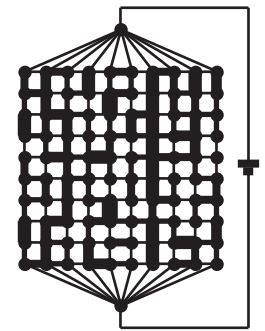
How does EPS affect fluid transport? How does the biology affect the physics?



Krembs, Eicken, Deming, PNAS 2011

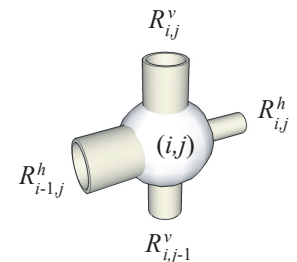


**RANDOM
PIPE
MODEL**



- 2D random pipe model with bimodal distribution of pore sizes
- Rigorous bound on permeability k ; results predict observed drop in k

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018



3D extension, effect of EPS clogging, blockage

Anna Hyde, Jingyi Zhu, Ken Golden

Zhu, Jabini, Golden,
Eicken, Morris
Ann. Glac. 2006

higher threshold for fluid flow in granular sea ice

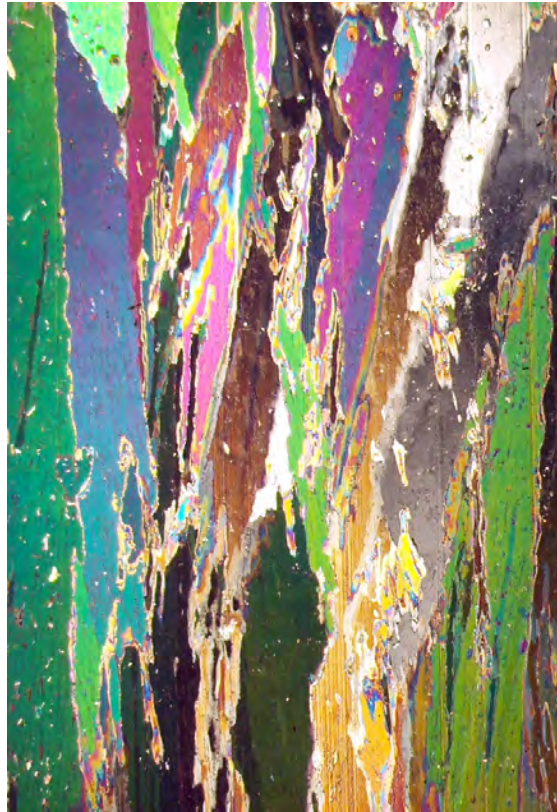
microscale details impact “mesoscale” processes

nutrient fluxes for microbes
melt pond drainage
snow-ice formation

columnar

granular

5%



10%



Golden, Sampson, Gully, Lubbers, Tison 2020

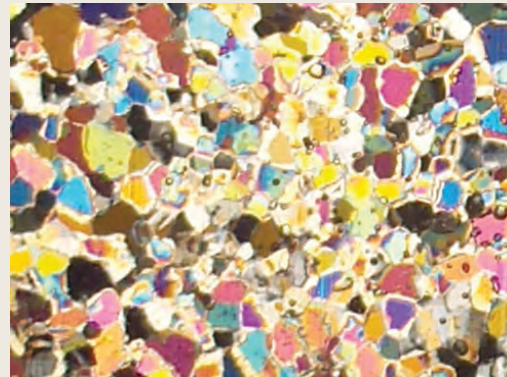
electromagnetically distinguishing ice types

Kitzel Lusted, Elena Cherkaev, Ken Golden

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**
Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



PROCEEDINGS A

350 YEARS
OF SCIENTIFIC
PUBLISHING

An invited review
commemorating 350 years
of scientific publishing at the
Royal Society

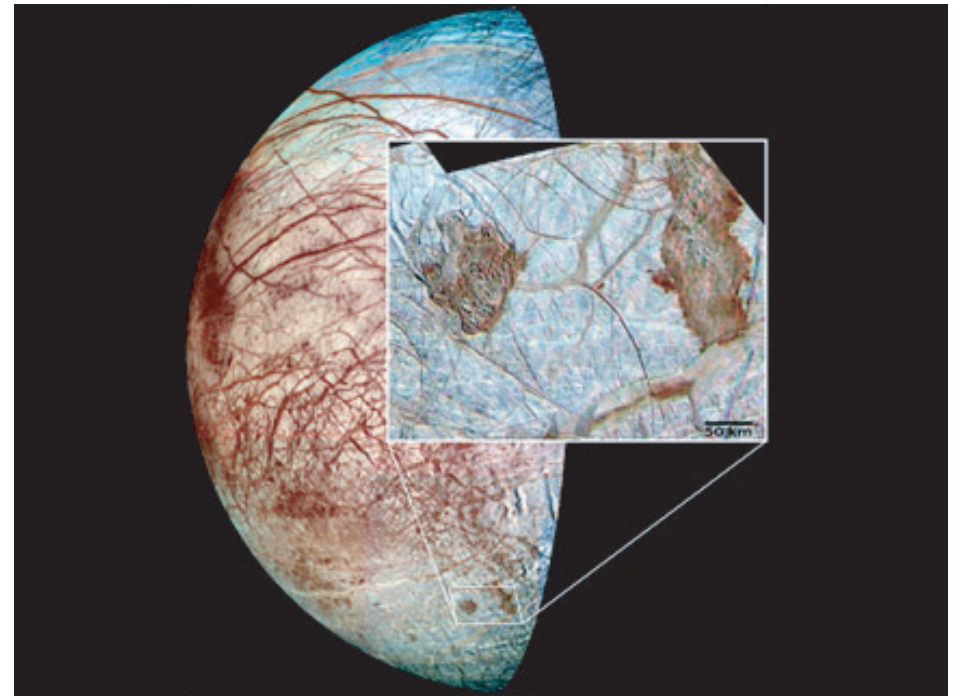
A method to distinguish
between different types
of sea ice using remote
sensing techniques

A computer model to
determine how a human
should walk so as to expend
the least energy

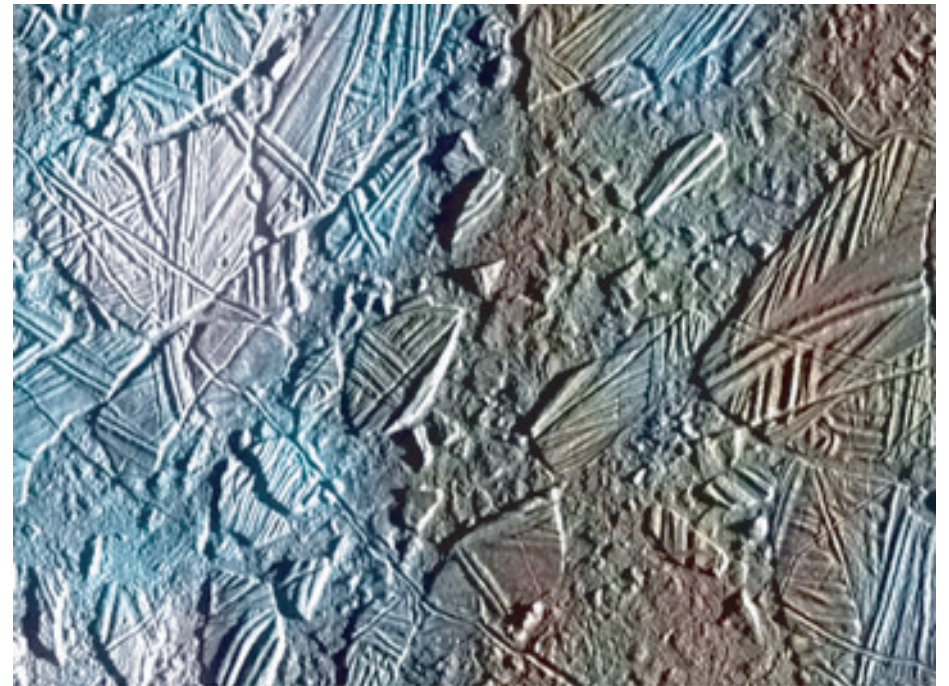


THE
ROYAL
SOCIETY
PUBLISHING

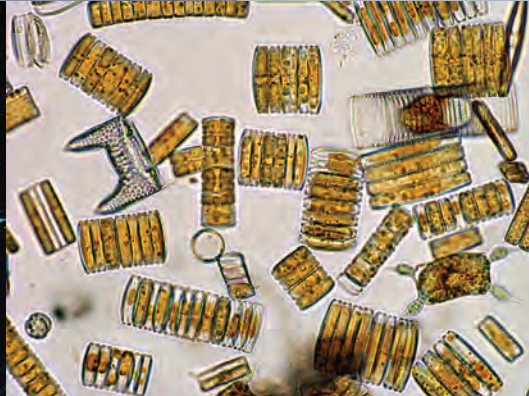
Are sea ice algae and bacteria
proxies for possible life forms
on extraterrestrial, icy bodies?
(Thomas, Dieckmann, *Science*, 2002)



EUROPA - believed covered by deep
briny ocean, with thick icy crust

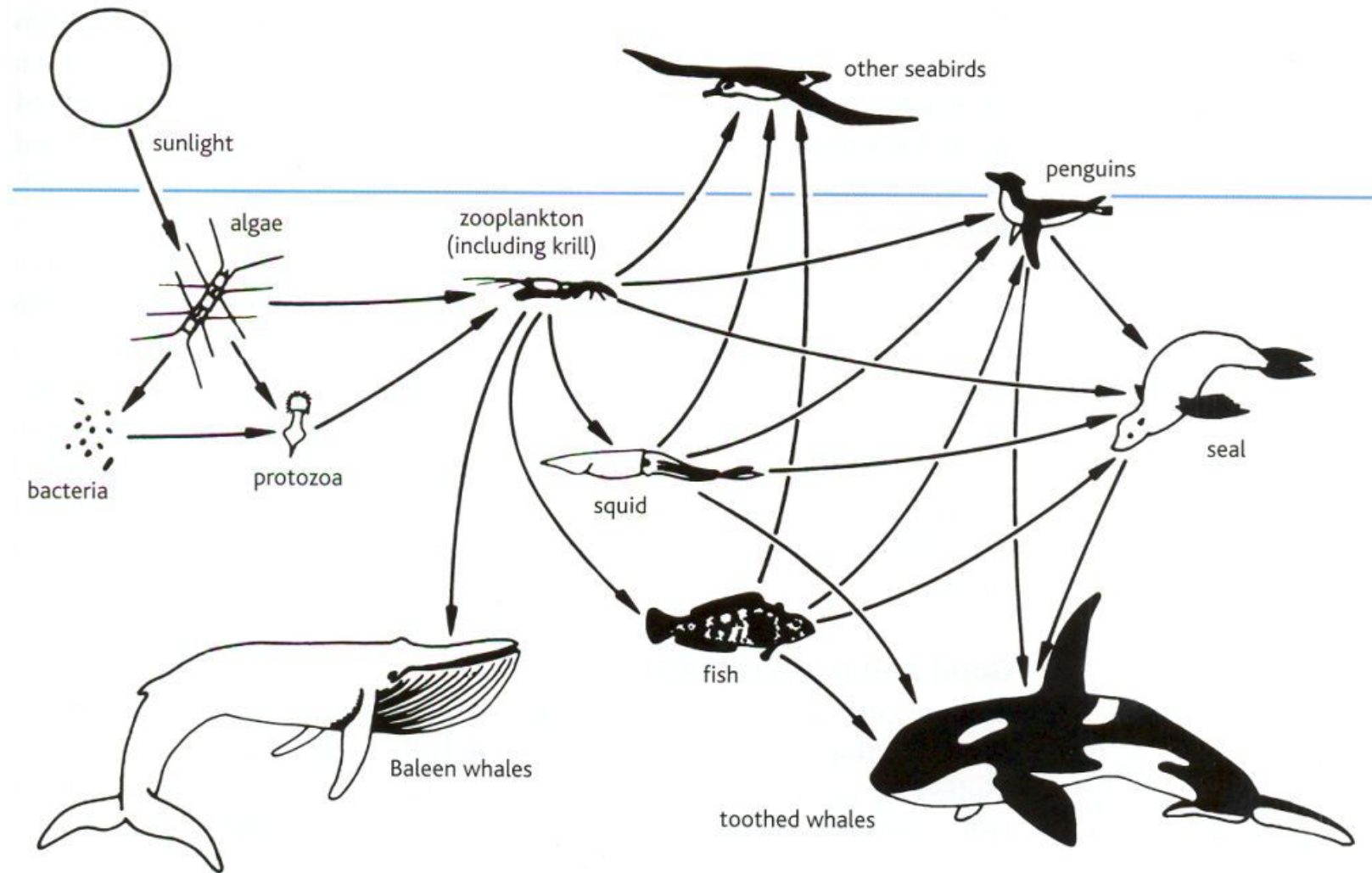


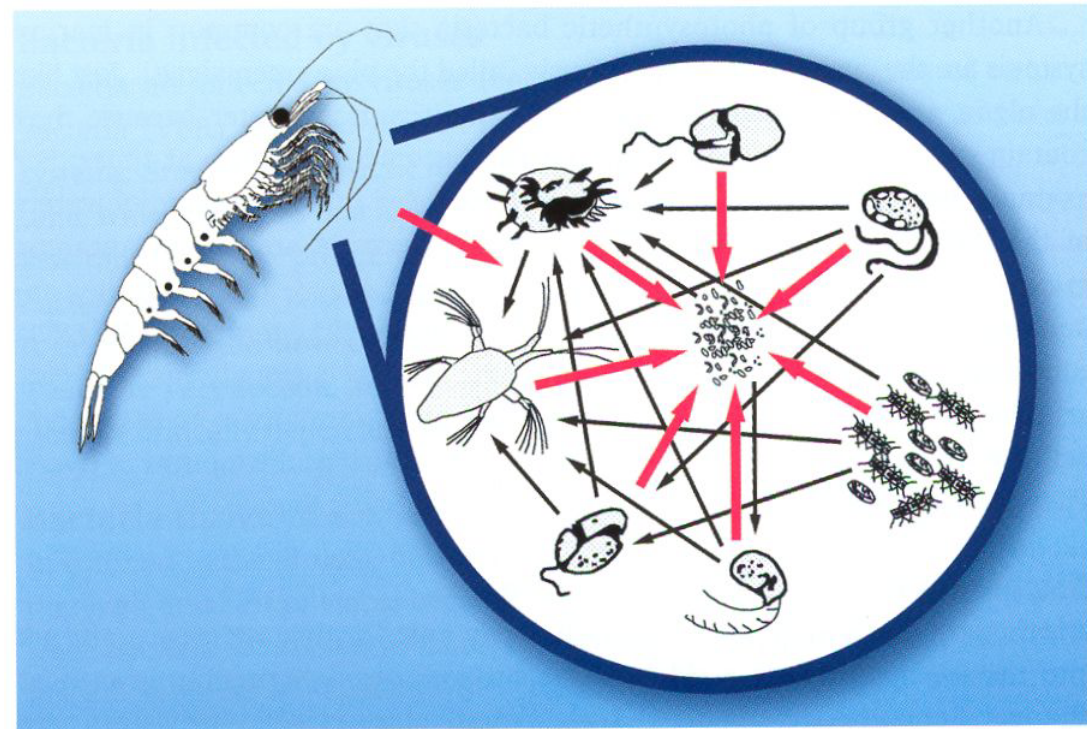
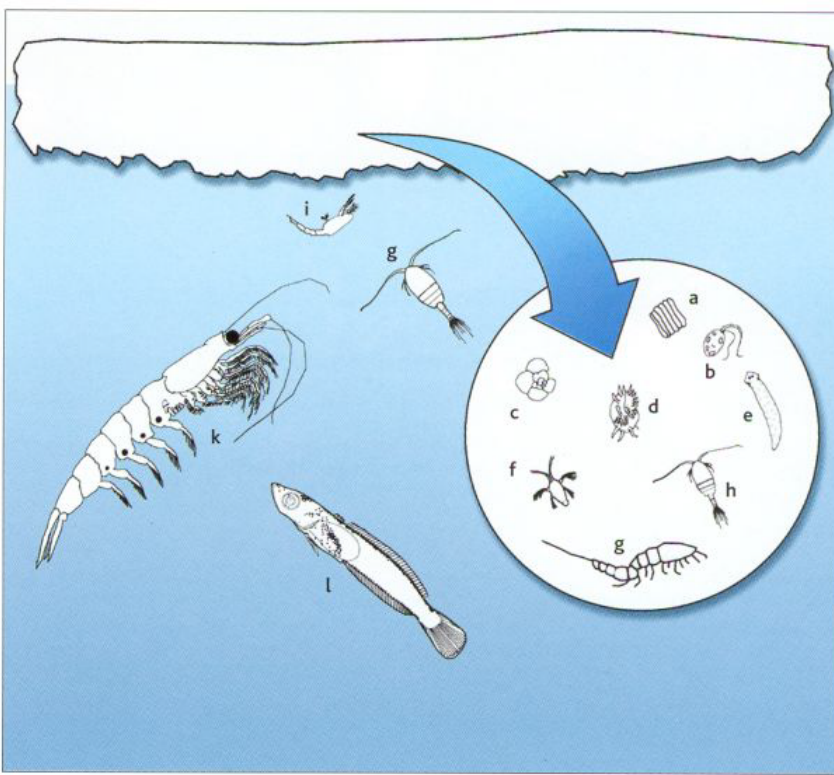
sea ice ecosystem



sea ice algae
support life in the polar oceans

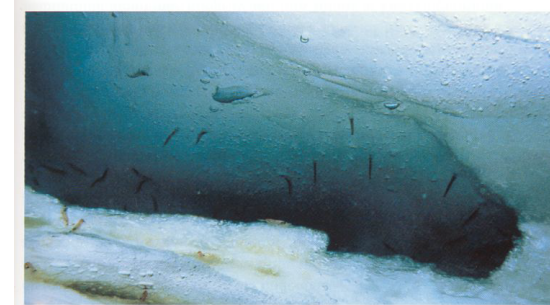
Antarctic marine food web





all organisms in food web produce dissolved organic matter which fuels bacterial growth in the ice

life within and at the edges of Antarctic sea ice floes

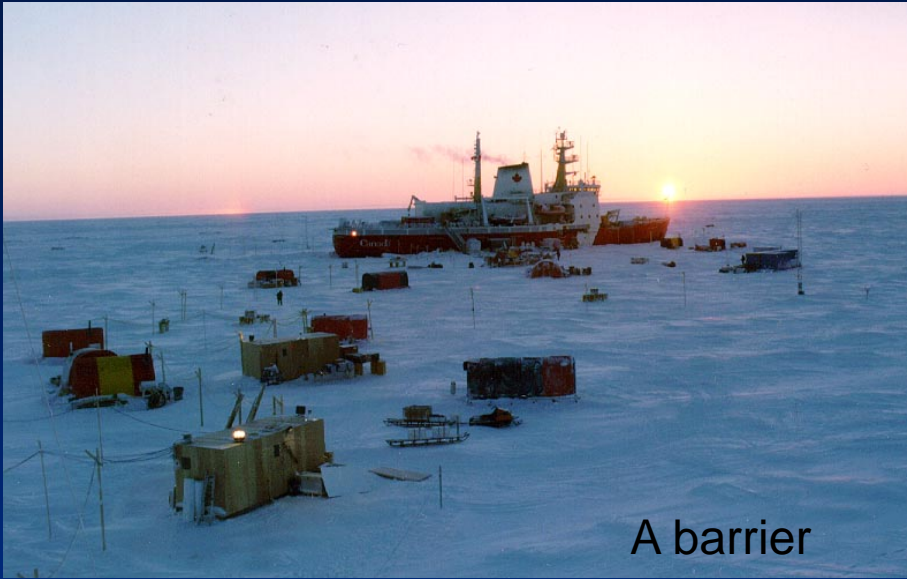


krill



(a) diatoms (b) flagellates (c) foraminiferans (d) ciliates
(e) turbellarians (f) crustaceans (g,h) copepods (i) crustacean larvae (k) krill (l) young fish (bottom) amphipod *Gammarus wilkitzkii*

The Arctic sea ice cover



A barrier



Blowing snow



Comes together



Breaks apart



Rich ecosystem

Of scientific, economic, societal, strategic importance

“Dynamic” duo

- Fast
- Rough



- Slow
- Smooth

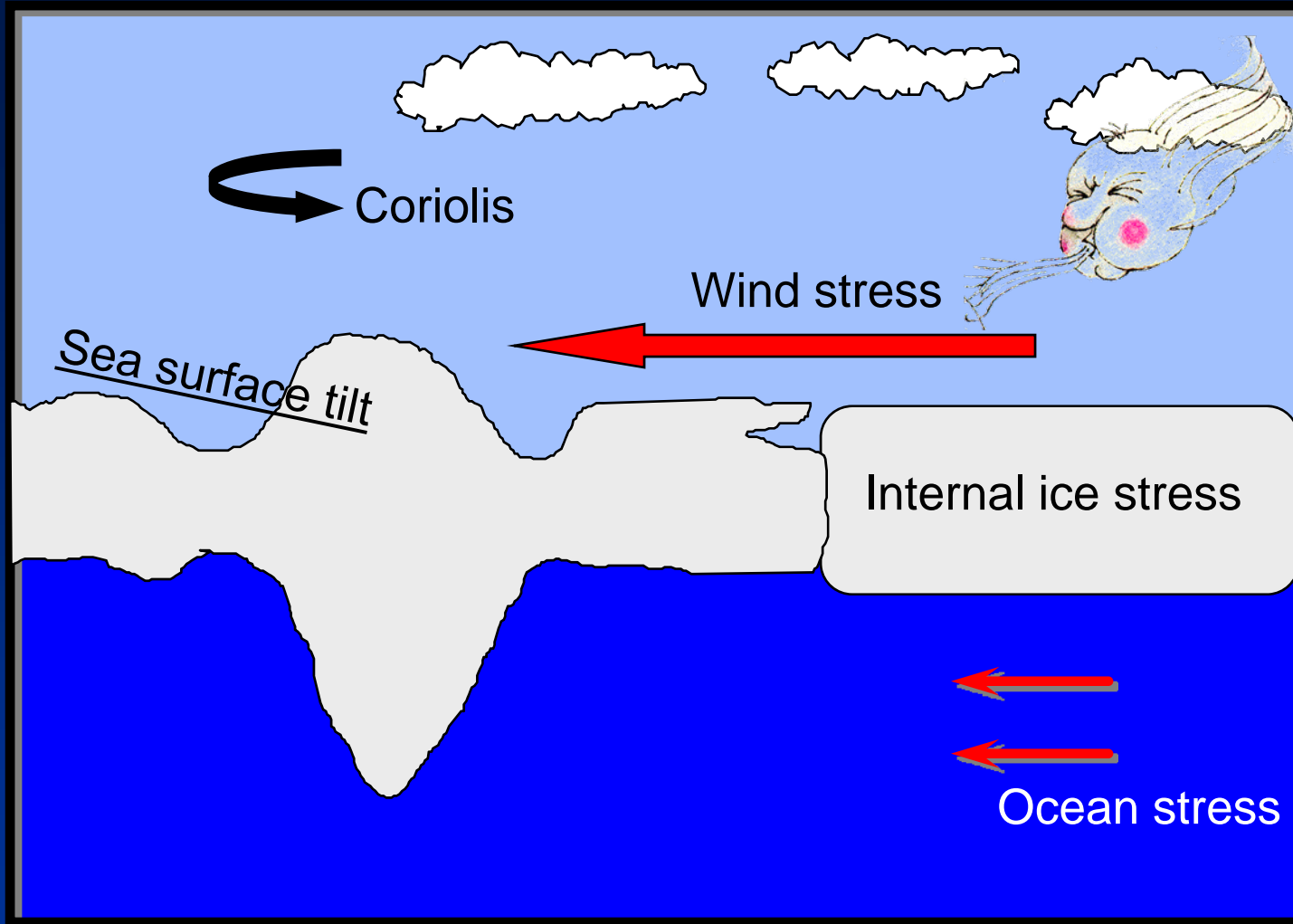


Dynamics



Thermodynamics

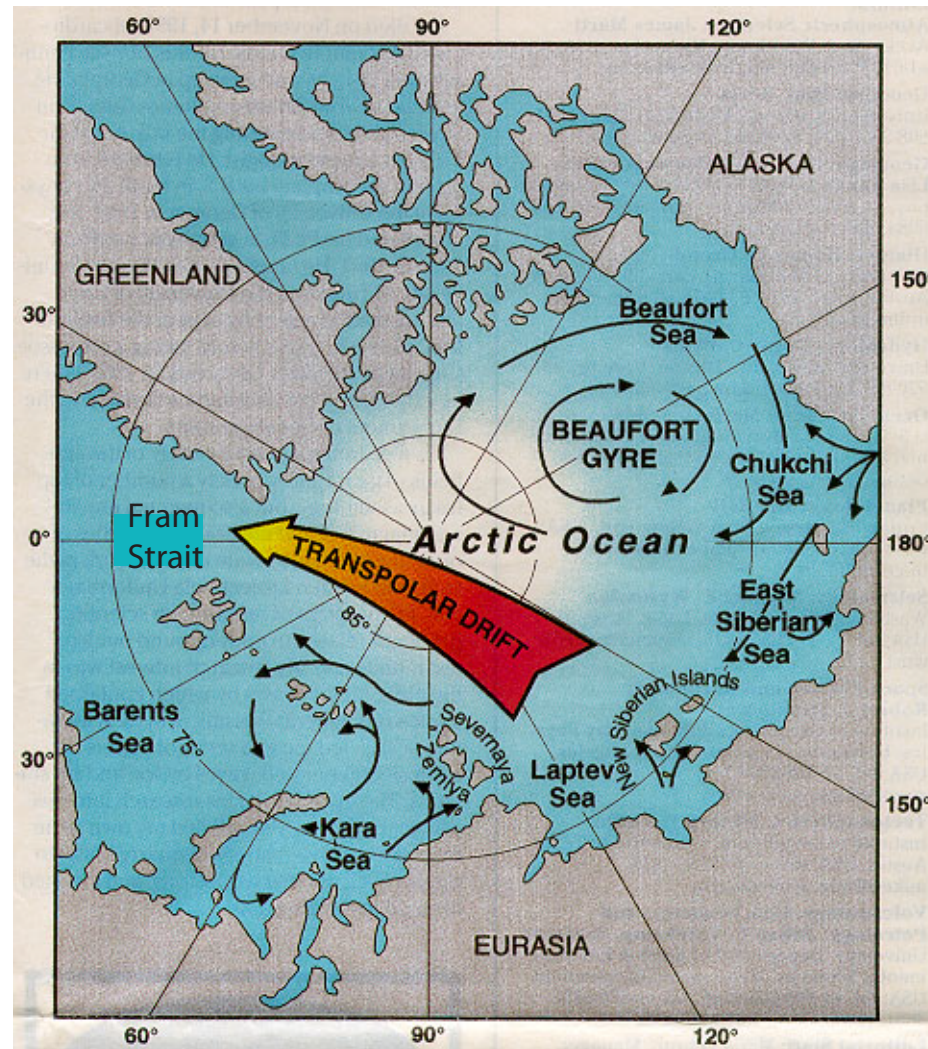
Dynamics



Momentum equation:

Ice acceleration = wind stress + ocean stress - Coriolis force
- sea surface tilt + internal ice stress

Arctic Ice Dynamics



large scale ice dynamics and ocean circulation



Ridges



measuring ice depth in ridges off Barrow, AK

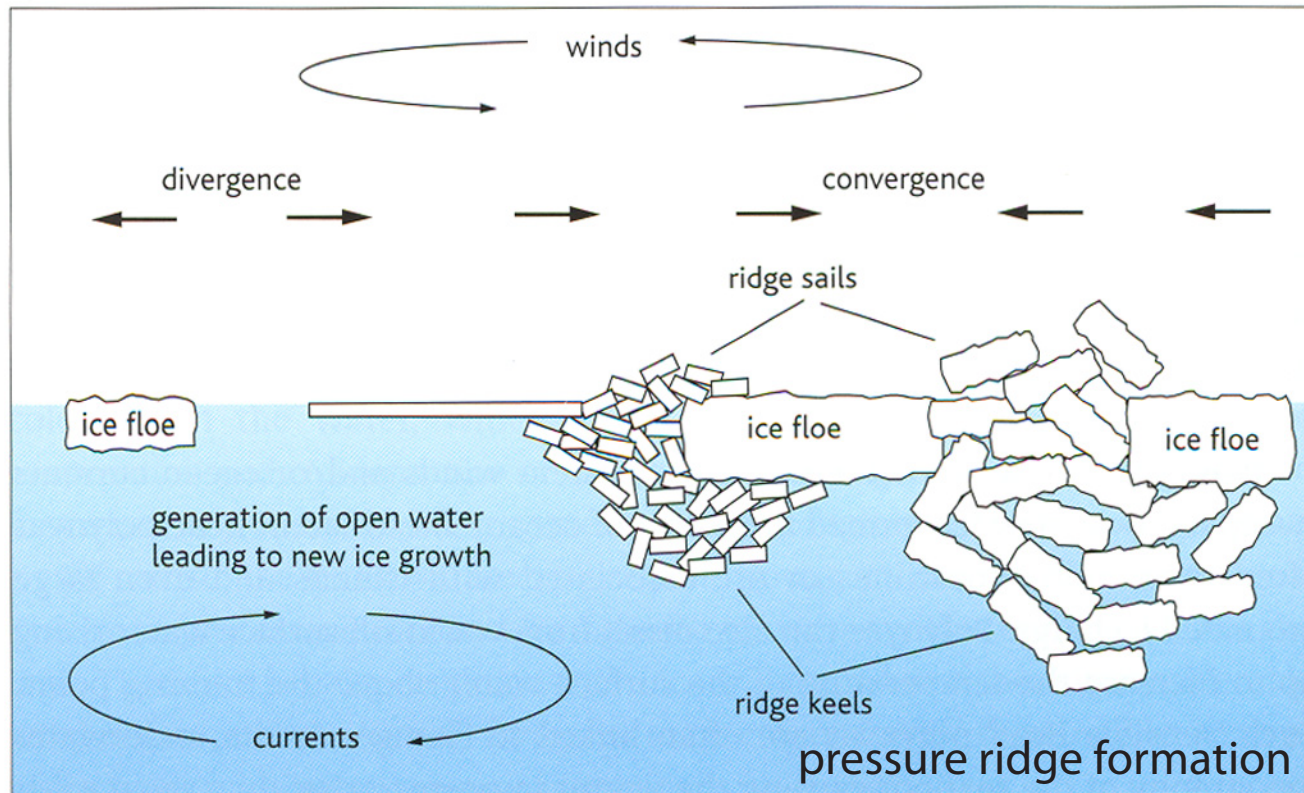


dynamic sea ice

sea ice dynamics
plate tectonics on a fast time scale



dynamically modifying the ice thickness distribution



thinning

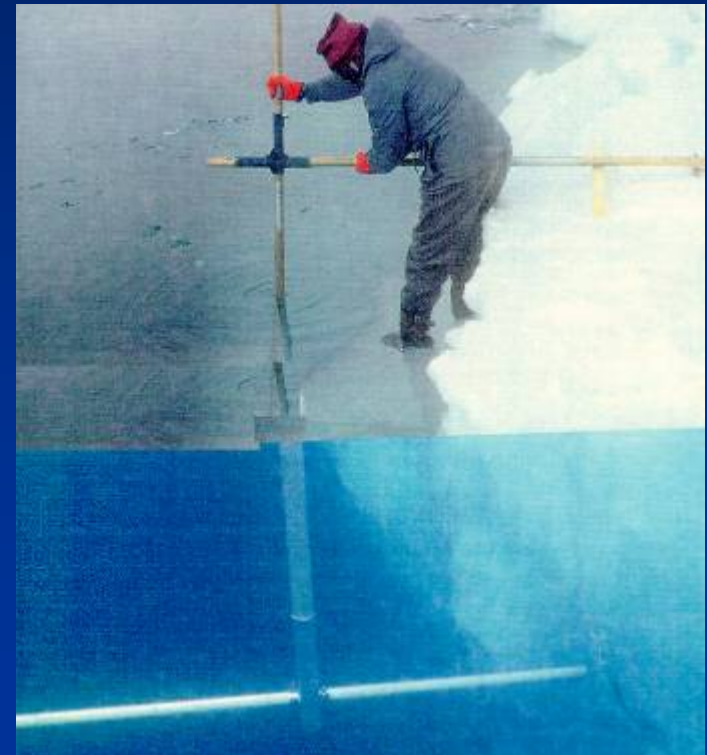
thickening

leads



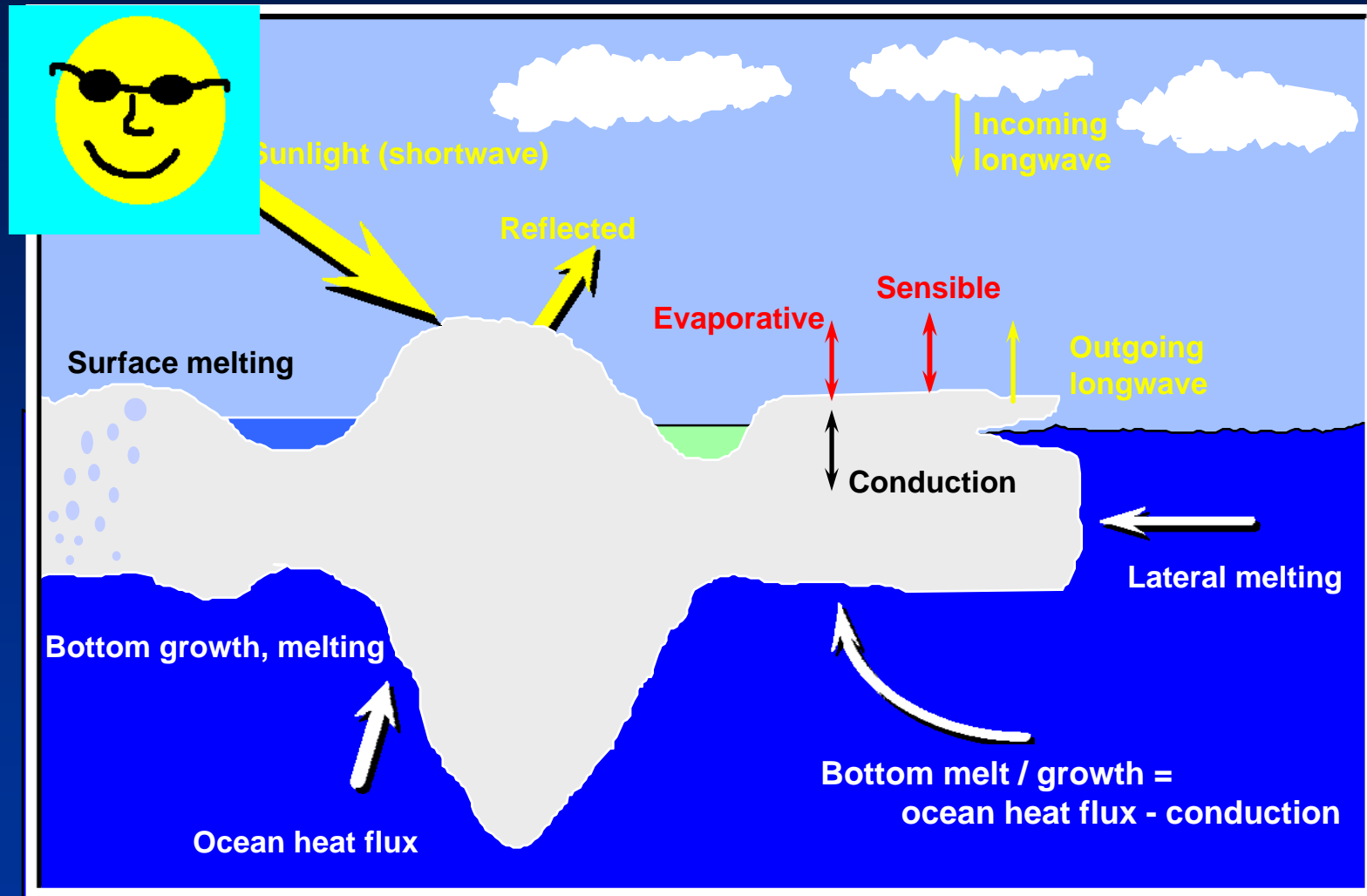
heat flows directly from ocean to atmosphere

Thermodynamics: 4 ways to melt



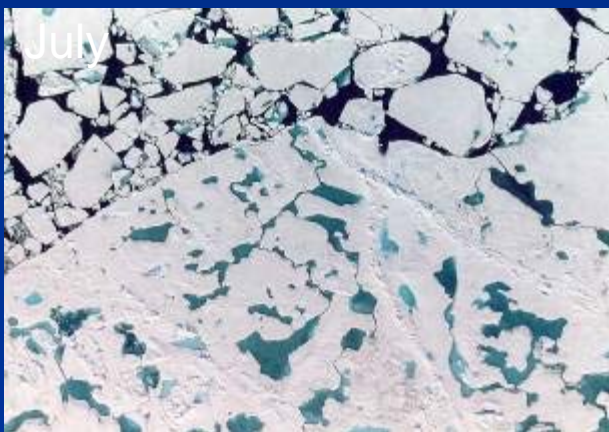
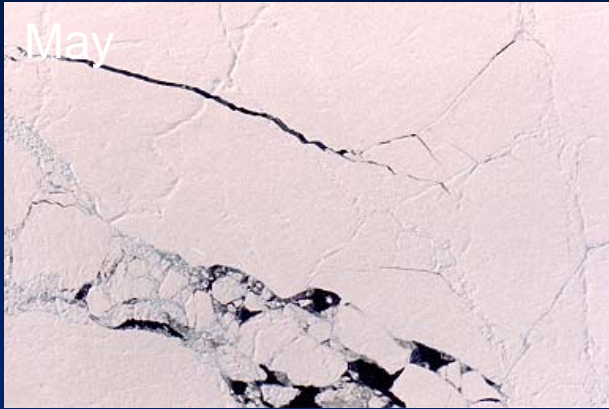
Top, bottom, lateral, internal

Heat budgets

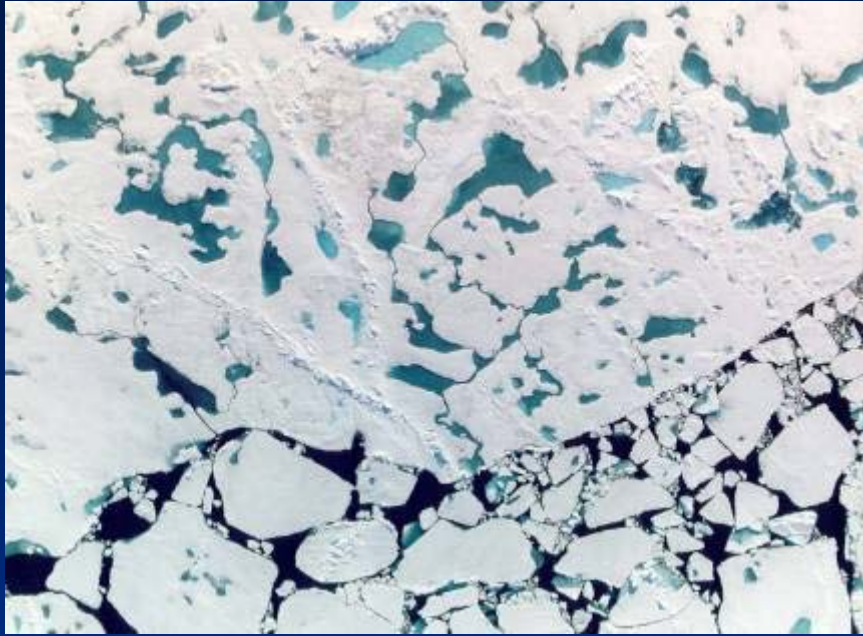


$$\text{Net shortwave} + \text{incoming longwave} + \text{outgoing longwave} + \text{sensible} + \text{evaporative} + \text{conduction} = \text{melt / freeze}$$

Seasonal changes in sea ice



Melt ponds

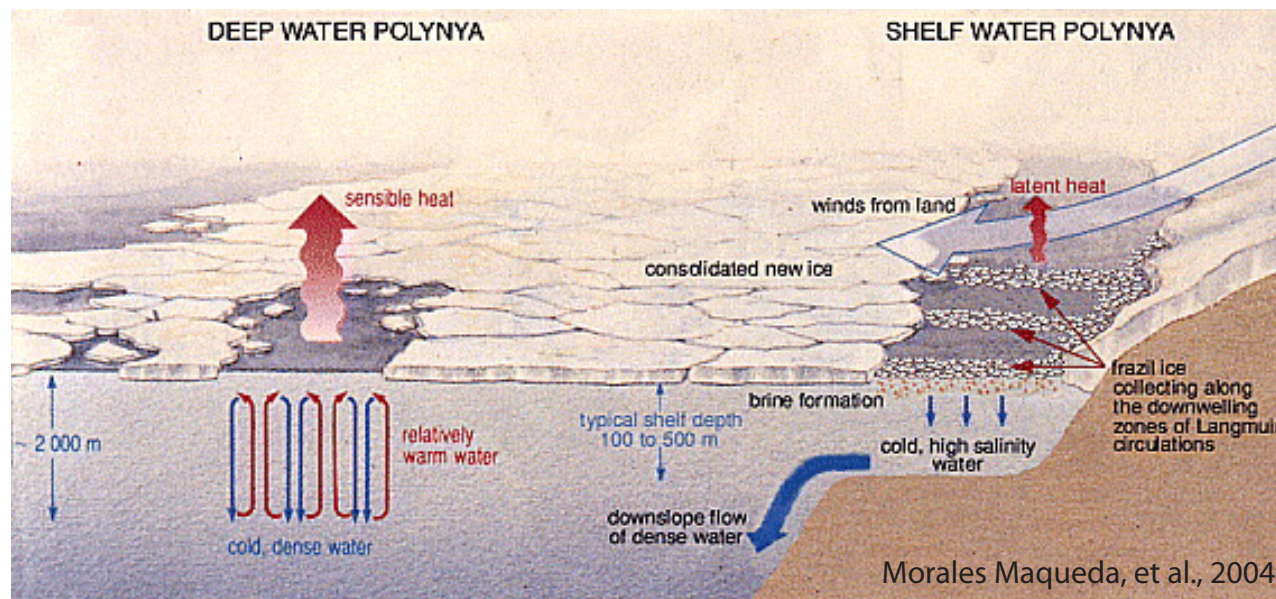


Polynyas

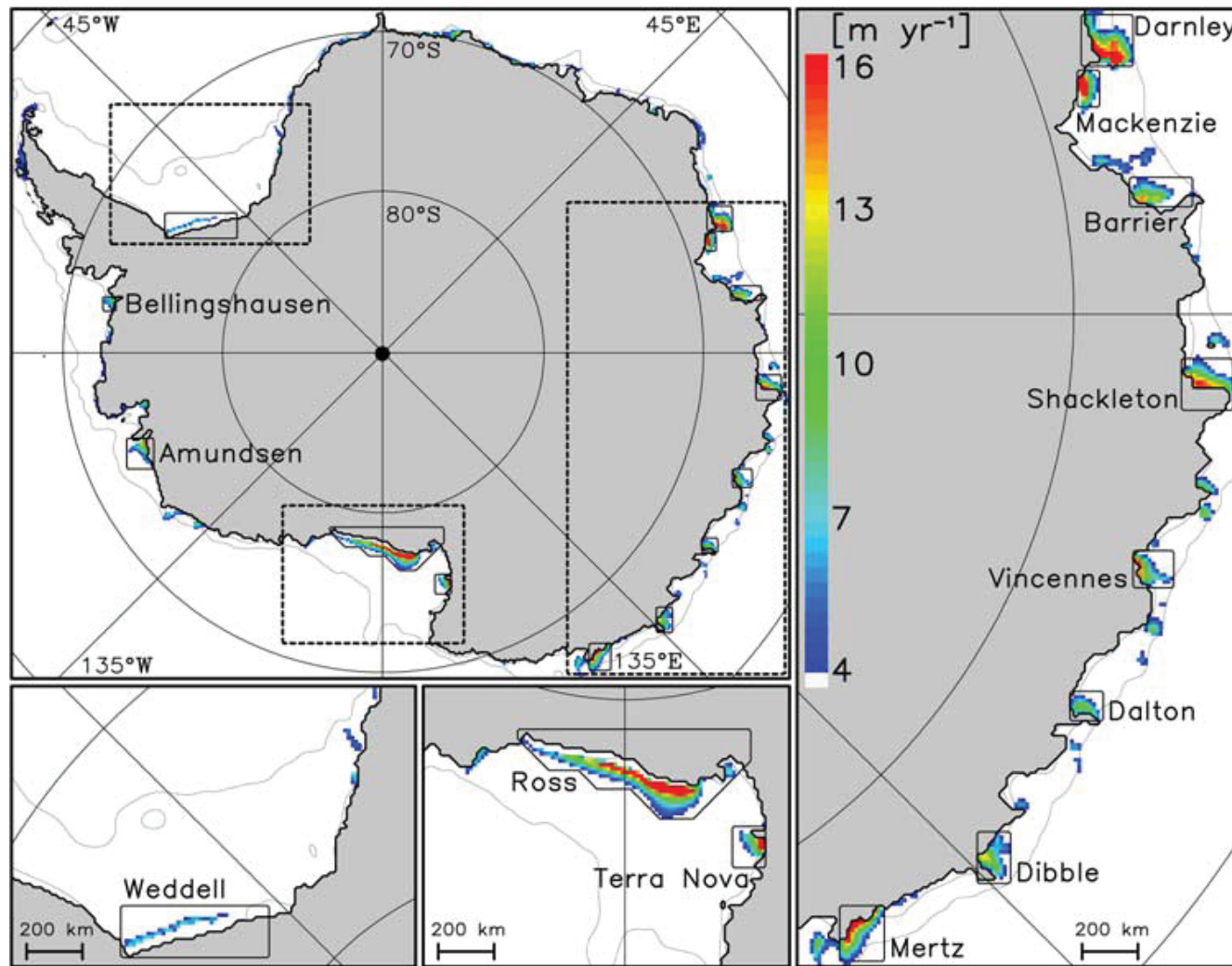
Size: 100 m - 1000 km

Two mechanisms can contribute to keeping polynyas open:

1. **Latent heat (or coastal) polynyas:** **Mertz Glacier Polynya**
Sea ice grows in open-water and is continually removed by winds and currents (e.g. katabatic winds)
 - latent heat released to the ocean during ice formation perpetuates the process
2. **Sensible heat (or open-ocean) polynyas:** **Weddell Polynya**
Upwelling warm waters, vertical heat diffusion, or convection may provide enough oceanic heat flux to maintain ice-free region



Antarctic coastal polynyas = ice factories



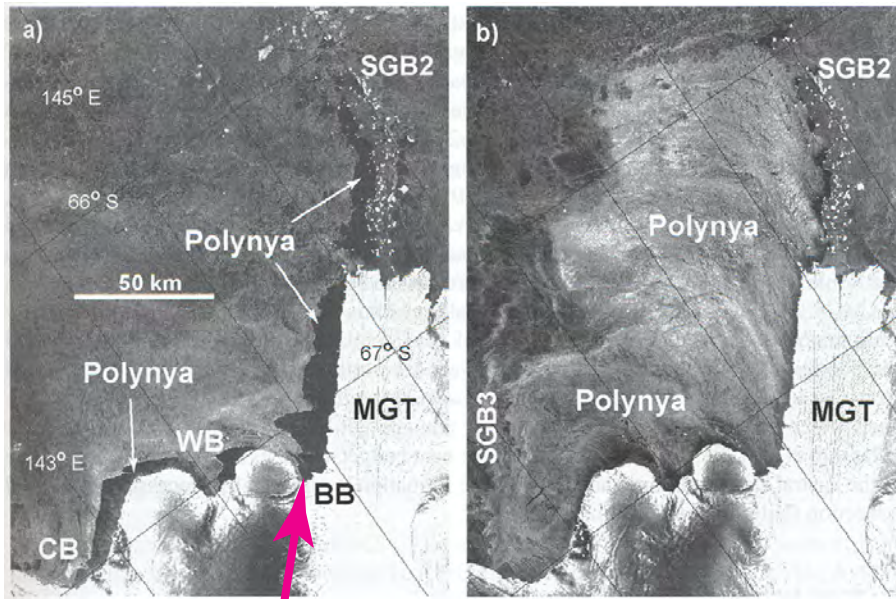
around 10% of Southern Ocean sea ice is produced in the major Antarctic coastal polynyas
ice production in Ross Ice Shelf Polynya decreased by about 30% from the 1990's to the 2000's
(caused by atmospheric warming or decreased polynya size from calving icebergs)

candidate for causing recent freshening of AABW

Tamura, Ohshima, Nihashi, GRL 2008

polynyas ice factories

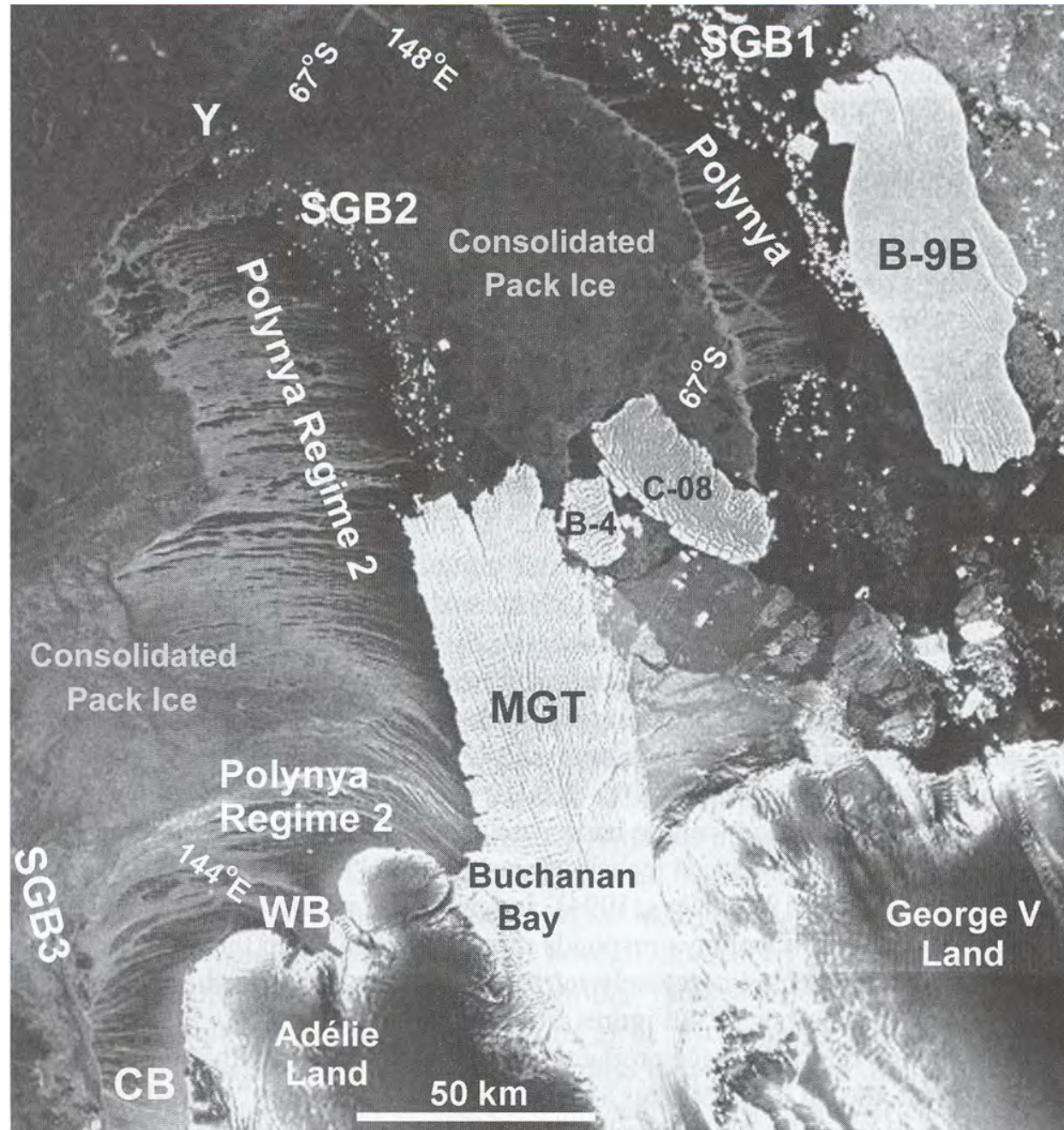
Mertz Glacier Polynya, located in East Antarctica, covers only 0.001% of the overall Antarctic sea ice zone at its maximum winter extent, but is responsible for 1% of the total sea ice production in the Southern Ocean.

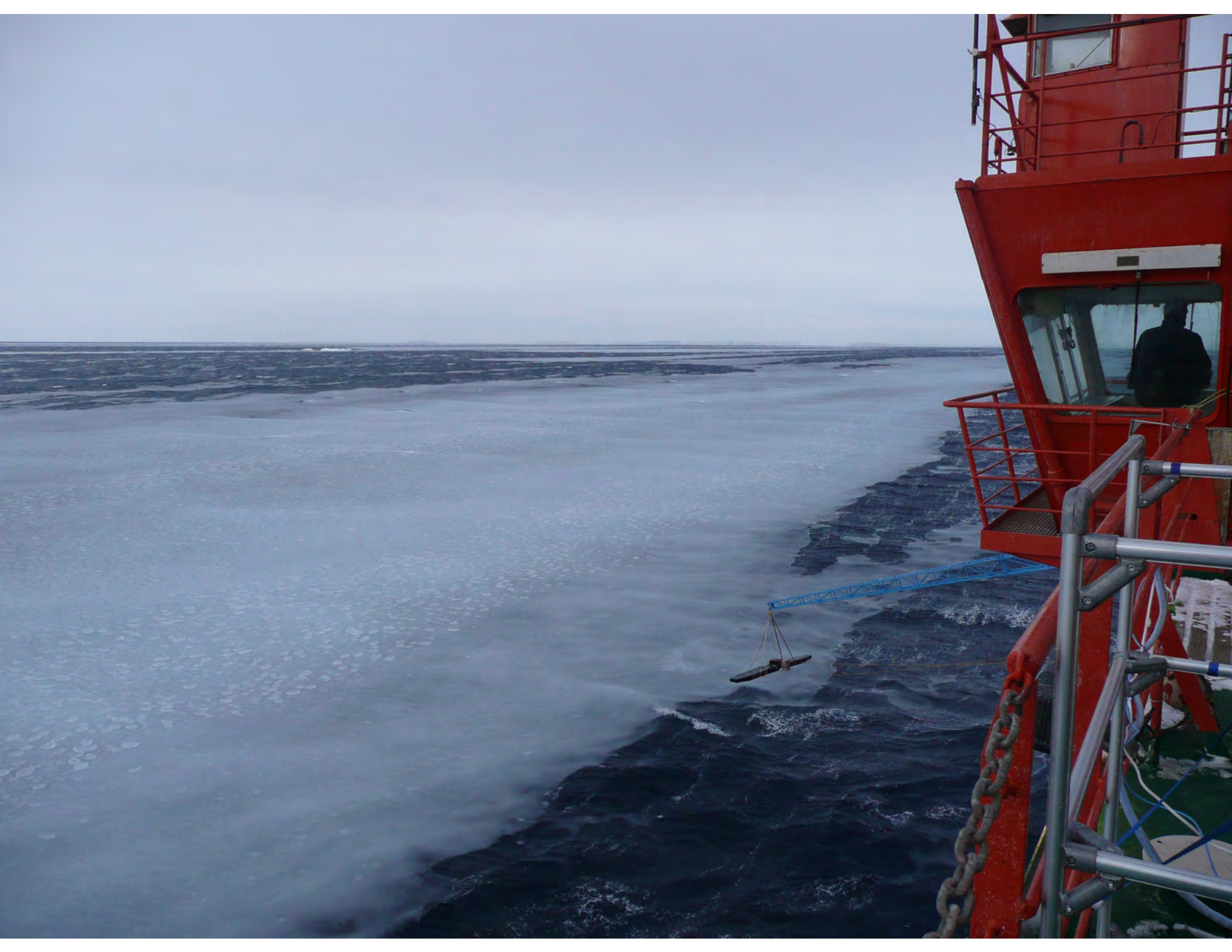


Buchanan Bay



Mertz Glacier Polynya -- third largest Antarctic sea ice producer

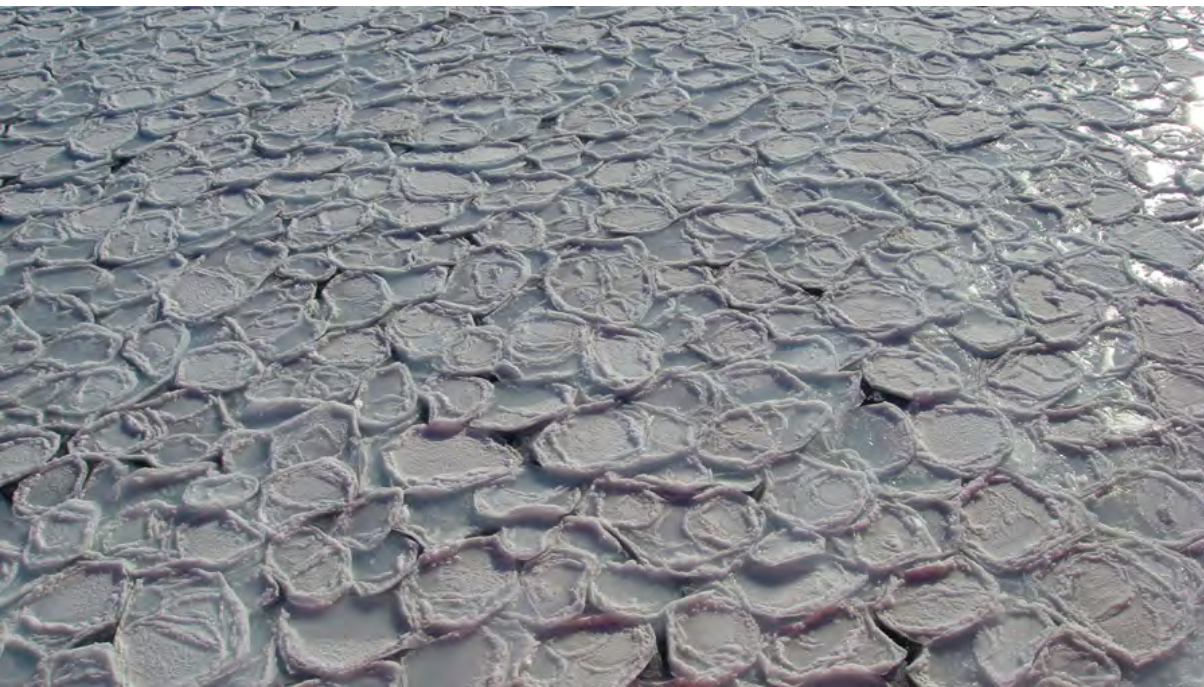


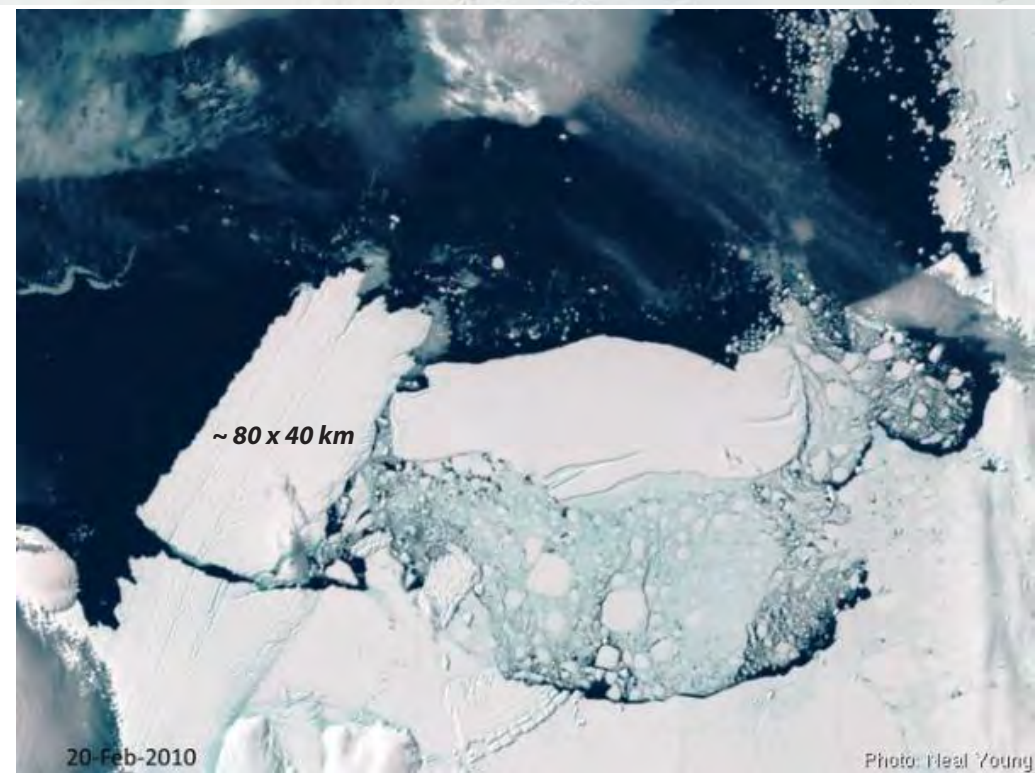
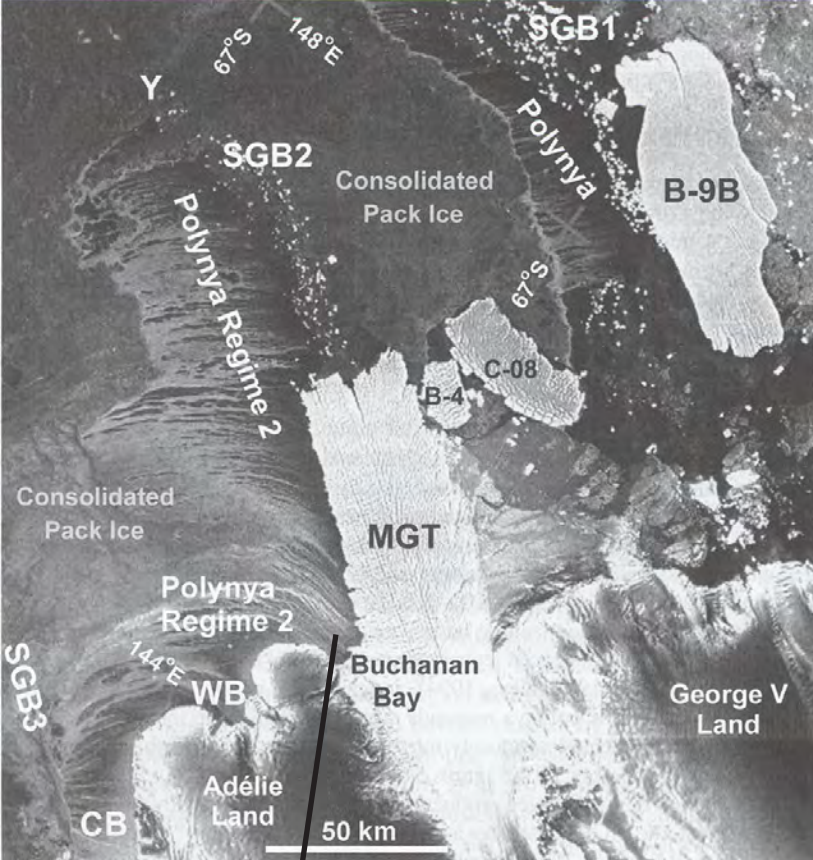


pancake ice forming in a wave field in the Southern Ocean

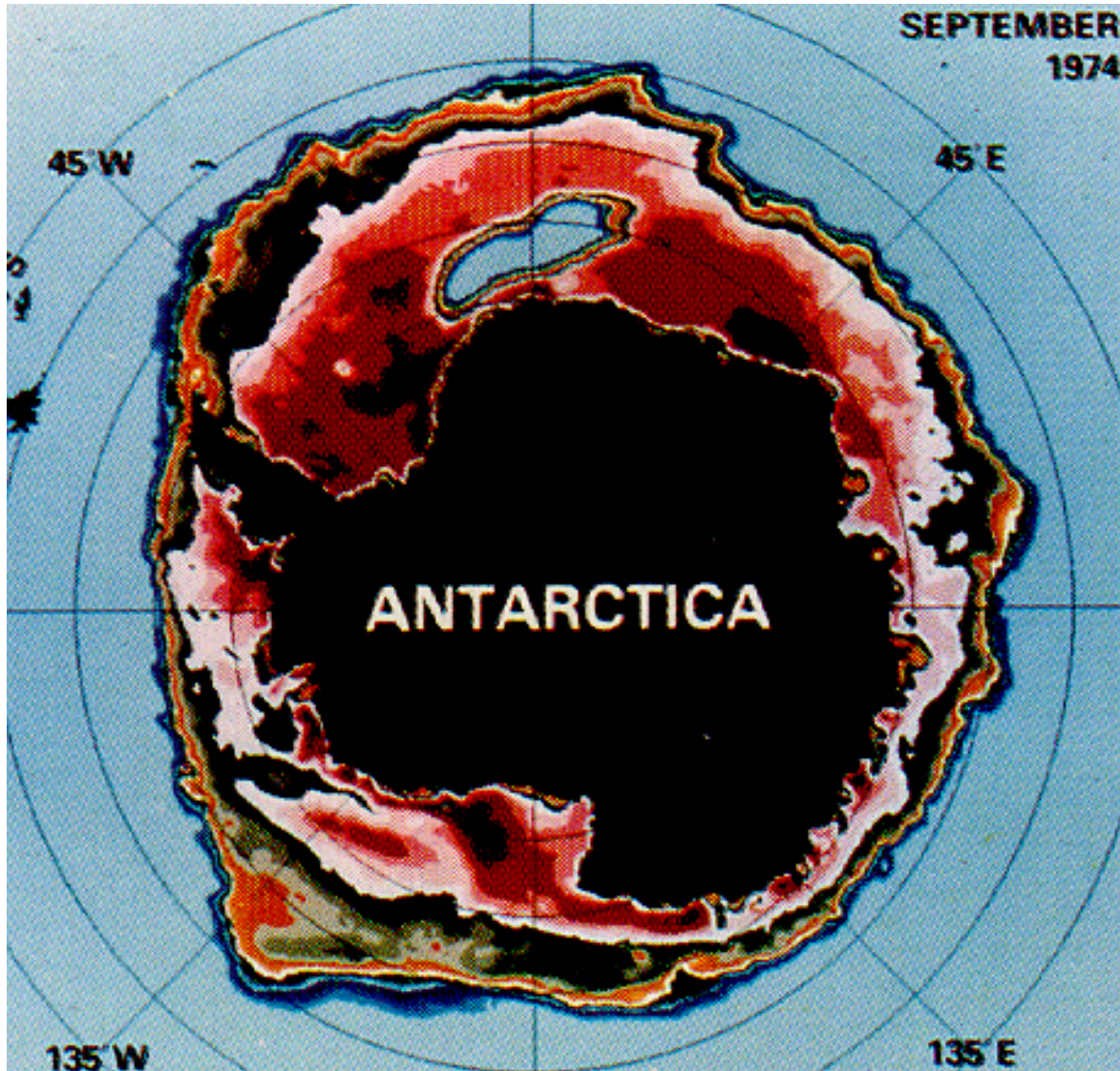


pancake ice





Weddell Sea Polynya



Antarctic Zone Flux Experiment **ANZFLUX 1994**

sea ice thickness in
dynamic equilibrium

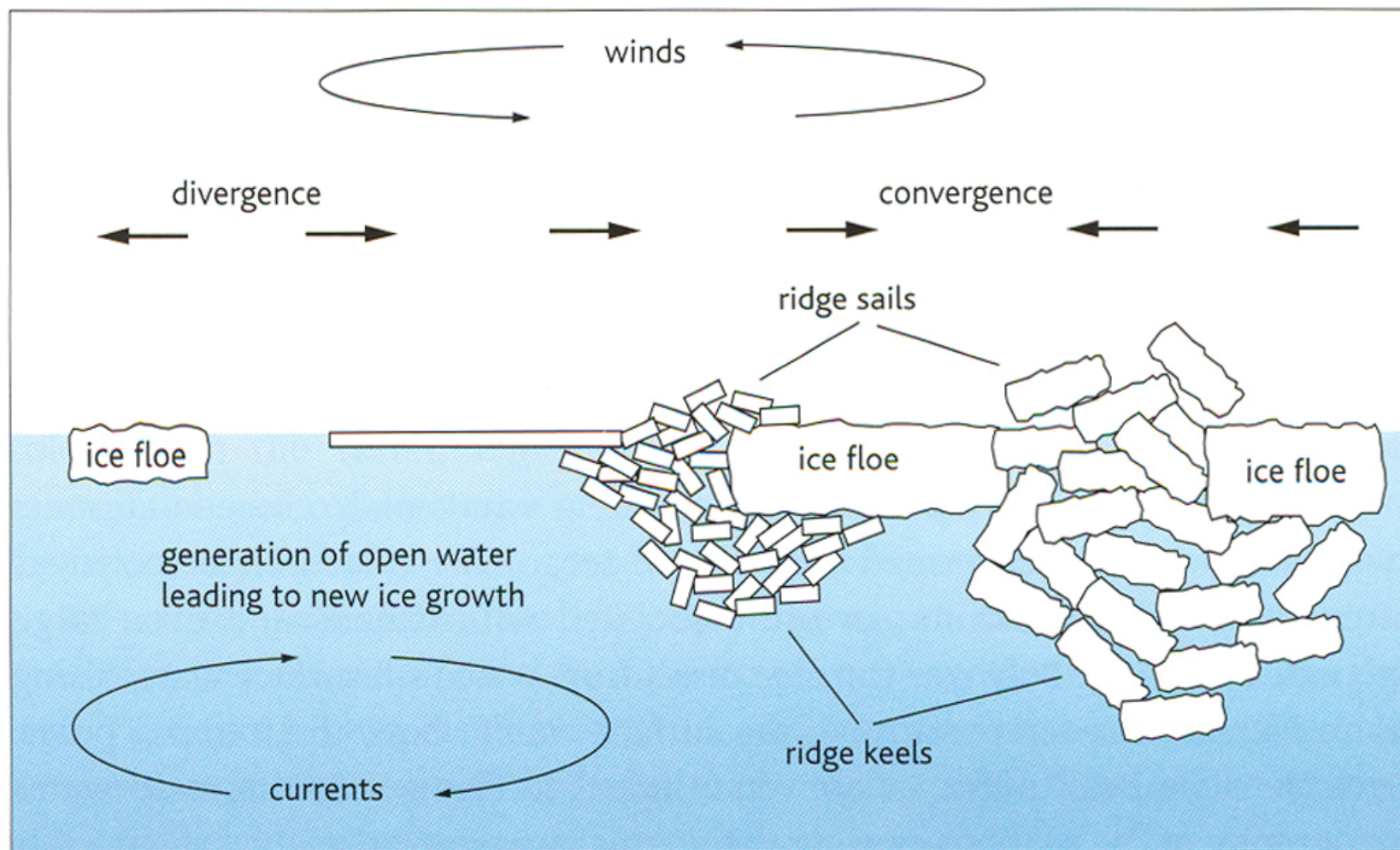
**surface flooding ->
snow-ice formation**

***controlled by
ice permeability***

Grow a little ice, descending brine plumes (+ storms) can help trigger deep convection, keep polynya going? Snow-ice formation shuts it down?

Doug Martinson

deeper, warmer waters deflected upwards by rise in sea floor (Maud Rise)

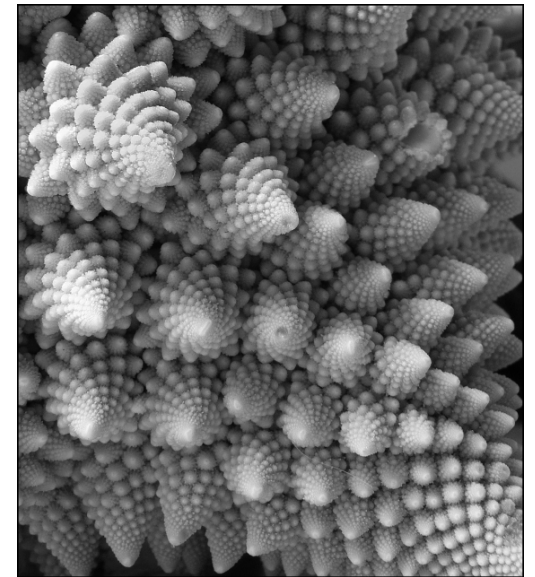
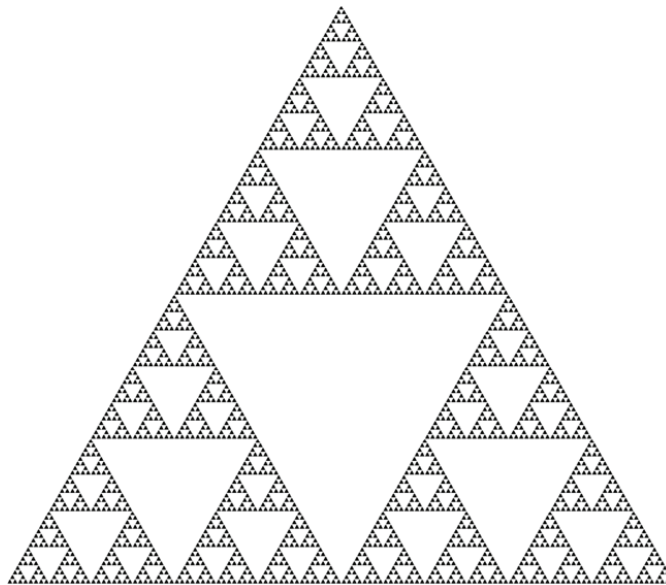
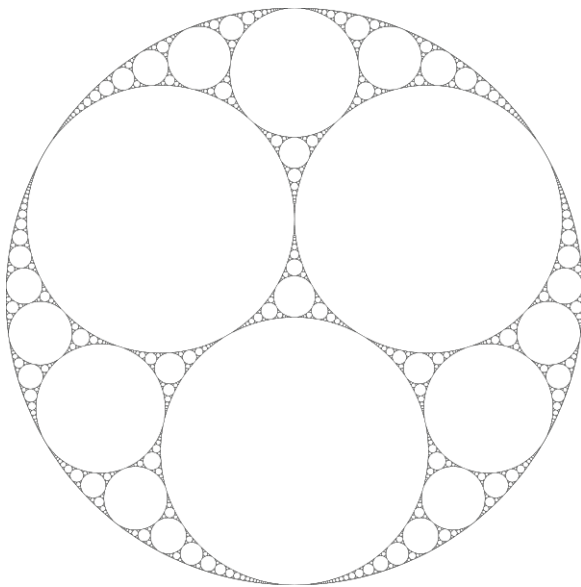


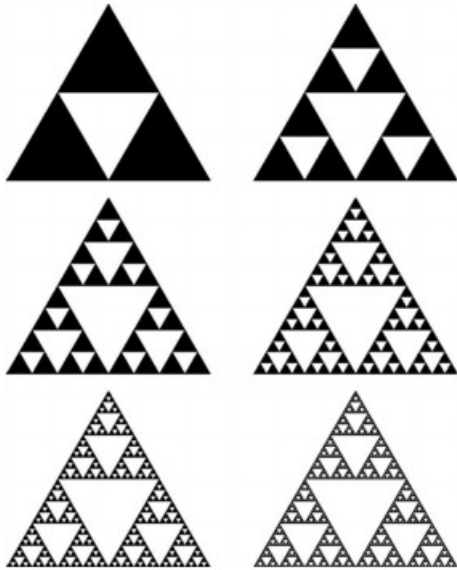
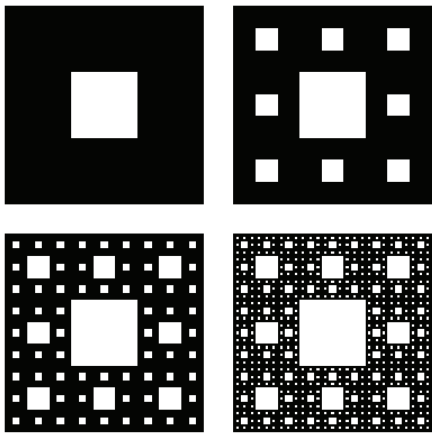
the sea ice pack is a *fractal*

displaying self-similar structure on many scales

floe size distribution important in
dynamics (fracture), thermodynamics (melting)

bigger floes easier to break, smaller floes easier to melt

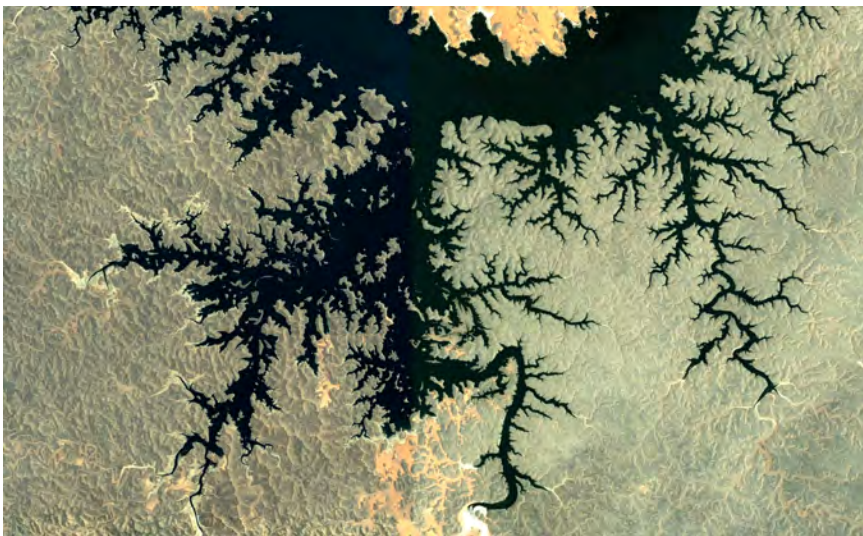




fractals

self-similar structure
non-integer dimension

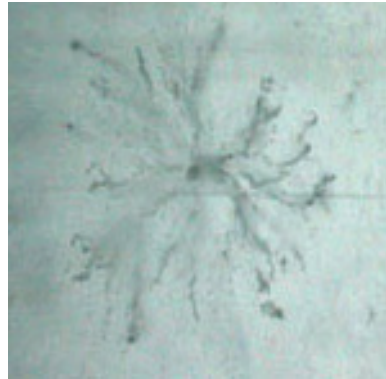
$$D = \frac{\log 3}{\log 2} = 1.585...$$



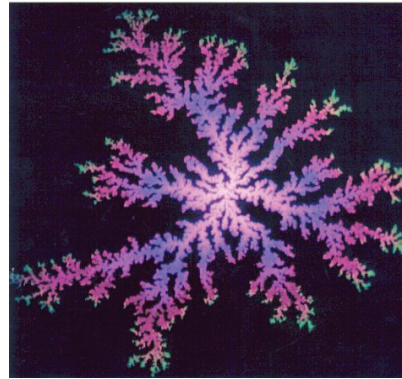
fractal microstructures



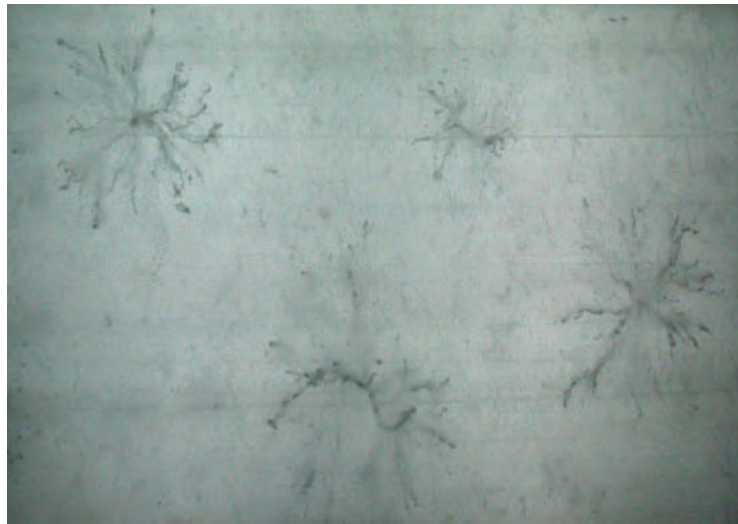
electrorheological fluid
with metal spheres



brine channel
in sea ice



diffusion limited
aggregation



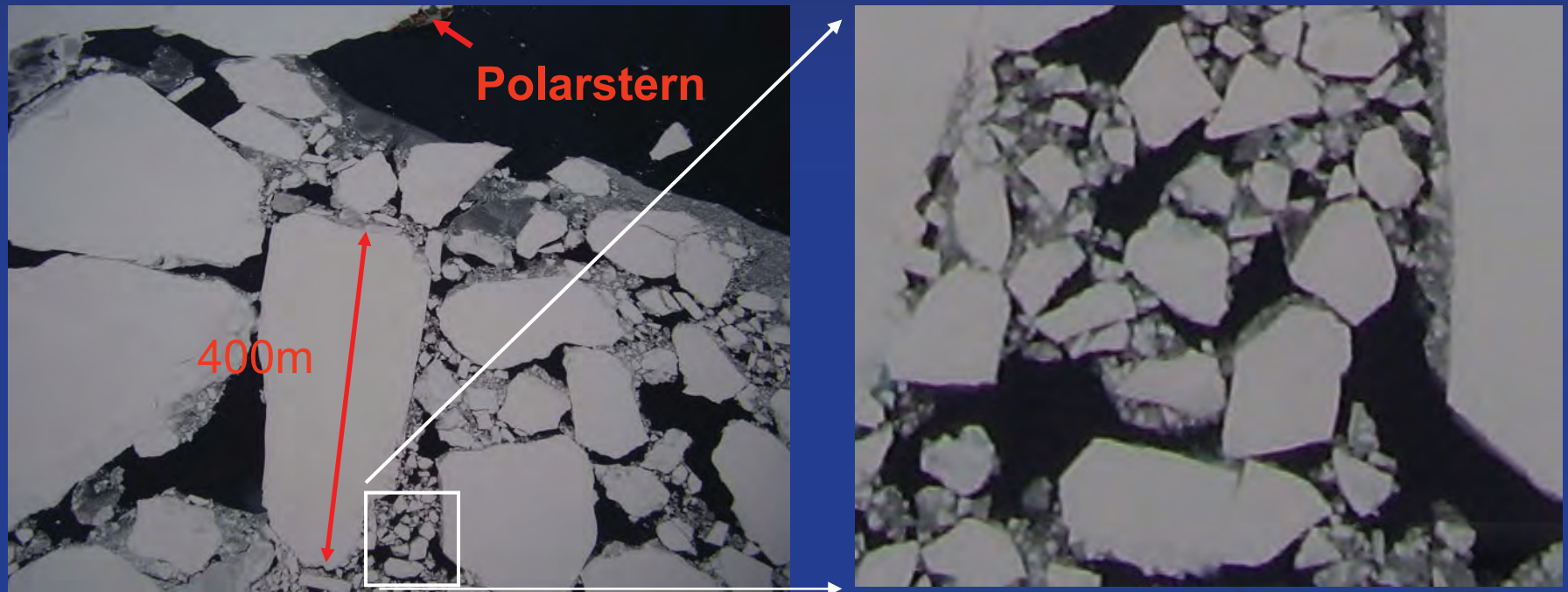
brine channels



The sea ice pack has fractal structure.

Self-similarity of sea ice floes

Weddell Sea, Antarctica

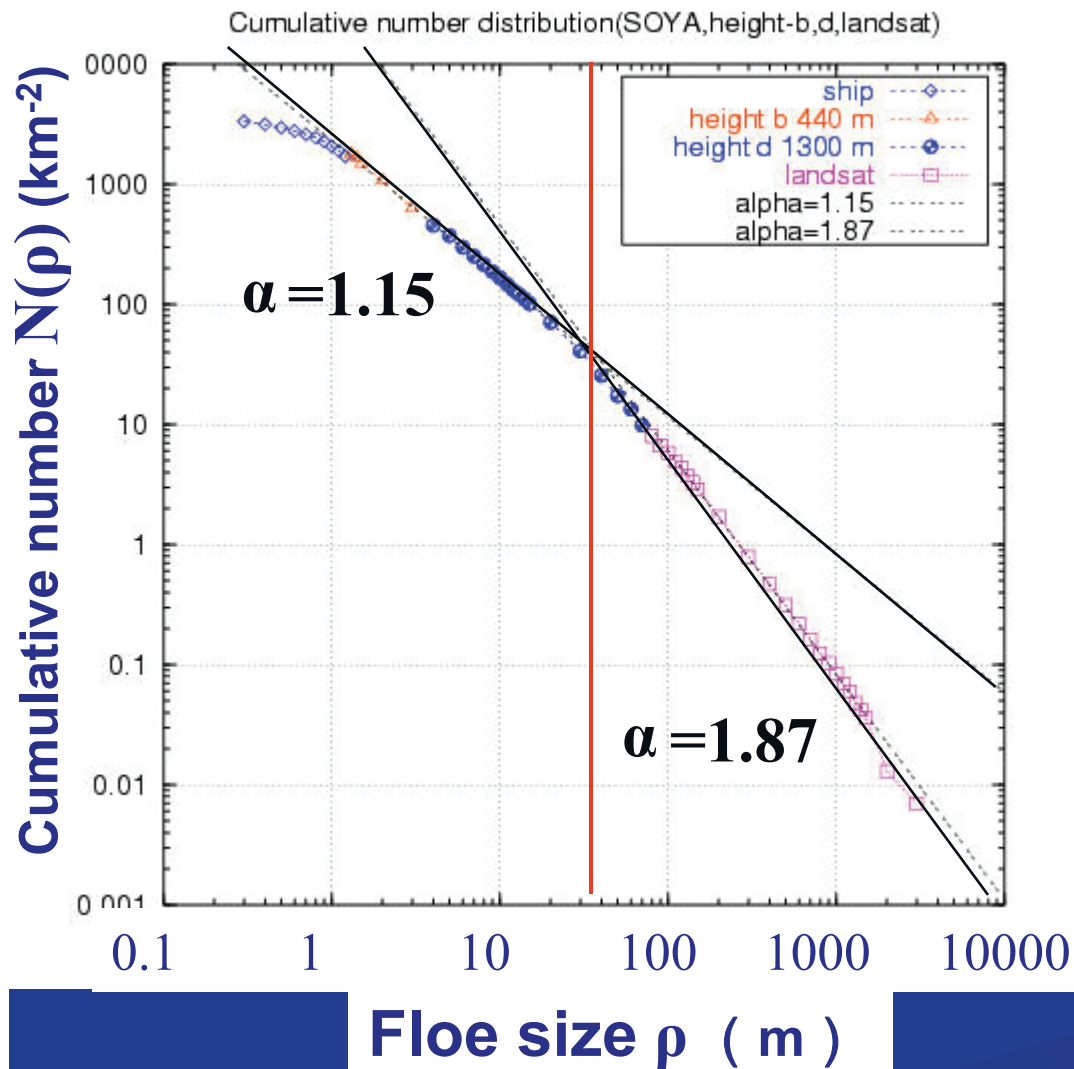


***fractal dimensions of Okhotsk Sea ice pack
smaller scales $D \sim 1.2$, larger scales $D \sim 1.9$***

Toyota, et al. *Geophys. Res. Lett.* 2006

Rothrock and Thorndike, *J. Geophys. Res.* 1984

Results from Okhotsk Sea ice



There are two regimes in the ice floe distribution.

Size

1 ~ 20 m : $\alpha = 1.15 \pm 0.02$

100 ~ 1500 m :

$\alpha = 1.87 \pm 0.02$

(Toyota, Takatsuji et al., 2006)

melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

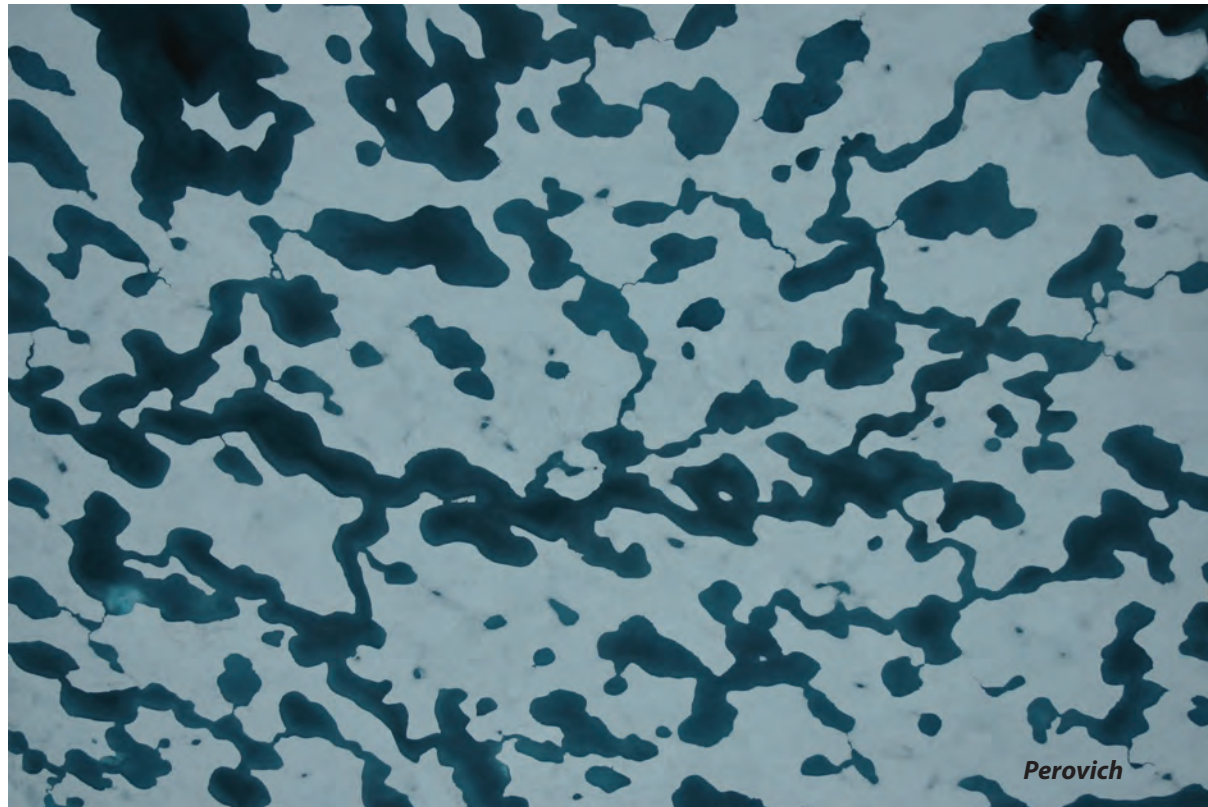
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson,
Perovich 2009

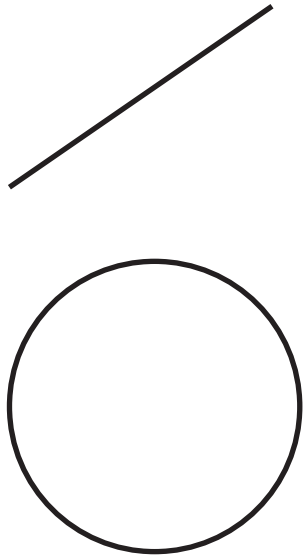
Flocco, Feltham,
Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

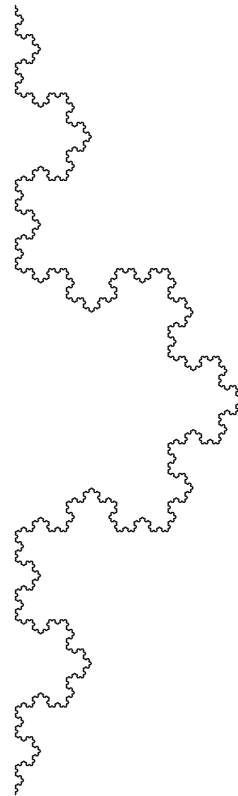
fractal curves in the plane

they wiggle so much that their dimension is >1



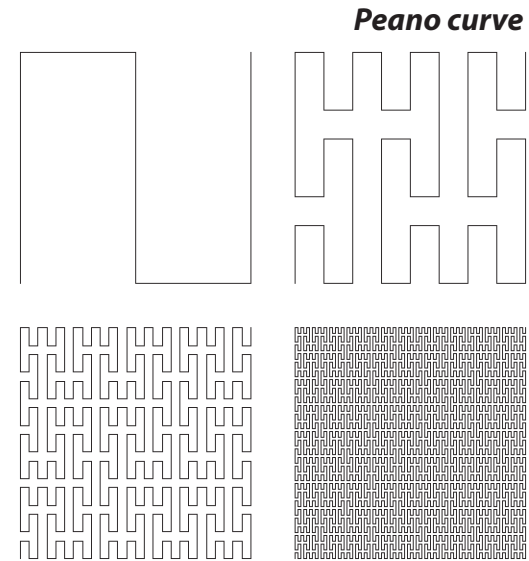
simple curves

$D = 1$



Koch snowflake

$D = 1.26$



Brownian motion

space filling curves

$D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

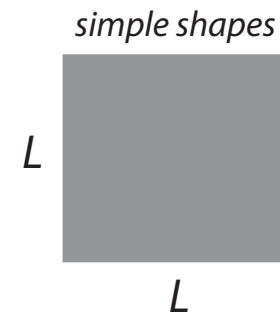
use **perimeter-area** data to find that
cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

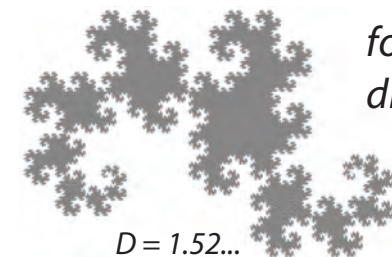


$$P \sim \sqrt{A}$$



$$A = L^2$$
$$P = 4L = 4\sqrt{A}$$

$$P \sim \sqrt{A}^D$$



for fractals with
dimension D

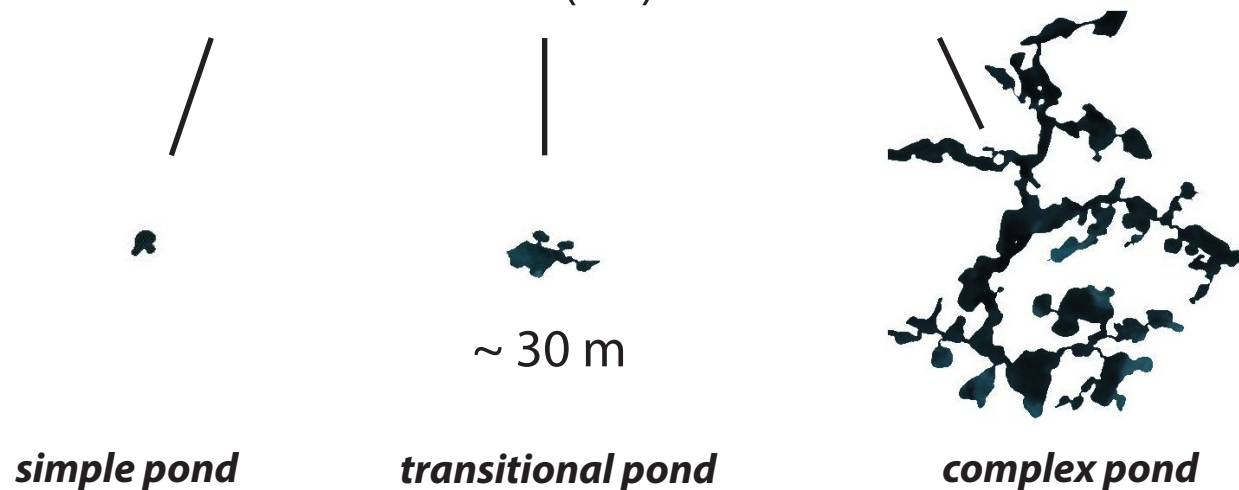
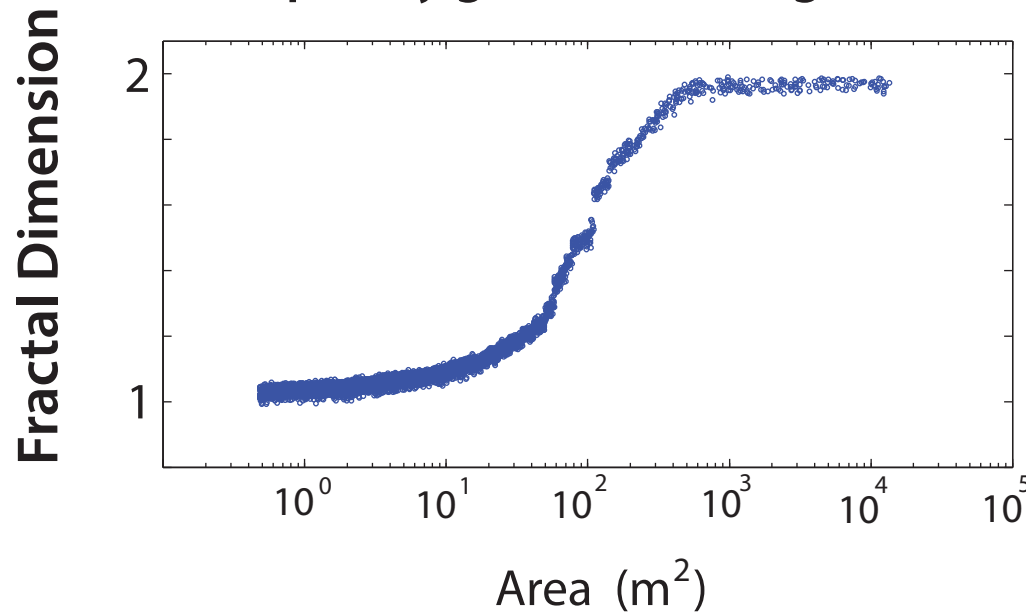
$D = 1.52...$

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale





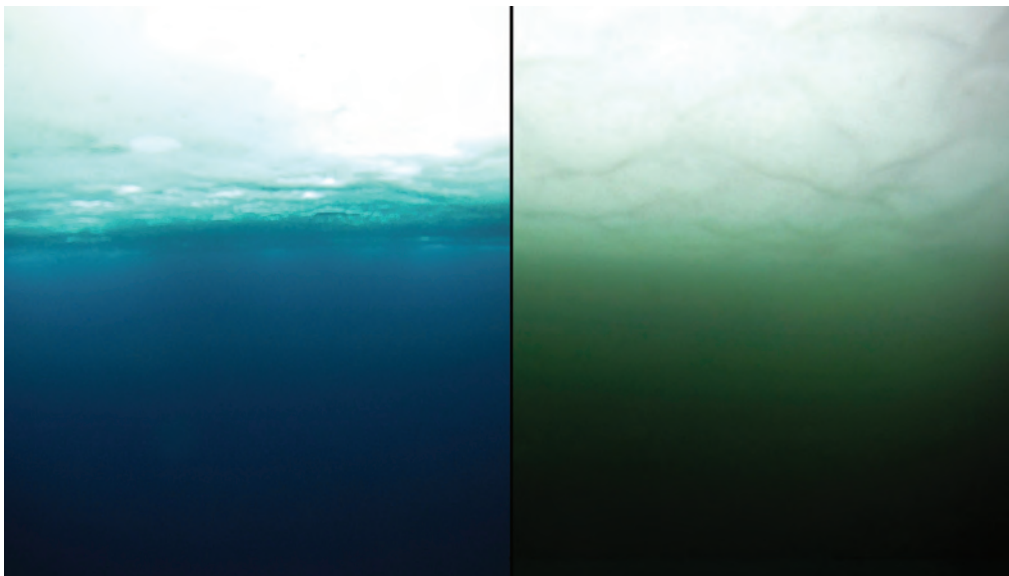
2011 massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light
through sea ice



no bloom

bloom

***Have we crossed into a
new ecological regime?***

The frequency and extent of sub-ice
phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder,
Flocco, Feltham, *Science Advances*, 2017

(2015 AMS MRC, Snowbird)