Modeling Sea Ice as a Multiscale Composite Material

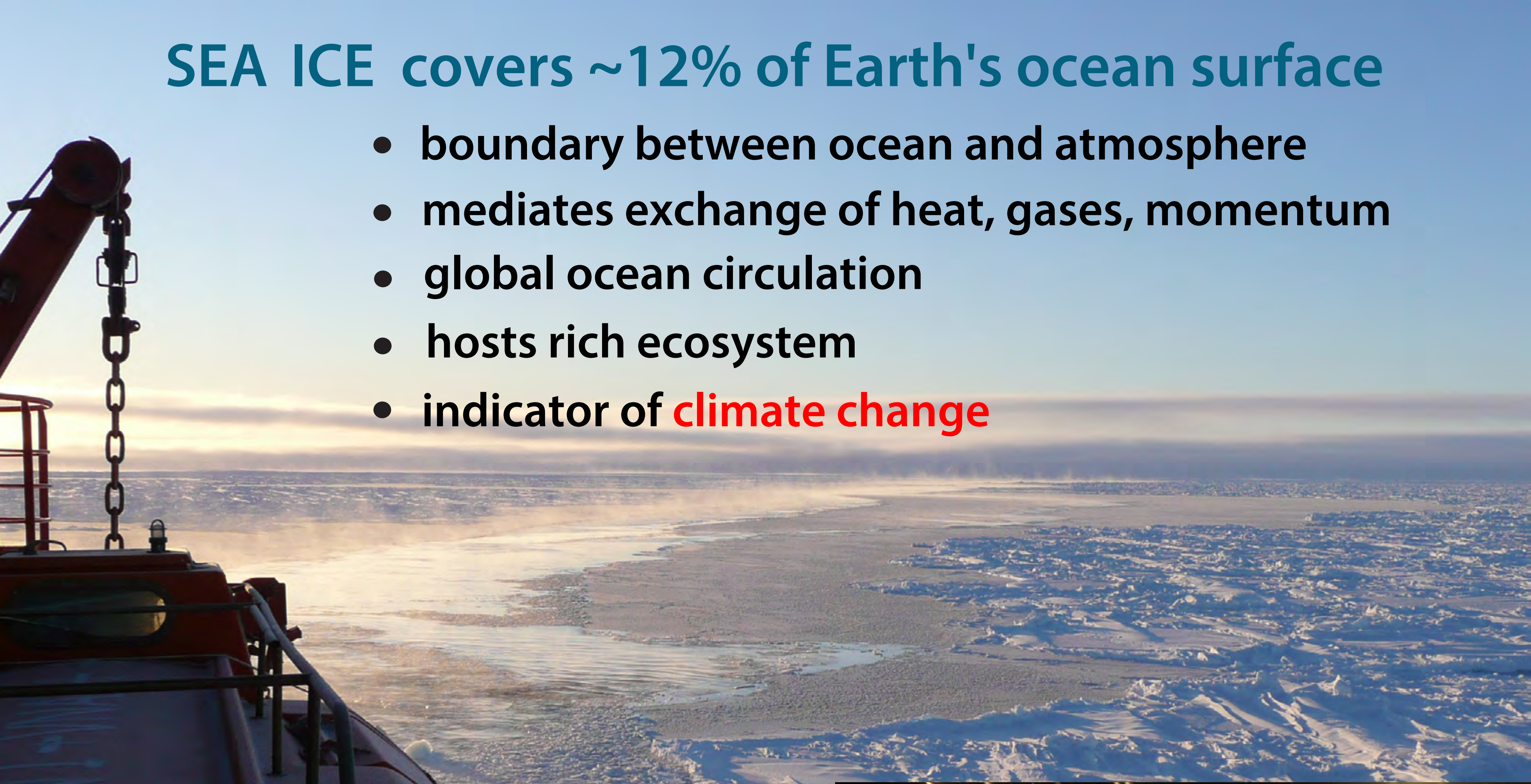
Kenneth M. Golden
Department of Mathematics
University of Utah

ONR Program Review
15 July 2020

Beaufort Sea
Golden

SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of climate change



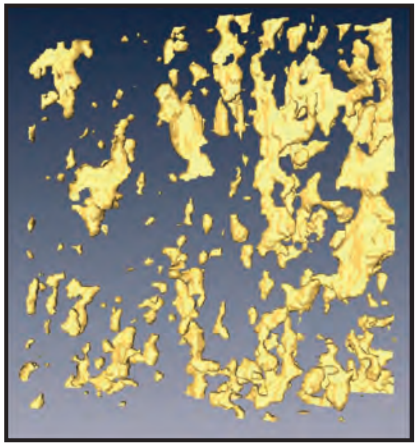
polar ice caps critical
to climate in reflecting
sunlight during summer

Sea Ice is a Multiscale Composite Material

MICROSCALE

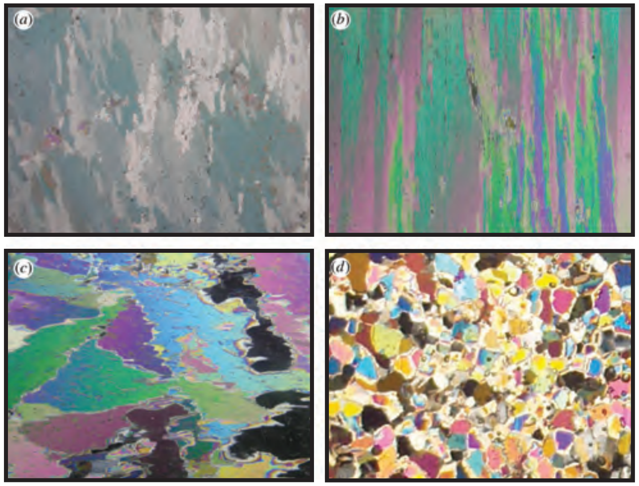
brine inclusions


Weeks & Assur 1969


H. Eicken

Golden *et al. GRL* 2007

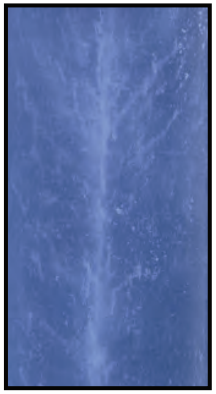
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels


D. Cole


K. Golden

millimeters

centimeters

MESOSCALE

Arctic melt ponds


K. Frey

pressure ridges


K. Golden

meters

MACROSCALE

sea ice floes

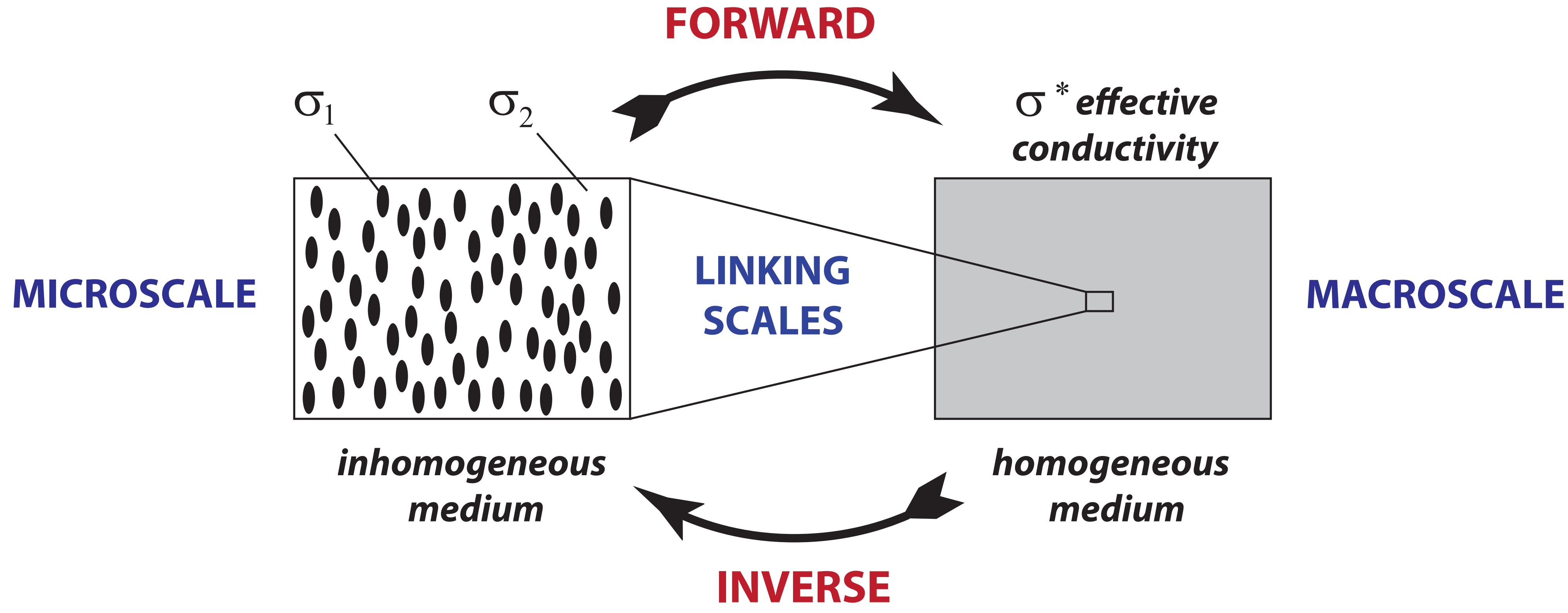

J. Weller

sea ice pack


NASA

kilometers

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about? homogenization for multiscale composites

the role of "microstructure" in determining sea ice effective properties

Using methods of **homogenization and statistical physics** to LINK SCALES in the sea ice system ... compute effective behavior on scales relevant to coarse-grained sea ice and climate models, process studies, ...

MICROSCALE: brine + polycrystalline structure; EM and fluid transport

MESOSCALE: advection diffusion, thermal transport, waves, melt ponds

MACROSCALE: ice transport, MIZ width and location, low order models

A tour of Stieltjes integrals in the study of sea ice and its role in climate.

Solving problems in physics of sea ice drives advances in theory of composite materials.

cross - pollination

bone, stealthy coatings
magnets, rat brains, RMT

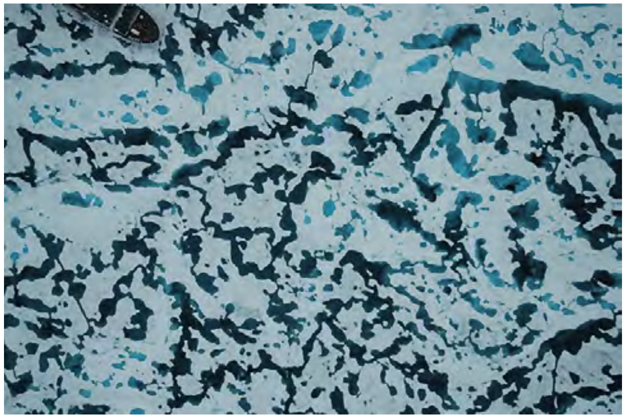
How do scales interact in the sea ice system?

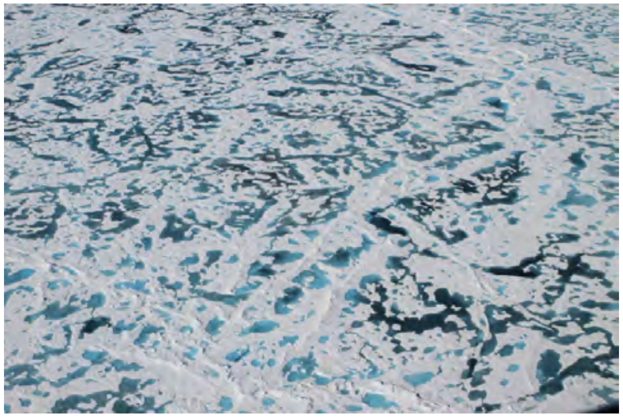
NASA

basin scale -
grid scale
albedo

Linking Scales

km
scale
melt
ponds





Perovich

km
scale
melt
ponds

Linking Scales

mm
scale
brine
inclusions

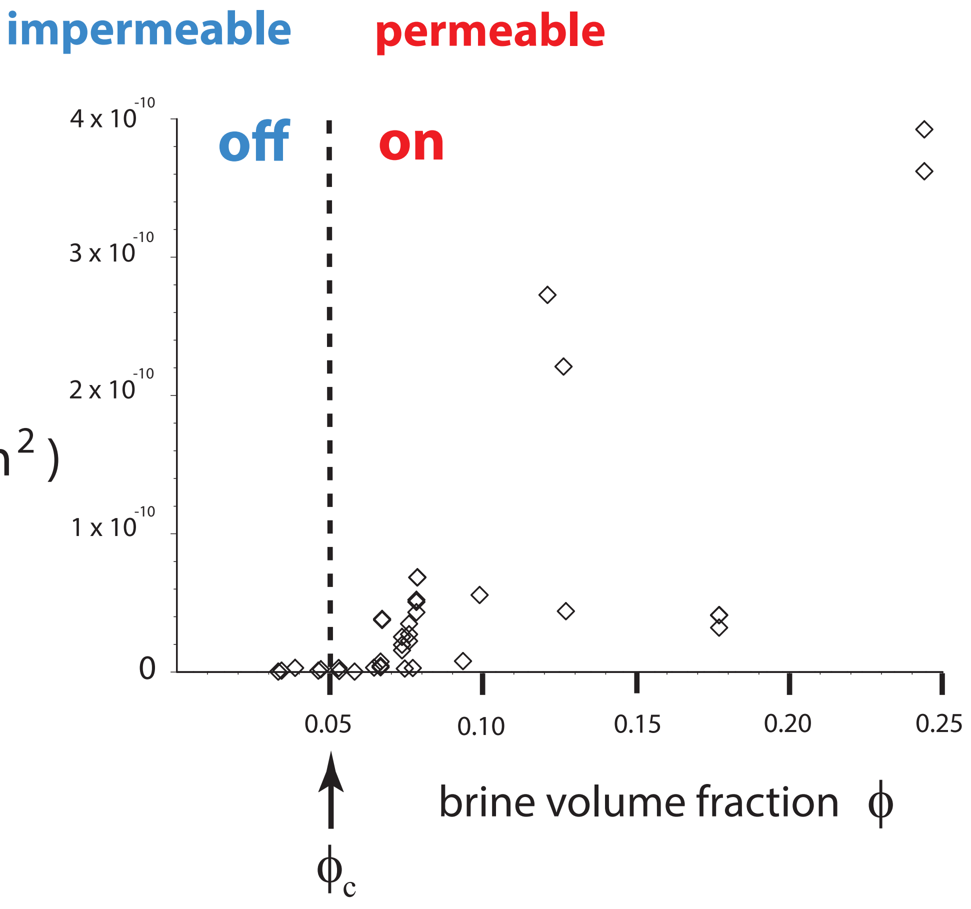
meter
scale
snow
topography

microscale

Critical behavior of fluid transport in sea ice

Arctic field data

vertical fluid permeability k (m^2)



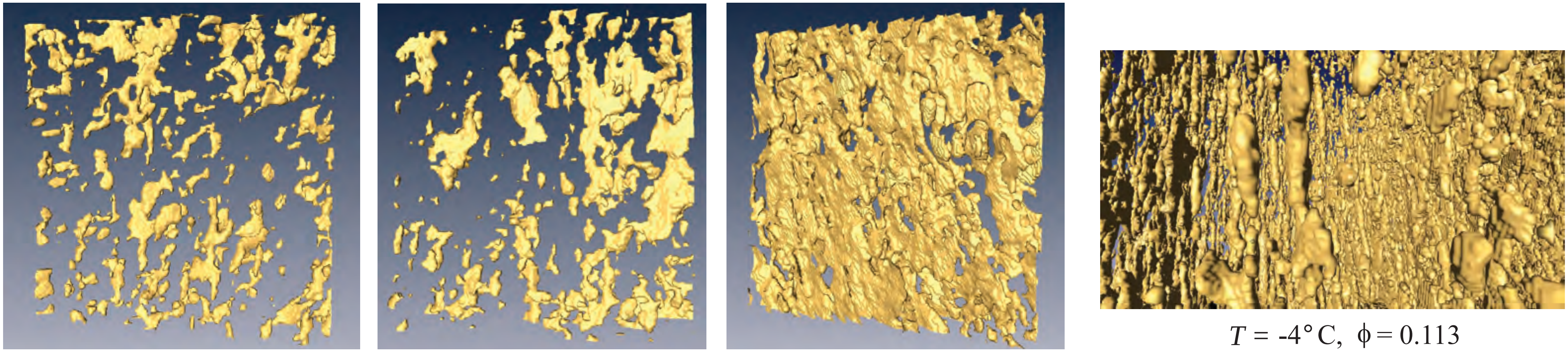
“on - off” switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C, \ S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007
Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

brine volume fraction and ***connectivity*** increase with temperature



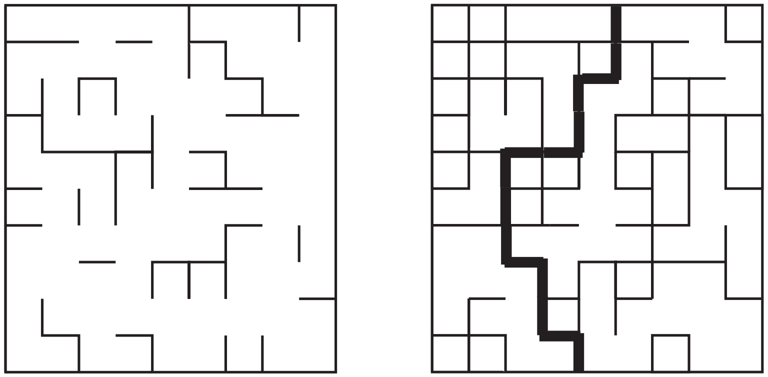
T = -15 °C, $\phi = 0.033$ T = -6 °C, $\phi = 0.075$ T = -3 °C, $\phi = 0.143$ T = -4 °C, $\phi = 0.113$

X-ray tomography for brine phase in sea ice

Golden, Eicken, *et al.*, *Geophysical Research Letters* 2007

PERCOLATION THRESHOLD $\phi_c \approx 5\ \%$

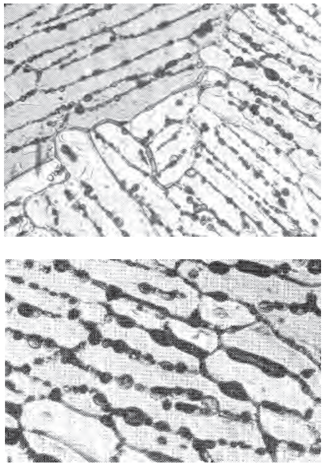
Golden, Ackley, Lytle, *Science* 1998



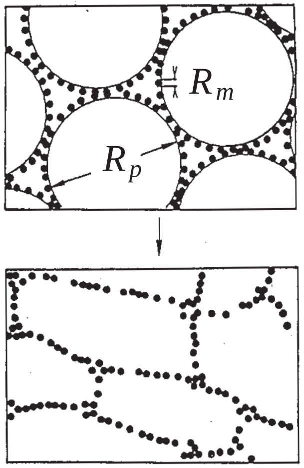
impermeable permeable

$p = 1/3$ $p = 2/3$

lattice percolation



sea ice



Kusy, Turner
***Nature* 1971**

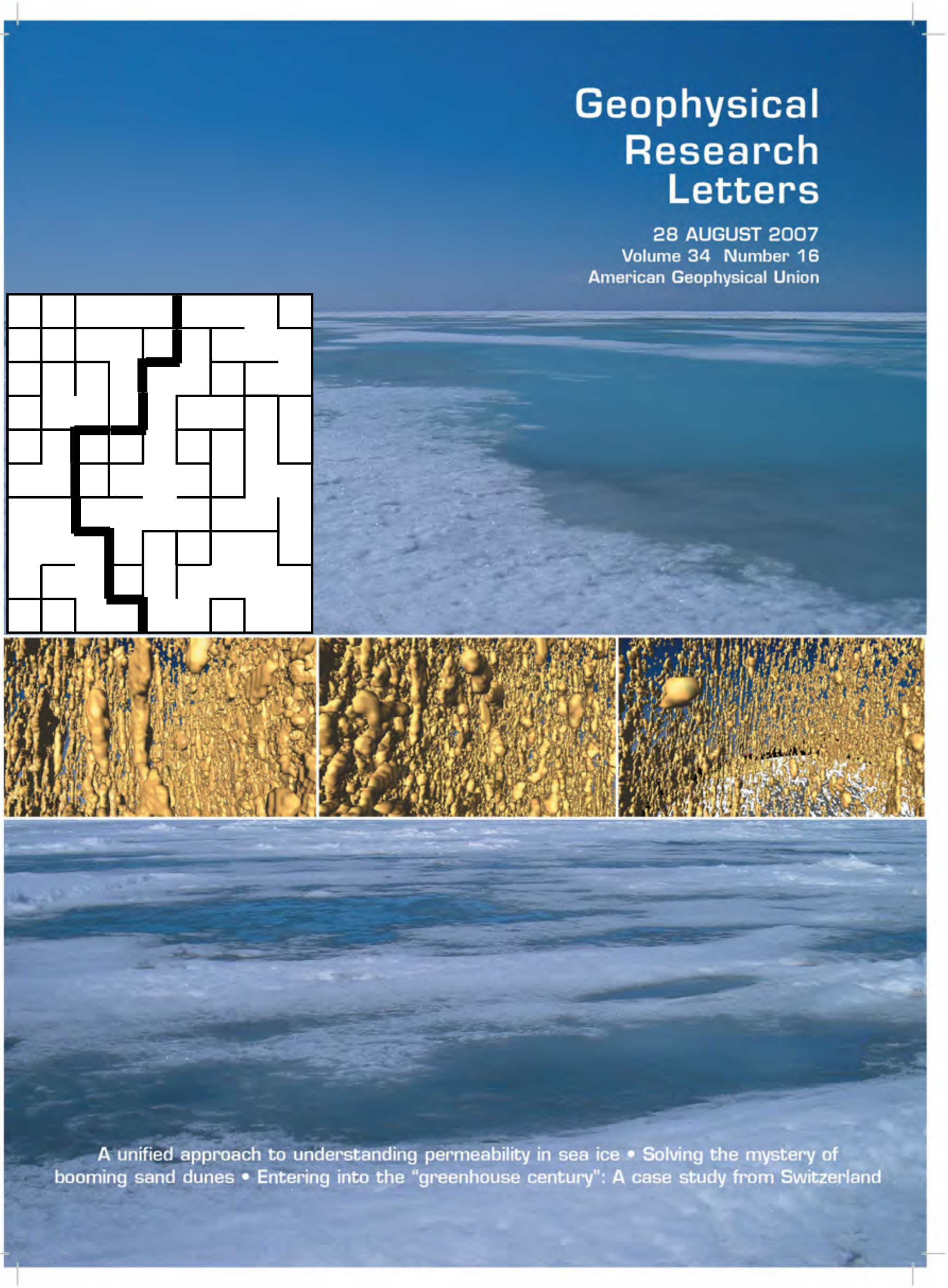
compressed powder

stealth

continuum percolation

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



Geophysical
Research
Letters

28 AUGUST 2007
Volume 34 Number 16
American Geophysical Union

A unified approach to understanding permeability in sea ice • Solving the mystery of booming sand dunes • Entering into the “greenhouse century”: A case study from Switzerland

micro-scale

controls

macro-scale

processes

percolation theory

$$k(\phi) = k_0(\phi - 0.05)^2$$

critical
exponent
t

$$k_0 = 3 \text{ x } 10^{-8} \text{ m}^2$$

hierarchical model
network model
rigorous bounds

agree closely with
field data

X-ray tomography for
brine inclusions

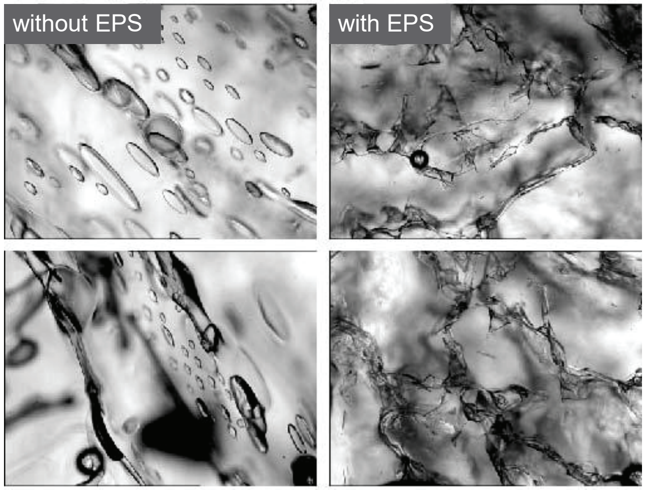
***unprecedented look
at thermal evolution
of brine phase and
its connectivity***

confirms rule of fives

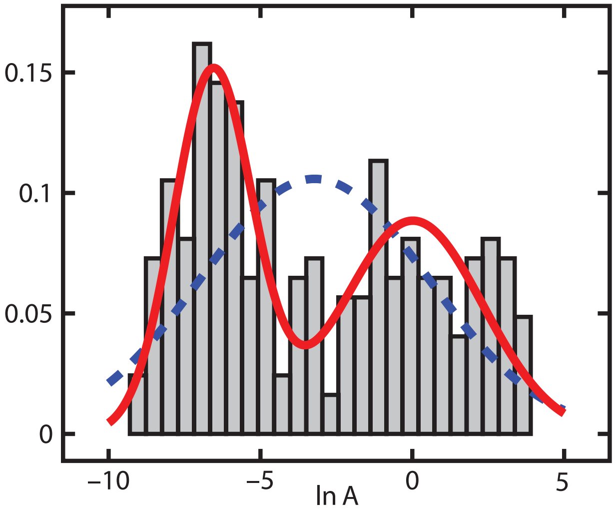
***Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009***

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

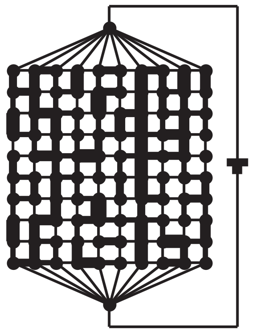
How does EPS affect fluid transport? **How does the biology affect the physics?**



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL

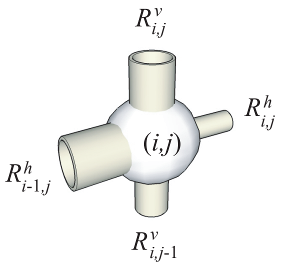


- 2D random pipe model with bimodal distribution of pore sizes
- **Rigorous bound on permeability *k*; results predict observed drop in *k***

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
***Multiscale Modeling and Simulation*, 2018**

3D extension, effect of EPS clogging, blockage

Anna Hyde, Jingyi Zhu, Ken Golden



Zhu, Jabini, Golden,
Eicken, Morris
***Ann. Glac.* 2006**

The Melt Pond Conundrum:

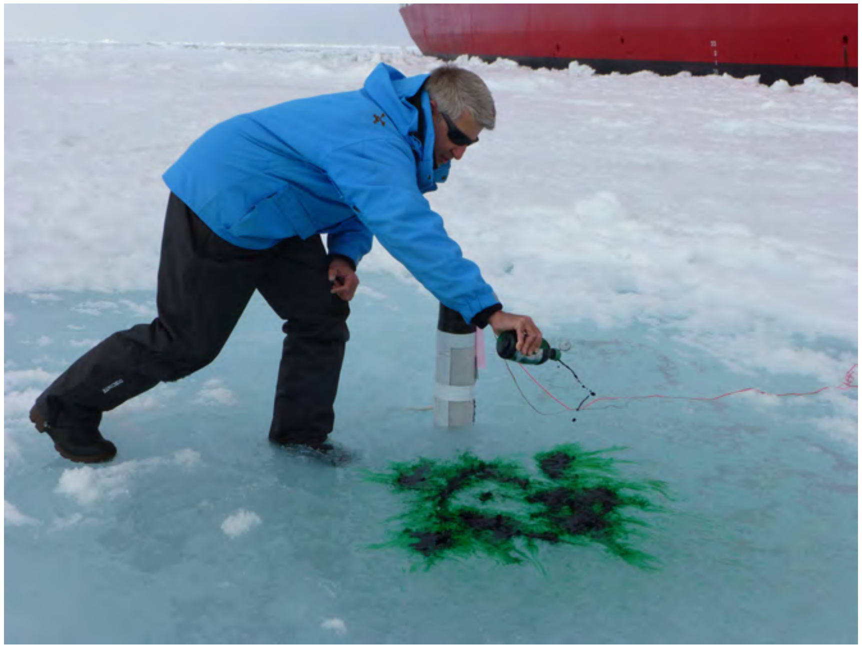
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy



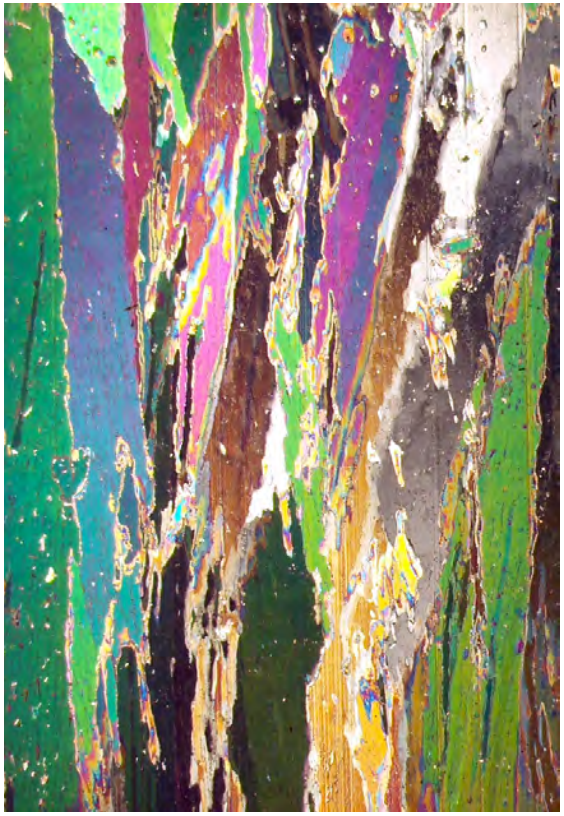
higher threshold for fluid flow in granular sea ice

microscale details impact "mesoscale" processes

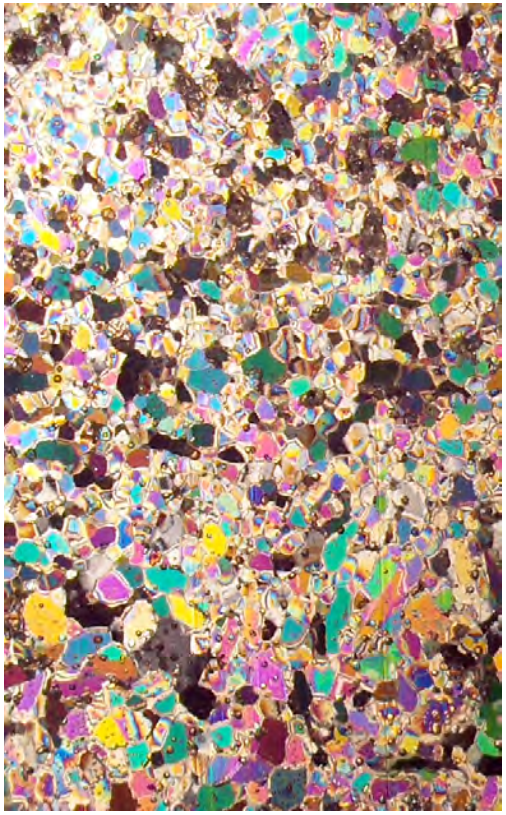
nutrient fluxes for microbes
melt pond drainage
snow-ice formation

columnar

5%



granular



10%

Golden, Sampson, Gully, Lubbers, Tison 2020

electromagnetically distinguishing ice types
Kitsel Lusted, Elena Cherkaev, Ken Golden

Remote sensing of sea ice

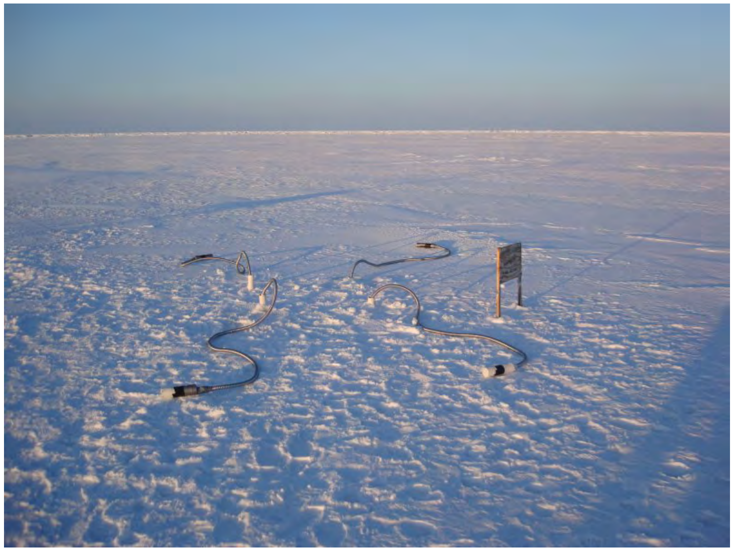
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice
properties from
electromagnetic
(EM) data

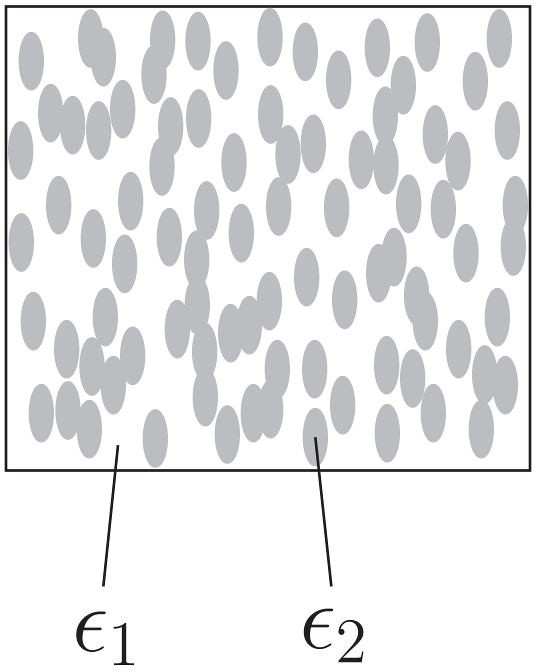
ε^*

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$$\Bigg\} \; \epsilon^*$$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \; , \text{ composite geometry } \right)$$

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method for Homogenization

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

separates geometry from parameters

geometry

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

material parameters

μ

- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n-point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$$

$\chi =$ characteristic function of the brine phase

resolvent

$$E = s\,(s + \Gamma\chi)^{-1} e_k$$

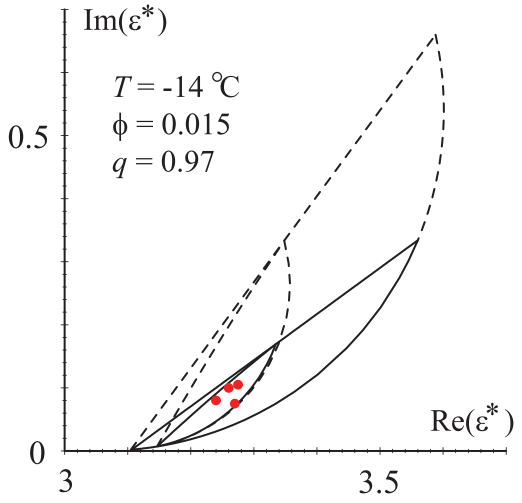
$\Gamma\chi$ **: microscale → macroscale**

$\Gamma\chi$ ***links scales***

Golden and Papanicolaou, Comm. Math. Phys. 1983

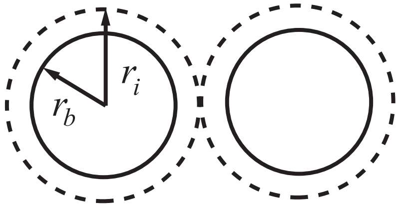
forward and inverse bounds on the complex permittivity of sea ice

forward bounds



4.75 GHz data

matrix particle

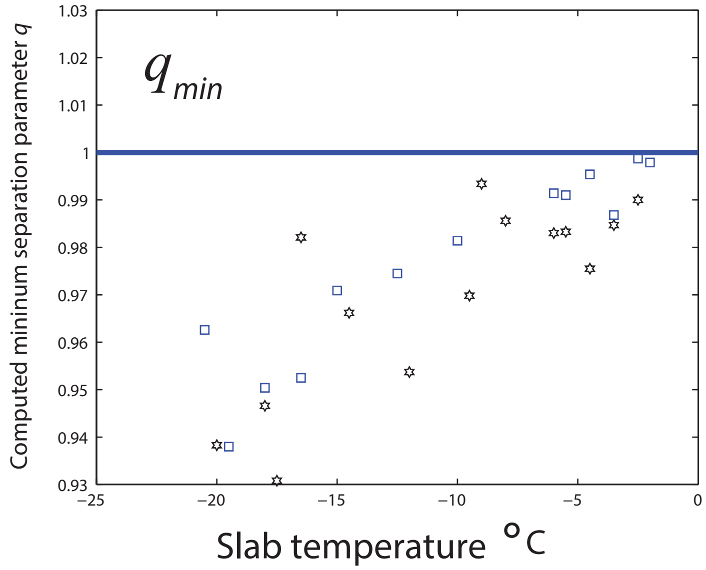


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



Inverse Homogenization

Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001), McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)

ε^* → ***composite geometry***
(spectral measure μ)

inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

rigorous inverse bound
on spectral gap

construct algebraic curves which bound
admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

direct calculation of spectral measures

Murphy, Hohenegger, Cherkaev, Golden, *Comm. Math. Sci.* 2015

- depends only on the composite geometry
- discretization of microstructural image gives binary network
- fundamental operator becomes a random matrix
- spectral measure computed from eigenvalues and eigenvectors

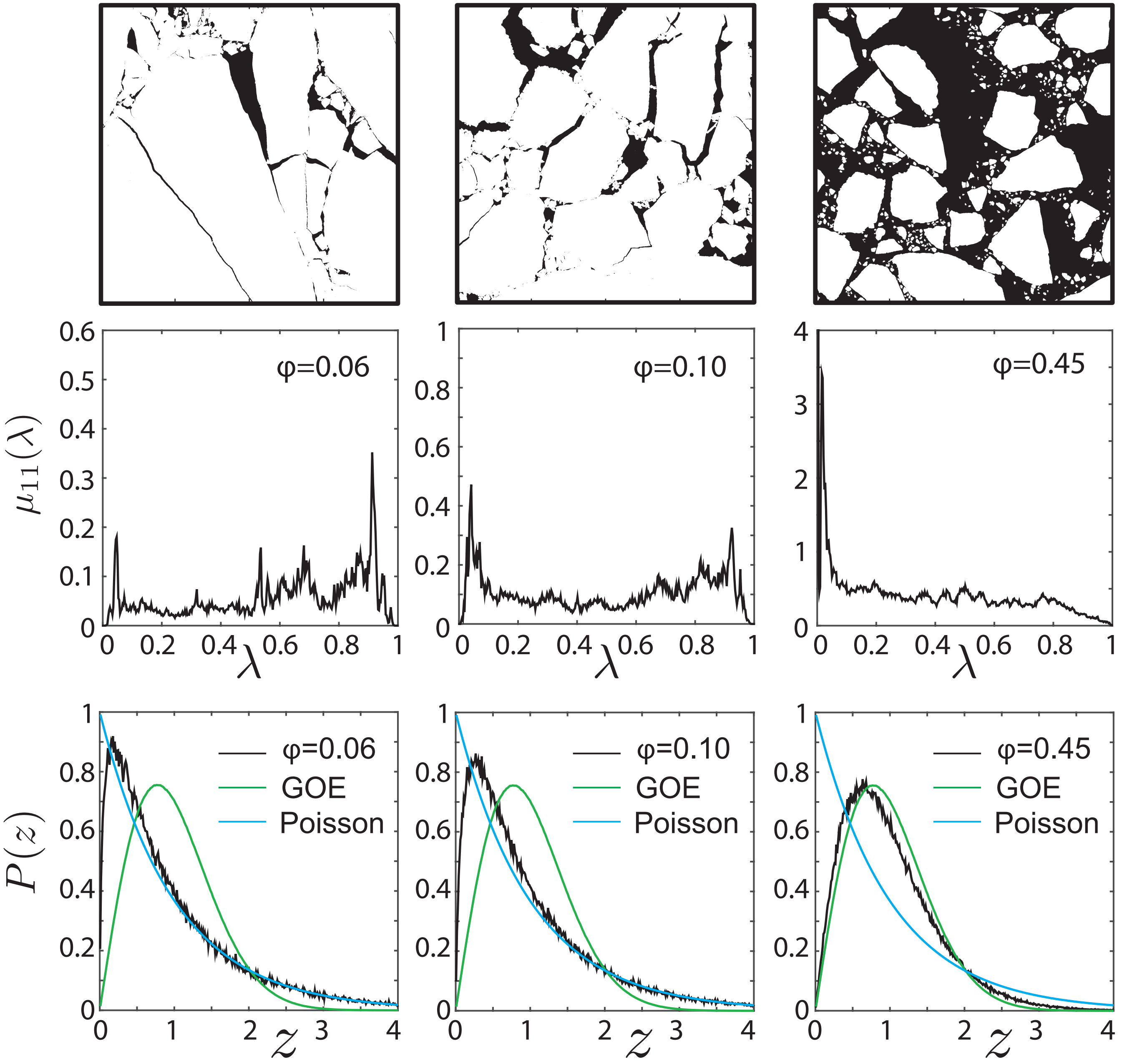
once we have the spectral measure μ it can be used in Stieltjes integrals for other transport coefficients:

electrical and thermal conductivity, complex permittivity, magnetic permeability, diffusion, fluid flow properties

earlier studies of spectral measures

Day and Thorpe 1996
Helsing, McPhedran, Milton 2011

Spectral computations for sea ice floe configurations



spectral measures

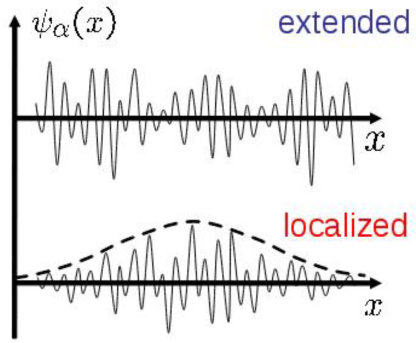
eigenvalue spacing distributions

UNIVERSAL Wigner-Dyson distribution

uncorrelated → **level repulsion**

ANDERSON TRANSITION

Murphy, Cherkaev, Golden
Phys. Rev. Lett. 2017



metal / insulator transition

localization

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

Anderson transition in wave physics:
quantum, optics, acoustics, water waves, ...

from analysis of spectral measures for brine, melt ponds, ice floes

we find percolation-driven

Anderson transition for classical transport in composites

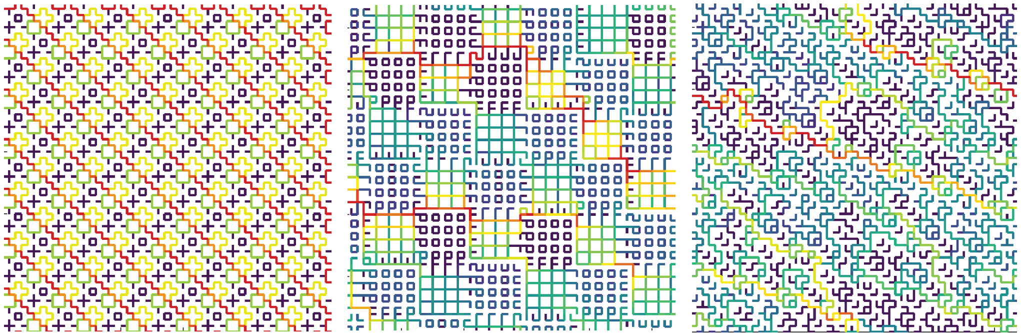
mobility edges, localization transition, universal spectral statistics

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

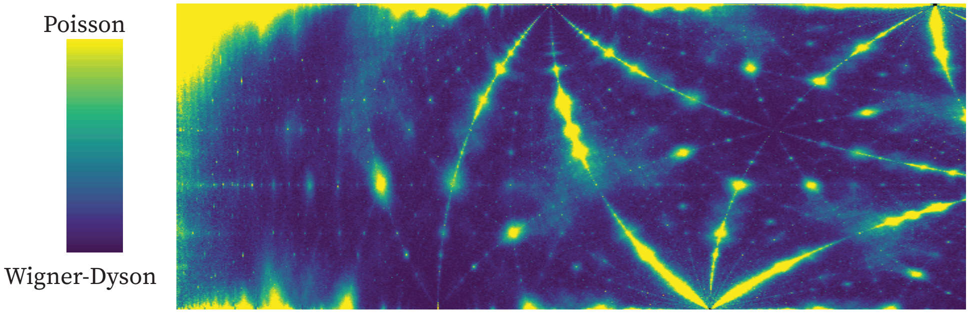
Order to disorder in quasiperiodic materials

Morison, Murphy, Cherkaev, Golden 2020

Parameterized Moiré Pattern Creates Tunable Microgeometry



Anderson transition as QP is tuned

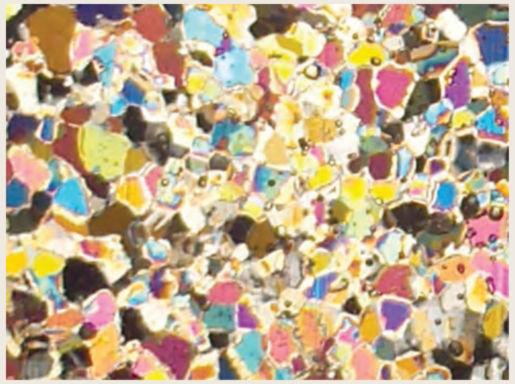


Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

Proc. Roy. Soc. A 8 Feb 2015

- **Stieltjes integral representation for effective complex permittivity**
  Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
  orientation statistics
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



Proc. R. Soc. A | Volume 471 | Issue 2174 | 8 February 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A

350 YEARS
OF SCIENTIFIC
PUBLISHING

An invited review commemorating 350 years of scientific publishing at the Royal Society

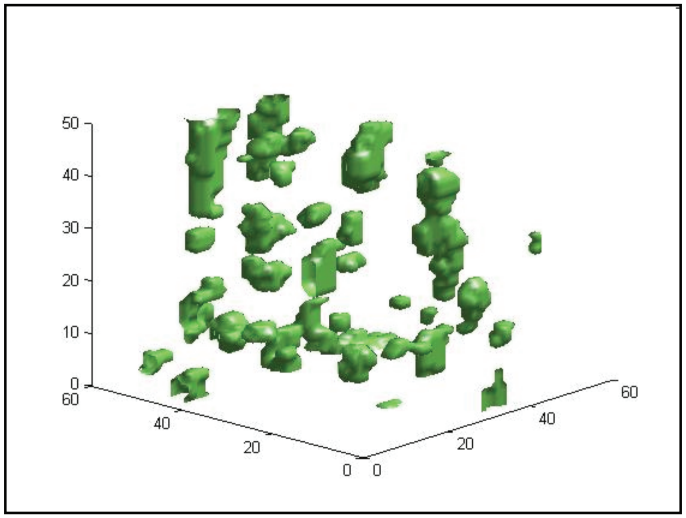
A method to distinguish between different types of sea ice using remote sensing techniques

A computer model to determine how a human should walk so as to expend the least energy

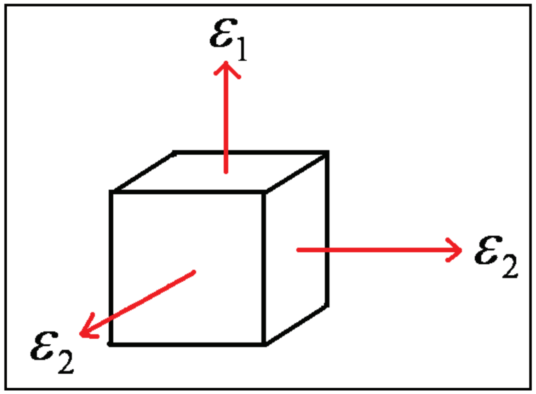
THE
ROYAL
SOCIETY
PUBLISHING

21/01/15 1:47 PM

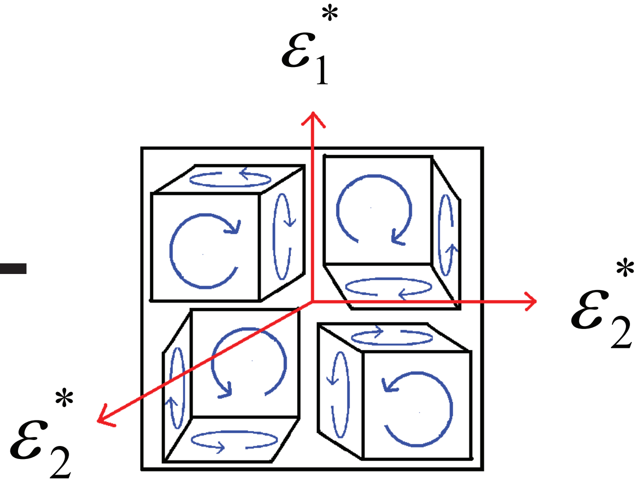
two scale homogenization for polycrystalline sea ice



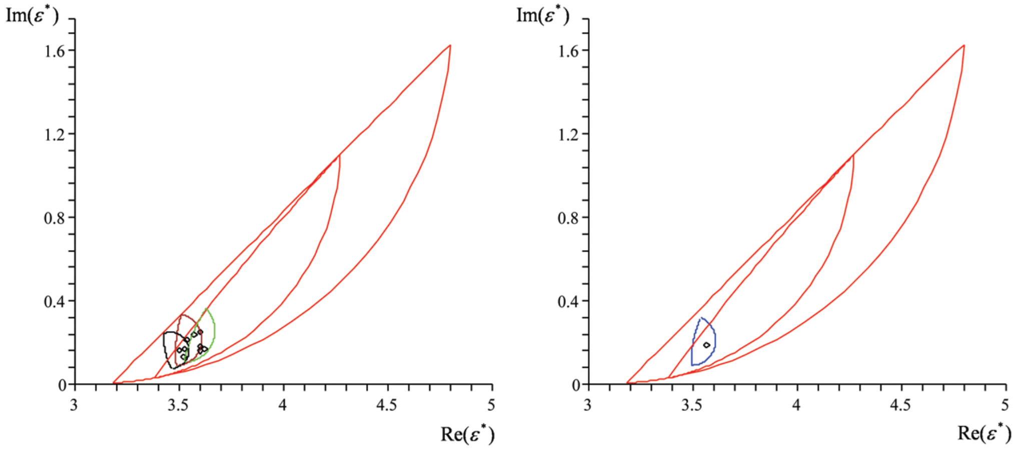
numerical homogenization for single crystal



analytic continuation for polycrystals



bounds



Gully, Lin, Cherkaev, Golden, Proc. Roy. Soc. A (and cover) 2015

Rigorous bounds on the complex permittivity tensor of sea ice with polycrystalline anisotropy within the horizontal plane

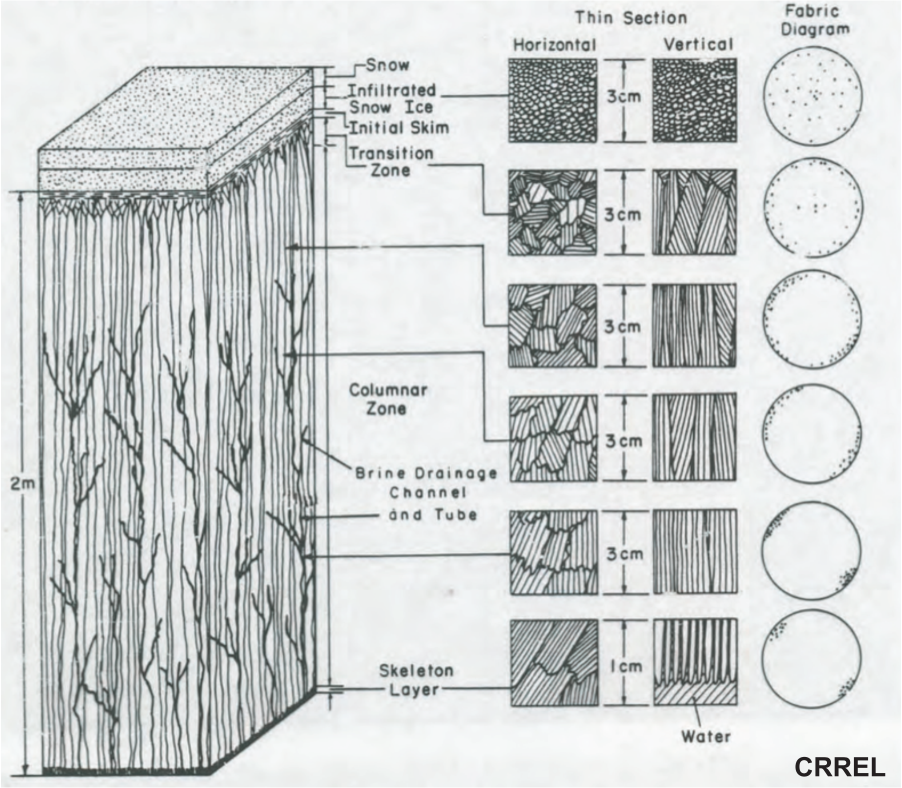
Kenzie McLean, Elena Cherkaev, Ken Golden 2020

motivated by

Weeks and Gow, *JGR* 1979: c-axis alignment in Arctic fast ice off Barrow

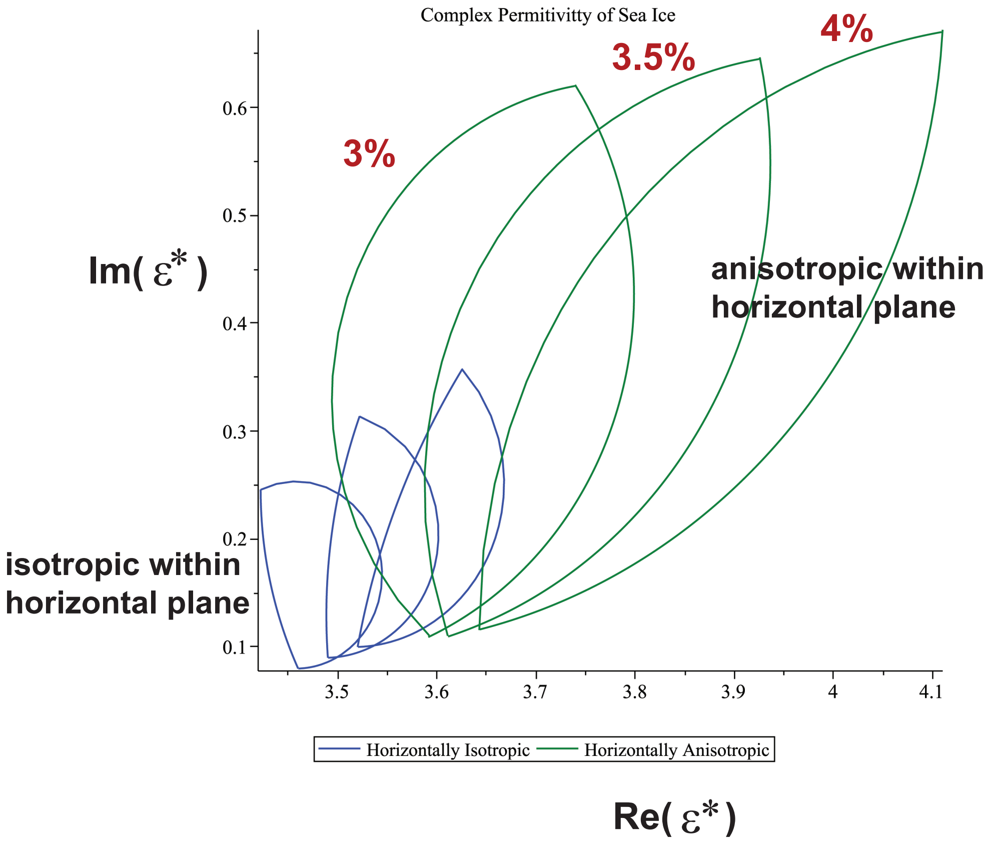
Golden and Ackley, *JGR* 1981: radar propagation model in aligned sea ice

input: orientation statistics



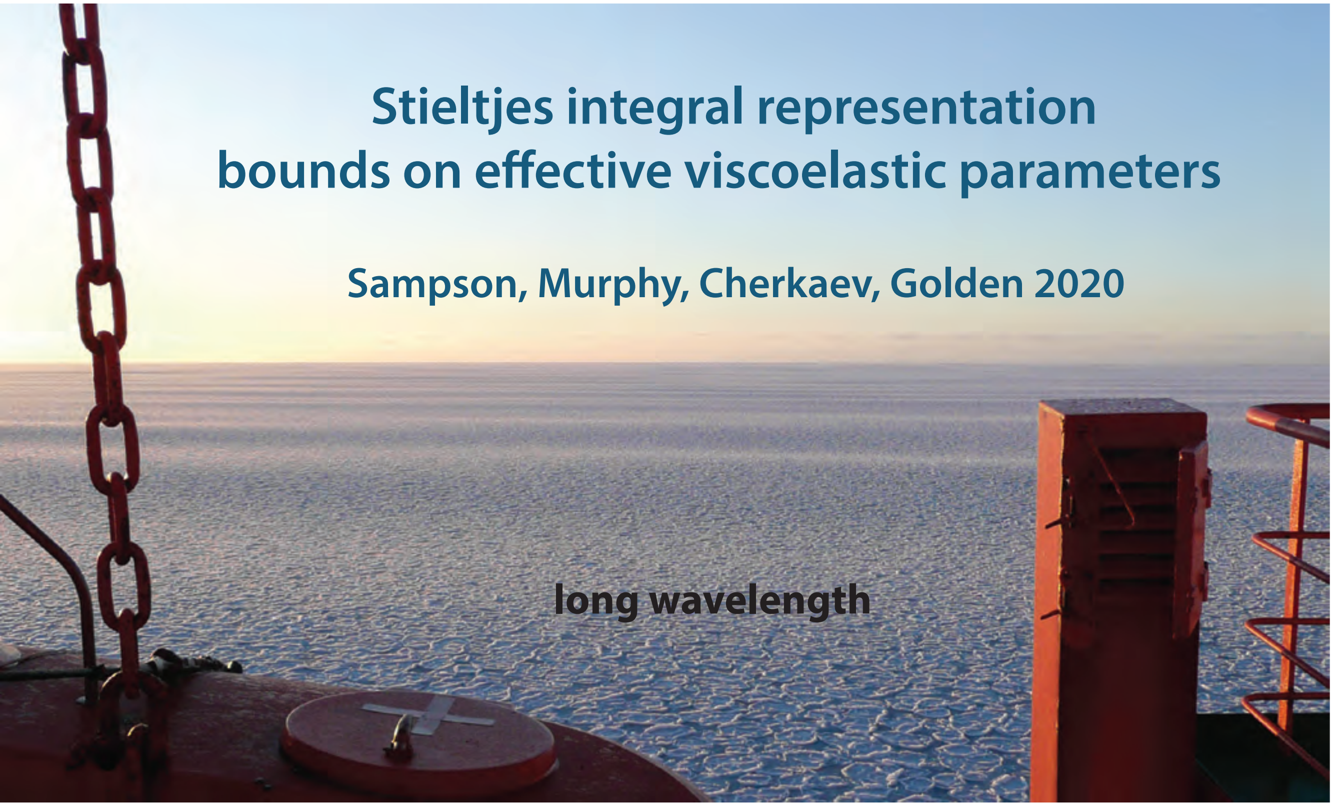
CRREL

output: bounds



mesoscale

wave propagation in the marginal ice zone



Stieltjes integral representation bounds on effective viscoelastic parameters

Sampson, Murphy, Cherkaev, Golden 2020

long wavelength

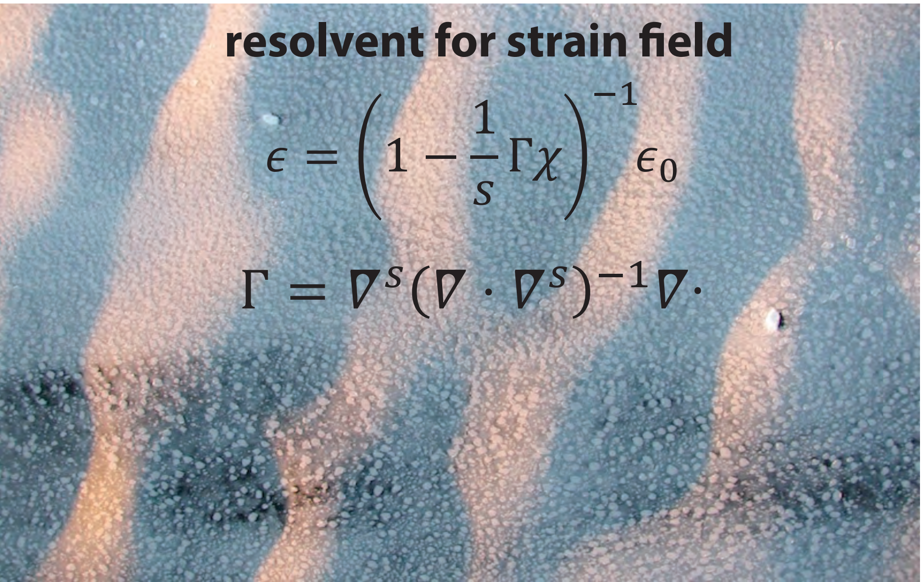
$$\langle\sigma_{ij}\rangle = C^*_{ijkl}\langle\epsilon_{kl}\rangle$$

ϵ_0 **avg strain**

$$C^*_{ijkl} = \nu^*\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right) = \nu^*\lambda_s$$

$$F(s) = 1 - \frac{\nu^*}{\nu_2} \qquad s = \frac{1}{1 - \frac{\nu_1}{\nu_2}}$$

$$F(s) = ||\epsilon_0||^{-2}\int_\Sigma \frac{d\mu(\lambda)}{s - \lambda}$$



resolvent for strain field

$$\epsilon = \left(1 - \frac{1}{s}\Gamma\chi\right)^{-1}\epsilon_0$$

$$\Gamma = \nabla^s(\nabla\cdot\nabla^s)^{-1}\nabla\cdot$$

local

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl}$$

quasistatic

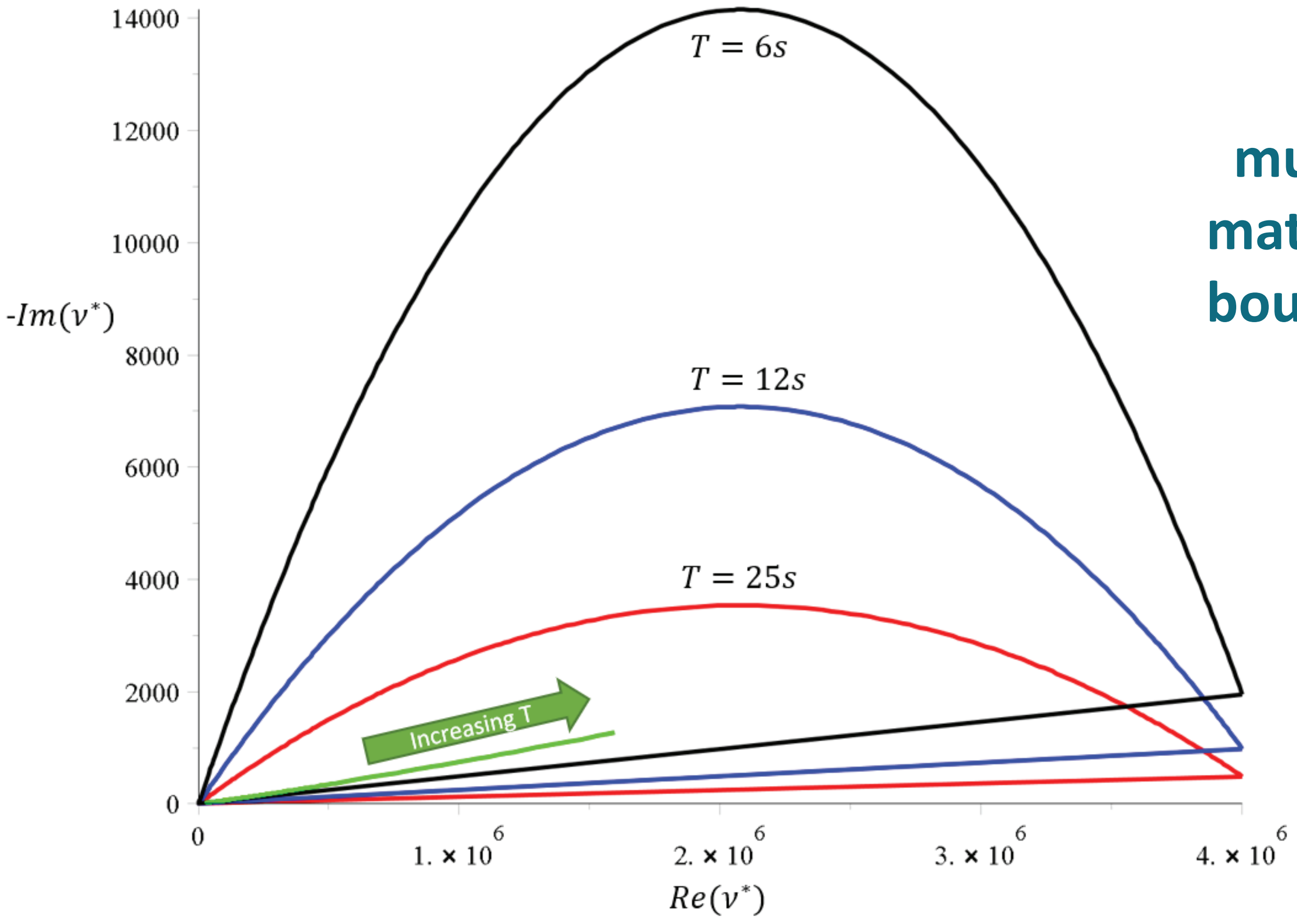
$$\nabla\cdot\sigma = 0$$

bounds on the effective complex viscoelasticity

complex elementary bounds
(fixed area fraction of floes)

$\nu_1 = 10^7 + i\,4875$ pancake ice

$\nu_2 = 5 + i\,0.0975$ slush / frazil

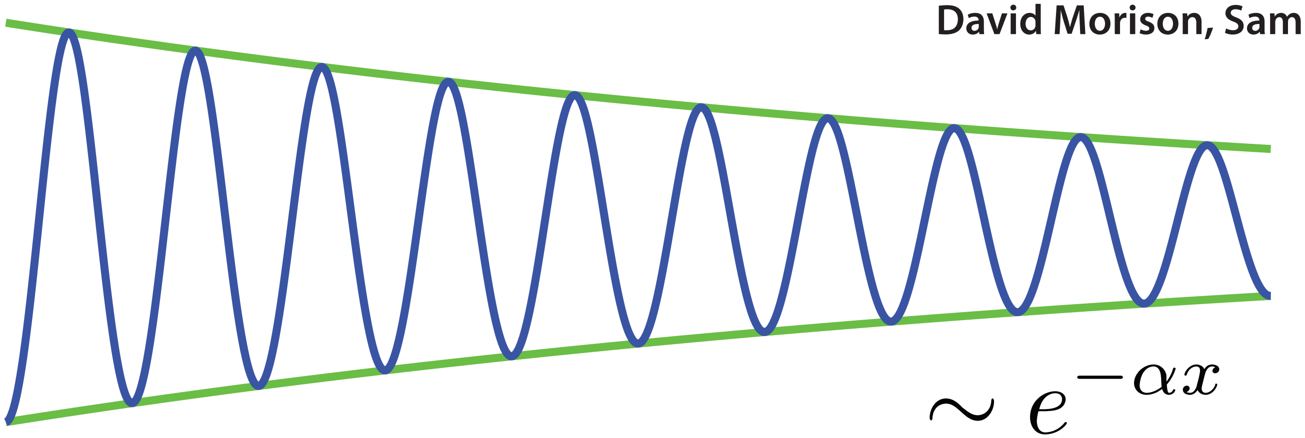


\+
much tighter
matrix particle
bounds + data

Sampson, Murphy, Cherkaev, Golden 2020

1D Model of Ocean Surface Wave Attenuation in Sea Ice

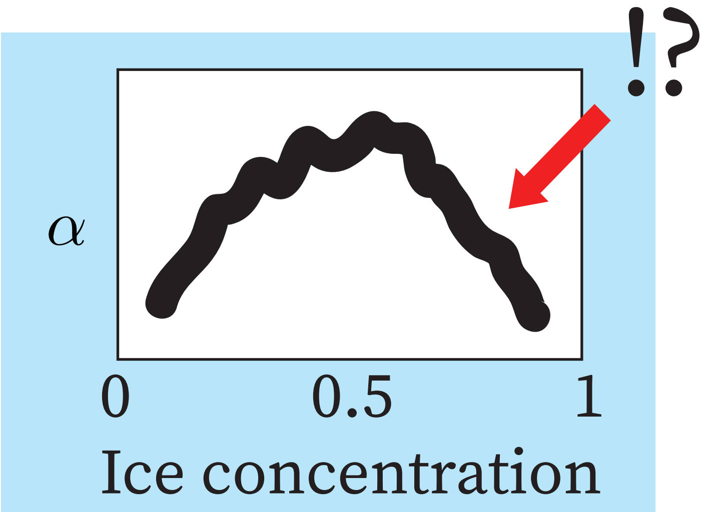
David Morison, Samir Suthar, Elena Cherkaev, Ken Golden



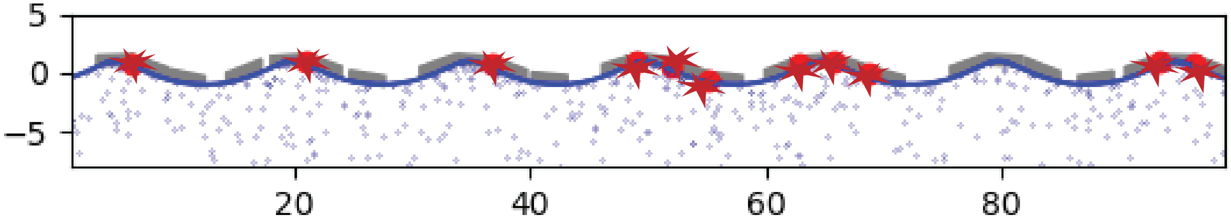
$\sim e^{-\alpha x}$

Observed in Ross Sea by
F. Montiel, T. Milne,
A. Kohout and L. Roach
Presented at KOZWaves 2020

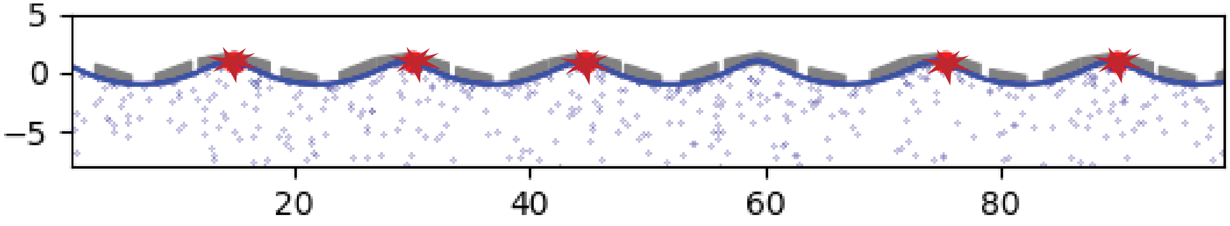
Simulate
- Airy waves
- Form drag
- Ice floe collisions



If ice concentration is high, randomly placed floes collide ✶ in groups of three or more.



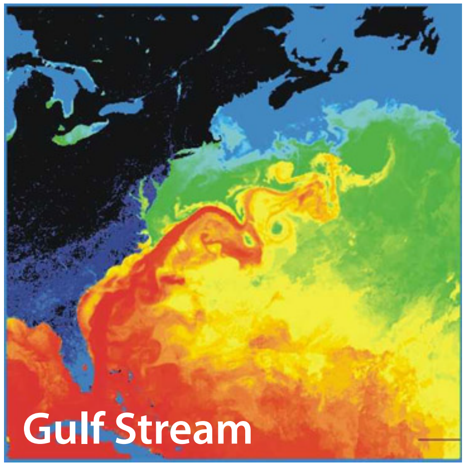
Floe spacing becomes more uniform and pairwise collisions predominate.

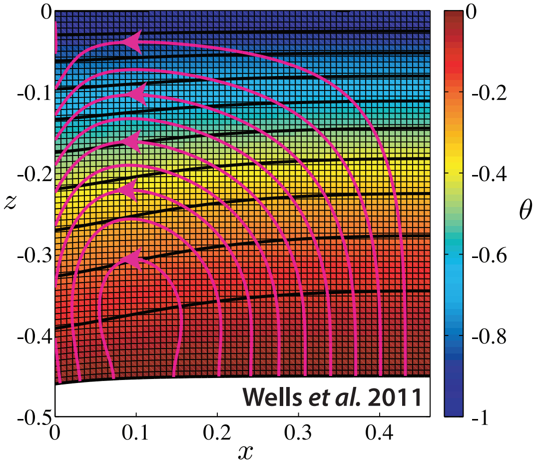


advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice
heat transport in sea ice with convection
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

$\big\downarrow$ ***homogenize***

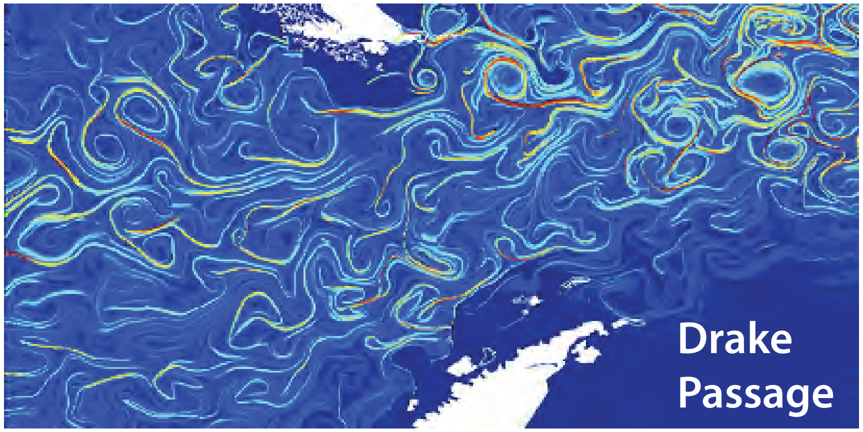
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity***

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017
Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020



Stieltjes Integral Representation for Advection Diffusion

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020

$$\kappa^* = \kappa\left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2+\tau^2}\right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2+\tau^2}$$

- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H\Gamma$
- $H =$ stream matrix , $\kappa =$ local diffusivity
- $\Gamma := -\nabla(-\Delta)^{-1}\nabla\cdot$, Δ is the Laplace operator
- $i\Gamma H\Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ-plane

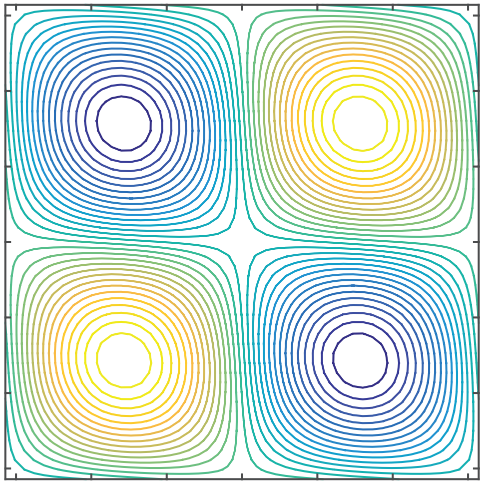
rigorous framework for numerical computations of spectral measures and effective diffusivity for model flows

new integral representations, theory of moment calculations

separation of material properties and flow field

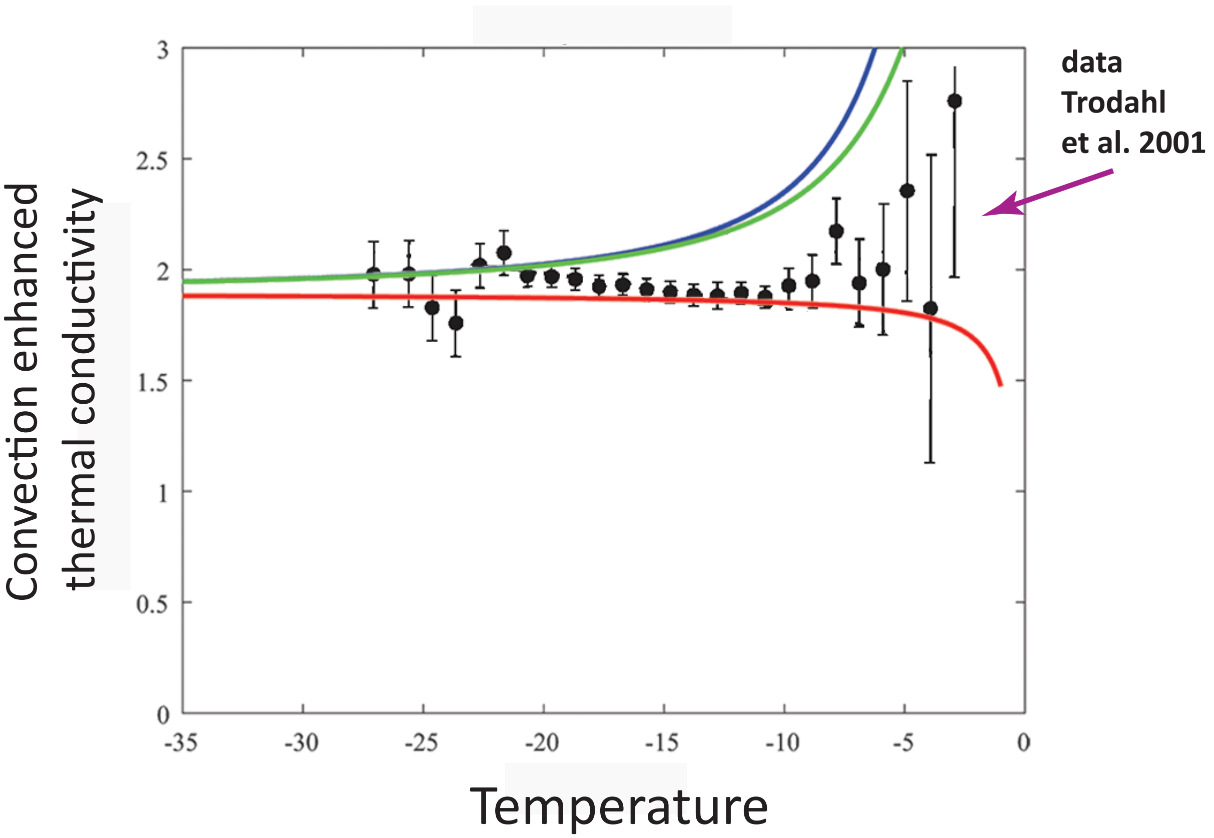
Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Murphy, Zhu, Cherkaev, Strong, Golden 2020



cat’s eye flow model for
brine convection cells

similar bounds
for shear flows



rigorous Padé bounds from Stieltjes integral +
analytical calculations of moments of measure

rigorous bounds assuming information on flow field INSIDE inclusions

Kraitzman, Cherkaev, Golden
***SIAM J. Appl. Math.* (in revision), 2020**

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

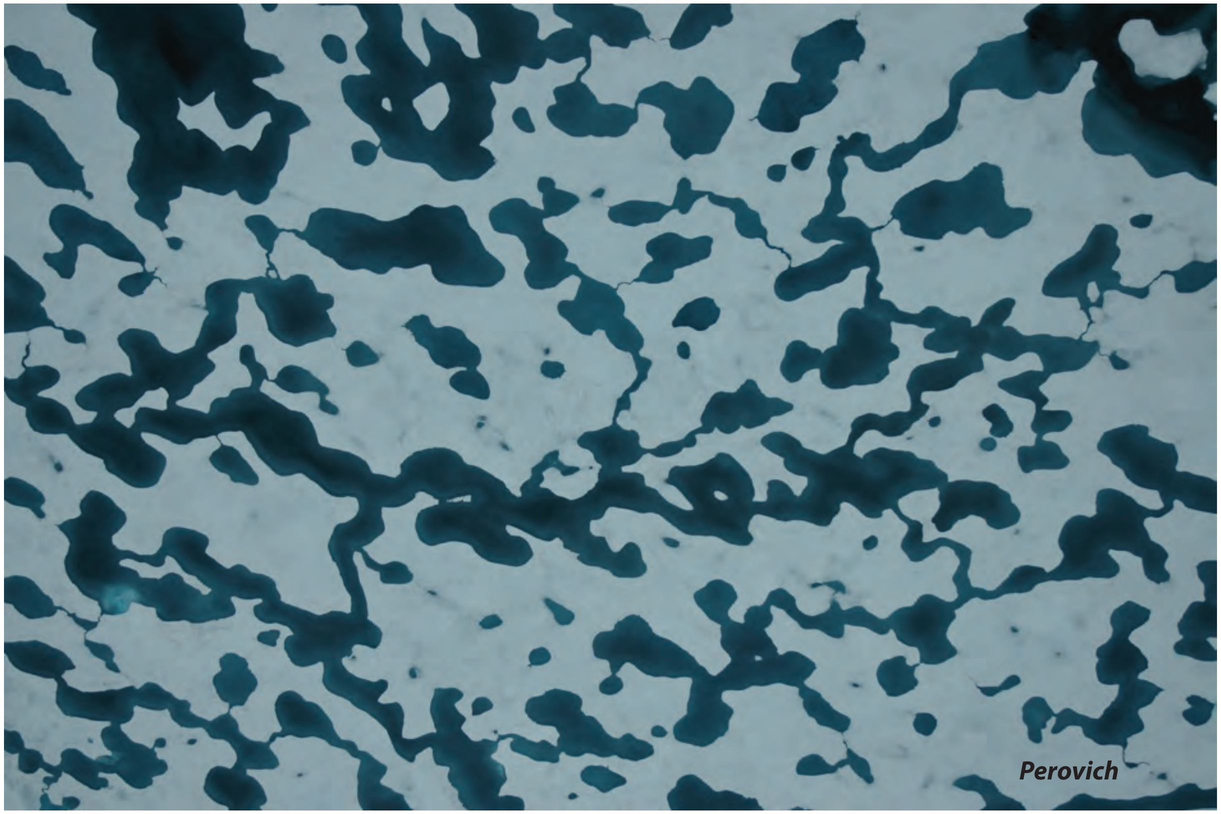
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

Flocco, Feltham, Hunke 2012



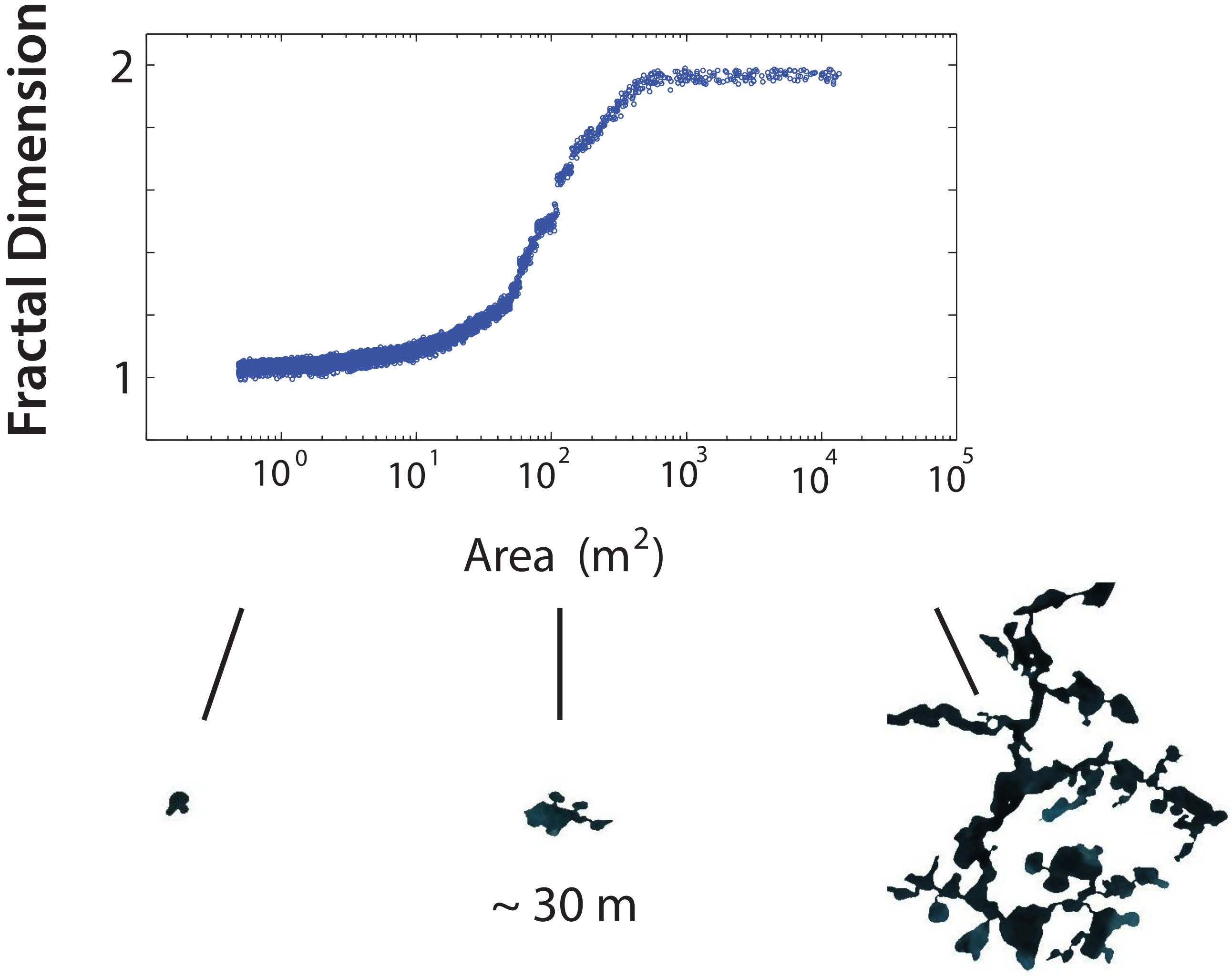
Are there universal features of the evolution similar to phase transitions in statistical physics?

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale

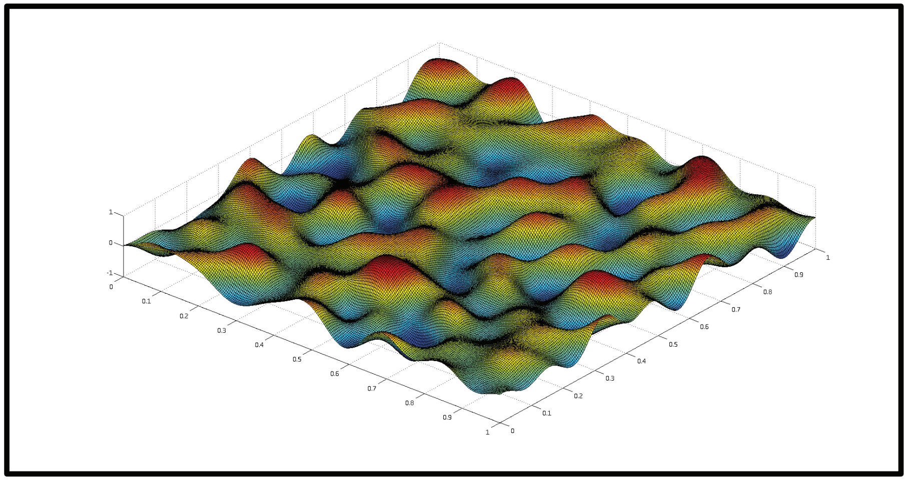


simple pond ***transitional pond*** ***complex pond***

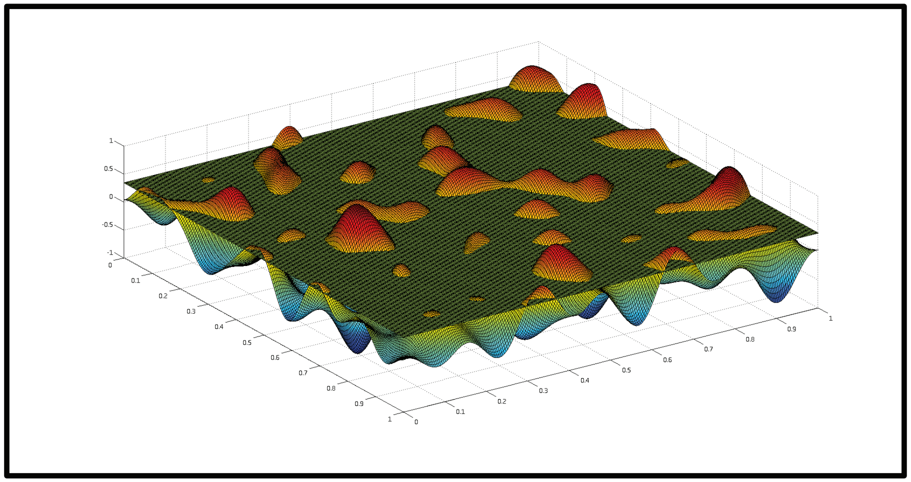
Continuum percolation model for melt pond evolution

level sets of random surfaces

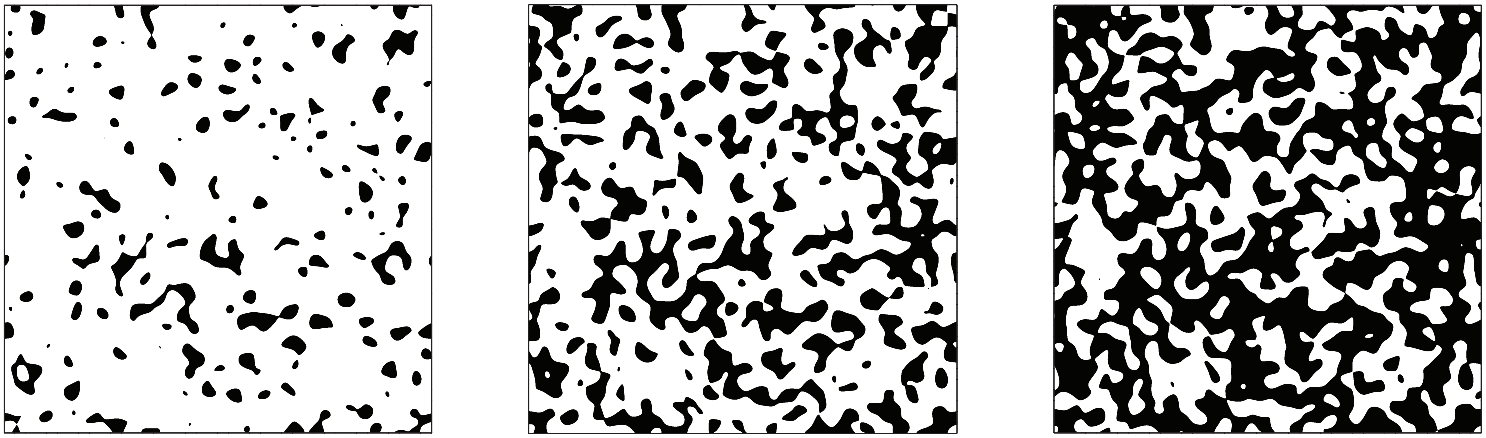
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

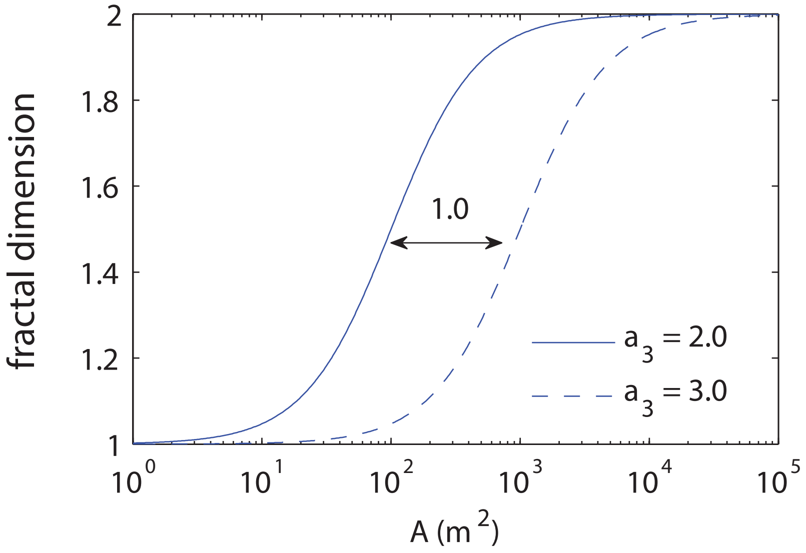


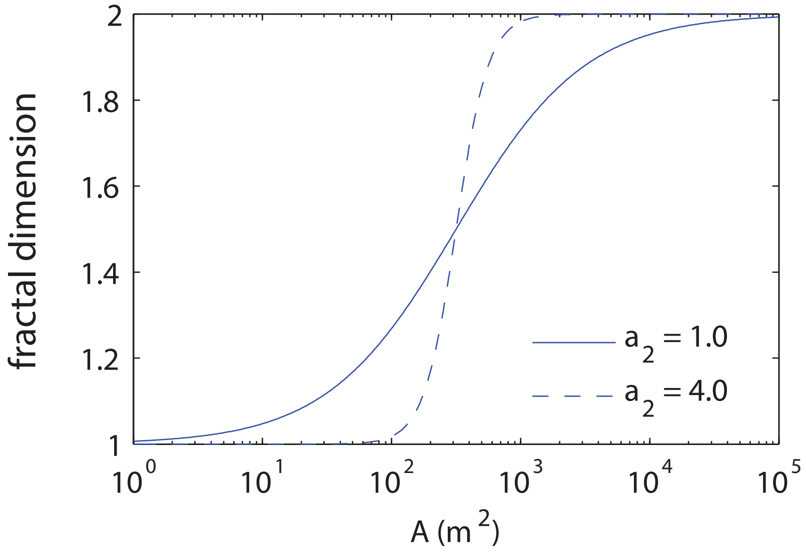
intersections of a plane with the surface define melt ponds



electronic transport in disordered media   *diffusion in turbulent plasmas*   *Isichenko, Rev. Mod. Phys., 1992*

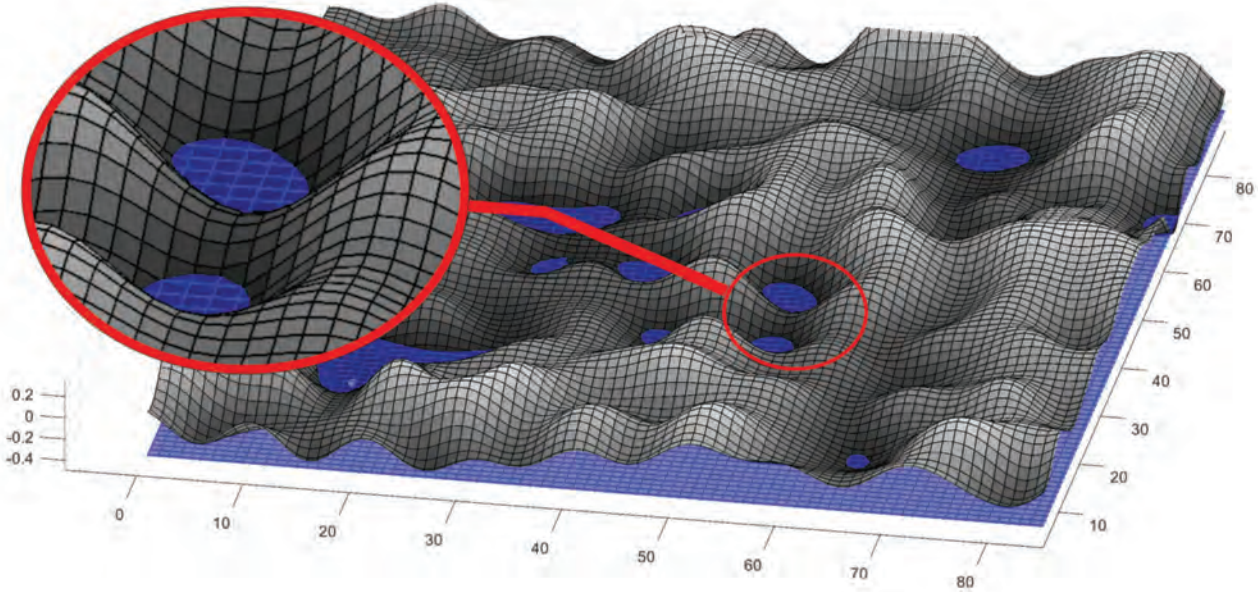
fractal dimension curves depend on statistical parameters defining random surface

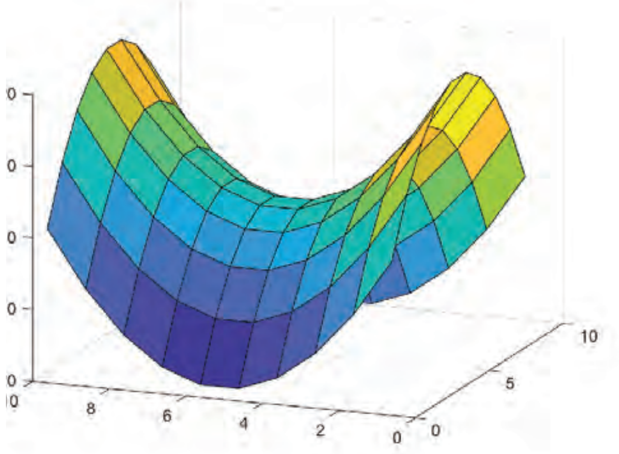




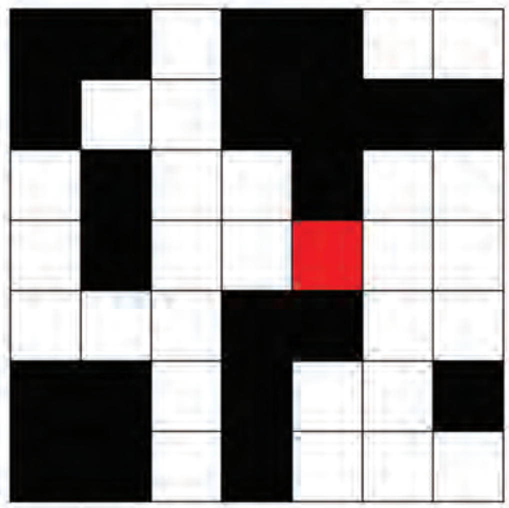
Saddle Points, Morse Theory and the Fractal Geometry of Melt Ponds

Ryleigh Moore, Jacob Jones, Dane Gollero, Court Strong, Ken Golden 2020

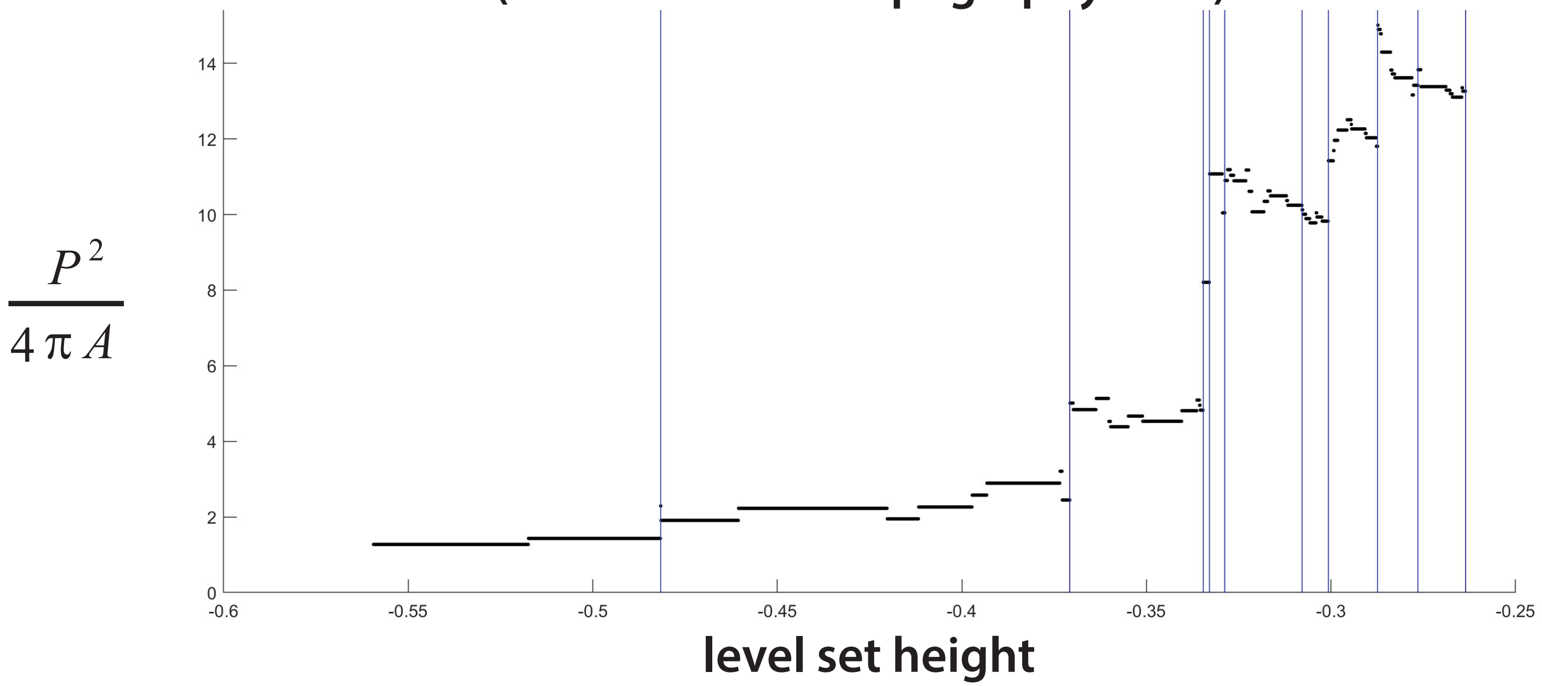




- Ponds connect through saddle points (Morse Theory).
- Red bond in lattice percolation theory ~ saddle point.



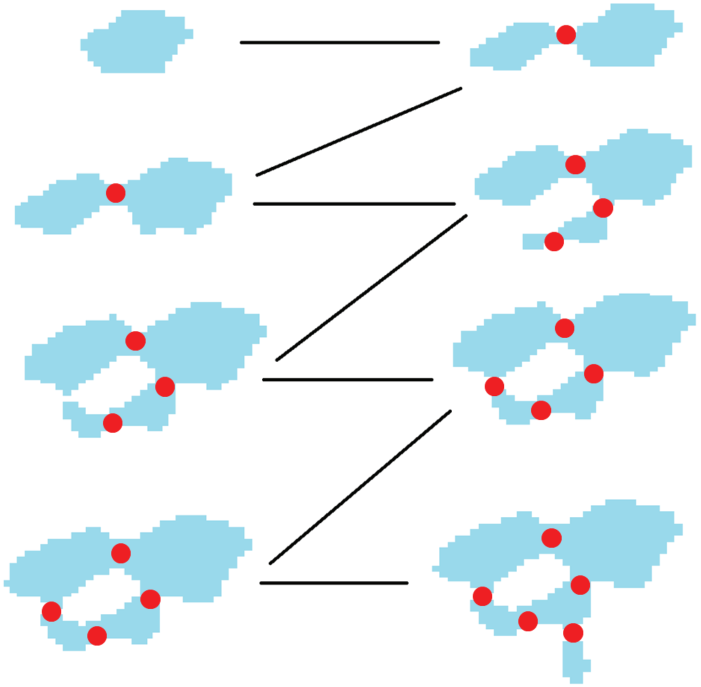
Evolution of Isoperimetric Quotient with Melt Pond Growth
(from real snow topography data)



In the graph, we follow a single pond's growth. The vertical lines denote when the pond goes through a saddle point.

We see that large jumps in fractal dimension occur through saddle points.

pond coalescence and thickening



Ryleigh Moore
Department of Mathematics
University of Utah

Multidisciplinary drifting Observatory for the Study of Arctic Climate (MOSAiC)

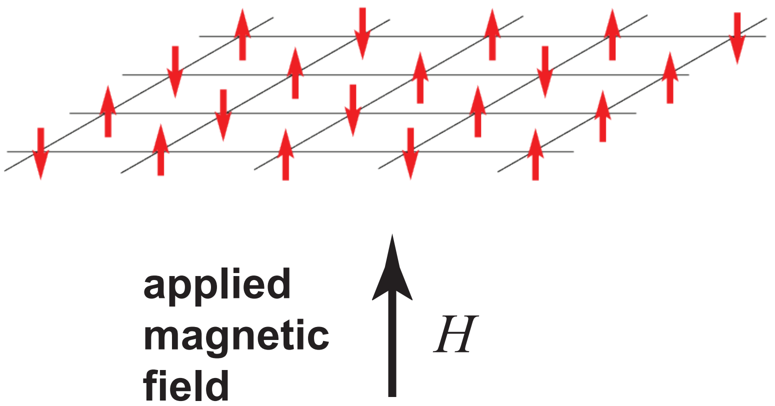
MOSAiC School
aboard the icebreaker ***RV Akademik Federov***

September 20 - October 28, 2019

20 grad students from around the world
(3 from U.S., 1 mathematician)

Ryleigh led successful installation of three seasonal ice mass balance (SIMB3) buoys in the Central Arctic.

Ising Model for a Ferromagnet



$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

blue

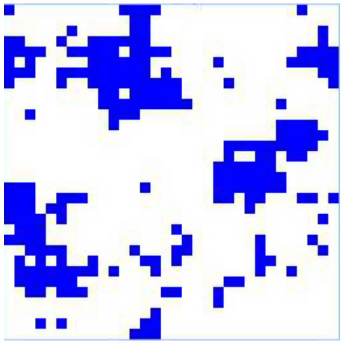
white

$$\mathcal{H} = -H \sum_i s_i - J \sum_{<i,j>} s_i s_j$$

nearest neighbor Ising Hamiltonian

ferromagnetic interaction $J \geq 0$

islands of like spins

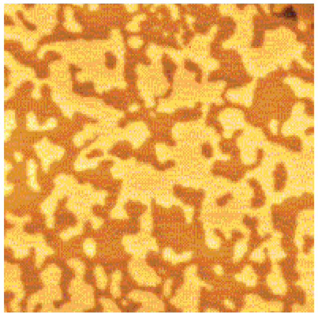


magnetization

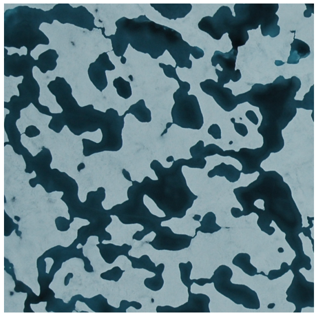
homogenized parameter like effective conductivity

$$M(T,H) = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

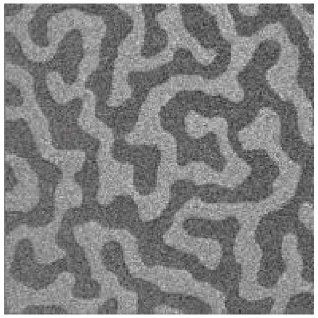
energy is lowered when nearby spins align with each other, forming **magnetic domains**



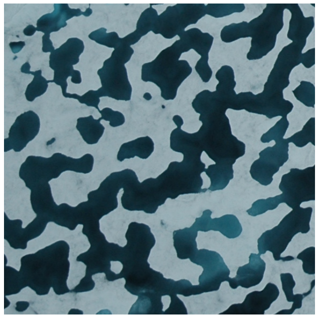
magnetic domains in cobalt



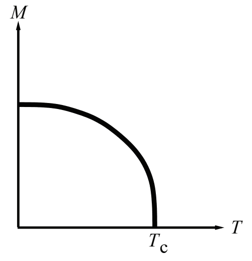
melt ponds (Perovich)



magnetic domains in cobalt-iron-boron



melt ponds (Perovich)



Curie point critical temperature

Ising model for ferromagnets ⟶ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

$$\mathcal{H} = -\sum_{i}^{N} H_i\, s_i - J \sum_{<i,j>}^{N} s_i s_j \qquad s_i = \begin{cases} \uparrow \quad +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow \quad -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

random magnetic field
represents snow topography

magnetization M

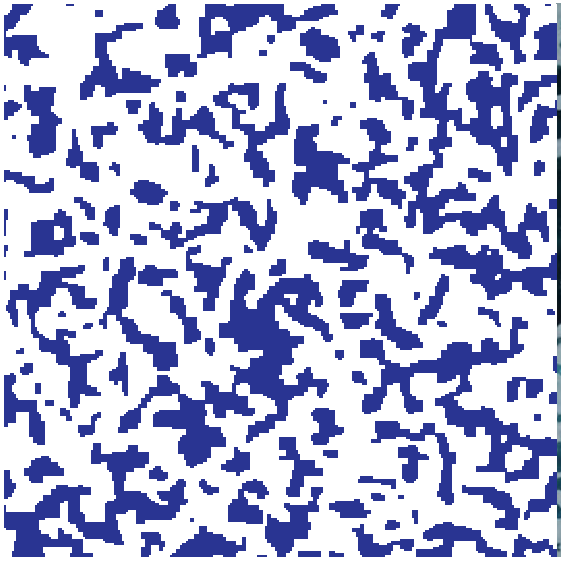
pond area fraction
~ albedo

$$F = \frac{(M+1)}{2}$$

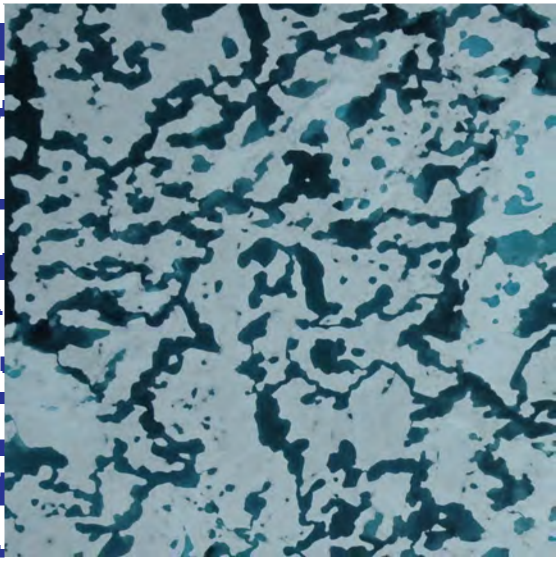
only nearest neighbor
patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

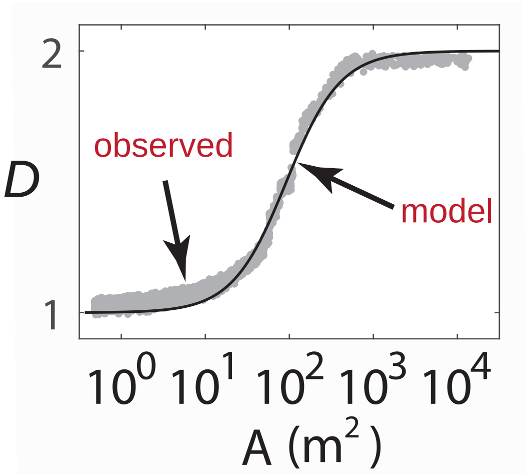
Order from Disorder



Ising model



melt pond photo (Perovich)



pond size
distribution exponent

observed -1.5
(Perovich, *et al.* 2002)

model -1.58

Scientific American
EOS, PhysicsWorld, ...

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data

The effect of melt pond geometry on the distribution of solar energy under first-year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, *Geophys. Res. Lett.* 2020

- Model for 3D light field under ponded sea ice.
- Distribution of solar energy at depth influenced by ***shape and connectivity*** of melt ponds, as well as area fraction.
- Aggregate properties of the sub-ice light field, such as a significant enhancement of available solar energy under the ice, are controlled by parameter closely related to pond fractal geometry.
- Model and analysis explain how melt pond geometry ***homogenizes*** under-ice light field, affecting habitability.

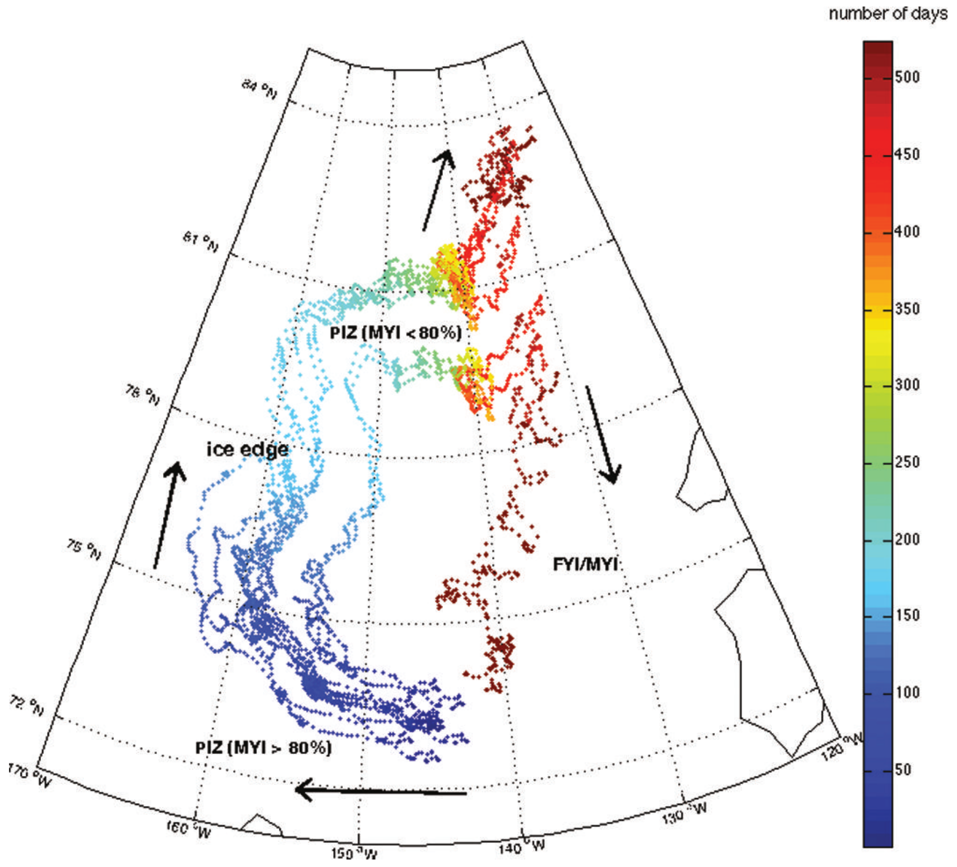
Pond geometry affects ecology and partitioning of solar energy in the upper Arctic Ocean.

macroscale

Anomalous diffusion in sea ice dynamics

Ice floe diffusion in winds and currents

Jennifer Lukovich, Jennifer Hutchings,
David Barber, *Ann. Glac.* 2015



- On short time scales floes observed (buoy data) to exhibit Brownian-like behavior, but they are also being advected by winds and currents.
- Effective behavior is purely diffusive, sub-diffusive or super-diffusive depending on ice pack and advective conditions - Hurst exponent.

Floe Scale Model of Anomalous Diffusion in Sea Ice Dynamics

Huy Dinh, Elena Cherkaev, Court Strong, Ken Golden 2020

$$\left\langle \left| \mathbf{x}(t) - \mathbf{x}(0) - \langle \mathbf{x}(t) - \mathbf{x}(0) \rangle \right|^2 \right\rangle \sim t^\alpha$$

$\alpha =$ Hurst exponent, a measure of anomalous diffusion.

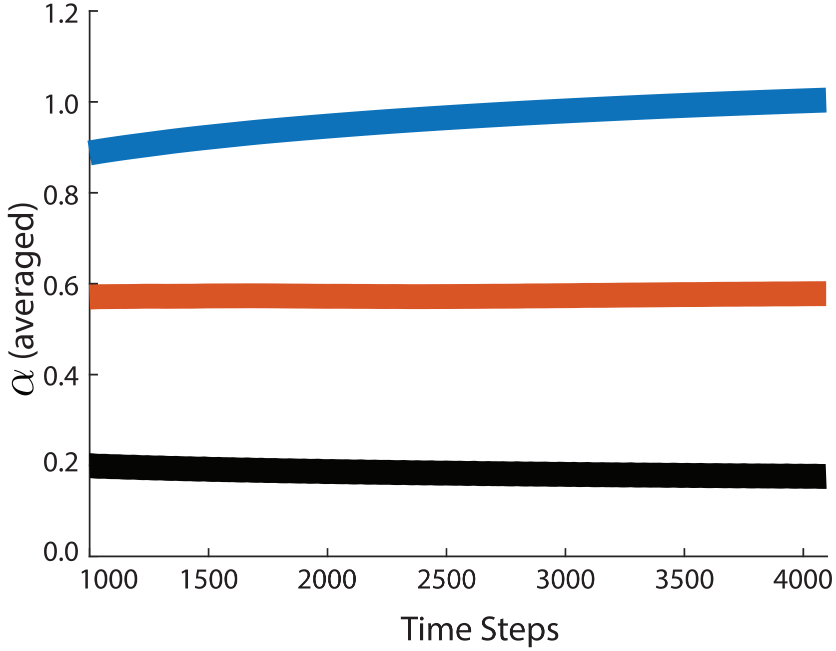
Measured from bouy position data. Detects ice pack crowding and advective forcing.

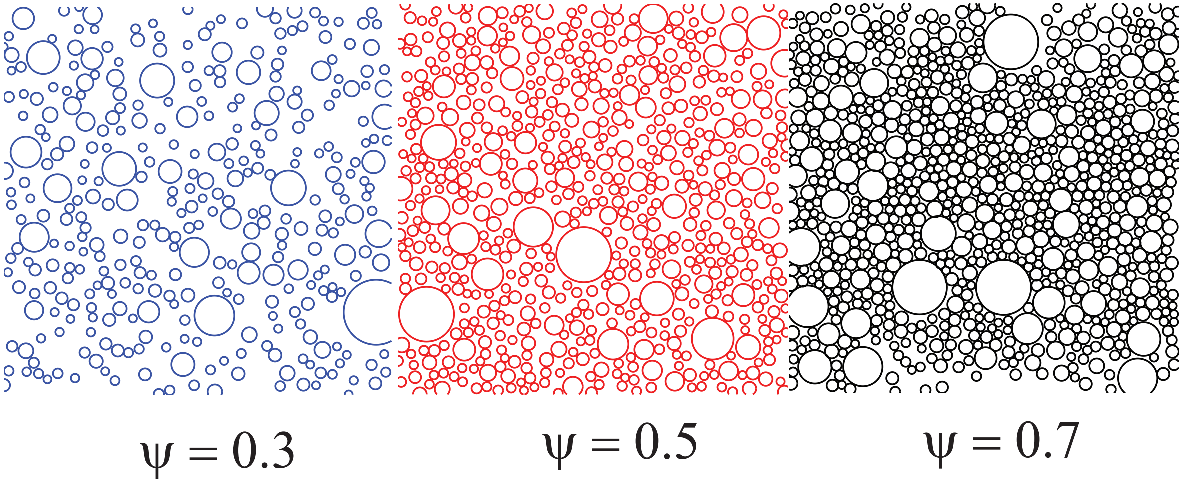
J.V. Lukovich, J.K. Hutchings, D.G. Barber *Annals of Glaciology* **2015**

diffusive	$\alpha = 1$	Sparse packing, uncorrelated advective field.
sub-diffusive	$\alpha < 1$	Dense packing, crowding dominates advection.
super-diffusive	$\alpha = 5/4$	Sparse packing, shear dominates advection.
	$\alpha = 5/3$	Sparse packing, vorticity dominates advection.

Goal: Develop numerical model to analyze regimes of transport in terms of ice pack crowding and advective conditions.

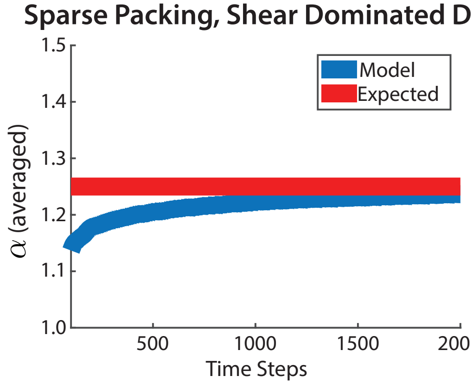
Model Results

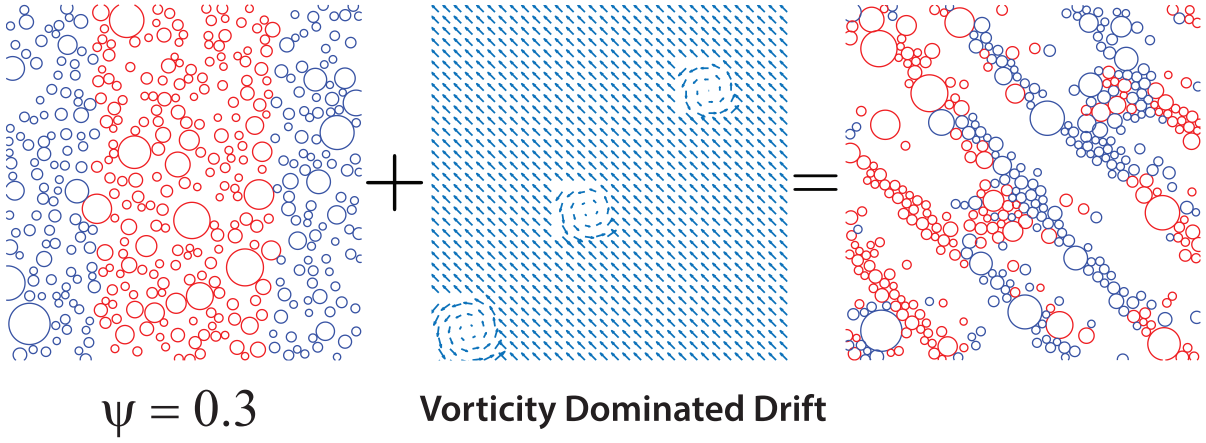

Crowding in Diffusive Drift


ψ = 0.3

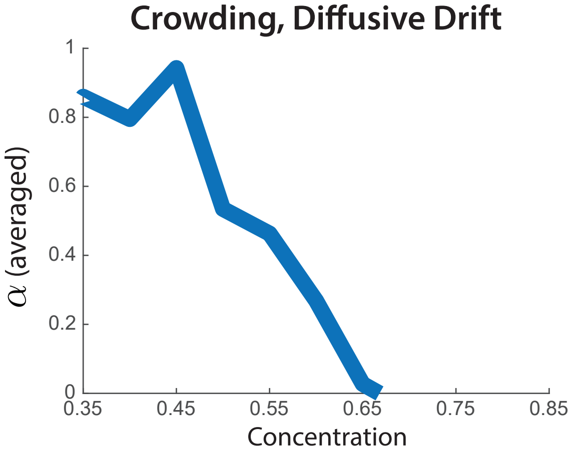
ψ = 0.5

ψ = 0.7


Sparse Packing, Shear Dominated Drift


ψ = 0.3

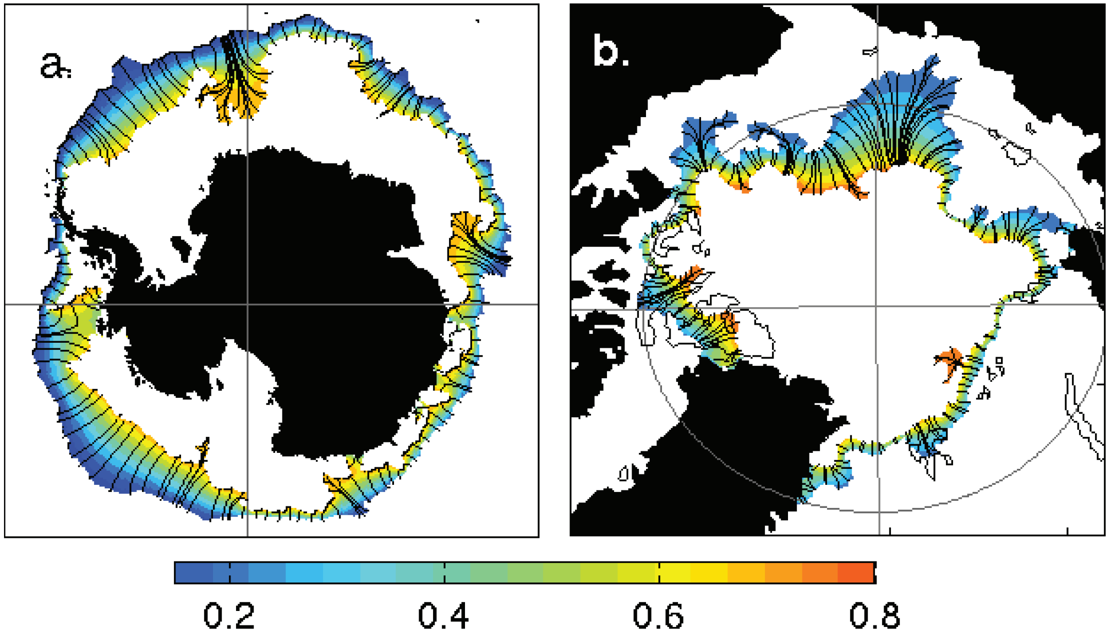
Vorticity Dominated Drift


Crowding, Diffusive Drift

Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



transitional region between dense interior pack ($c > 80\%$) sparse outer fringes ($c < 15\%$)

MIZ WIDTH
fundamental length scale of ecological and climate dynamics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

How to objectively measure the "width" of this complex, non-convex region?

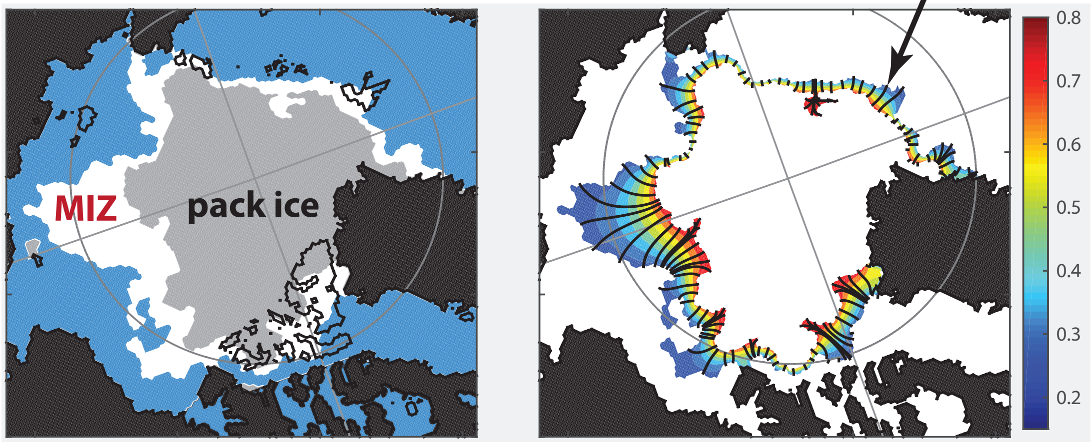
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

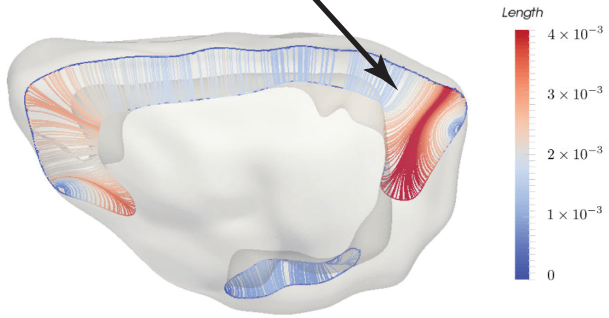
39% widening
1979 - 2012

streamlines of a solution to Laplace's equation

“average” lengths of streamlines



Arctic Marginal Ice Zone



crossection of the cerebral cortex of a rodent brain

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

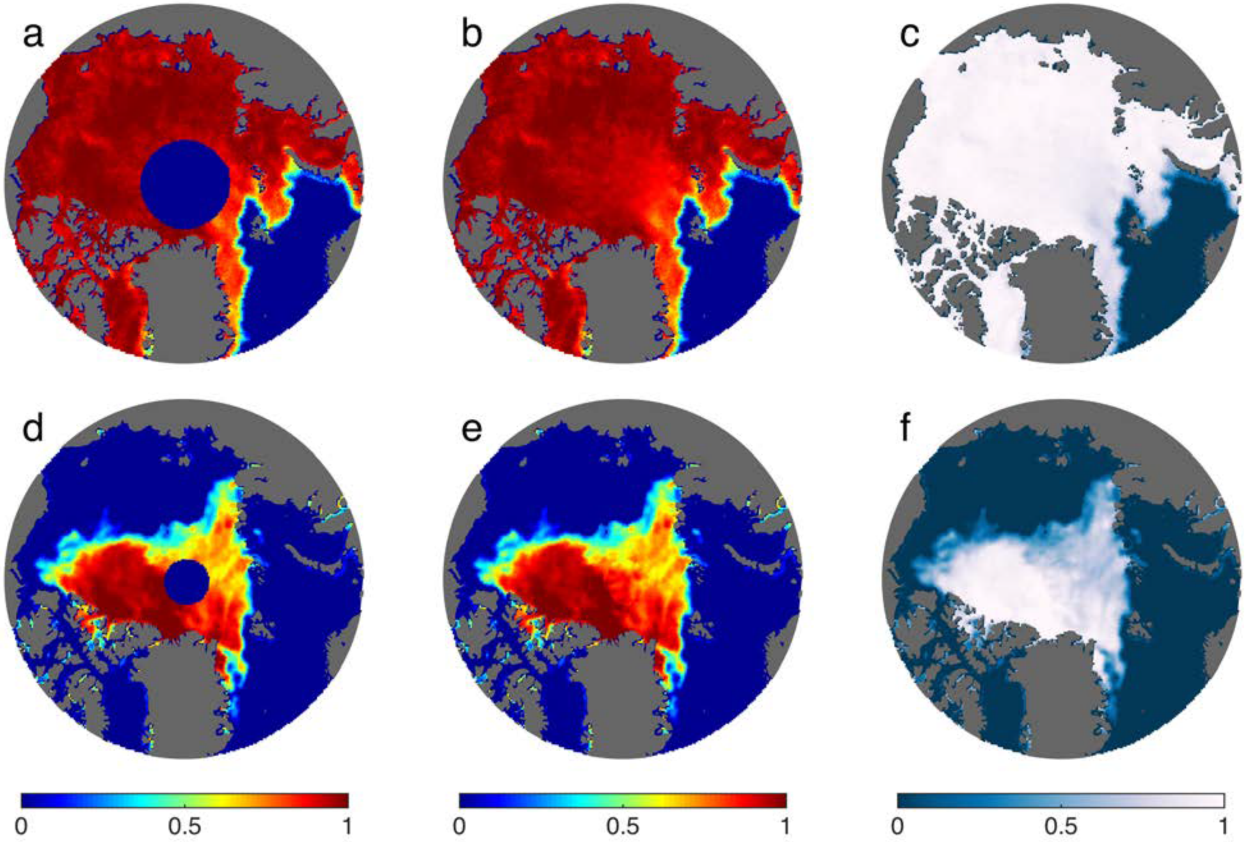
Filling the polar data gap with partial differential equations

hole in satellite coverage of sea ice concentration field

previously assumed ice covered

Gap radius: 611 km
06 January 1985

Gap radius: 311 km
30 August 2007



$$\Delta\psi = 0$$

fill with harmonic function satisfying satellite BC's plus stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

NOAA/NSIDC Sea Ice Concentration CDR product update will use our PDE method.

Partial differential equation models for sea ice concentration

Delaney Mosier, Court Strong, Jingyi Zhu, Elena Cherkaev, Ken Golden

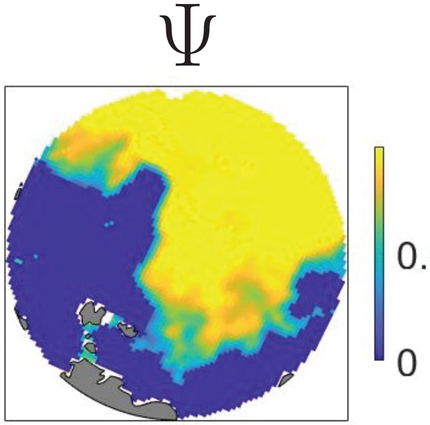
$$\Delta \Psi = 0$$

Generalize simplistic Laplace equation (steady state heat equation) model for Ψ

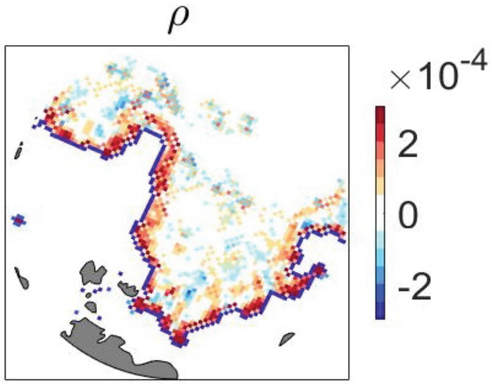
$$\nabla \cdot (D \, \nabla \Psi) = 0 \qquad \Delta \Psi = \rho$$

Advection diffusion equation model to generate more realistic dynamics of Ψ

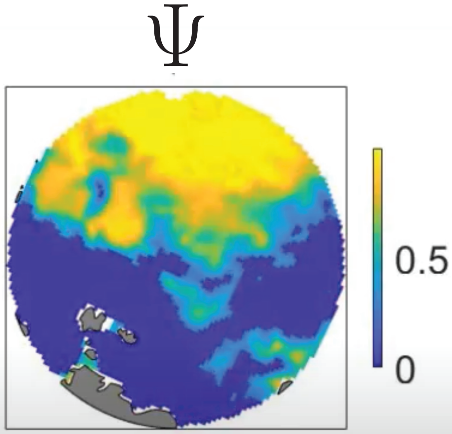
$$\frac{\partial \Psi}{\partial t} = \nabla \cdot (D \nabla \Psi) - \vec{v} \cdot \nabla \Psi \qquad \nabla \cdot \vec{v} = 0$$



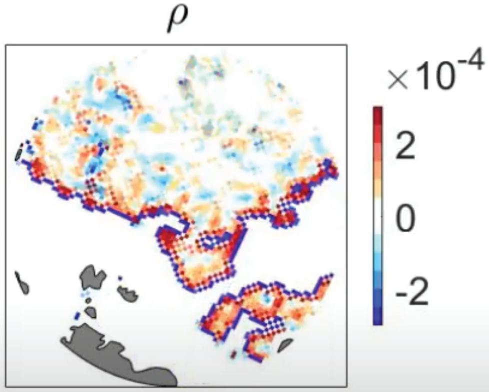
Siberia
ice concentration field



Siberia
source/sink field



ice concentration field



source/sink field

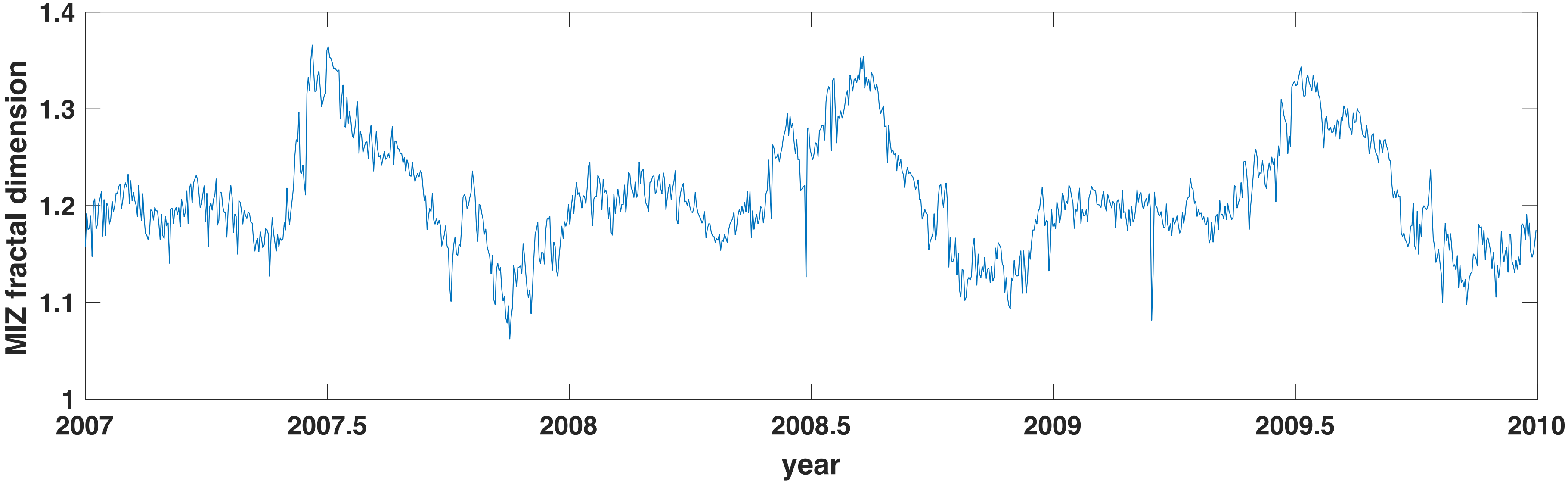
NSIDC

Poisson equation to identify sources and sinks of ice concentration

MIZ fractal dimension

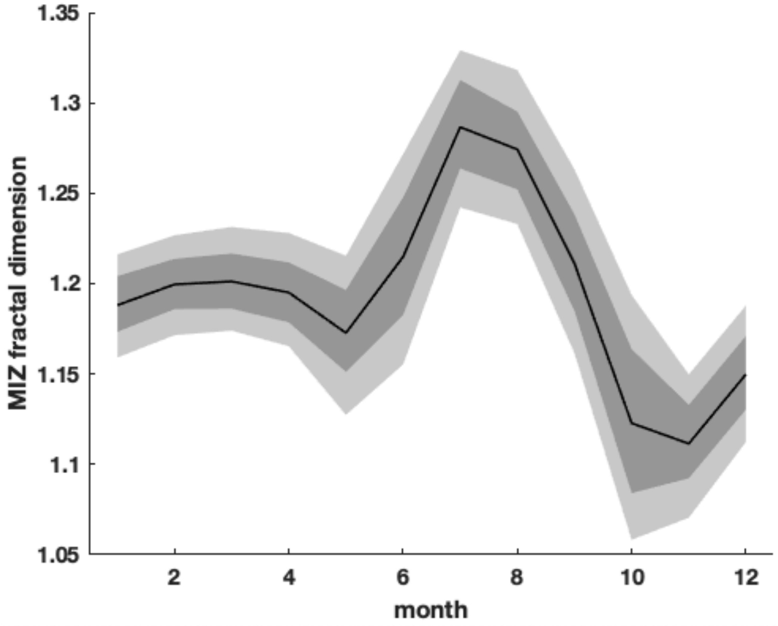
Jerry Zhang, Court Strong, Ken Golden

The shape of the MIZ becomes more complex during the melt season, increasing its fractal dimension.



- MIZ fractal dimension undergoes a pronounced seasonal cycle, maximizing around July
- We have preliminary evidence of decadal trends in MIZ fractal dimension

MIZ fractal dimension



For daily Arctic values 1979-2019
Bold curve: mean
Shading: 10th-90th and 25th -75th percentiles

- MIZ fractal dimension undergoes a pronounced seasonal cycle, maximizing around July
- We have preliminary evidence of decadal trends in MIZ fractal dimension

MIZ model: macroscale horizontal Stefan problem with mushy layer - composite of ice and ocean

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + S$$

$$S = [\rho(c_l - c_s)T + \rho L] \frac{\partial \psi}{\partial t}$$

$$= 1 - \left(\frac{T - T_s}{T_l - T_s} \right)^{\alpha}$$

ρ effective density

T temperature

c specific heat

L latent heat of fusion

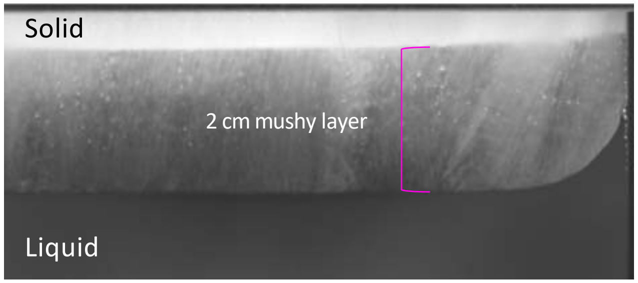
ψ sea ice concentration

k effective diffusivity

l liquid, s solid

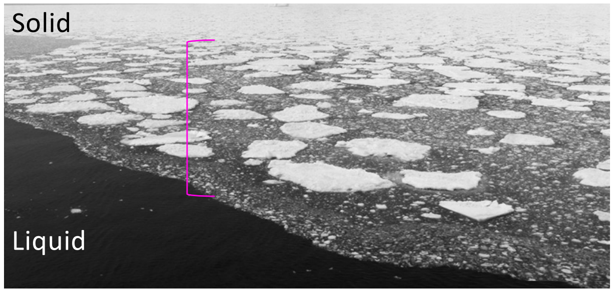
HOMOGENIZED PARAMETERS

Classical small-scale application



NaCl-H_2O in lab
(Peppin et al., 2007;, J. Fluid Mech.)

Macroscale application

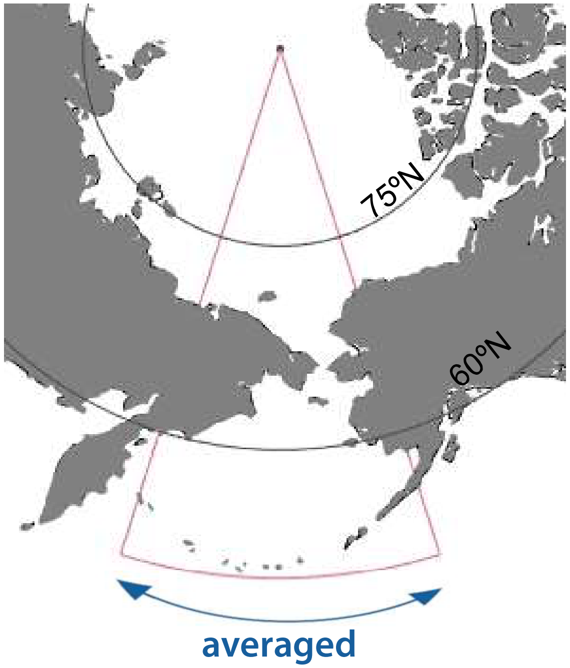


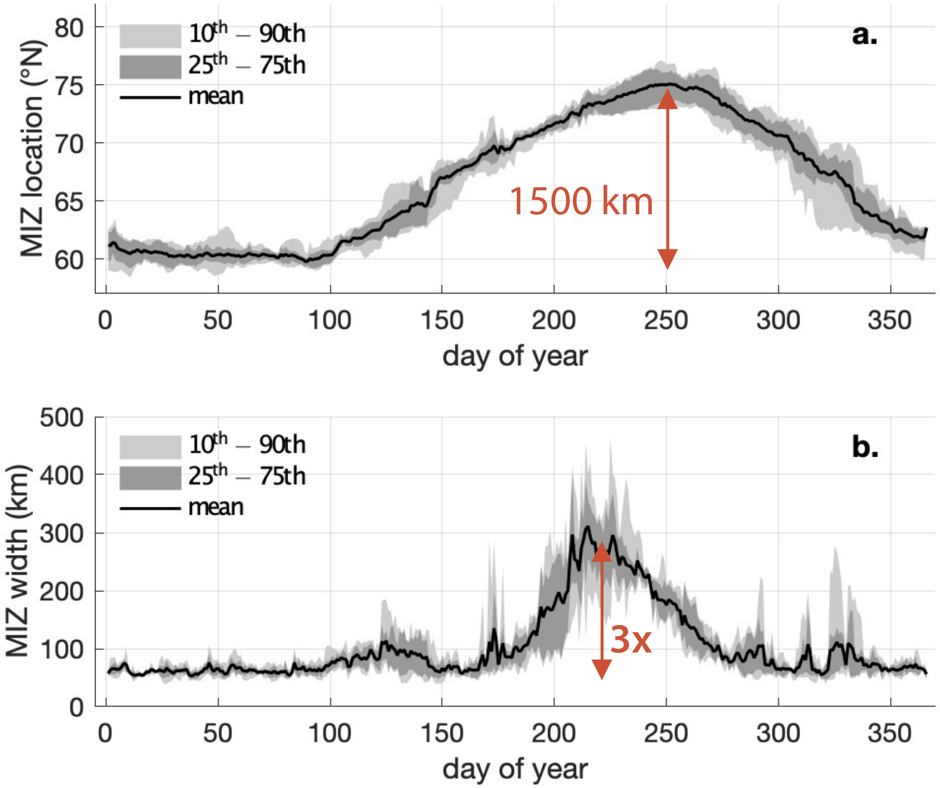
- Eigenmodes of solution skillfully capture seasonal cycle of MIZ location and width
- Model could ultimately be used to explain trends toward wider and more poleward MIZ

Court Strong, Elena Cherkaev, Ken Golden 2020

Observed location and width

Observational analysis 2000-2004 in Bering-Chukchi Sea sector





The first principal component of simulated temperature (PC1) accounts for 98% of the MIZ location seasonal cycle.

Poleward MIZ migration follows large scale warming.

Third component (PC3) accounts for 95% of the MIZ width annual cycle.

University of Utah Sea Ice Modeling Group (2017-2020)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Ph.D.

Postdoctoral Researcher: Noa Kraitzman (now at Australian National University)

Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)
Christian Sampson (now at UNC Chapel Hill with Chris Jones)
Huy Dinh (starting sea ice MURI Postdoc at NYU/Courant)
Rebecca Hardenbrook
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier + incoming

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich, Dane Gollero,
Samir Suthar, Anna Hyde, Kitsel Lusted, Ruby Bowers
Kimball Johnston, Jerry Zhang

High School Students: Jeremiah Chapman, Titus Quah, Dylan Webb

---

Sea Ice Ecology Group Postdoc Jody Reimer, Grad Student Julie Sherman,
Undergrads Anna Hyde, Kayla Stewart + incoming

Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.

2. Mathematical methods developed for sea ice advance the theory of composites and inverse problems in general.

2. Homogenization and statistical physics help ***link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.

3. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.

5. Field experiments are essential to developing relevant mathematics.

6. Our research will help to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

THANK YOU

Office of Naval Research

Applied and Computational Analysis Program

Arctic and Global Prediction Program

UNITED STATES COAST GUARD

DEPARTMENT OF THE NAVY ONR Science & Technology

MCRN Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

ace CRC
ANTARCTIC CLIMATE & ECOSYSTEMS
COOPERATIVE RESEARCH CENTRE

NSF

Antarctica New Zealand

UAF UNIVERSITY OF ALASKA FAIRBANKS

THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999

Most promising extensions of current research program:

Partial differential equation models of the large scale behavior of sea ice

1. **MIZ location, width, geometry** - seasonal and long term trends

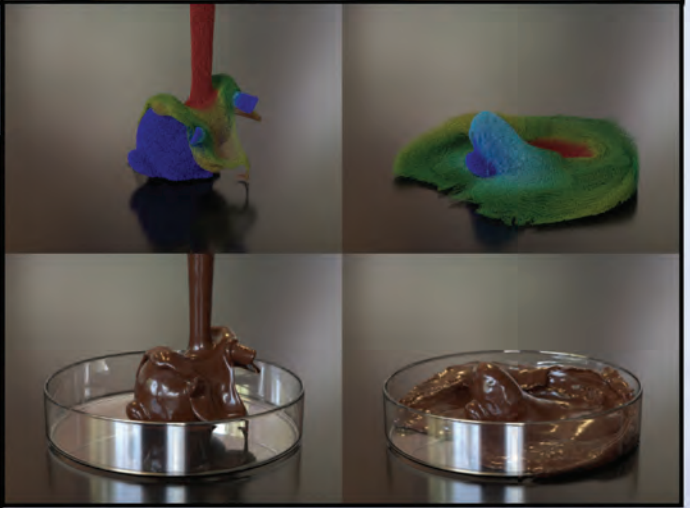
2. **Advection diffusion model for sea ice concentration** - use floe scale model to study advective forcing on ice concentration, use homogenization on sub-grad scale to upscale to coarse grid, "learn" the effective advection field from data. (thickness, ...)

3. **Anomalous diffusion in large scale ice mass transport** - effects of crowding, jamming, ice "diffusion coefficient"; connection to fractional PDE.

4. **Anderson transition for waves in the sea ice pack?** effects of floe configurations and ice concentration

Ultra-Realistic Modeling and Simulation of Arctic Sea Ice

Kenneth Golden (Univ. Utah), Jennifer Hutchings (Oregon State), Joseph Teran (UCLA)



MPM simulation of thermomechanical behavior. Top left: avalanche simulation of a novel elastoplastic constitutive model for weak layer driven avalanches (Gaume et al., Nature Comm. 2019). Top right: Baking bread - MPM simulation of thermomechanical porous media. Bottom right: Melting chocolate - MPM simulation of thermomechanical phase change.

- We seek to develop advances in the ability to simulate, predict and visualize key sea ice processes:
  - dynamics of ice cover on intermediate spatiotemporal scales: simulate critical processes in formation, ridging, melting, and break-up of sea ice.
  - novel thermo-mechanical models for multi-species interactions with sea ice, snow and water mixtures that are accurate at macro- and micro-scales.
  - GPU and HPC implementations of MPM discretizations that allow for simulations with ultra high spatial and temporal resolution.

Long term “blue sky” research program, new perspective on sea ice modeling:

Learning the statistical mechanics of sea ice

Develop a statistical physics framework to analyze and model collective sea ice phenomena.
Treat sea ice from a “systems” perspective, with a large number of interacting components,
as in statistical mechanics of gases, liquids, solids, phase transitions, complex materials, etc.

1. What are the Hamiltonians, order parameters, entropy, ... ?
   Ising model and statistical mechanics of granular media point way; evolution models based on minimization over energy landscapes.
2. Gas, liquid, solid description of ice pack; phase transitions; phase diagrams, mass transport transitions.
3. Develop algorithms to learn the parameters of the Hamiltonian, ...
4. Statistical mechanics framework opens renormalization schemes; hierarchical appraches, employ multiscale homogenization.
5. Statistical framework enables new techniques for data assimilation, studying the effects of advective forcing , waves, etc.