Network Modeling of Arctic Melt Ponds

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Abstract

The recent precipitous losses of summer Arctic sea ice have outpaced the projections of most climate models. Efforts to improve these models have focused in part on a more accurate accounting of sea ice albedo or reflectance. In late spring and summer, the albedo of the ice pack is determined primarily by melt ponds that form on the sea ice surface. The transition of pond configurations from isolated structures to interconnected networks is critical in allowing the lateral flow of melt water toward drainage features such as large brine channels, fractures, and seal holes, which can significantly alter the albedo. Moreover, pond connectivity can also influence their effectiveness in breaking up an ice floe as the melt season progresses. Here we develop algorithmic techniques for mapping photographic images of melt ponds onto discrete conductance networks which represent the geometry of pond configurations and approximate the ease of lateral flow. We implement an image processing algorithm with mathematical morphology operations to produce a conductance matrix representation of the melt ponds. Basic clustering and edge elimination using graph theory is then used to reduce the conductance matrix to include only direct connections. The results for images taken during different times of the year are visually inspected and the number of mislabels is used to evaluate performance.

Keywords: Melt ponds, Horizontal Conductivity, Mathematical Morphology, Graph Theory

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1. Introduction

Sea ice is a critical component of Earth's climate system, and a sensitive indicator of climate change. The dramatic losses of summer Arctic sea ice observed in the past few decades have a substantial impact on Earth's climate system, yet most global climate models have significantly underestimated the rate of decline [1, 2, 3]. One of the fundamental challenges of climate science is to develop more rigorous representations of sea ice in climate models, and incorporate important small scale processes and structures into these large scale models. For example, during the melt season the Arctic sea ice cover becomes

- ¹⁰ a complex, evolving mosaic of ice, melt ponds on the sea ice surface, and open water. While white snow and ice reflect most incident sunlight, melt ponds and ocean absorb most of it. The overall reflectance or albedo of sea ice floes – the ratio of reflected to incident sunlight – is determined by the evolution of melt pond coverage and geometry [4, 5, 6]. As melting increases, the albedo
- is lowered, which increases solar absorption, leading to more melting, and so on. This critical mechanism is called *ice-albedo feedback* [7], and has played a significant role in the decline of the summer Arctic ice pack [8]. Sea ice albedo is a significant source of uncertainty in climate projections and one of the most important parameters in climate modeling [9, 5, 10, 6].
- ²⁰ While melt ponds form a key component of the Arctic marine environment, comprehensive observations or theories of their formation, coverage, and evolution remain relatively sparse. Available observations of melt ponds show that their areal coverage is highly variable, particularly for first year ice early in the melt season, with rates of change as high as 35% per day [11, 6]. Such
- variability, as well as the influence of many competing factors controlling melt pond and ice floe evolution, makes the incorporation of realistic treatments of albedo into climate models quite challenging [6]. Small and medium scale models of melt ponds which include some of these mechanisms have been developed [12, 13, 5], and melt pond parameterizations are being incorporated into global
 climate models [9, 14, 10].

As melting progresses during the season, the evolution of melt ponds from small isolated structures into large interconnected networks is responsible for a number of processes that help control the rate at which the ice pack melts. It is believed [15] that this evolution of connectedness is an example of a percolation

- transition [16, 17]. Such a transition occurs when one phase in the microstructure of a composite material, for example, becomes connected on macroscopic scales as some parameter exceeds a critical value, called the *percolation threshold* [16, 17]. Percolation theory was initiated in 1957 by Broadbent and Hammersly [18] with the introduction of a simple lattice network model to study the flow
 of air through permeable sandstones used in miner's gas masks. In subsequent
- decades, this theory has been used to successfully model a broad array of disordered materials and processes. In the case of melt ponds, the critical threshold is thought to be related to the area fraction of sea ice surface covered by the ponds.
- ⁴⁵ An important example of this percolation phenomenon in the microphysics of sea ice, which itself is fundamental to the process of melt pond drainage and changes in sea ice albedo, is the percolation transition exhibited by the brine phase in sea ice, known as the *rule of fives* [19, 20, 21]. When the brine volume fraction of columnar sea ice is below about 5%, it is effectively impermeable
- to fluid flow. However, for brine volume fractions above 5%, the brine phase becomes macroscopically connected so that fluid pathways enable flow through the porous microstructure of the ice. For a typical bulk sea ice salinity of 5 parts per thousand, the 5% volume fraction corresponds to a critical temperature of about -5° C; hence the term "rule of fives." Similarly, even casual inspection of
- ⁵⁵ aerial photos shows that the melt pond phase of the sea ice surface undergoes a percolation transition where disconnected ponds evolve into much larger scale connected structures with complex boundaries [15]. Connectivity of melt ponds promotes further melting and break-up of floes, as well as horizontal transport of meltwater and drainage through large vertical brine channels, cracks, leads,
- and seal holes [11, 6].

Establishing that the brine phase in sea ice actually exhibits a percolation

transition, attended by critical behavior of the vertical fluid permeability, was accomplished through the development of X-ray computed tomography for sea ice, and subsequent mapping of the data onto random graphs of nodes and

edges [20, 21]. The connectivity of these graphs was analyzed as a function of temperature and sample size, and found to display a percolation threshold in the vertical direction around the 5% critical value conjectured in [19]. Furthermore, the theory of fluid and electrical transport through lattice percolation models [16, 17] was used to predict the dependence of the vertical component of the fluid permeability of sea ice as a function of brine volume fraction [20].

Other types of network models have been used to quantitatively describe the behavior of fluid flow through the porous sea ice microstructure. For example, in the random pipe model, the diameters of the pipes (assigned to the edges in a square lattice) are chosen from lognormal probability distributions that ⁷⁵ describe the cross-sectional areas of the brine inclusions in sea ice [22]. The fluid permeability of the model is computed by using a random resistor network representation of the system and employing a fast multigrid method to find its effective conductivity. This approach has also been used to directly model the electrical conductivity of sea ice [23], an important parameter in remote sensing of sea ice thickness, transport properties, and microstructural transitions [24, 25, 26, 27, 28, 29].

Here we begin to develop techniques for network modeling of melt ponds, their connectivity, and horizontal flow characteristics. Some of the groundwork for this type of modeling was laid in [15]. Images of melting Arctic sea ice collected during two Arctic expeditions – the 2005 Healy-Oden TRans Arctic EXpedition (HOTRAX) [30] and the 1998 Surface Heat Budget of the Arctic Ocean (SHEBA) expedition [4] – were analyzed for area-perimeter data on thousands of individual melt ponds. Algorithmic methods of distinguishing melt ponds from the ocean in leads between the sea ice floes were developed. This

⁹⁰ data was used to discover that pond fractal dimension transitions from 1 to 2 around a critical length scale of 100 square meters in area [15]. Pond complexity was found to increase rapidly through the transition as smaller ponds coalesce to form large connected regions, reaching a maximum for ponds larger than about 1000 square meters whose boundaries resemble space filling curves.

In earlier work on melt ponds and sea ice albedo, image processing has been used to measure the area fractions of melt ponds and leads from aerial and satellite images. In [4] these area fractions from June to October, using SHEBA images taken in 1998 [4], show how the area fraction of melt ponds increases as summer progresses, and starts decreasing again at the end of summer as new ice forms. A probability distribution for the size of melt ponds is also derived from the data, which depends on the progress of the melt season.

In the work reported here, the connectivity of these melt pond networks is determined using aerial images of Arctic sea ice from the HOTRAX database. We develop an algorithmic method of mapping a configuration of melt ponds

- onto a graph of nodes and edges. These configurations may be disconnected into individual components, or partially or completely connected across an image. The edges are assigned values which indicate the width of the "bottlenecks" separating larger pools of melt water, which are identified with the nodes of the graph.
- The volume of water in a melt pond results from the net balance of melt water accumulation, water in-flux from and out-flux to neighboring ponds, drainage due to ice porosity, fluid permeability, and larger cracks in the pack ice. Some melt ponds may have large sink holes with high drainage rate. The flow of water between melt ponds depends on the narrowest bottlenecks between them
- and the width of these bottlenecks is inversely proportional to the fluid conductance between them. A conductance graph of the melt pond networks can help model the evolution of the melt pond configurations. Mathematical morphology based image processing techniques [31] are used with a clustering algorithm and graph theory to find a conductance graph associated with each melt pond con-
- ¹²⁰ figuration studied. Further work will explore the relationship of these graphs and associated conductance networks with the actual flow of fluid in the pond network, and the effect on sea ice albedo.

2. Method

The images provided by the SHEBA and HOTRAX expeditions are in color. The intensity and color of each pixel in the image is encoded using the intensities of the Red, Green and Blue colors that make up the pixel. The image is a matrix of pixels, with each pixel being a vector of three variables - red, green and blue color values. These are respectively called the red, green and blue channels of the image.

These images are converted to gray-scale because it reduces each pixel to only one intensity and hence fewer computations are required. This is done by using only the red channel as it shows the most clear cut difference between ice and water intensities. A simple thresholding operation is sufficient to segment the melt pond water from ice and get a binary image. Otsu's method [31] is used to determine this threshold individually for each image, which is then segmented

based on this threshold. Figure 1 shows a histogram of the intensity levels of a gray-scale aerial image with Otsu's threshold.

The images used are cropped from those in the SHEBA and HOTRAX databases, which have dimensions around 865 × 770 pixels. The size of the ¹⁴⁰ images does not affect the algorithm as long as the resolution, i.e., the number of pixels per unit physical area covered, remains the same. Only the processing time varies with image size.



Figure 1: An aerial image of melt ponds from HOTRAX is shown on the left. A histogram of the image is shown on the right

2.1. Preprocessing the image

The binary image produced by Otsu's method has small pieces of ice floating ¹⁴⁵ in the melt ponds, melt ponds that are too small to provide much information, and other small artifacts due to noise. These can clutter up the final connectivity graph with unnecessary data. Basic mathematical morphology operations involving erosion and dilation, as described in [31] are used to clean up the image. A predetermined mask or structuring element of fixed size is centered at each pixel of the image and only those pixels, at which the structuring element fits inside the original image, are set to one. So, if a 3×3 structuring element is used, it will remove the outermost layer of pixels from the foreground, a 5×5 structuring element would remove two layers and so on. Morphological dilation is a complementary process where all those pixels, at which the intersection be-

- tween the structuring element and the image is non-zero, are set as one. Dilation by a 3×3 structuring element would cause the foreground to grow another layer of pixels. Opening involves erosion followed by dilation with the same structuring element and is used to remove smaller structures from the foreground like protrusions, narrow connections, etc. Closing on the other hand is dilation fol-
- lowed by erosion and it fills in small gaps in the foreground. Geodesic opening or closing involves finding the intersection of the result of opening or closing with the original image to preserve the shape of the image. The image is first cleaned up using geodesic opening of melt ponds to remove inconsequential melt ponds and geodesic closing to remove floating ice. Circular masks are used for
- these processes to maintain the curvy shapes of ponds. The mask size can be adjusted as desired. Here a 3×3 mask is used. Note that care should be taken to ensure that the mask size is at least smaller than the narrowest bottleneck in the image, otherwise this connection will be lost.

2.2. Isolating melt ponds

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The next step is to find individual melt ponds. The previous step results in large interconnected melt pond networks. First, connected components are used to find all the separate unconnected melt pond networks and label them. Each of these networks is then eroded progressively with a 3×3 circular mask. At each erosion, some ponds might break away from the main network. These

- ¹⁷⁵ can be identified from an increase in the number of unconnected regions in the image, which are found using connected components. The connection strength of the separated melt pond, to the full network is proportional to the number of iterations at which it breaks away. If a region is split into multiple regions, the separated regions will form smaller networks of melt ponds, which will all be connected to each other. This step is repeated until a pre-defined maximum
- ¹⁸⁰ be connected to each other. This step is repeated until a pre-defined maximum bottleneck size is reached.

Also, a minimum pond size is set and if a region reaches this size, it is no longer split into smaller regions. This minimum pond size is increased with increasing erosion iterations. The minimum pond size is scaled to maintain a
¹⁸⁵ minimum ratio pond area bottleneck size. This is done to avoid labeling connections between ponds as melt ponds themselves. An example of this is shown in Figure 2.



Figure 2: The connection between melt ponds is incorrectly labeled in the image on the left. Image on the right uses pond area scaling to correctly label melt ponds.

2.3. Connections between melt ponds

The last part of the problem is finding the conductances between the individual melt ponds. This is done in parallel as the interconnected melt ponds ¹⁹⁰ are being separated into smaller melt pond networks. Each erosion with a 3×3 mask removes the outermost pixel layer (perimeter-wise). Thus two layers of pixels, one from each side of the bottleneck, are removed. Hence at each step, when a region splits into multiple regions, the conductance between these regions will be $2 \times i$. Here *i* is the iteration number. The problem also requires

that we find only direct connections between ponds. This means that if each pond is a node, we must ignore connections that contain intermediate nodes in their paths. Consider the images in Figure 4. The interconnected pond splits into a number of smaller ponds in the same erosion step. The next step is to find out which ponds are directly connected to each other. Two simple methods of doing this would involve the following operations

1. morphological dilation,

2. a simple clustering approach followed by a graph theory method.

In the first method, at each iteration, the eroded image is subtracted from the original image to get only the bottlenecks that were eroded away. This resulting ²⁰⁵ image is then dilated and a simple overlapping operation (using the logical OR function) is performed to check which ponds form a direct connection with each other. This is illustrated in Figure 3. A major problem with this approach is that sometimes the dilation is not sufficient to cause an overlap with the expected ponds and this leads to incorrect or missing connections.

In the second method, the center of each melt pond pixel-cluster is located using the mean of the cluster with Euclidean distances. One may try to use kmeans clustering on the initial image to separate the ponds, but as this only uses euclidean distances between pixels and needs a fixed estimate of the number of clusters at the output, it will assign more than one cluster center to larger ponds and may ignore the smaller ponds. The geodesic distances between these cluster centers are calculated. The distance between unconnected ponds is considered to be an arbitrarily large number, which is larger than the maximum distance between two ponds. These distances are then used along with the conductance strengths calculated in section 2.2 to construct a graph of the melt pond network. Initially, the nodes of the graph are the cluster centers found above, and the all the nodes belonging to connected melt ponds are connected to each other with



Figure 3: Mapping connections using morphological dilation.

edges. Note that the conductance strength here only refers to the width of the channel connecting different ponds and gives a basis for relative comparison of ease of flow of fluid between these channels. Let this conductance strength be denoted by σ_{ij} and the geodesic distance be d_{ij} . The weight of each edge is the ratio $\frac{\sigma_{ij}}{d_{ij}}$. Between two nodes, the direct path and all paths involving only one intermediate connection are considered. For any node, there are (n-1) possible paths to another node, or (n-2) indirect paths with one intermediate node and one direct path. The weight of the *k*th indirect path connecting two nodes is calculated as,

$$w_{ij}^{(k)} = \left(\frac{\sigma_{ik}}{d_{ik}} + \frac{\sigma_{kj}}{d_{kj}}\right), \quad \forall k \neq i, j.$$

$$\tag{1}$$

Here $\frac{\sigma_{ik}}{d_{ik}}$ is the weight of the edge from node *i* to node *k*. The weight of the edge which directly connects nodes *i* and *j* is

$$w_{ij} = \frac{\sigma_{ij}}{d_{ij}} . (2)$$

²¹⁰ Only the path corresponding to the maximum weight between two nodes is retained and all the edges corresponding to other paths are dissolved. This favors paths which are either very short or have large conductances. At each step, one pair of nodes in the graph is considered. For the next pair, the previously updated connection graph is used so that the edges that no longer exist are

not reconsidered. The final step of the algorithm is for node deletion, where the algorithm searches for very small nodes that lie between two or more much larger nodes, and eliminates these small nodes based on a predetermined ratio. For the results presented later, this ratio is set to 20.

The latter graph method performs much better for mapping connections than the dilation method. Figure 5 shows the results obtained using the two different approaches. Consider the nodes 5 and 6 at the bottom right corner in the first figure. The connection between the two nodes is not detected because dilation of the connection shown in Figure 3 is not sufficient to overlap with ponds 5 and 6. Thus, pond 6 is shown connected directly to pond 1. This issue is solved in the second figure by using the clustering and graph method.

2.4. Conductivity factor calculations

To calculate the horizontal fluid "conductivity," first two battery nodes are added to the left and right of the image. The left battery node is connected to all the ponds touching the left edge of the image with a conductance value of 1 for each connection. The right battery node is similarly connected. The purpose of the battery nodes is to simulate the computation of the effective or equivalent conductivity of a conductor network, which must be subjected to a potential difference, most easily visualized by connecting a battery. The conductivity across the network, between these battery nodes, is then measured. The conductivity of very large networks can be calculated approximately by

considering smaller sections and then replacing these subsections with their equivalent conductivities. The conductivity of each section could be calculated to create a new, simpler graph model.

All the melt pond nodes which are not directly connected to a battery node in the graph are removed as they do not contribute to conductivity. To calculate the conductivity between battery nodes, let c_{ij} be the conductivity of the edge between nodes *i* and *j*. Here, each c_{ij} is the normalized edge weight, w_{ij} , as



Figure 4: The figure on right at the top shows geodesic distances between melt pond nodes. The figure at the bottom shows the final connections obtained after edge elimination.

described in the equation below.

$$c_{ij} = \frac{w_{ij}}{\max_{i,j}(w_{ij})} \forall i, j \tag{3}$$

Let the M be the total number of nodes in the graph, including the two battery



Figure 5: The image on the left results from using morphological dilation for mapping pond connections. The image on the right uses the clustering and graph method approach.

nodes. We define the $M \times M$ matrix A such that

$$A_{ij} = -c_{ij} \qquad \qquad i, j = 1 \dots M, i \neq j \qquad (4)$$

$$A_{ii} = \sum_{\forall j: j \neq i} c_{ij} \qquad \qquad i = 1 \dots M \tag{5}$$

The matrix A' is the $(M-1) \times (M-1)$ array obtained by removing the first row and column of A, which corresponds to the left battery node. Removing the last row and column of matrix A', corresponding to right battery node, gives the (M-2)(M-2) matrix A''. The conductivity factor of the image represented by matrix A, between the battery nodes, is given by

$$\sigma(A) = \frac{\det(A')}{\det(A'')}.$$
(6)

It should be noted that the conductivity factor obtained is then related to the fluid permeability of the network, but not equal to the effective conductivity of the network, due to the length scale involved. As noted in the Introduction, further work will explore the relationship of this computed network conductivity to the horizontal fluid flow properties of melt pond configurations.

Set	Month	Number	Database
1	June	5	SHEBA
2	July	10	SHEBA
3	August	10	HOTRAX

Table 1: List of images considered

3. Results

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The above method is used to generate the conductance graphs for different sets of images as described in Table 1. MATLAB is used to implement the method summarized above for each of these images.

This method was found to be most useful for images obtained in midsummer, i.e. July, as the melt ponds are large and interconnected. The average time taken for different sets of images was calculated and is shown in table 250 2. The SHEBA images taken in July were processed the quickest, because the images consist of larger and fewer melt ponds. Consequently, the operations involving connected components and the calculation of geodesic distances, do not occupy the processor for too long. When these times are compared to the August melt pond images from SHEBA, which have many more melt ponds 255 per image, the computations take much longer. This can be easily rectified by selecting a smaller area of the image to give a faster and more accurate result. When images have a large number of melt ponds, the resolution of the calcu-

is spent in the calculation of geodesic distances and using graph methods to 260 eliminate all but the direct connections between melt ponds. A major part of the computation time is spent in iteratively eroding the image, finding all the connected components and updating the bottleneck widths at each iteration. This can be sped up by using parallel processing for different connected compo-

lated conductance values is reduced. Only about 10% of the computation time

nents. Another step in reducing the time latency would be to ignore all ponds 265 that have no other connections. However, this choice would be application spe-

Set	Month	Database	Number of iterations	Average Time(minutes)						
1	June	SHEBA	8	31.66						
2	July	SHEBA	20	9.06						
3	August	HOTRAX	20	18.04						

Table 2: Average time to process each image

Image1	Image2	Image3	Image4	Image5
0	0	0	0	0

Table 3: Conductivities for image set 1

cific, as even the isolated ponds may be used to study the evolution of networks with time.

Due to lack of any ground truth for these images, they are visually inspected to ascertain the performance of the method used. The processed images from July, August and June are shown in Figures 6, 7, 8, 9 and 10 respectively. Figure 11 shows the conductance graph obtained for the 3rd image in Figure 8. The conductivity factors for these figures are shown in Tables 3, 4 and 5. The images shown in Figure 10 do not have any complete connections that go across

²⁷⁵ the image from left to right. For this reason, the images are shown without removing nodes which are unconnected to the battery nodes. The conductivity factor values for all these images are zero.

Image1	Image2	Image3	Image4	Image5
0	0	0	0	0
Image6	Image7	Image8	Image9	Image10
0	0.0546	0.0283	0.0443	0.2062

Table 4: Conductivities for image set 2

Image1	Image2	Image3	Image4	Image5
0	0.0542	0.1353	0.1216	0.0563
Image6	Image7	Image8	Image9	Image10
0.1778	0.1003	0.1078	0.0718	0.1127

Table 5: Conductivities for image set 3



Figure 6: Melt ponds in July from SHEBA, continued on next page...

4. Conclusion

After visual inspection, it can be concluded that the algorithm does a very good job of identifying melt ponds, labeling their connections and creating the conductance matrix. More work can be done to improve the speed of algorithm and remove the few mislabeling errors. The edge elimination method used assigns weights to the edges between nodes (melt pond centers) based on geodesic



Figure 7: ...continued, Melt ponds in July from SHEBA

distance and widths of the connections. The function assigning weights to the edges can be modified and the weights of the nodes (areas of melt ponds) can also be used in this function.

The conductivity factors calculated can help to determine the rate at which melt water might drain from ponds to a sink node, which might be a sink hole



Figure 8: Melt ponds in August from HOTRAX, continued on next page...

in the ice pack. This water drainage influences ice pack albedo, and hence a calculation of the rate of drainage could prove to be an important factor in climate models.

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Figure 9: ...continued, Melt ponds in August from HOTRAX

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Figure 10: Melt ponds in June from SHEBA

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	Α	В	С	D	E	F	G	Н	1	J	K	L	М	Ν	0	Р	Q	R	S	Т	U	V	W	Х	Y	Z	AA	AB	AC	AD	AE
1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.18	0	0	0	0	0	0	0	0	0	0	0	0	0	0
з	1	0	0	0.47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0.47	0	1	0	0	0	0	0	0	0	0	0	0	0	0.24	0	0	0	0.06	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0.65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0.65	0	0.71	0.41	0	0	0	0	0	0	0	0	0	0.24	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0.71	0	0	0	0	0	0	0	0	0	0.29	0	0	0.18	0	0	0	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0.41	0	0	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0.71	0	0.47	0	0	0	0	0	0	0	0	0	0.12	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0.47	0	0	0	0	0.35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0.35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0.35	0	0.76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0.76	0	0.76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0.35	0	0	0.76	0	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0.29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0.18	0	0.24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0.24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0.18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0.12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0.06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.06	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.06	0	0.12	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12	0	0.12	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12	0	0	0	0	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
31	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0

Figure 11: Conductance values for August (HOTRAX 3rd photograph)

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