Math 5750 / 6880
Mathematics and Climate

Kenneth M. Golden

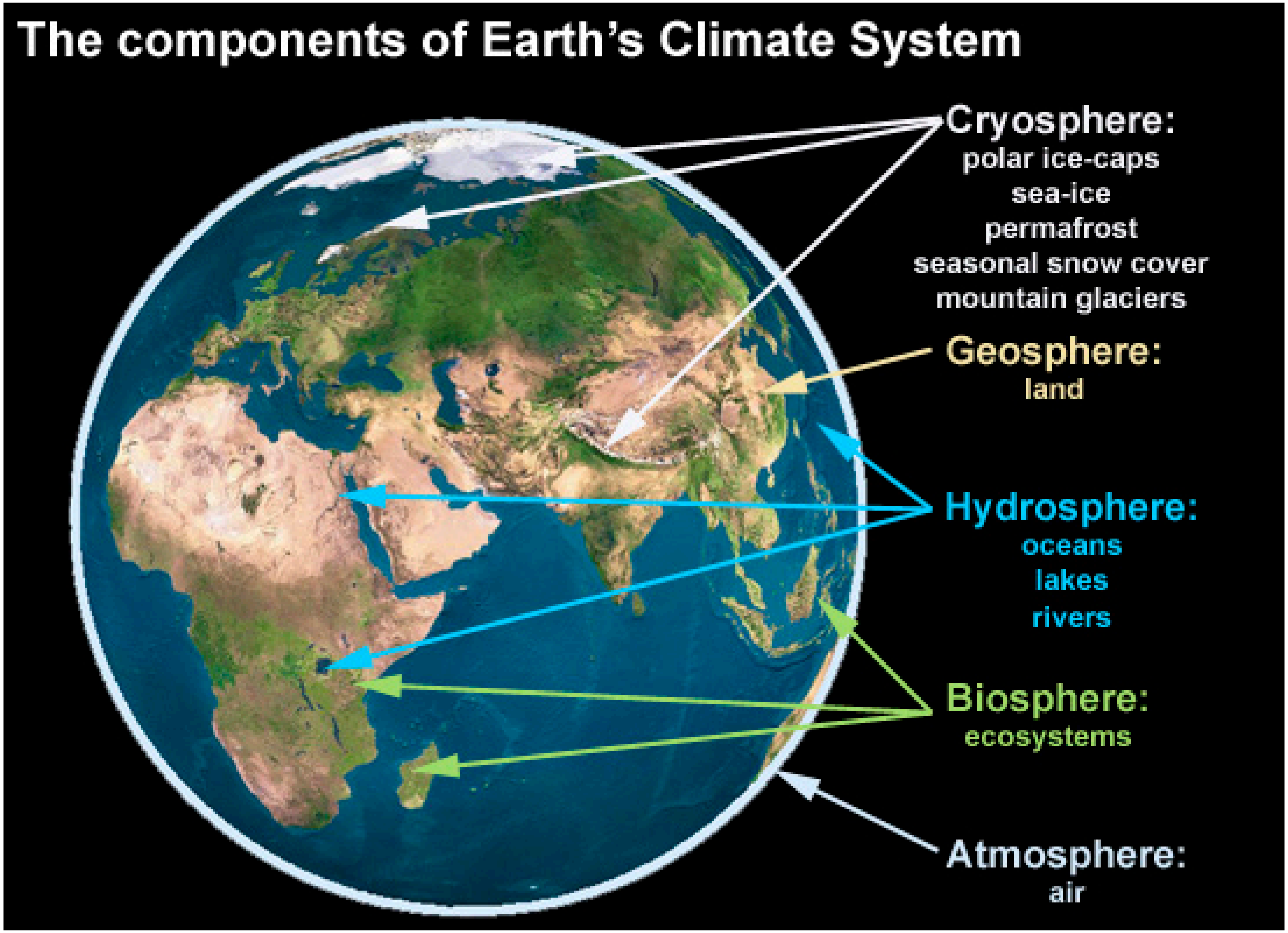
Earthrise December 24, 1968 William Anders NASA

Introduction to Modeling Earth's Climate System

Energy Balance Models

Ken Golden
Math 5750 / 6880
Spring 2023

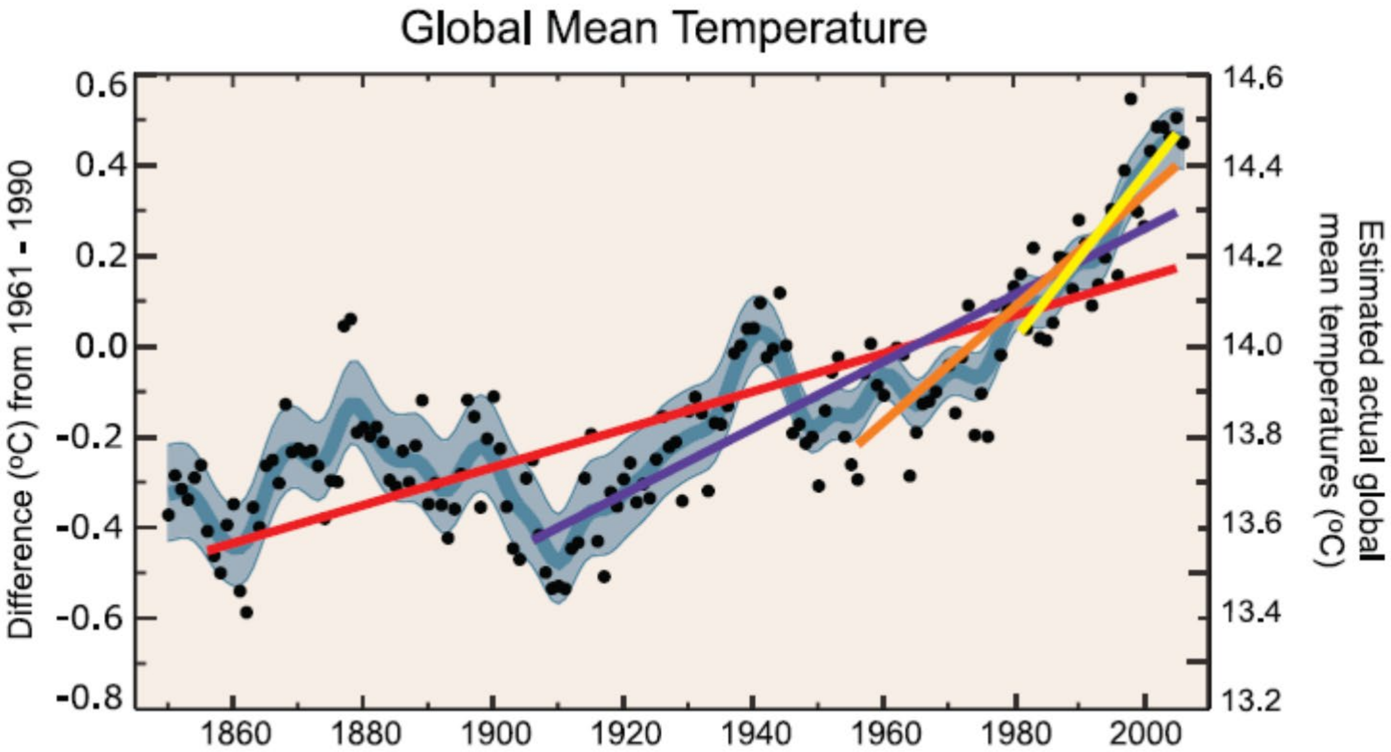
Earthrise, NASA
December 24, 1968



The components of Earth's Climate System
Cryosphere:
polar ice-caps
sea-ice
permafrost
seasonal snow cover
mountain glaciers
Geosphere:
land
Hydrosphere:
oceans
lakes
rivers
Biosphere:
ecosystems
Atmosphere:
air

Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):
Warming is “unequivocal”



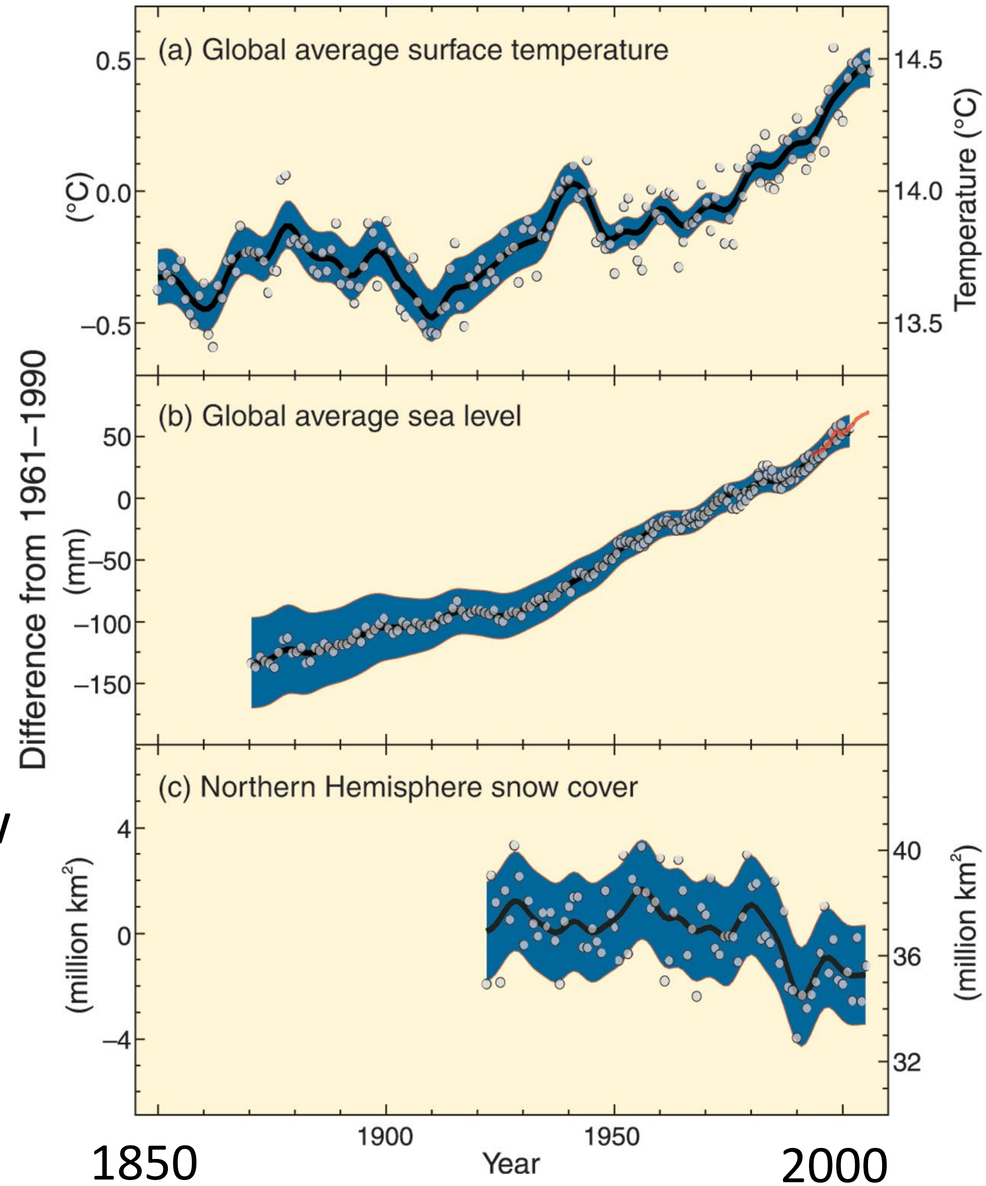
Dots: yearly average
Curve: decadal average
Blue: uncertainty interval

IPCC: Warming is “unequivocal”

Global mean surface temp

Global mean sea level

Northern Hemisphere snow cover (March-April)



Dots: yearly average
Curve: decadal average
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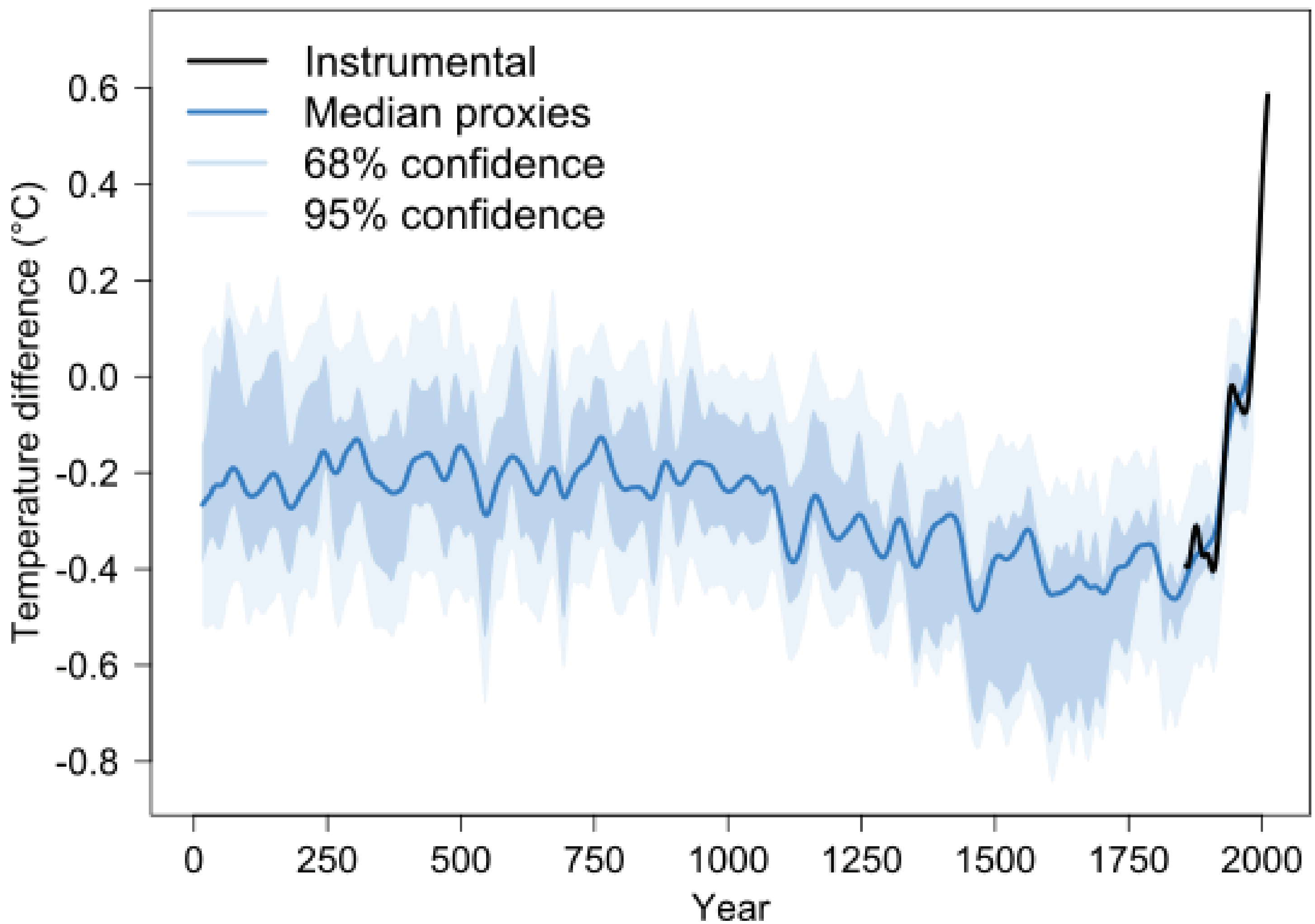
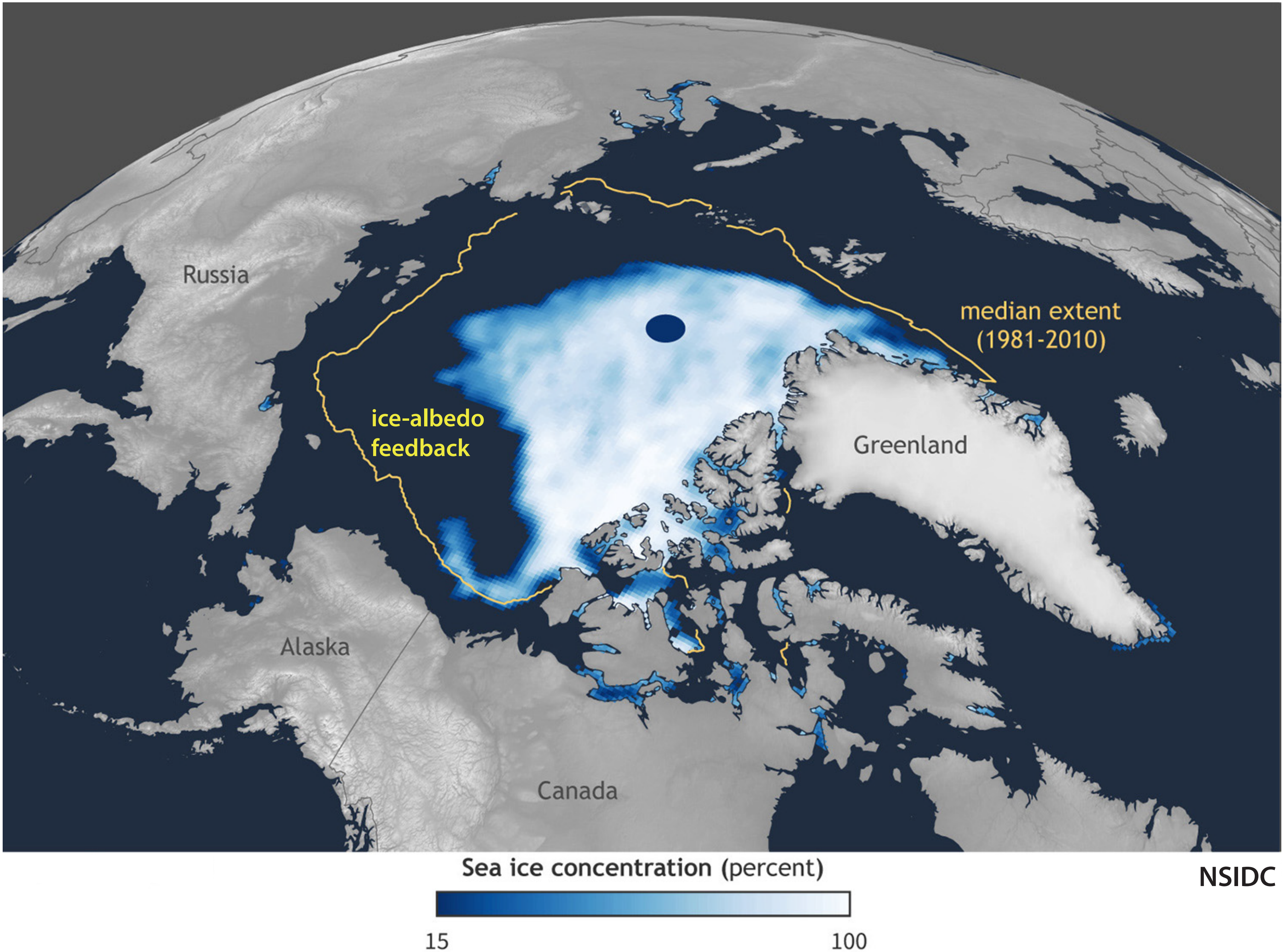
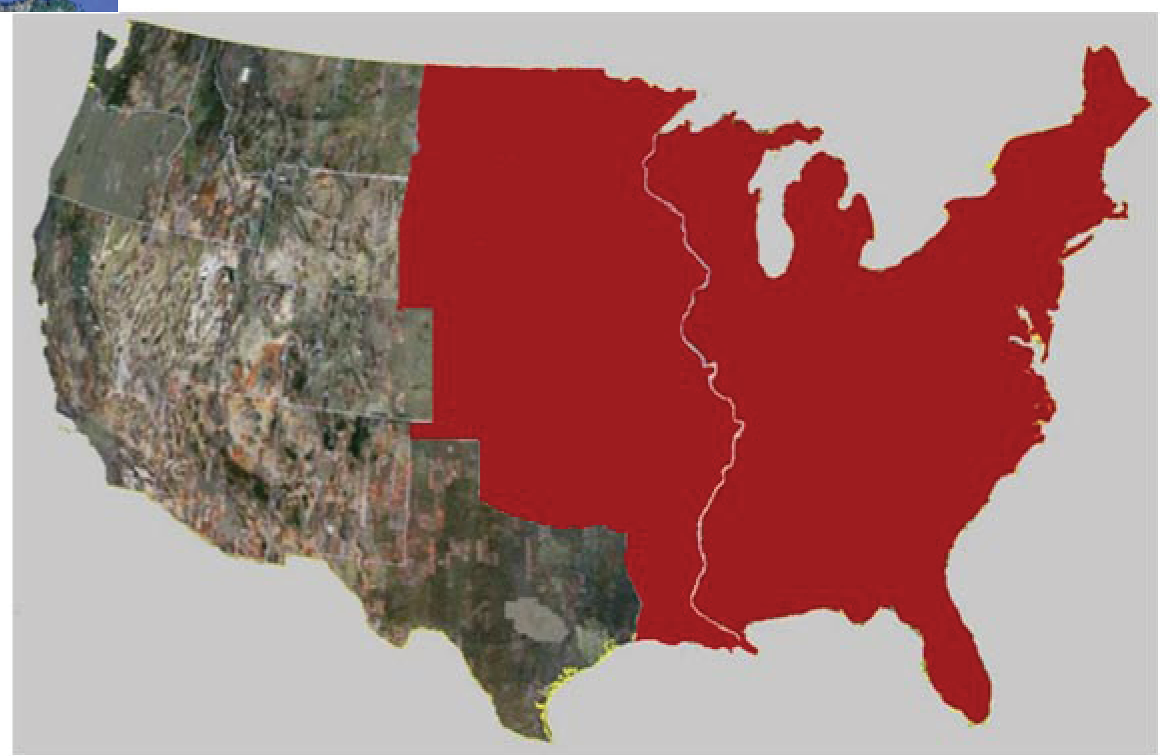
Mean Global Temperature over the past 2000 years



Arctic sea ice extent September 15, 2020

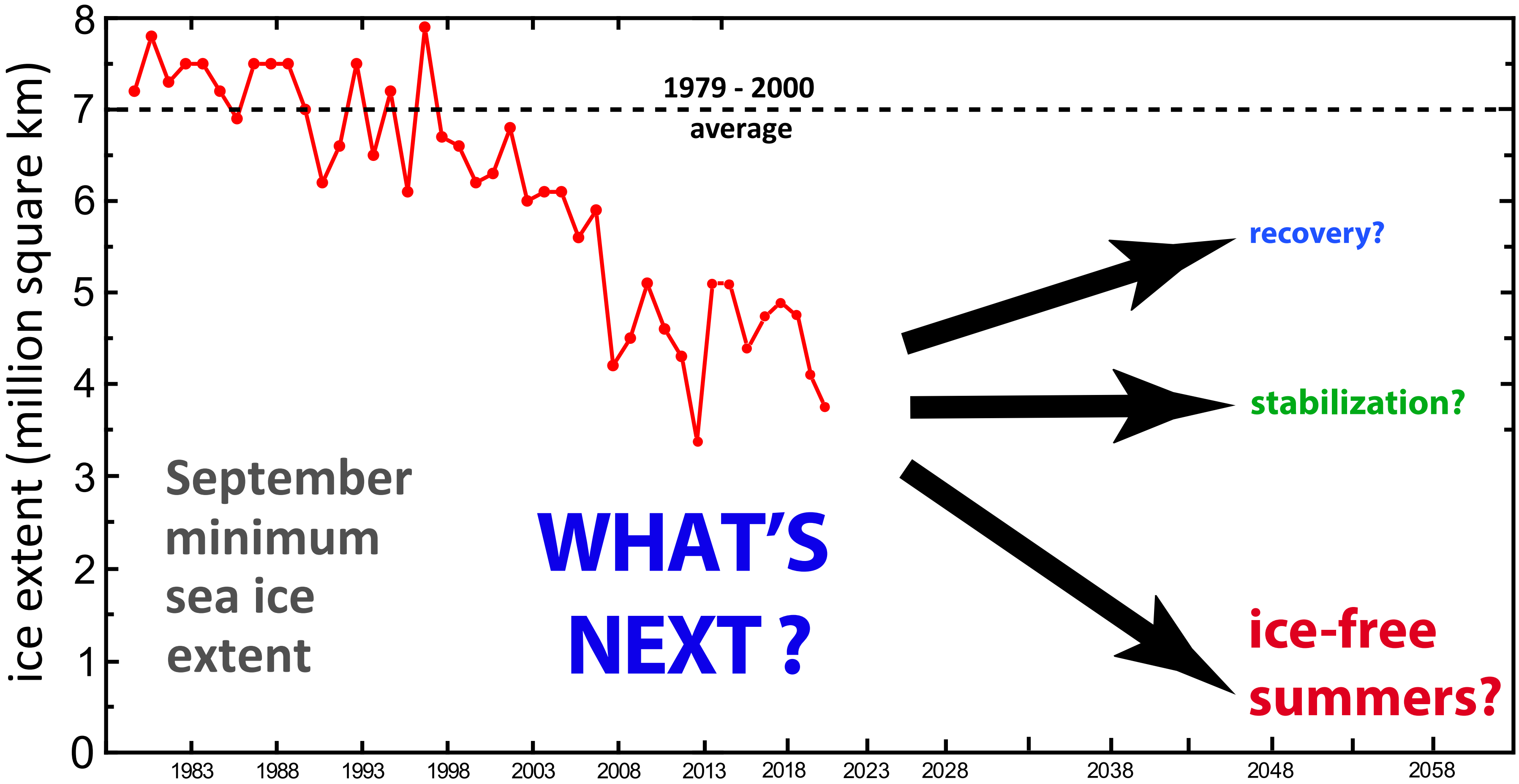


recent losses
in comparison to
the United States



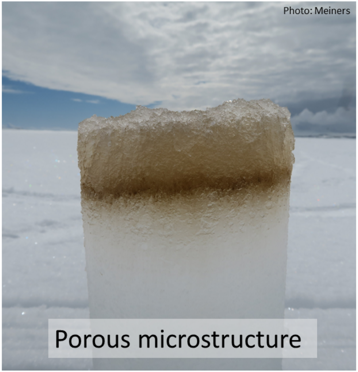
Perovich

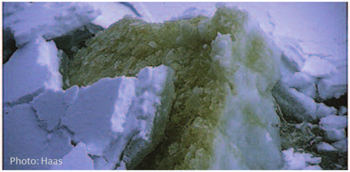
Predicting what may come next requires lots of math modeling.

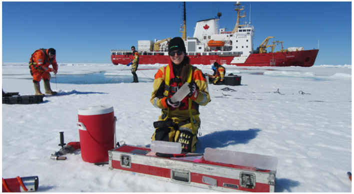


National Snow and Ice Data Center (NSIDC)

Sea ice algae







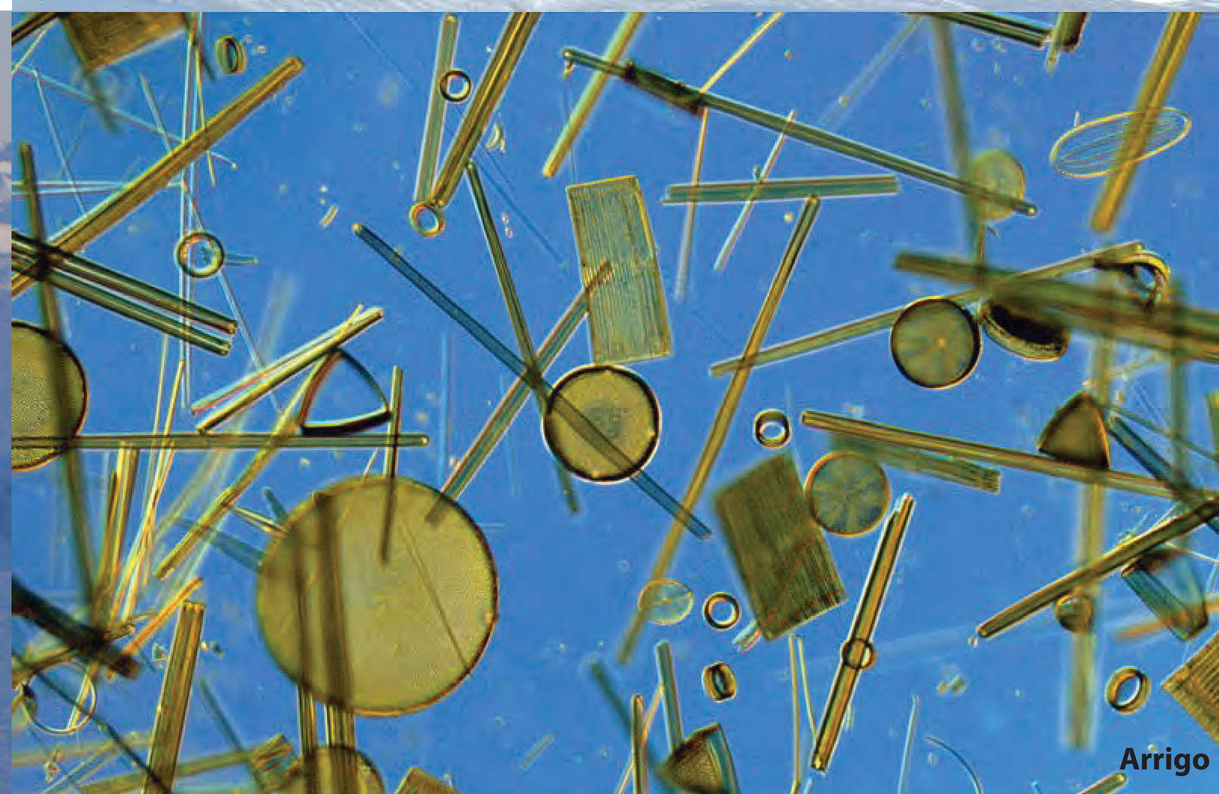
80% of polar bear diet can be traced to ice algae*.

* Brown TA, et al. (2018). *PloS one*, 13(1), e0191631

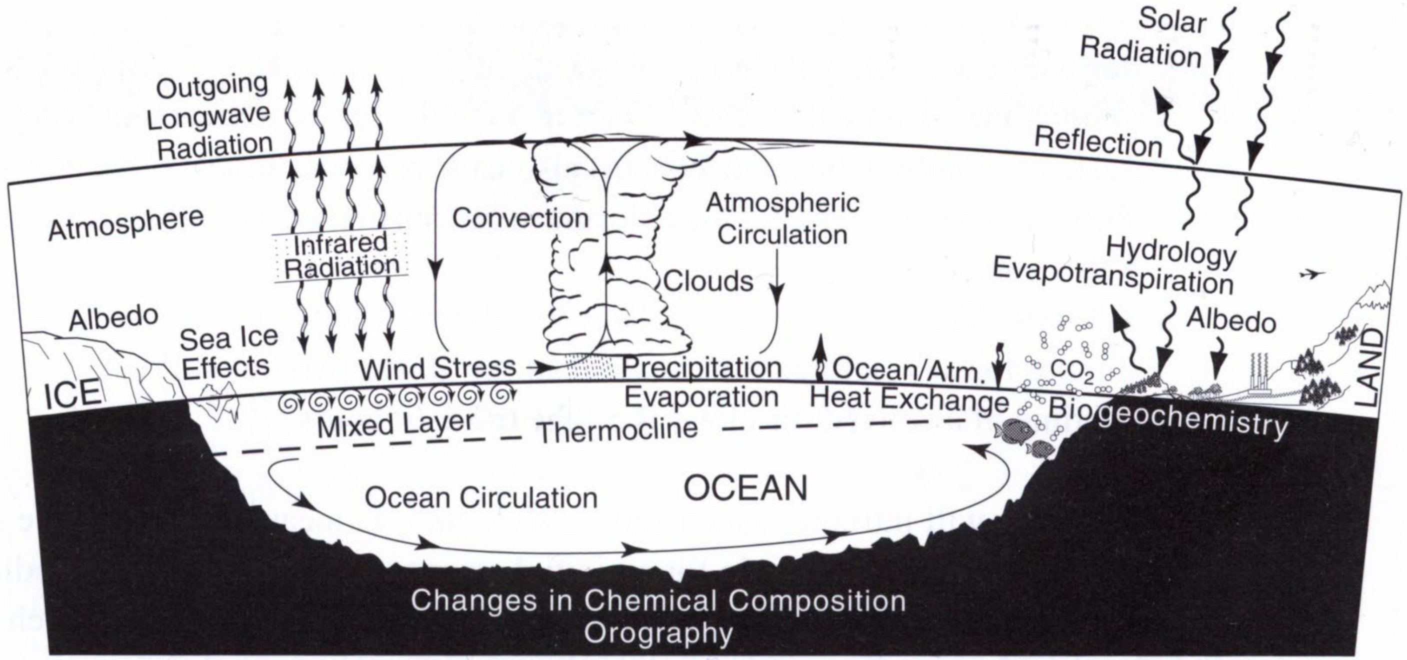
Polar Ecology in a Changing Climate

How does the changing polar marine environment affect life - from extremophile microbes to charismatic megafauna ?

How does microbial life in sea ice affect its properties?



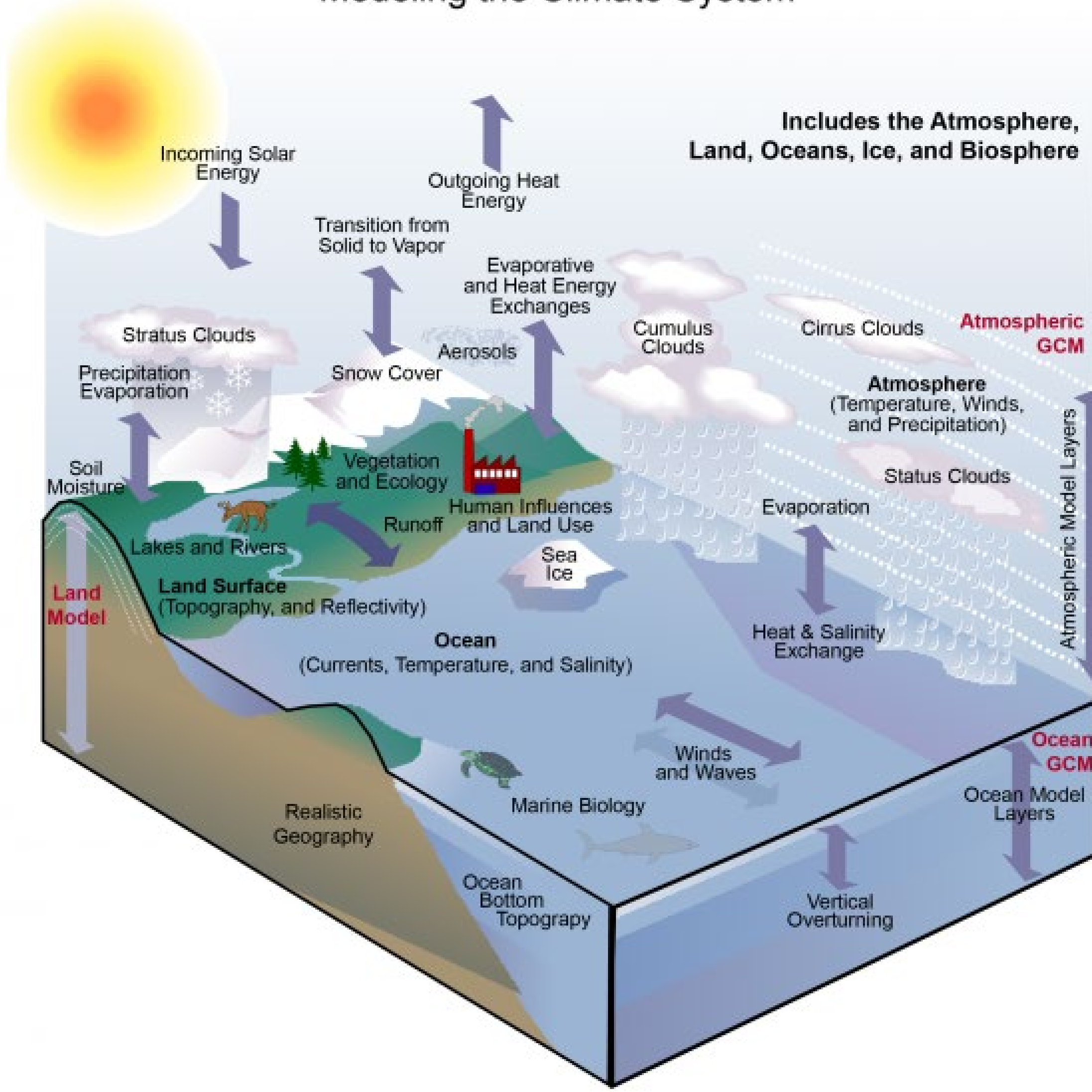
Basics of global climate



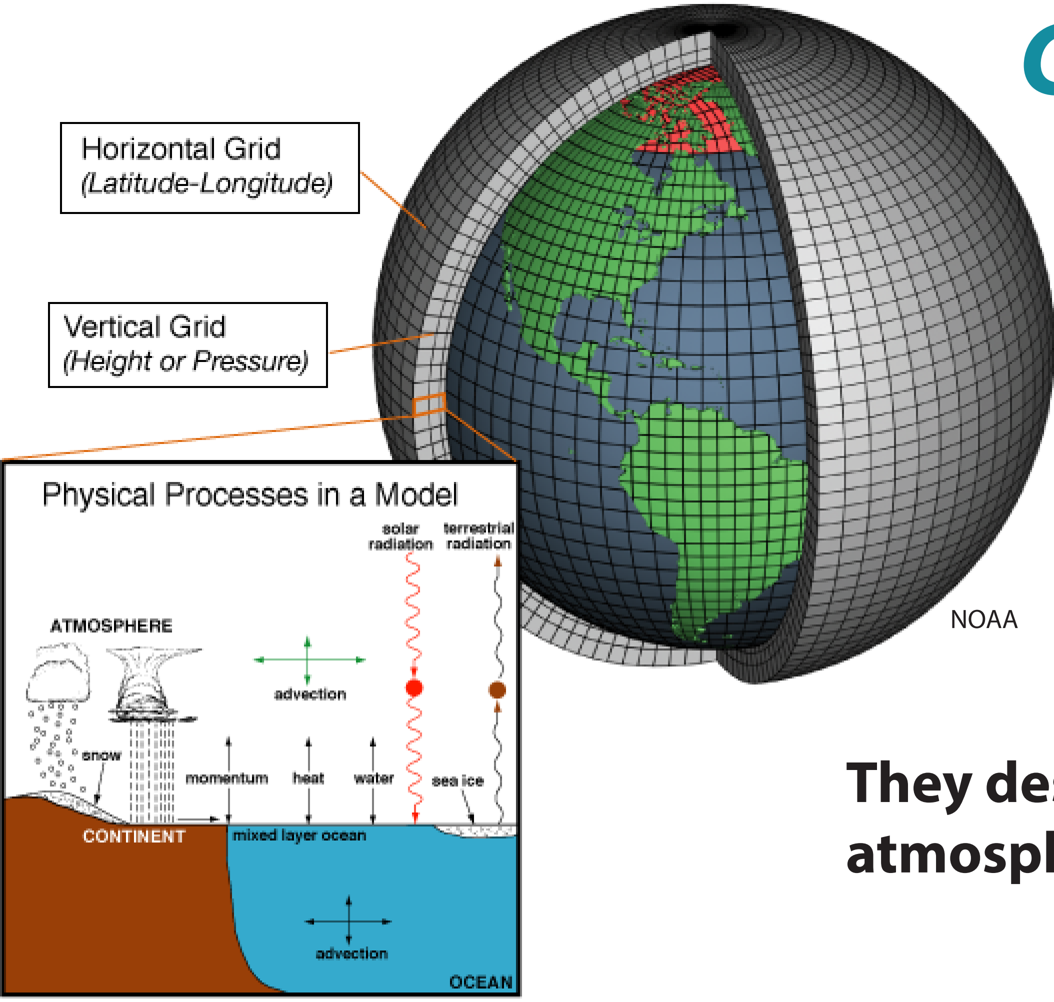
Schematic of some important processes within each component of the climate system.

Basic components: ***cryosphere*** – land ice, snow, sea ice, permafrost (melt -> ?)
biosphere – sum total of all living things
lithosphere – solid Earth – ocean basins, mountain ranges
atmosphere, ***hydrosphere*** (oceans, seas, rivers, …)

Modeling the Climate System



Global Climate Models



NOAA

climate fueled by the nonuniform spatial distribution of incoming solar radiation.

Stute et al., PNAS 2001

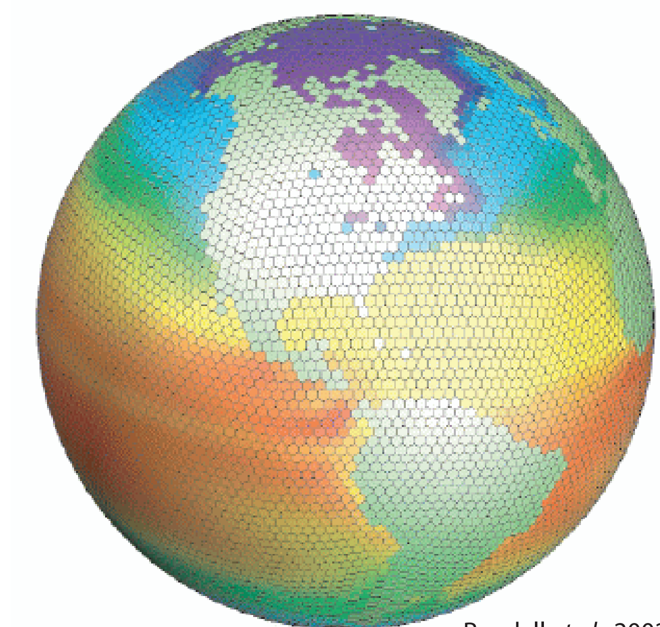
Climate models are coupled systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

They describe the state of the ocean, land, ice atmosphere, biosphere, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 50 km), using very powerful computers.

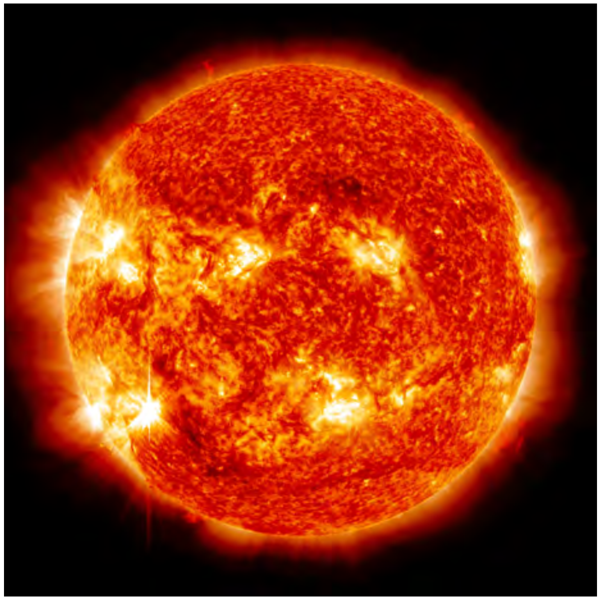
key challenge :

incorporating sub - grid scale processes



Randall *et al.*, 2002

climate system parameters



$$S_0 = 1,368\,\mathrm{Wm}^{-2}$$

solar enery flux per unit area

solar "constant"

depends on sun's strength and average distance from Earth to Sun

MODEL INPUT

$$T(t) = \langle \mathrm{Temperature}(x,t) \rangle$$

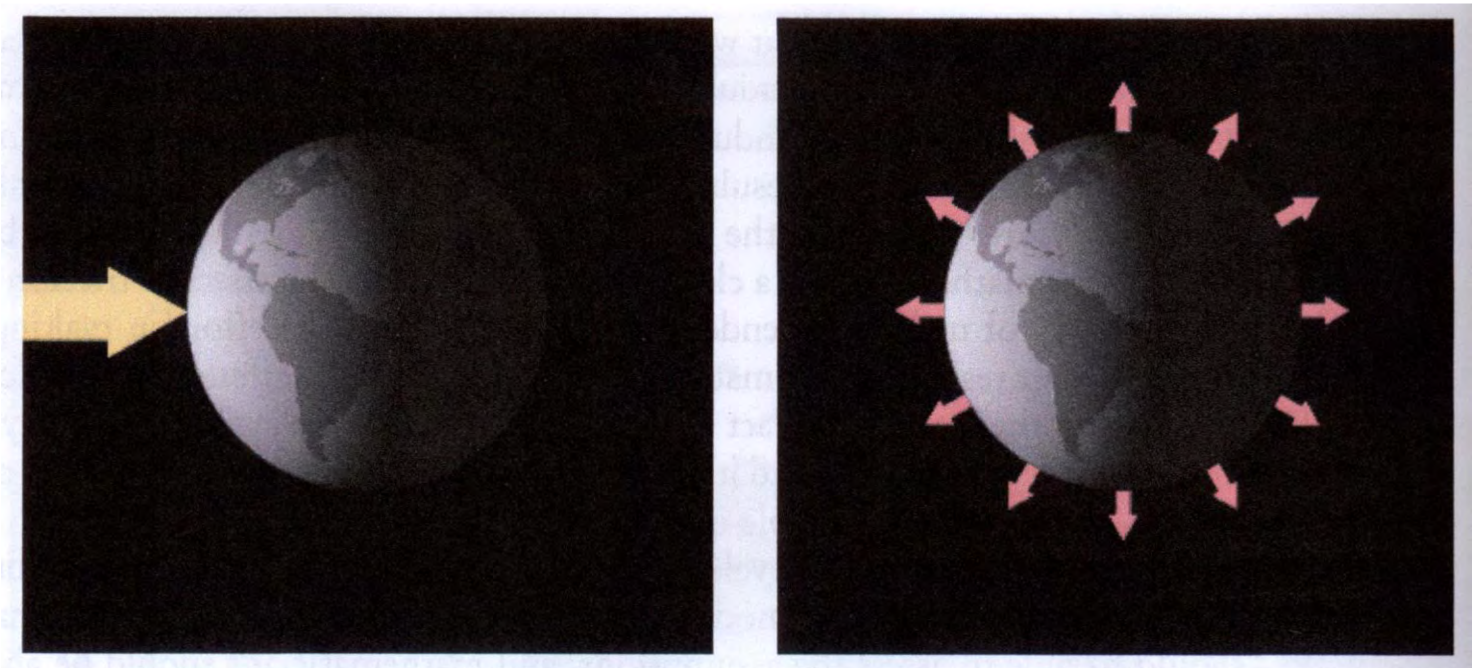
global mean surface temperature

How to predict ??

(How to define and measure??)

MODEL OUTPUT

Earth's energy balance



E_{in}

incoming shortwave radiation
sunlight

E_{out}

outgoing longwave radiation
heat

if $E_{out} > E_{in}$ **planet cools**

if $E_{out} < E_{in}$ **planet warms**

Energy Balance Model

$$C\,\frac{dT}{dt} = E_{in} - E_{out}$$

rate of change with time of mean global temperature

C = ***heat capacity***
= ***energy needed to raise temperature by one Kelvin***

$$E_{in} = \;?$$

$$E_{out} = \;?$$

E_{in}

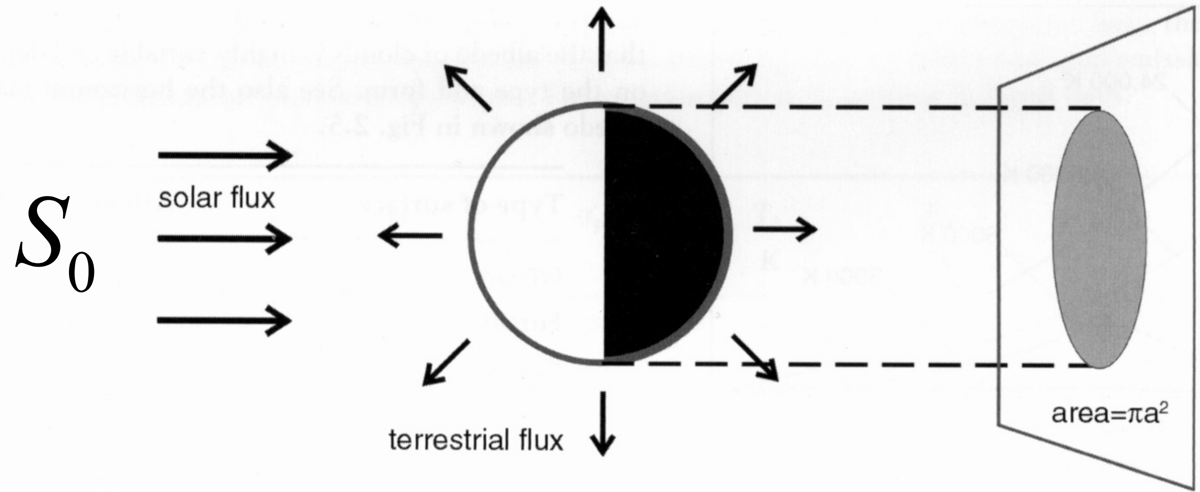


FIGURE 2.4. The spinning Earth is imagined to intercept solar energy over a disk of radius a and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

solar power incident on Earth

$$S_0 \pi a^2 = 1.74 \times 10^{17}\,\mathrm{W}$$

most important climate parameter:
how much energy reflected *vs.* how much energy absorbed

reflection

albedo $\alpha = \dfrac{\text{reflected sunlight}}{\text{incident sunlight}}$

absorption

co- albedo $1-\alpha$

fraction of energy that reaches Earth's surface

amount of solar energy intercepted by Earth per unit time (as a flat disk)

$$\pi R^2 S_0$$

amount of solar energy reaching Earth's surface per unit time

$$(1-\alpha)\pi R^2 S_0$$

amount of solar energy reaching Earth's surface per unit time / unit area

$$E_{in} = \frac{(1-\alpha)\pi R^2 S_0}{4\pi R^2} = \frac{(1-\alpha)S_0}{4} = (1-\alpha)Q$$

albedo of Earth's surface

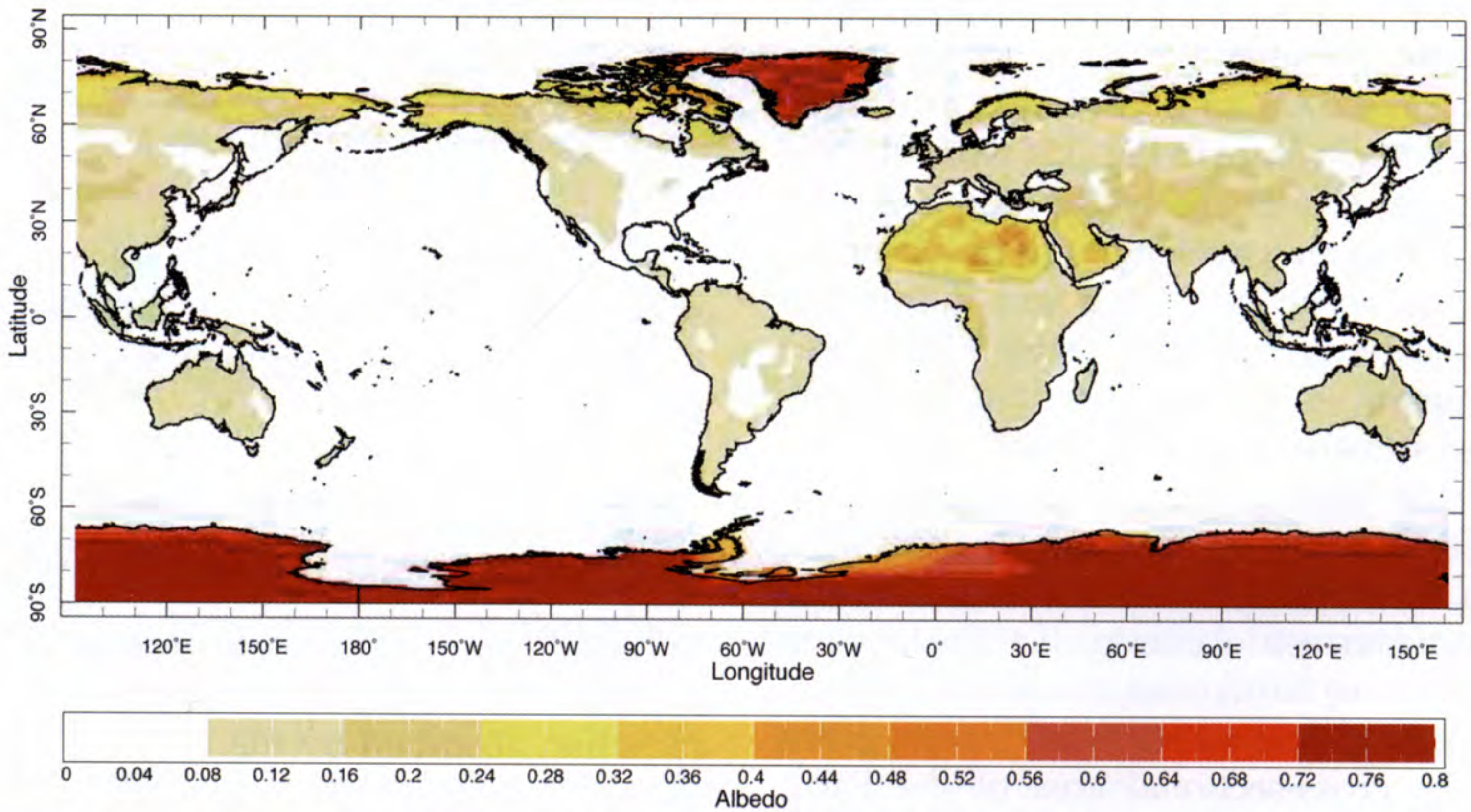


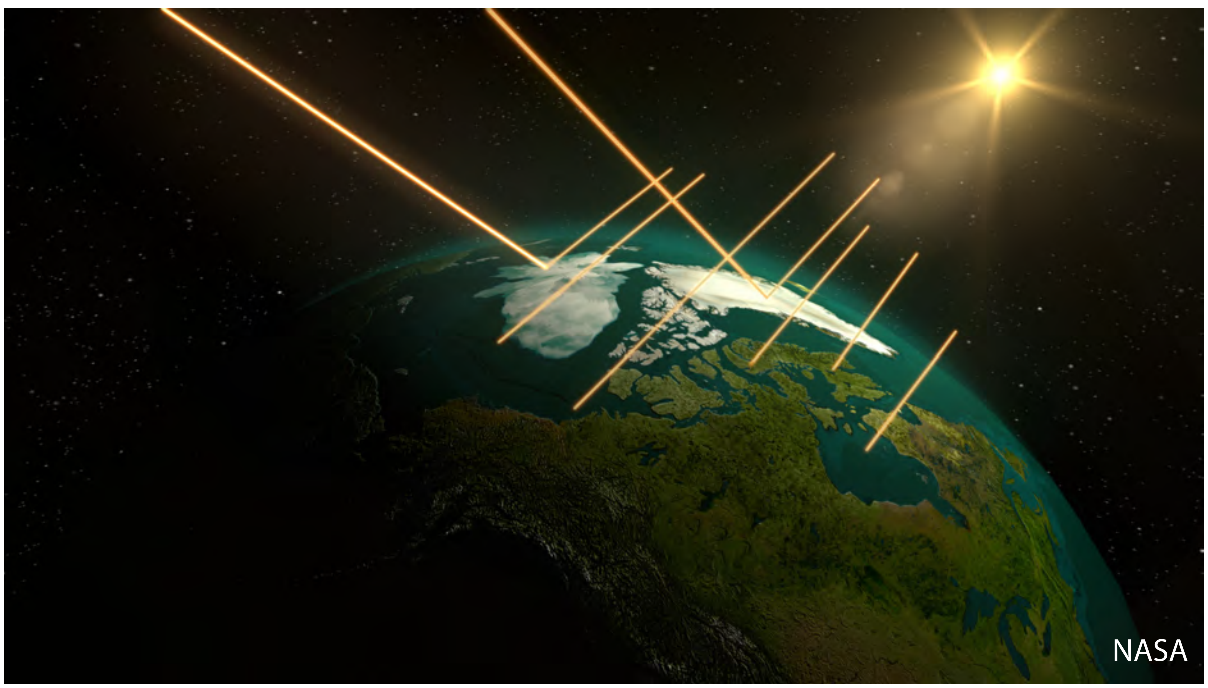
FIGURE 2.5. The albedo of the Earth's surface. Over the ocean the albedo is small (2–10%). It is larger over the land (typically 35–45% over desert regions) and is particularly high over snow and ice (~80%) (see Table 2.2).

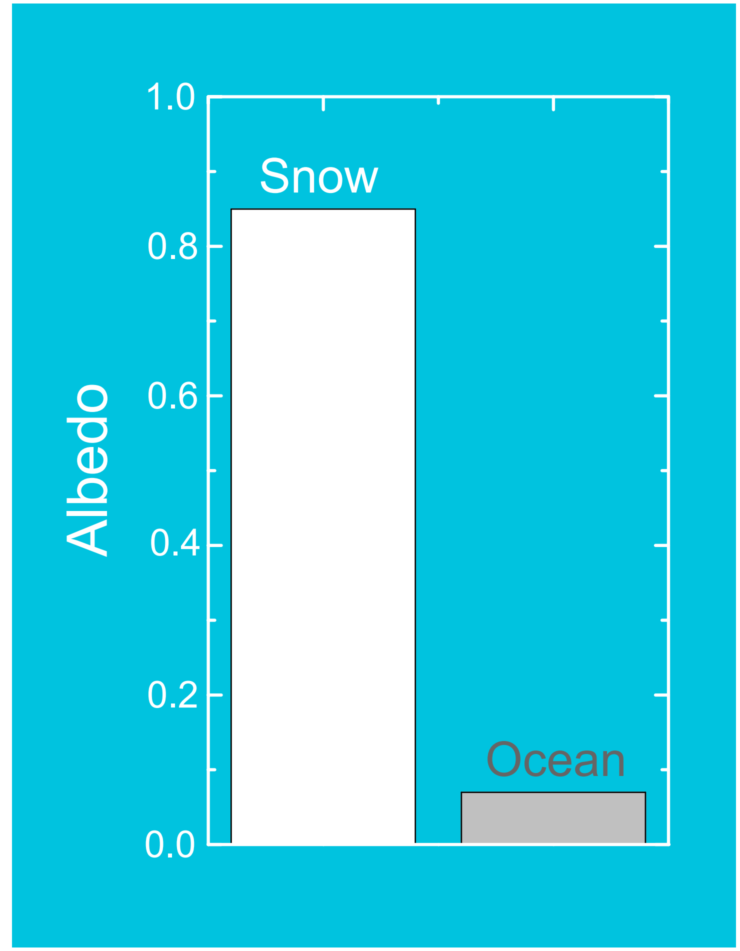
average albedo ~ 0.3

Marshall and Plumb 2008

polar ice caps critical to global climate in reflecting incoming solar radiation

white snow and ice reflect





dark water and land absorb

$$\textit{albedo}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

How much energy radiates out into space from a ball at temperature T?

Earth radiates mostly in infrared spectrum, energy radiated depends on T
(approximate as a "black body")

$$E_{out} = \sigma T^4$$

$$\sigma = 5.67 * 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$$

Stefan - Boltzmann constant

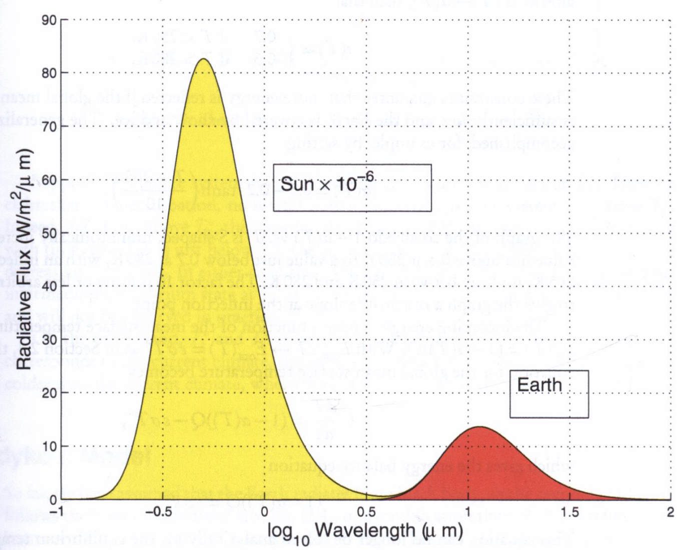


Figure 2.4. *Black-body emission curves of the Sun and the Earth. Reprinted courtesy of NASA.*

Energy Balance Model

$$C\,\frac{dT}{dt} = E_{in} - E_{out}$$

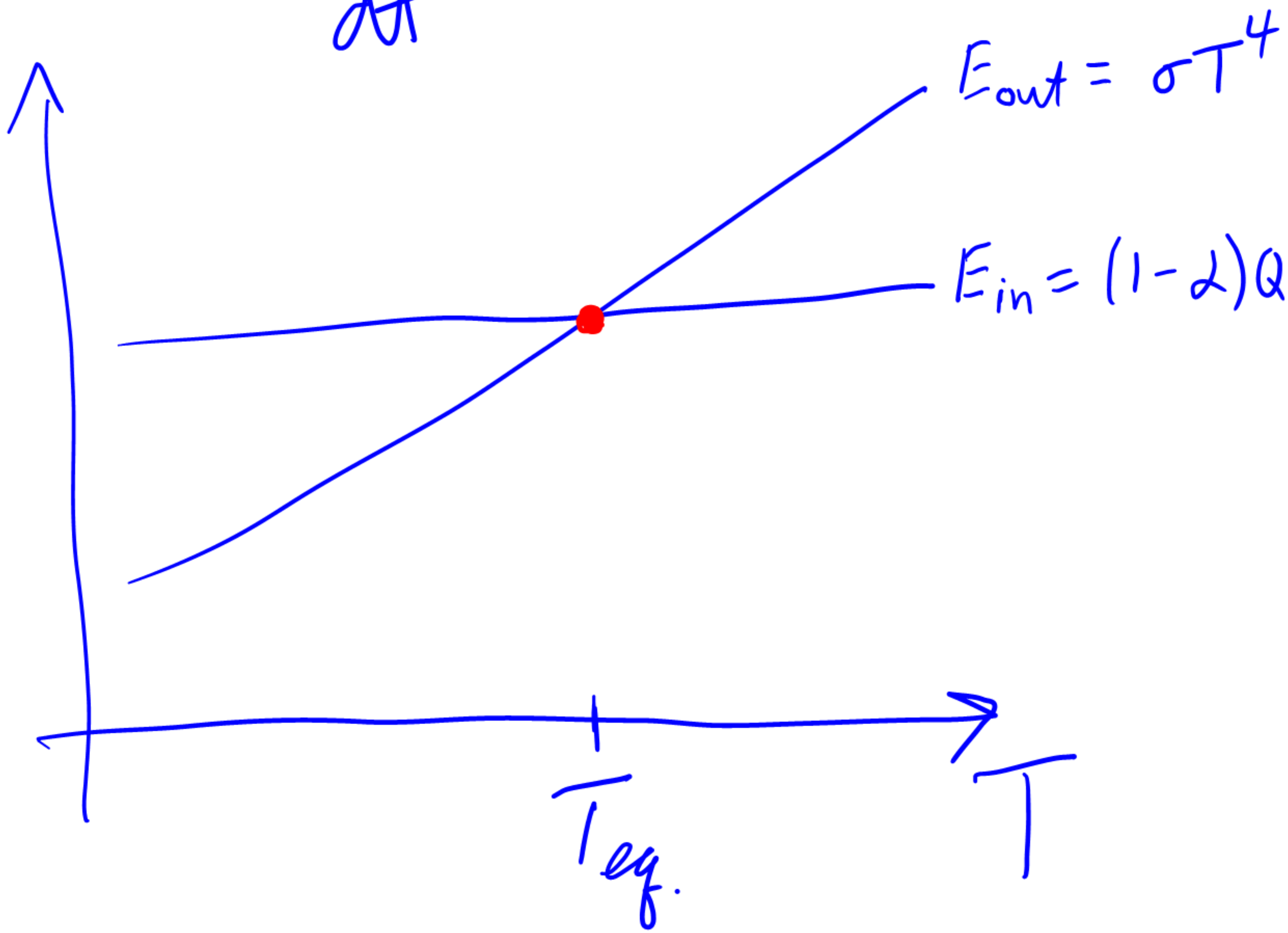
C = ***heat capacity***
= ***energy needed to raise temperature by one Kelvin***

rate of change of mean global temperature

$$C\,\frac{dT}{dt} = (1-\alpha)Q \; - \; \sigma T^4$$

$$C \frac{dT}{dt} = E_{in} - E_{out}$$

At equilibrium $\frac{dT}{dt} = 0 \implies E_{in} = E_{out}$



Find steady-state **equilibrium** temperature: $C\frac{dT}{dt} = E_{in} - E_{out}$

$$\frac{dT}{dt} = 0$$

constant temperature rate of change = 0

$$E_{in} = E_{out}$$

solve $$\sigma T^4 = (1-\alpha)Q$$

$$T_{eq} = \left(\frac{(1-\alpha)\,Q}{\sigma}\right)^{1/4}$$

$T_{eq} \approx 254.8\,\mathrm{K} = -1.03\,°\mathrm{F}$, which is quite cold!

The actual measured temperature of the surface of the Earth is $287.89\,\mathrm{K} = 58.5\,°\mathrm{F}$

Basics of global warming date back to the 1800's

HEAT EQUATION
FOURIER SERIES

1. Jean Baptiste Joseph Fourier (1768-1830), French mathematician and natural philosopher, did groundbreaking work in mathematics and the theory of heat. He was the first to propose that the Earth's atmosphere acts to raise the planet's temperature.

   "As a dam built across a river causes a local deepening of the stream, so our atmosphere, thrown as a barrier across the terrestrial rays, produces a local heightening of the temperature at the Earth's surface."

atmosphere traps heat escaping from Earth - acts like a greenhouse

Fourier, 1827

2. 1861 John Tyndall discovers in his laboratory that certain gases - water vapor and CO2 are opaque to heat rays. He understood that such gases high in the air help keep our planet warm by interfering with escaping radiation.

1856 Eunice Newton Foote gives paper at AAAS conference on *her* experiments

3. 1896 Svante Arrhenius (1903 Nobel Prize in Chemistry) proposed: relationship between atmospheric CO2 concentrations and temperature;

global warming may result from fossil fuel combustion (burning coal).

He and Thomas Chamberlin calculated that human activities could warm the earth by adding carbon dioxide to the atmosphere.

In a nutshell:

Scientific basis of CO2 greenhouse effect was given by Tyndall (1861); while the first extensive calculations of its magnitude were made by Arrhenius (1896).

Pretty good .... but what are we missing?

greenhouse effect

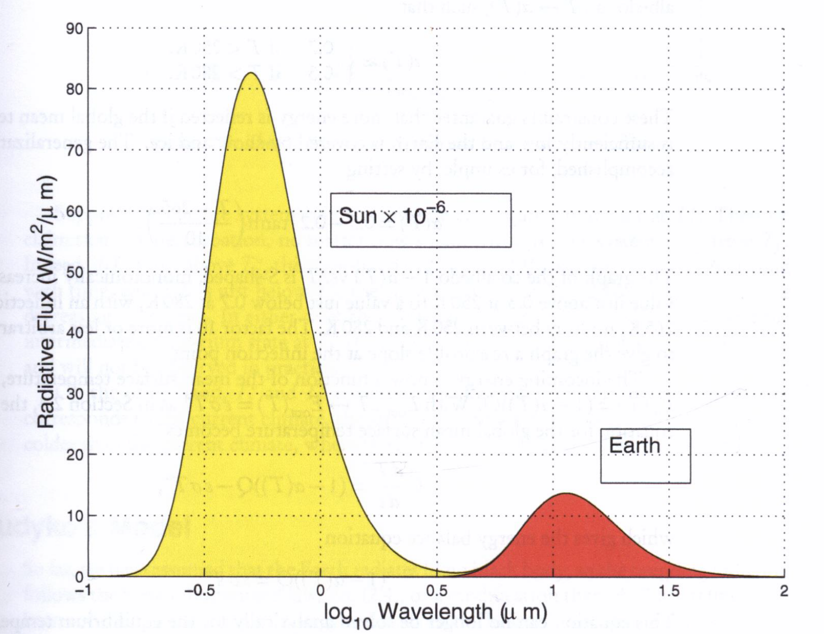
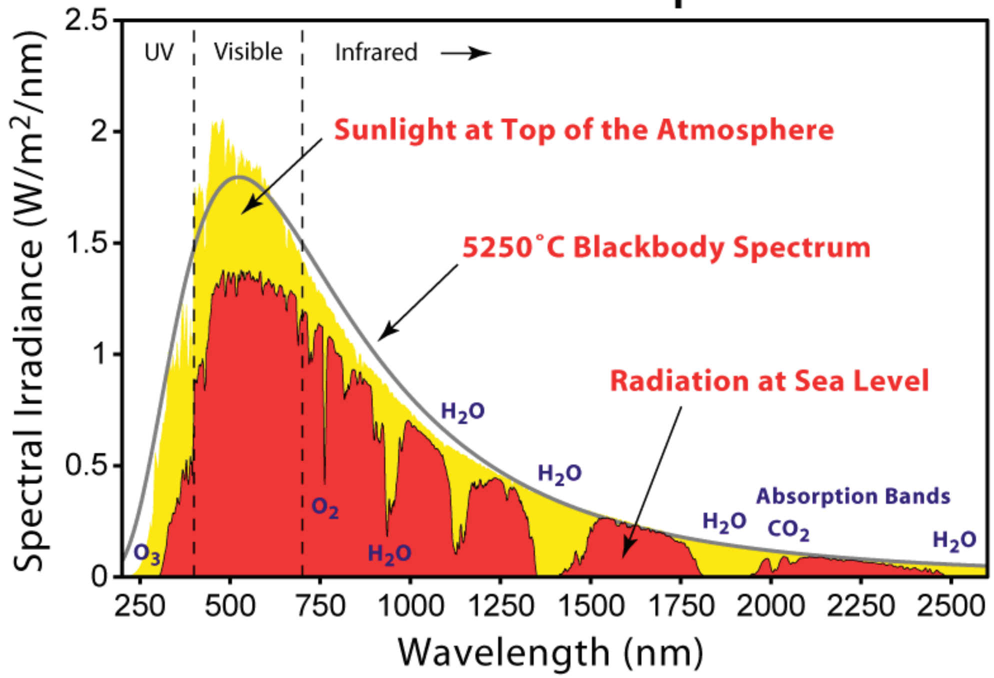
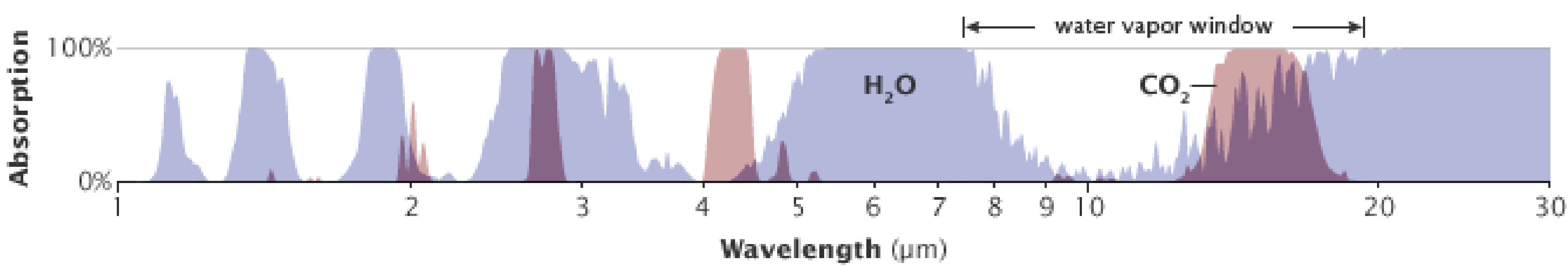


Figure 2.4. *Black-body emission curves of the Sun and the Earth. Reprinted courtesy of NASA.*

Solar Radiation Spectrum



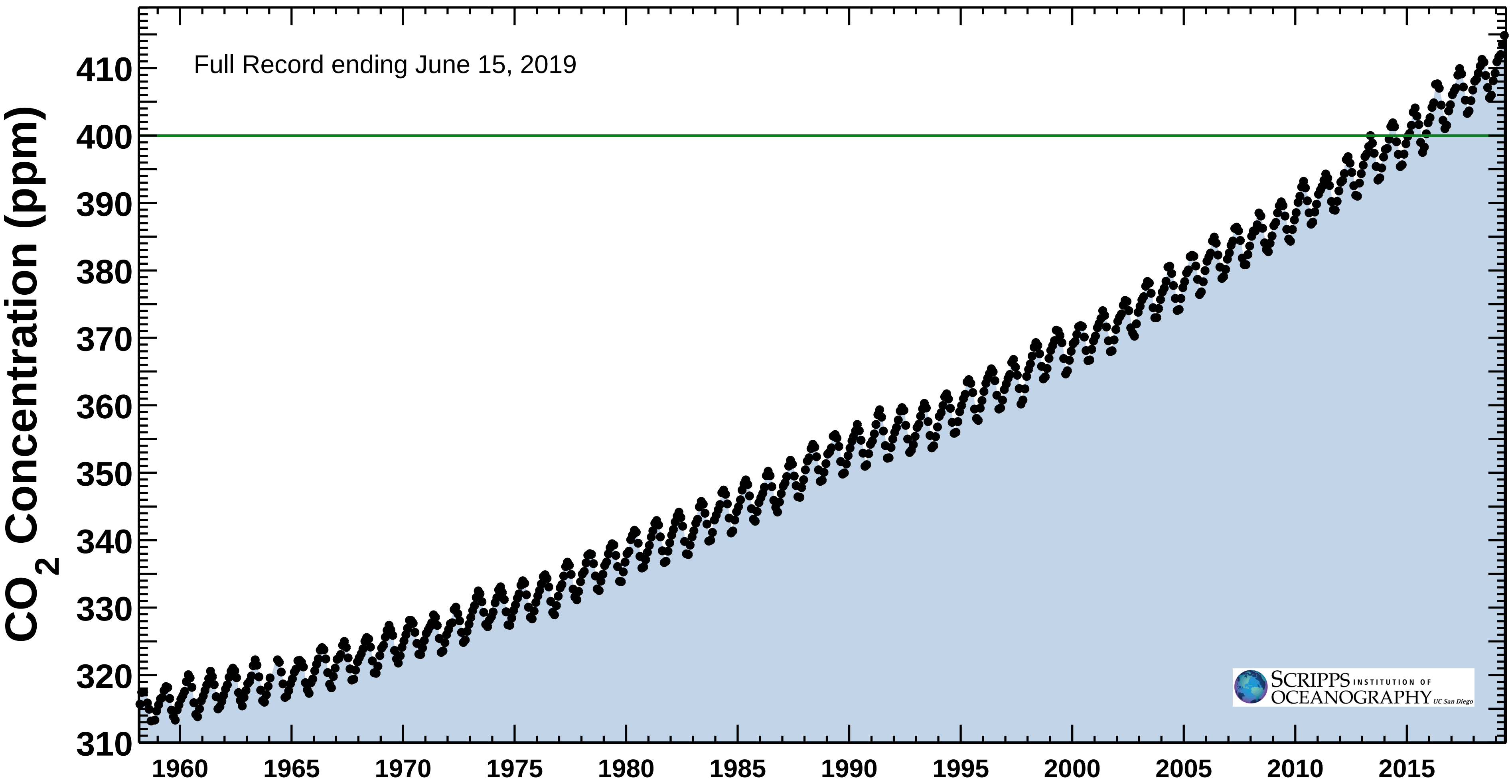


Earth's emission spectrum in far infrared > 5 μm

Latest CO_2 reading
June 13, 2019

414.71 ppm

Carbon dioxide concentration at Mauna Loa Observatory

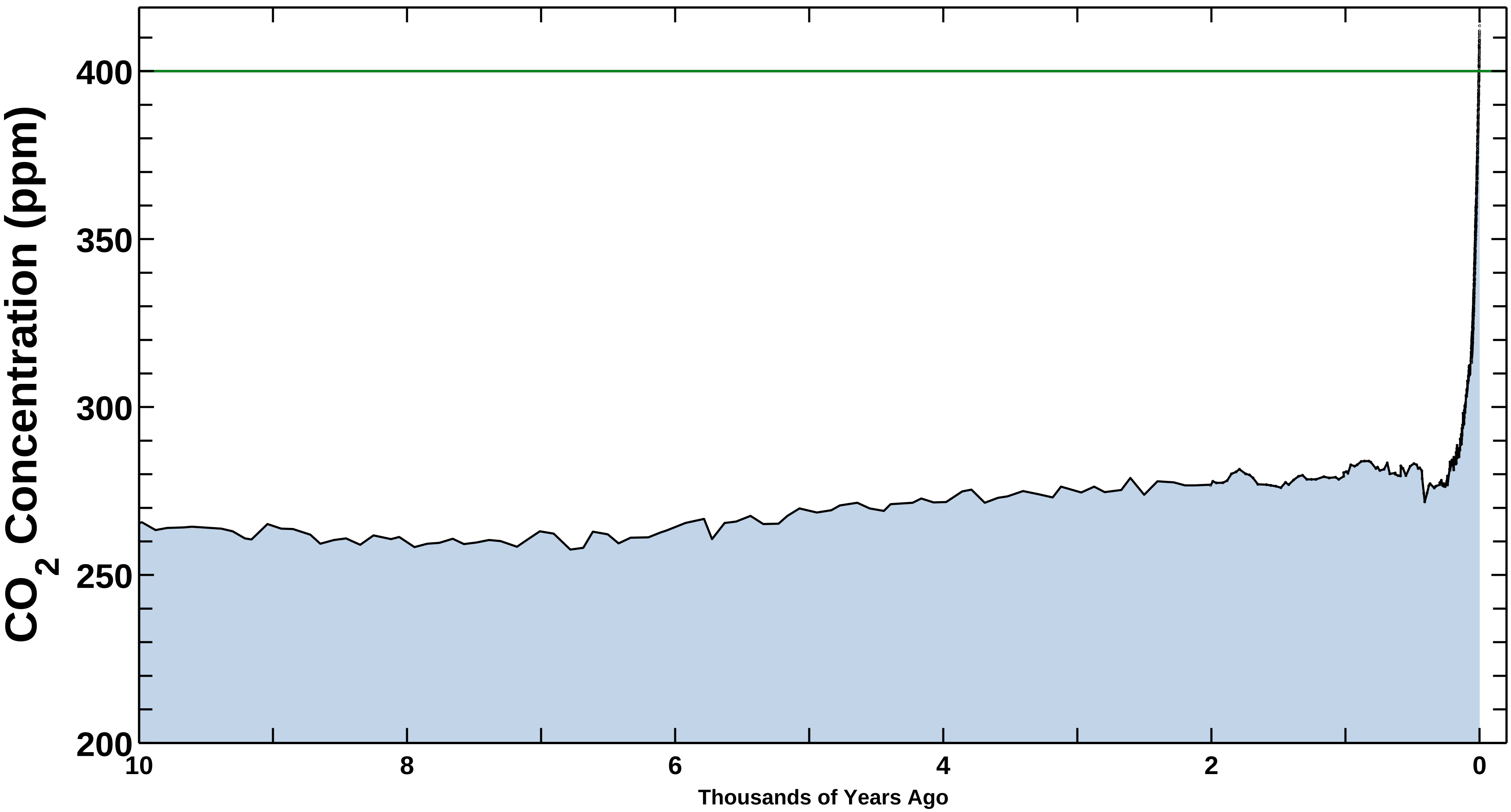


Latest CO_2 reading

June 13, 2019

414.71 ppm

Ice−core data before 1958. Mauna Loa data after 1958.



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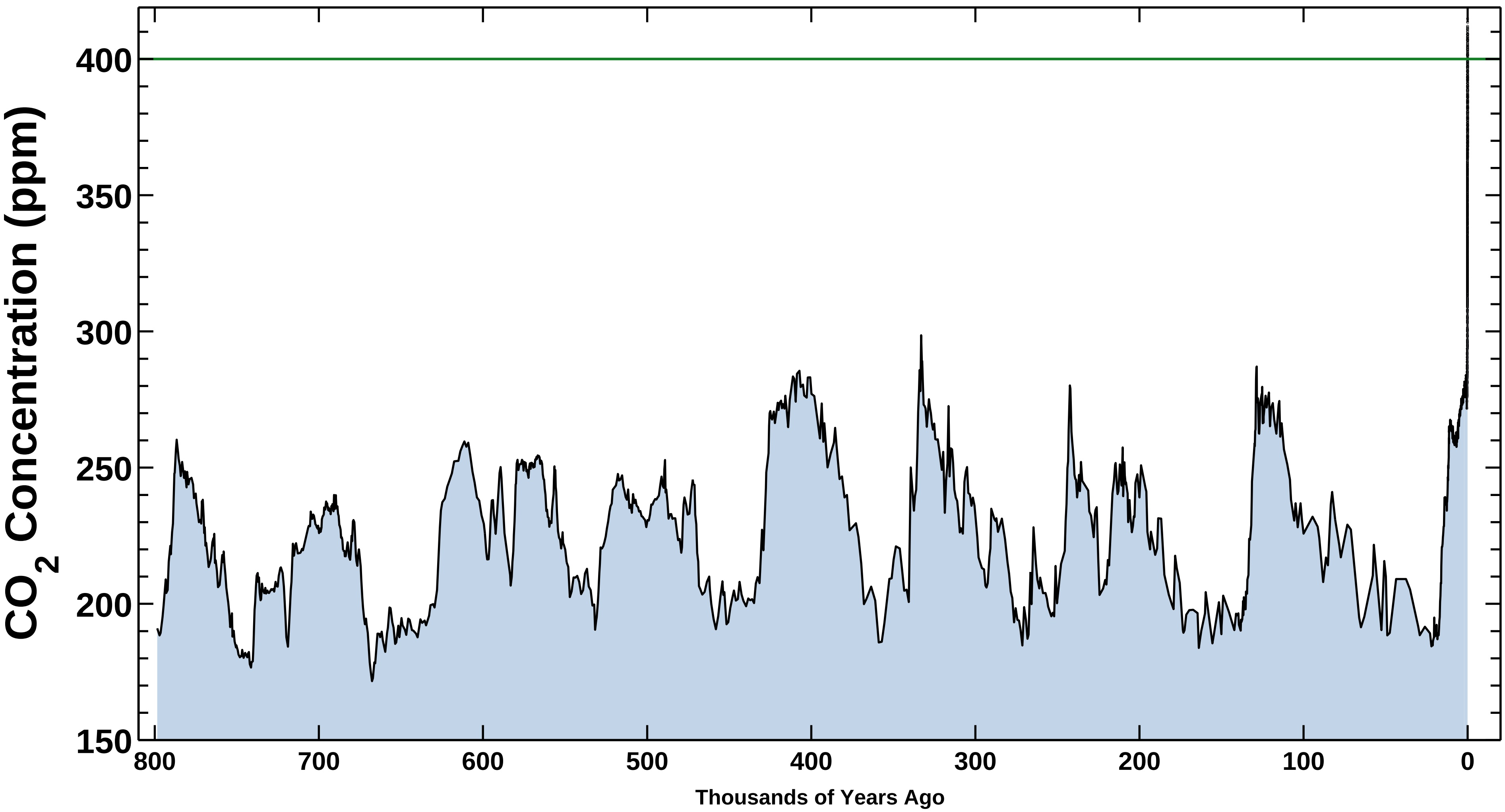
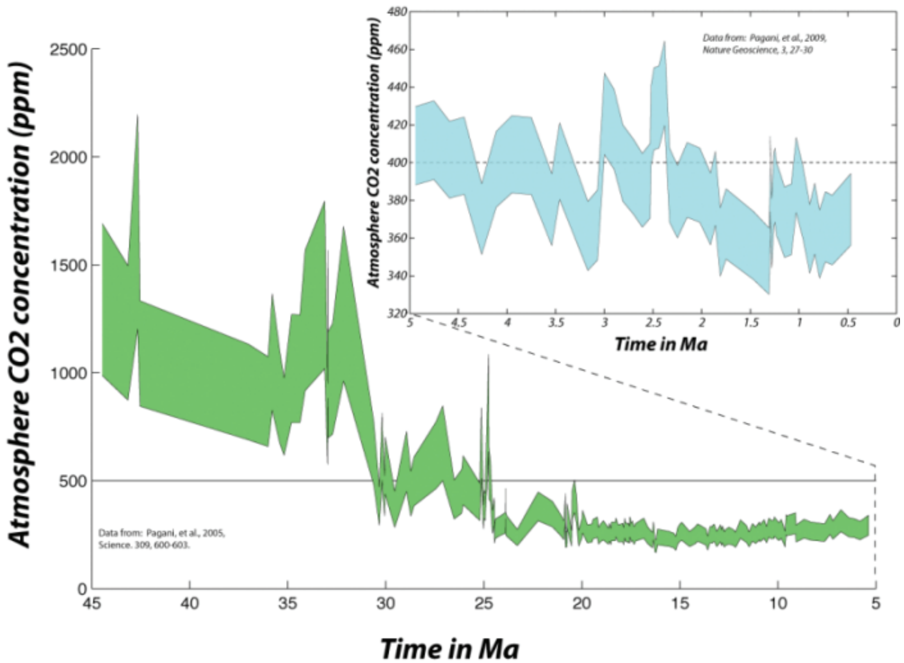


When was the last time CO_2 was above 400 ppm for a sustained period?



The PLIOCENE Era ~ 2.6 - 5.3 M years ago

Giant camels roamed the forested high Arctic.

Sea levels were at least 30 feet higher.

Global temperatures were ~ 3°C higher.

The atmosphere warms (on geophysical time scales) much faster than the great land ice sheets can melt.

The atmosphere and ice sheets are out of equilibrium.

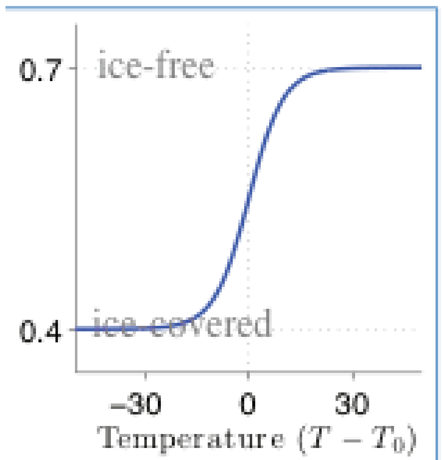
We've just put a quart of ice cream in the oven.
It's still frozen, but after a while it will melt!

Assuming albedo independent of Earth's temperature too simplified....

planetary co-albedo

$$1 - \alpha$$

warm
watery
absorptive



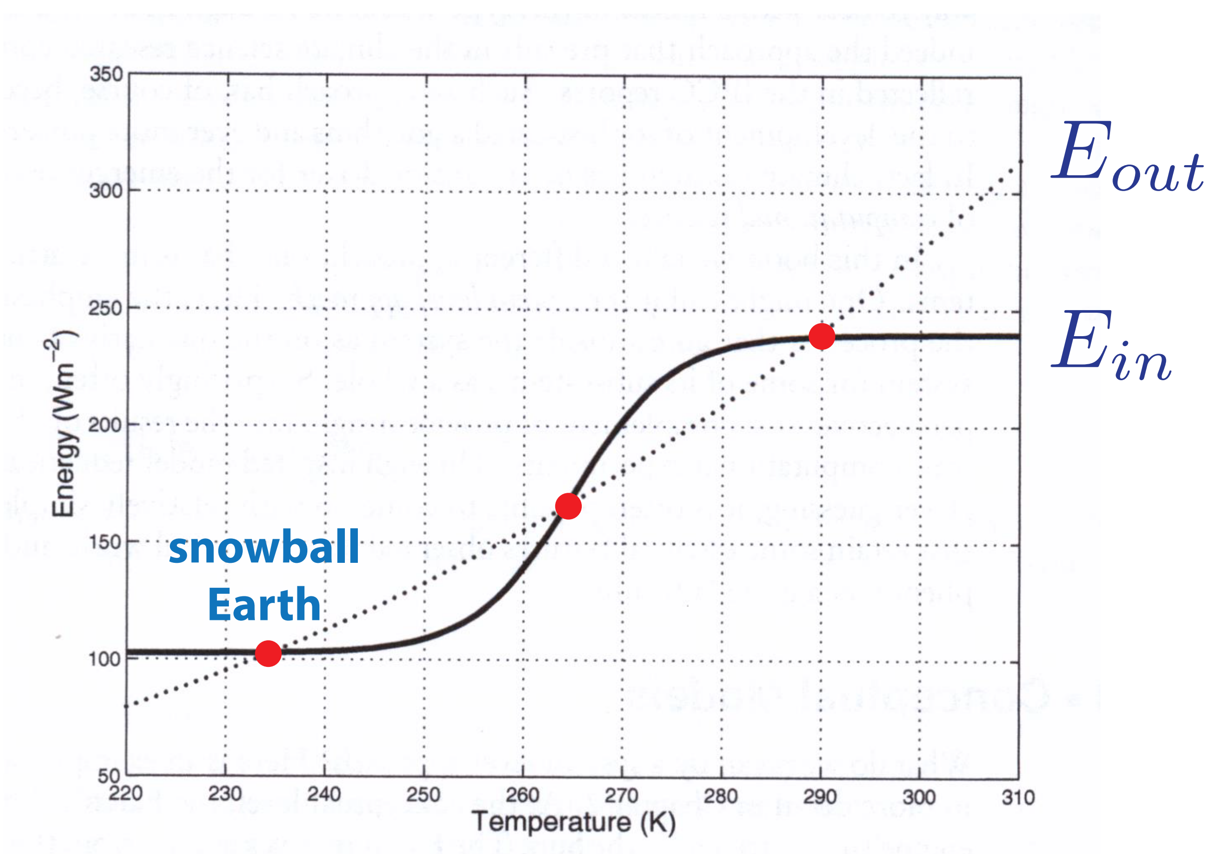
cold
icy
reflective

albedo nonlinear in temperature

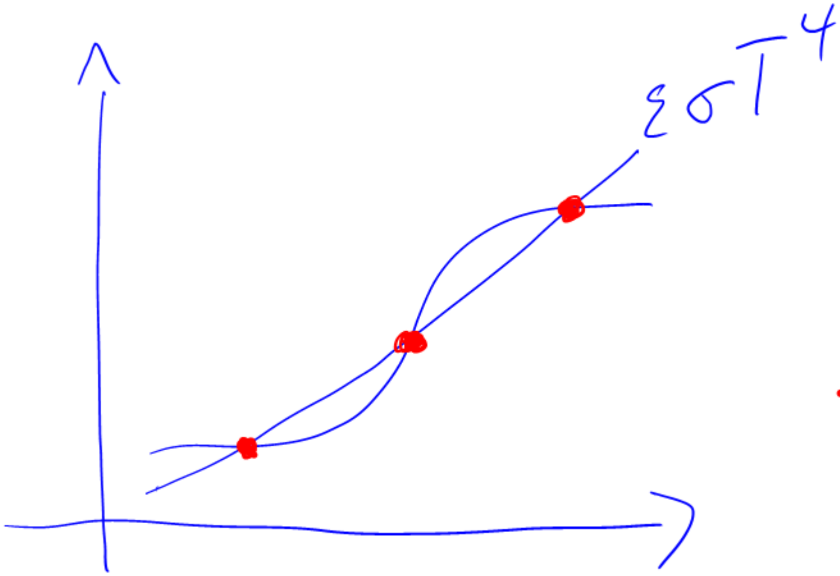
Now solve for equilibrium temperature $E_{in} = E_{out}$

$$\sigma T^4 = (1 - \alpha(T))Q$$

3 solutions!
multiple equilibria

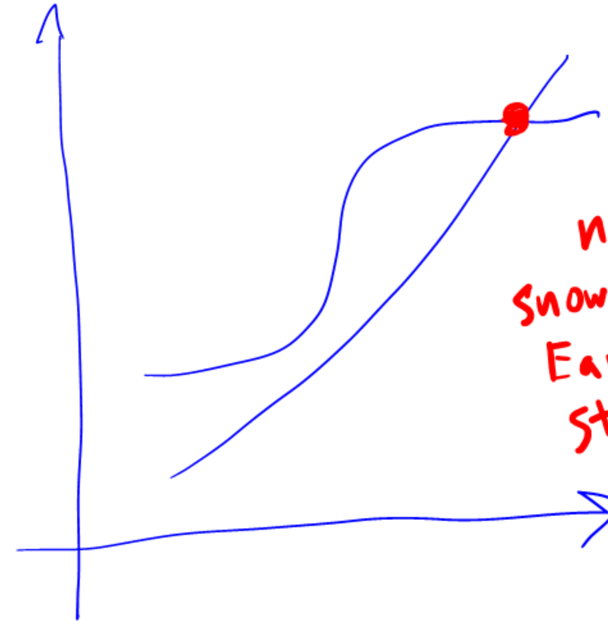


stable *vs.* unstable equilibria



decrease ε

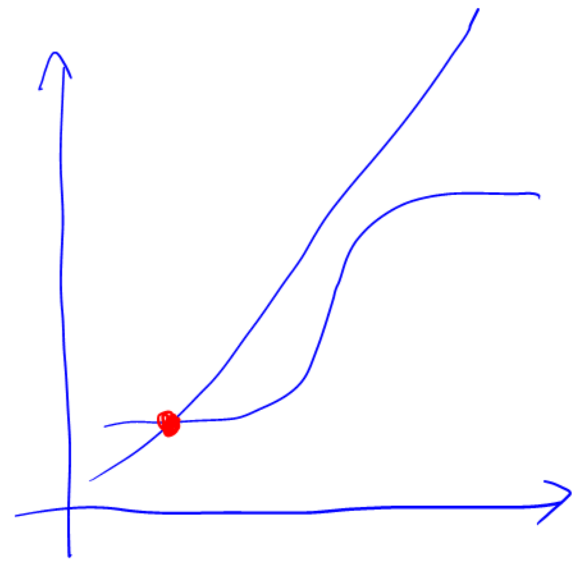
$\uparrow$ greenhouse



no snowball Earth state

decrease Q

increase ε



Bifurcation Diagram

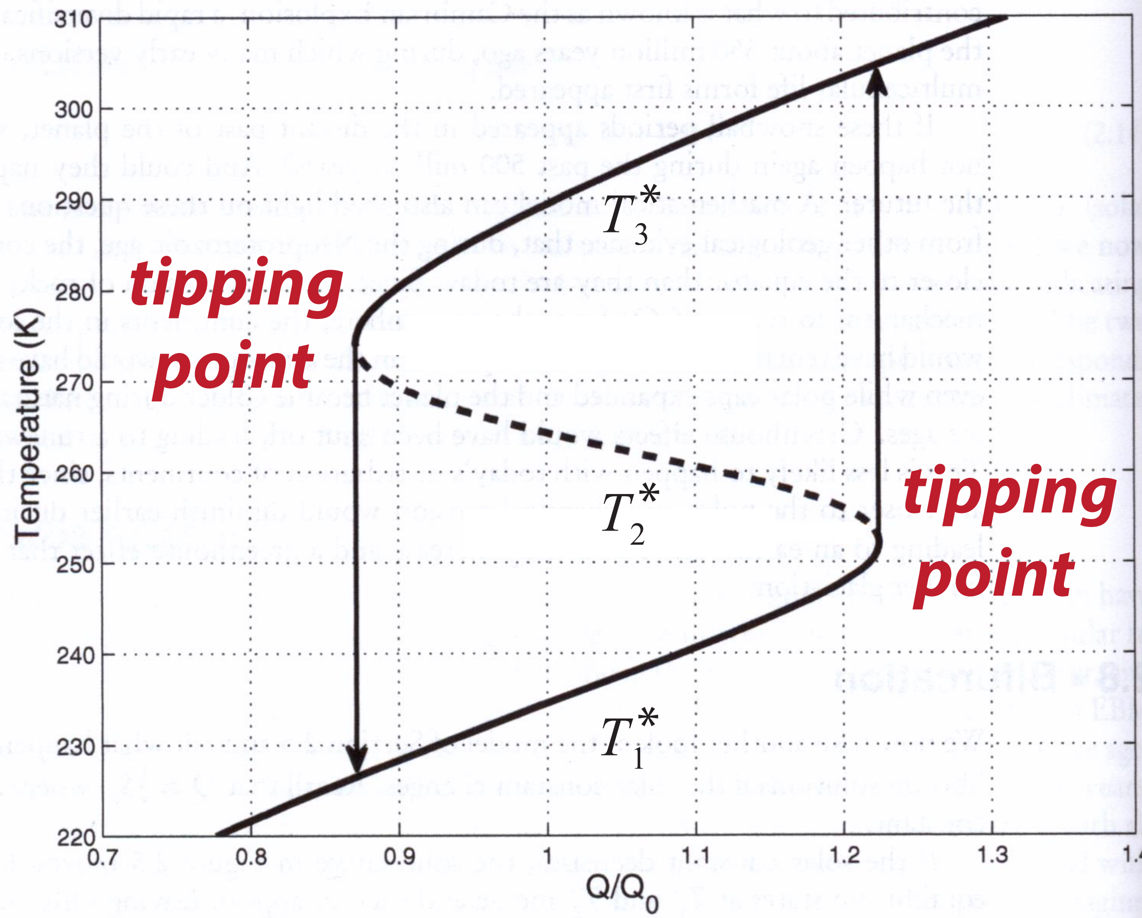
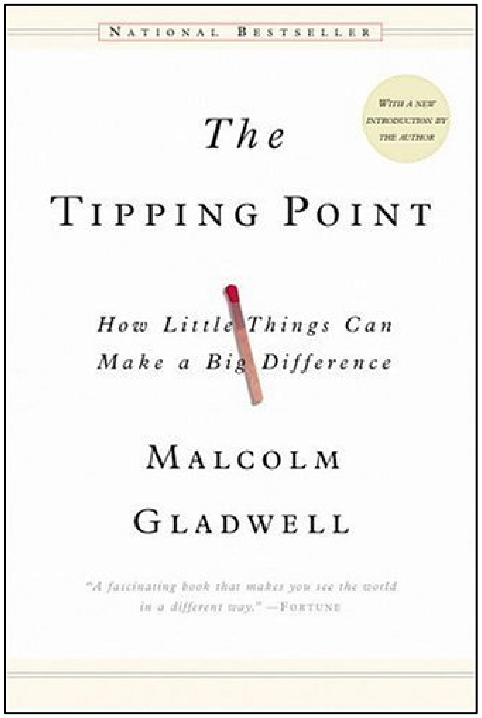
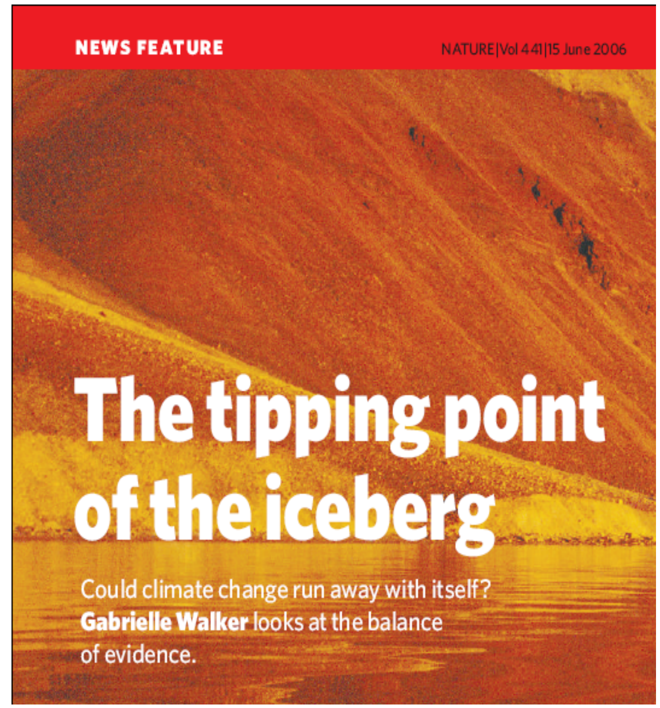


Figure 2.6. *Mean surface temperatures at equilibrium as a function of the solar constant (in units of its present value).*

tipping points in the mainstream

Increasing emphasis in recent years on idea of climate tipping points, with September Arctic sea ice cover receiving much of the attention.





Melting of the Greenland ice sheet
Melting of the West Antarctic ice sheet
Permafrost and tundra loss, leading to the release of methane
Formation of Atlantic deep water near the Arctic ocean **•••**

Lenton, et al., PNAS 2008

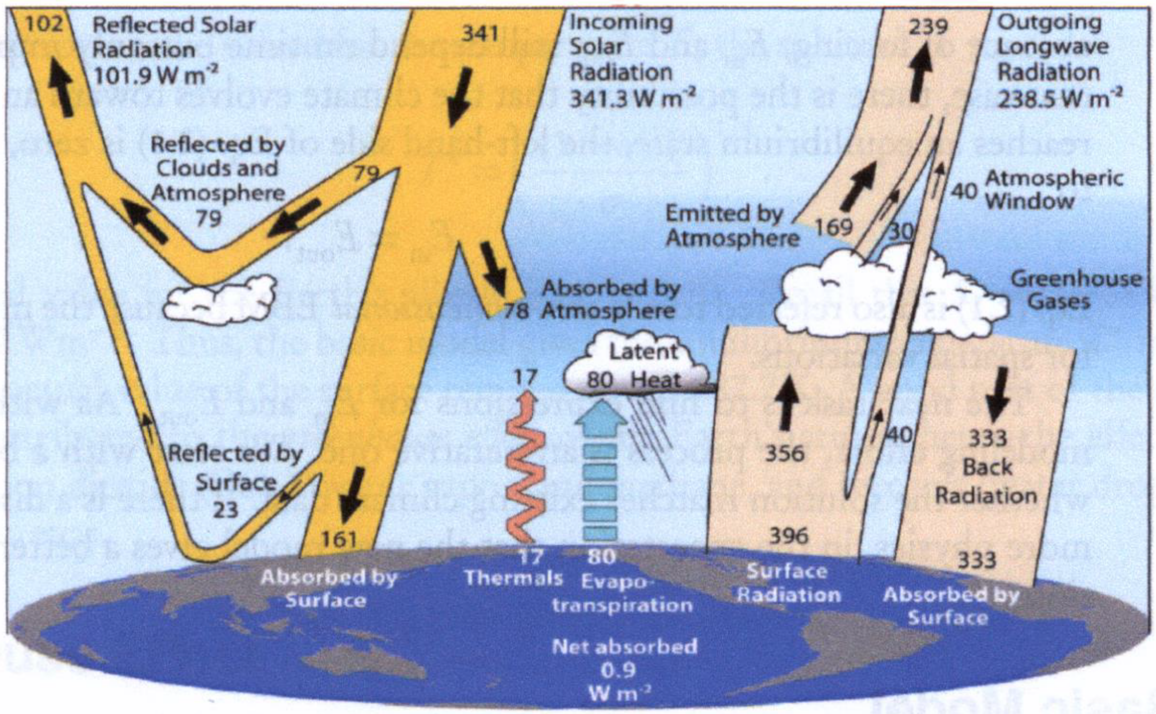
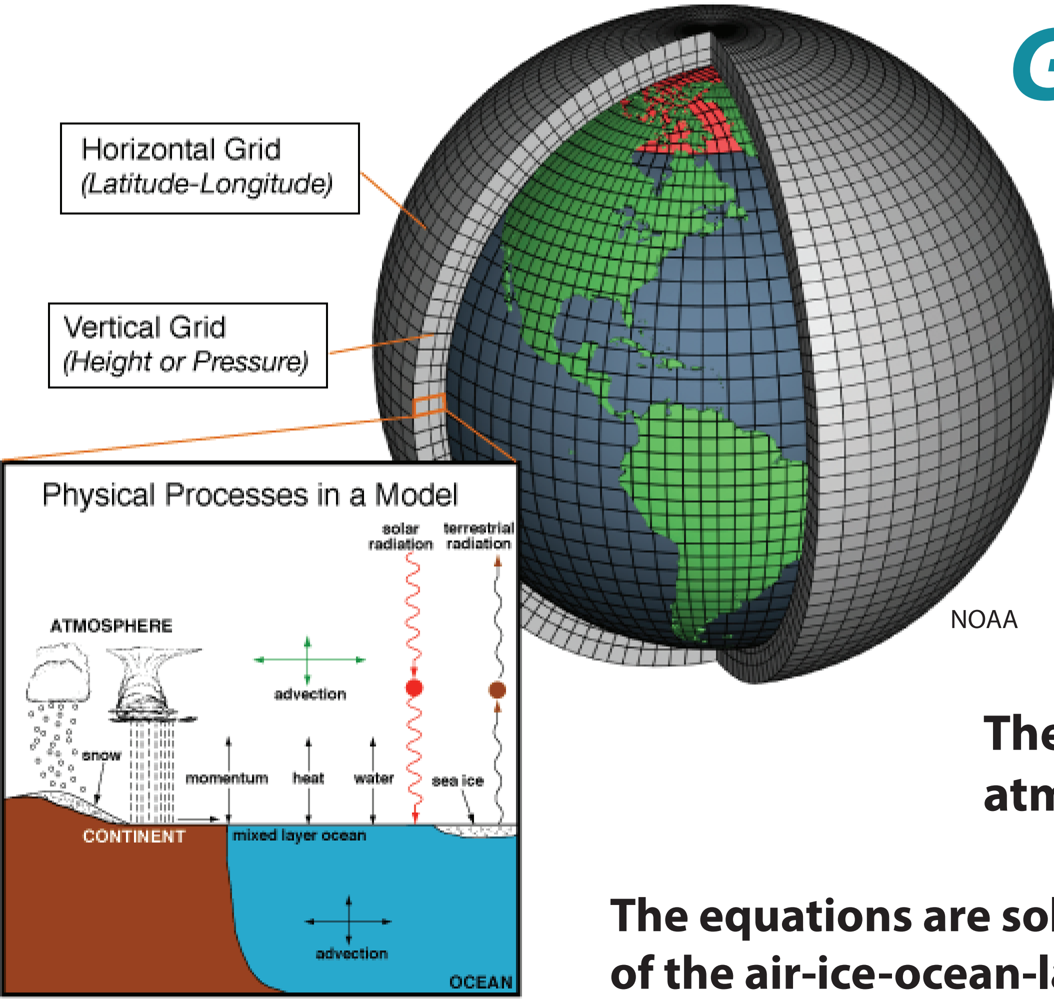


Figure 2.3. *Detailed radiative energy balance* [112].

Global Climate Models



NOAA

climate fueled by the nonuniform spatial distribution of incoming solar radiation.

Stute et al., PNAS 2001

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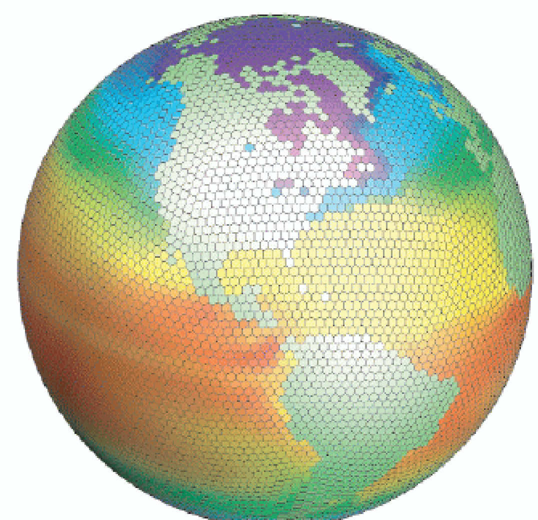
They describe the state of the ocean, land, ice atmosphere, biosphere, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

key challenge :

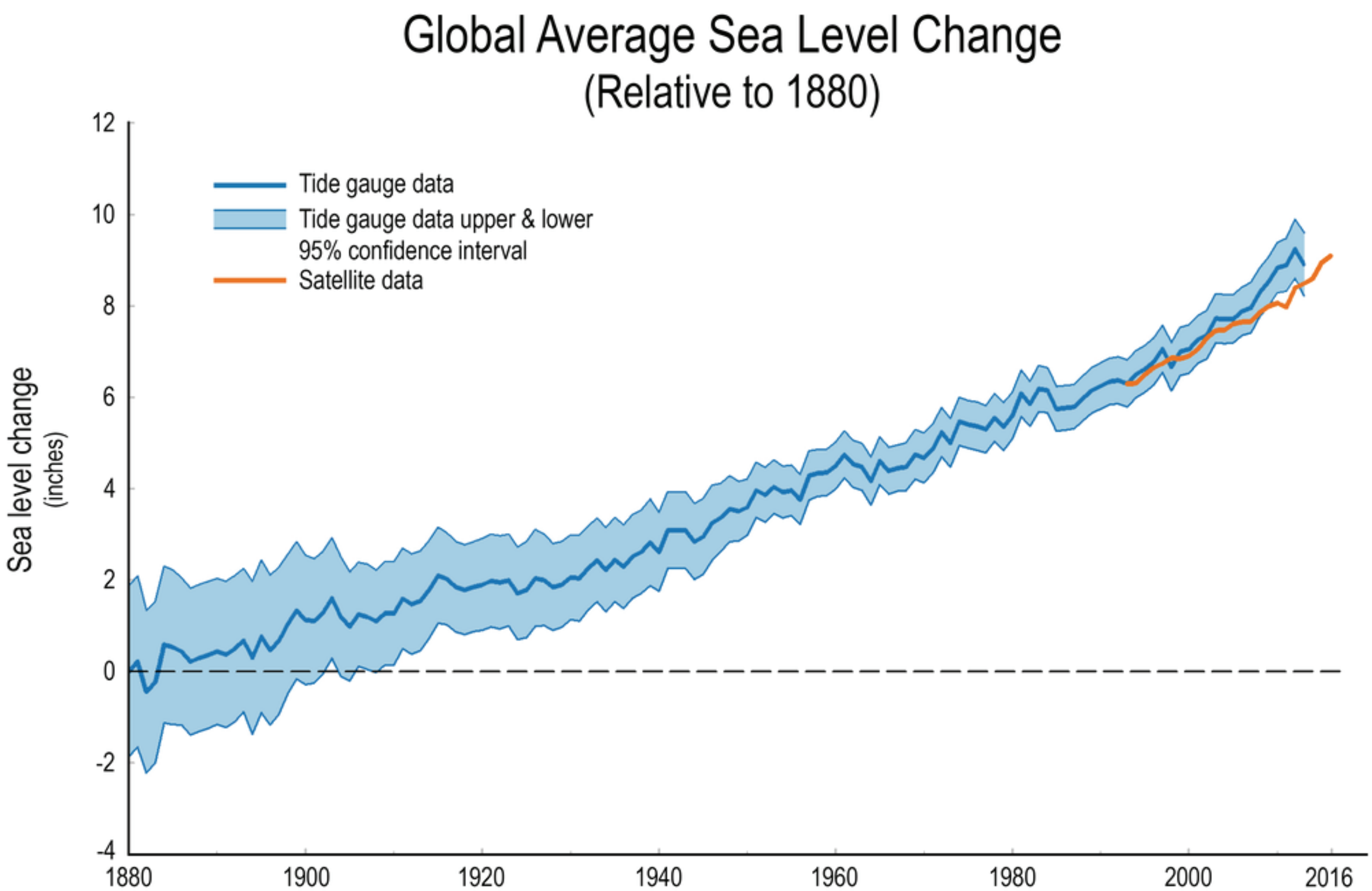
incorporating sub - grid scale processes

linking scales

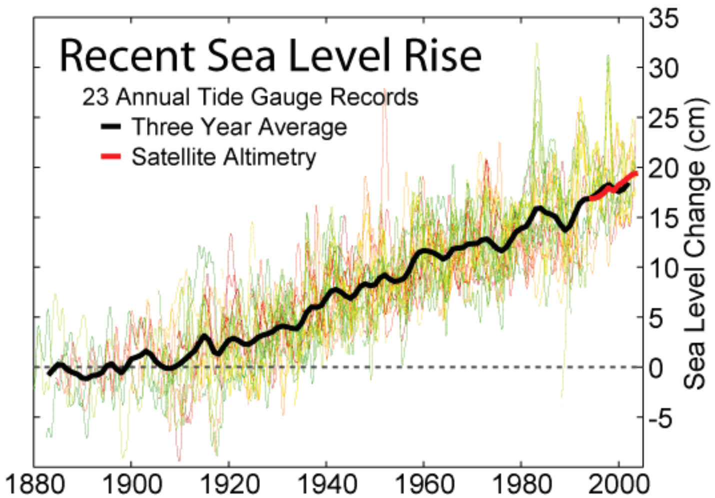


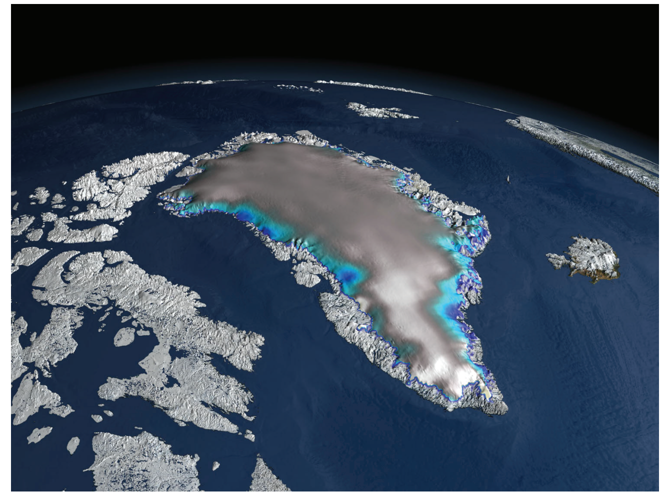
Randall *et al.*, 2002

Why is sea level rising?



As Earth's climate warms, why does sea level rise?





- ***melting land ice: Antarctica, Greenland, mountain glaciers***
  (if all melted : 70 m or ~230 ft of sea level rise)

- ***thermal expansion of warming ocean***

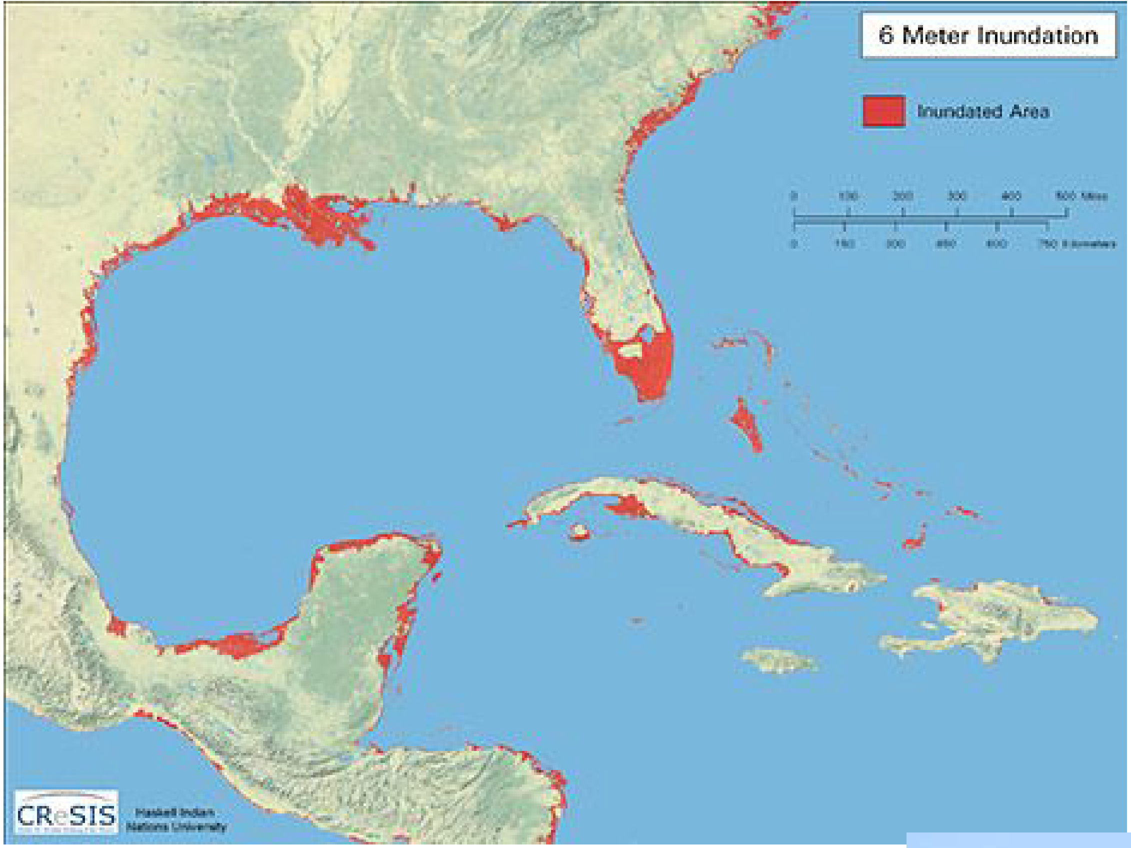
  accounts for about 25% of rise in last half of 20th century, rate ~ tripled in 21st century

- ***continental rebound***

  rise of land masses that were depressed by the huge weight of ice sheets

The interaction of warm waters with the periphery of the large ice sheets represents one of the most significant possibilities for abrupt change in the climate sysytem.

no coupling yet of ice sheets and ocean in climate models - no feedback effects


6 Meter Inundation
Inundated Area
0 100 200 300 400 500 Miles
CReSIS
Haskell Indian Nations University


EUROPE AT RISK FROM RISING SEA LEVELS
0 60 100 150 200 250 Miles
0 75 160 225 300 375 kilometers
5 Meter Inundation
Inundated area