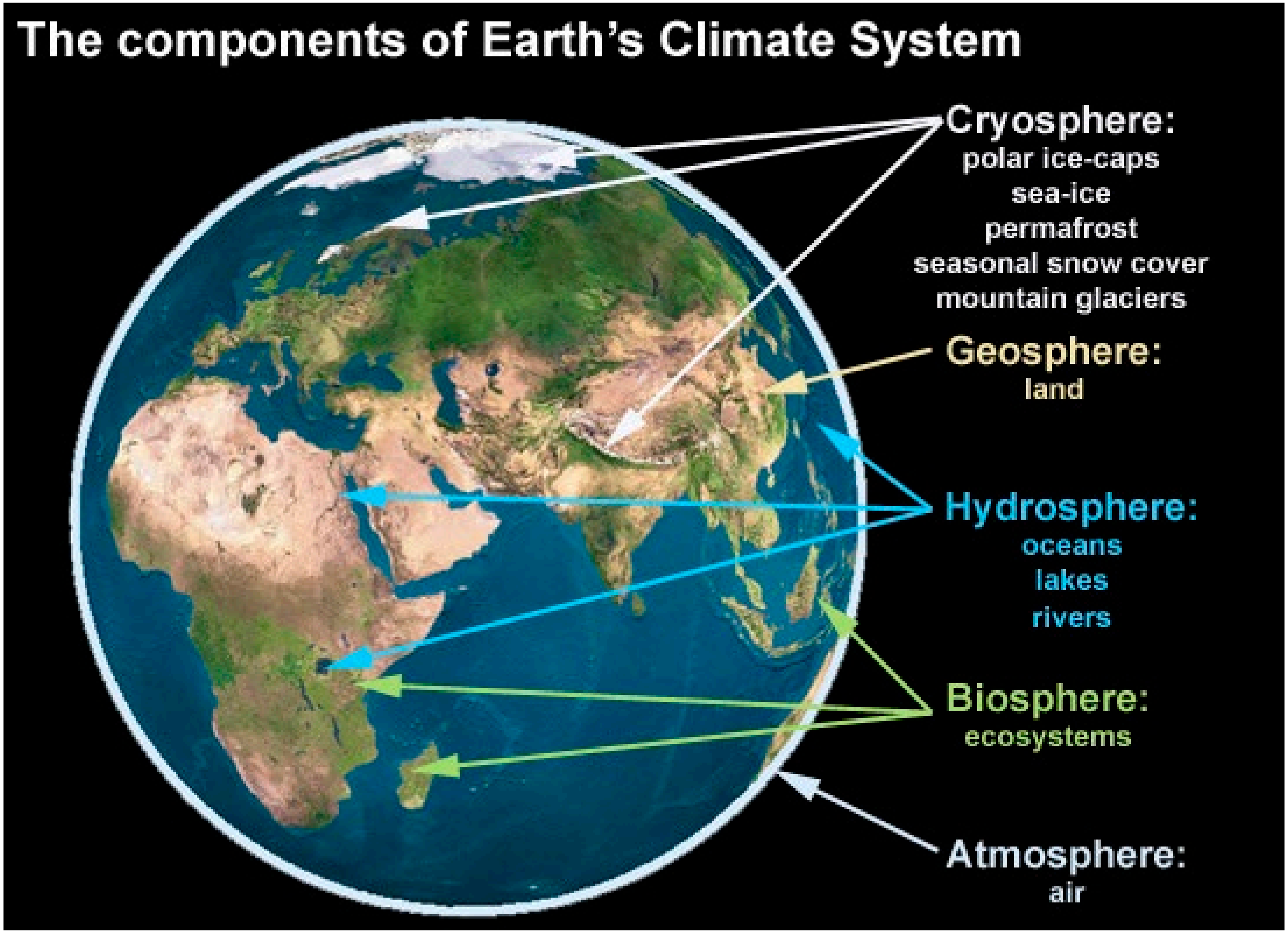
Math 5750 / 6880
Mathematics and Climate

Kenneth M. Golden

Earthrise   December 24, 1968   William Anders   NASA

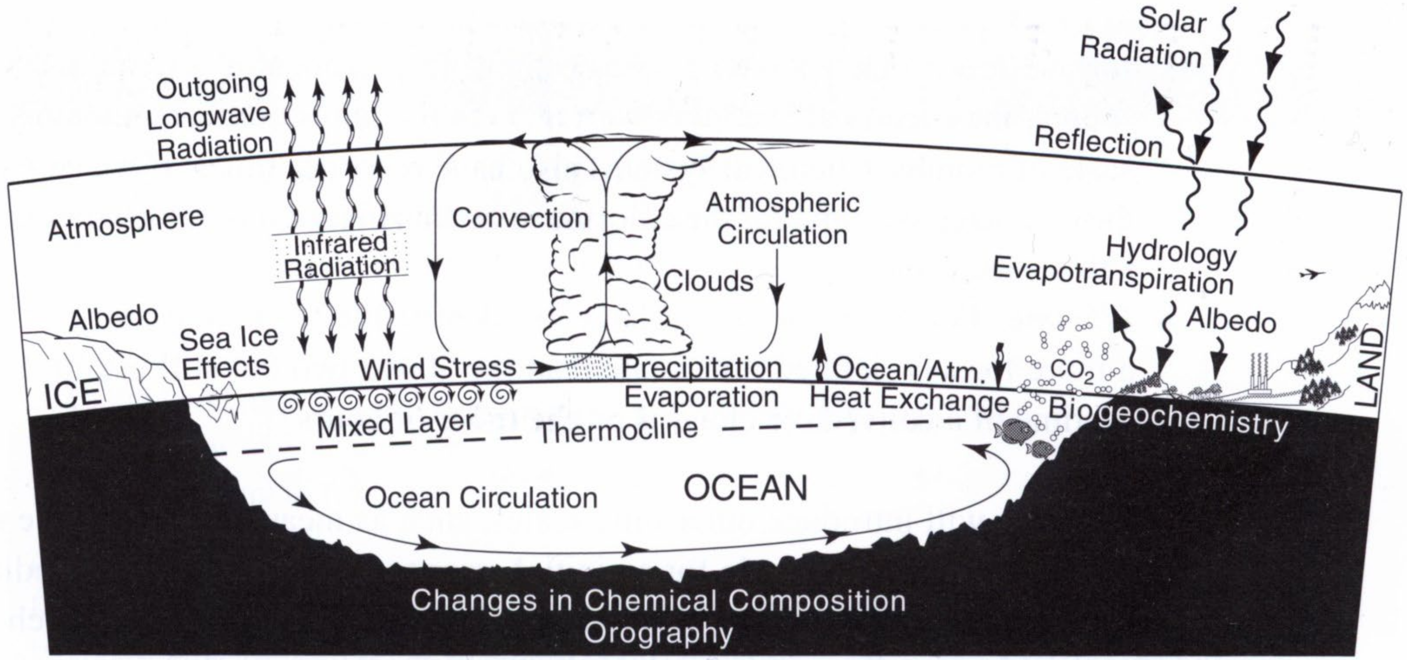

The components of Earth's Climate System
Cryosphere:
polar ice-caps
sea-ice
permafrost
seasonal snow cover
mountain glaciers
Geosphere:
land
Hydrosphere:
oceans
lakes
rivers
Biosphere:
ecosystems
Atmosphere:
air

Mathematics of Planet Earth 2013

A Worldwide initiative, involving:

- North America: US and Canadian Math Institutes, Societies, etc.
- Partners worldwide: Europe, Africa, India…

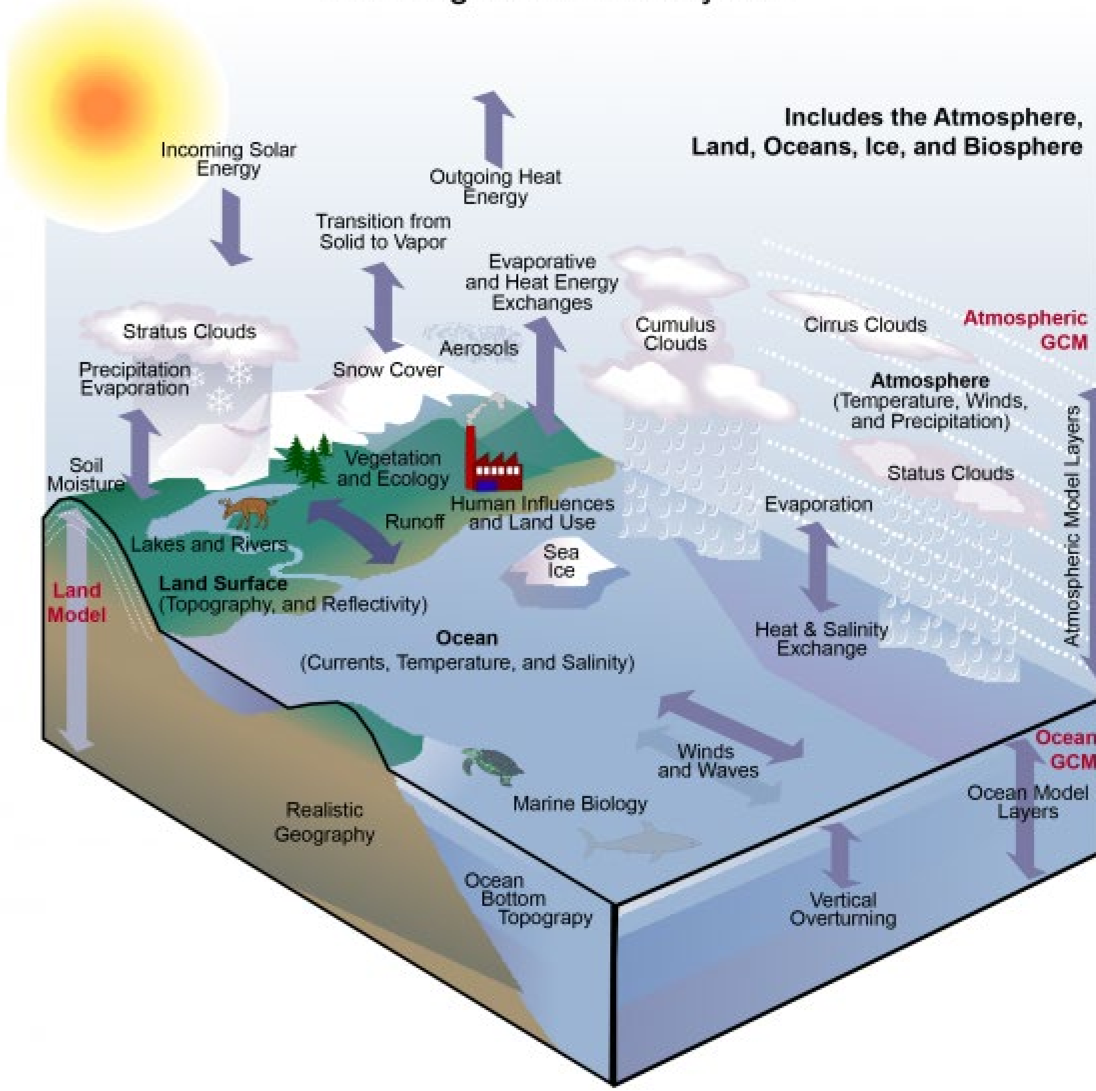
Basics of global climate



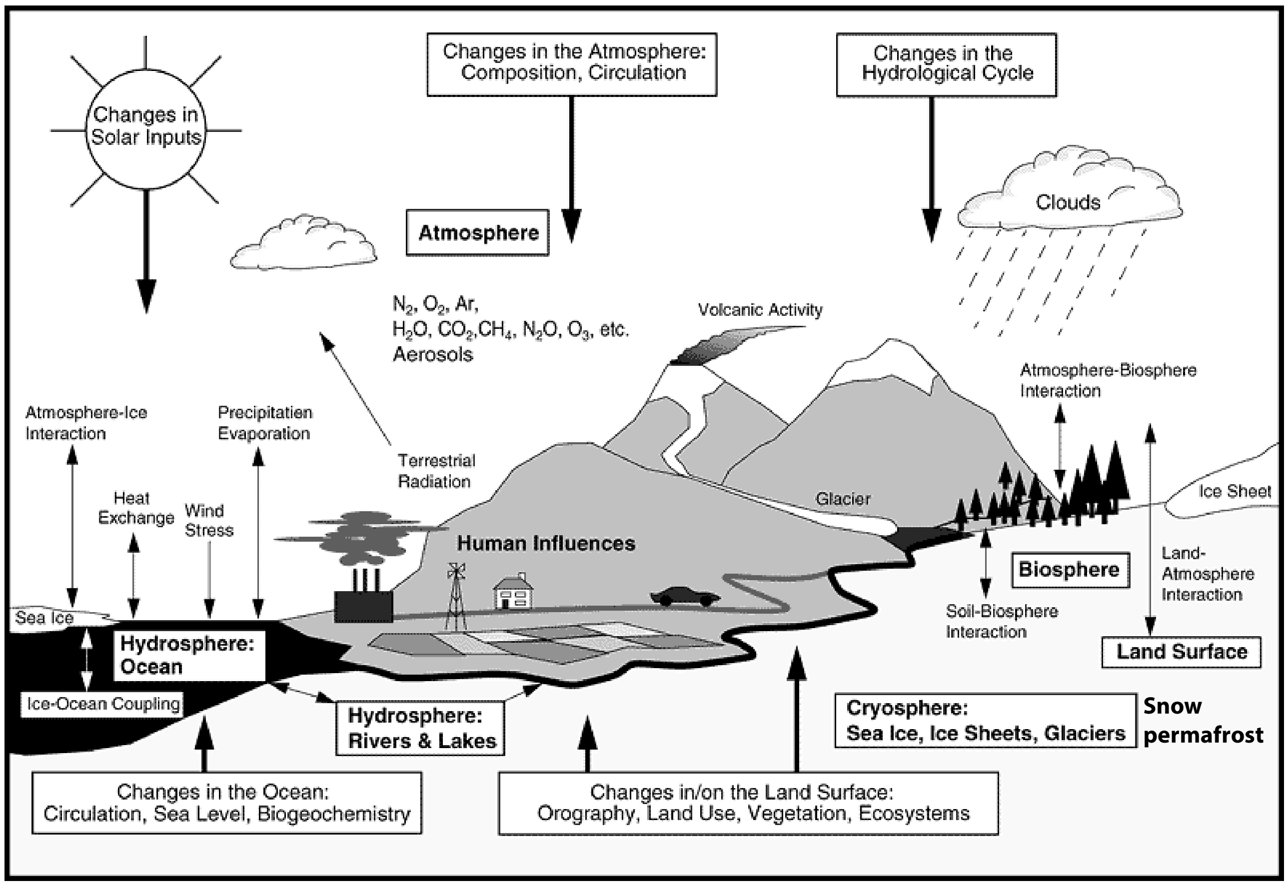
Schematic of some important processes within each component of the climate system.

Basic components: ***cryosphere*** – land ice, snow, sea ice, permafrost
biosphere – sum total of all living things
lithosphere – solid Earth – ocean basins, mountain ranges
atmosphere, ***hydrosphere*** (oceans, seas, rivers, …)

Modeling the Climate System

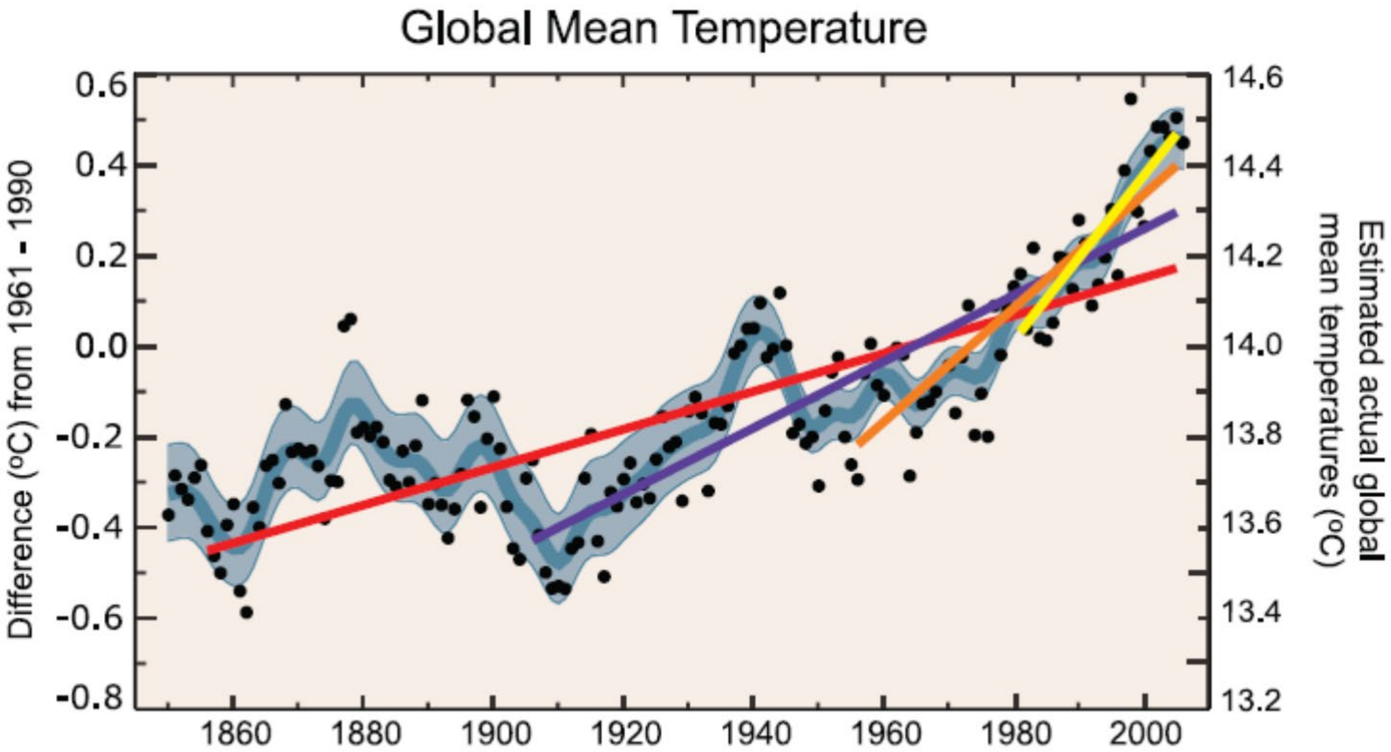


Schematic view of Earth's climate system (IPCC).



Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):
Warming is "unequivocal"



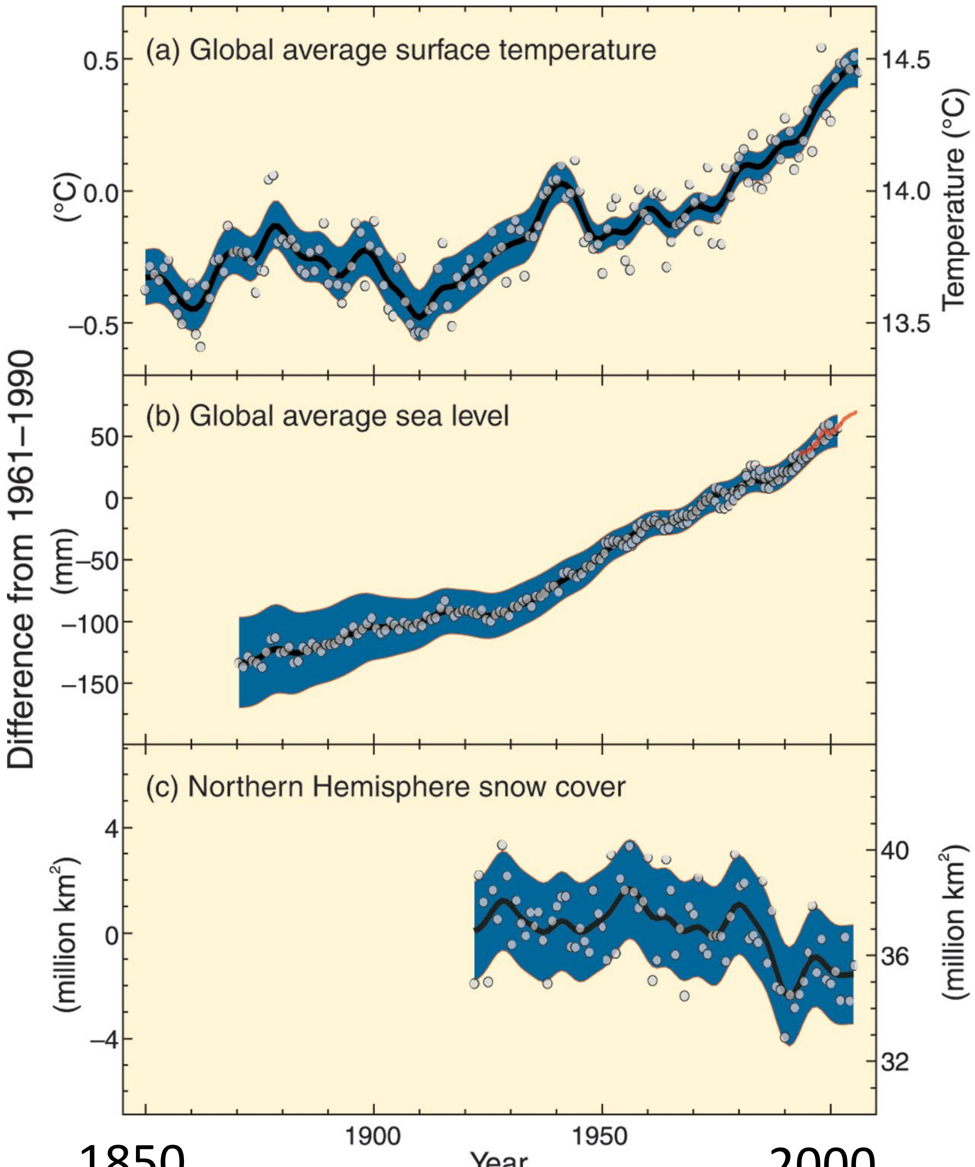
Dots: yearly average
Curve: decadal average
Blue: uncertainty interval

IPCC: Warming is "unequivocal"

Global mean surface temp

Global mean sea level

Northern Hemisphere snow cover (March-April)



1850

2000

Dots: yearly average
Curve: decadal average
Blue: uncertainty interval


[redacted]
Open 102.17
High 103.81
Low 102.00
Close 103.07
Volume 13.82M
% Change -10.85%
yahoo!finance
117.50
115.00
112.50
110.00
107.50
105.00
103.07
100.00
111.24M
95.00
92.50
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7
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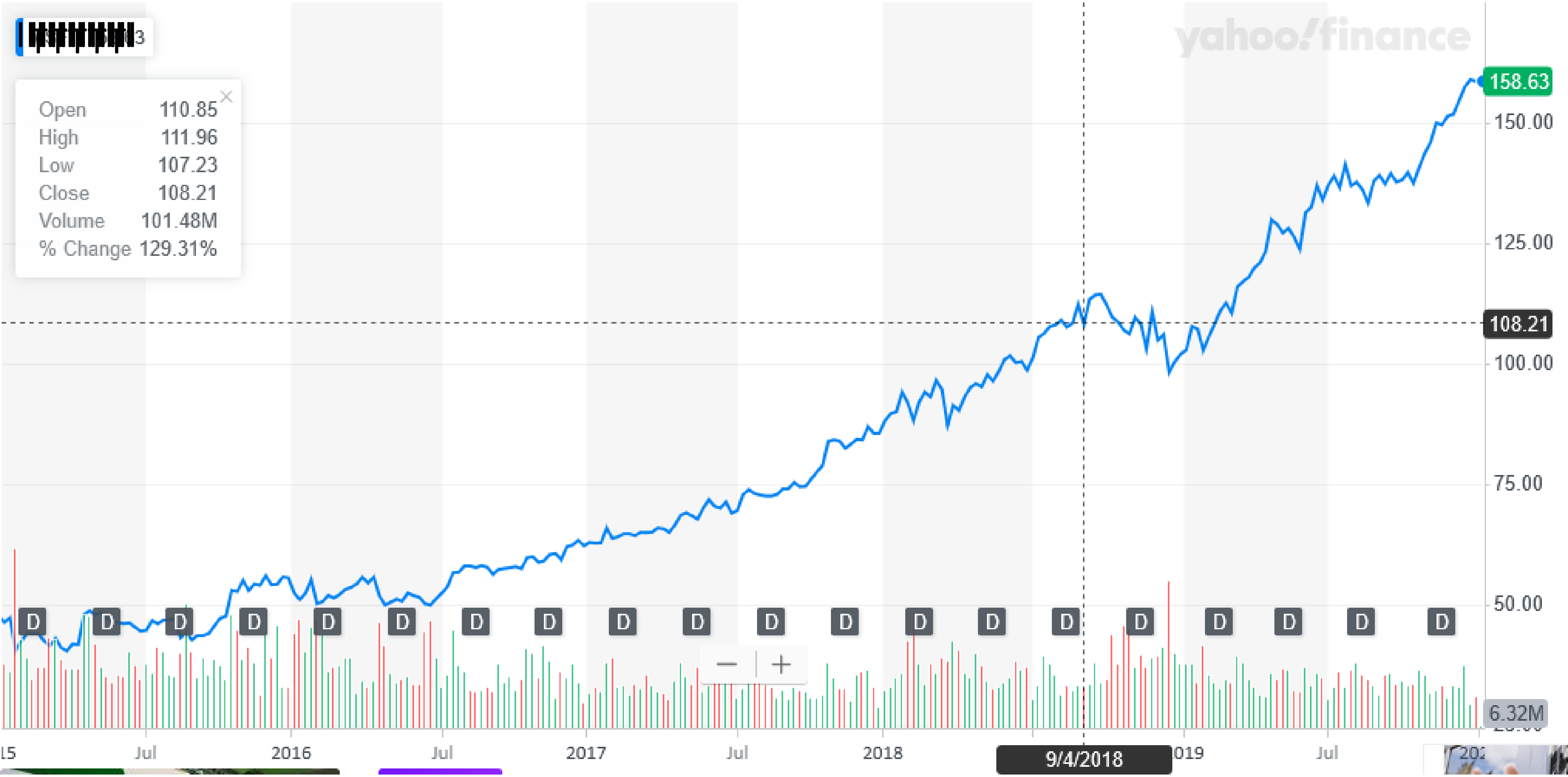
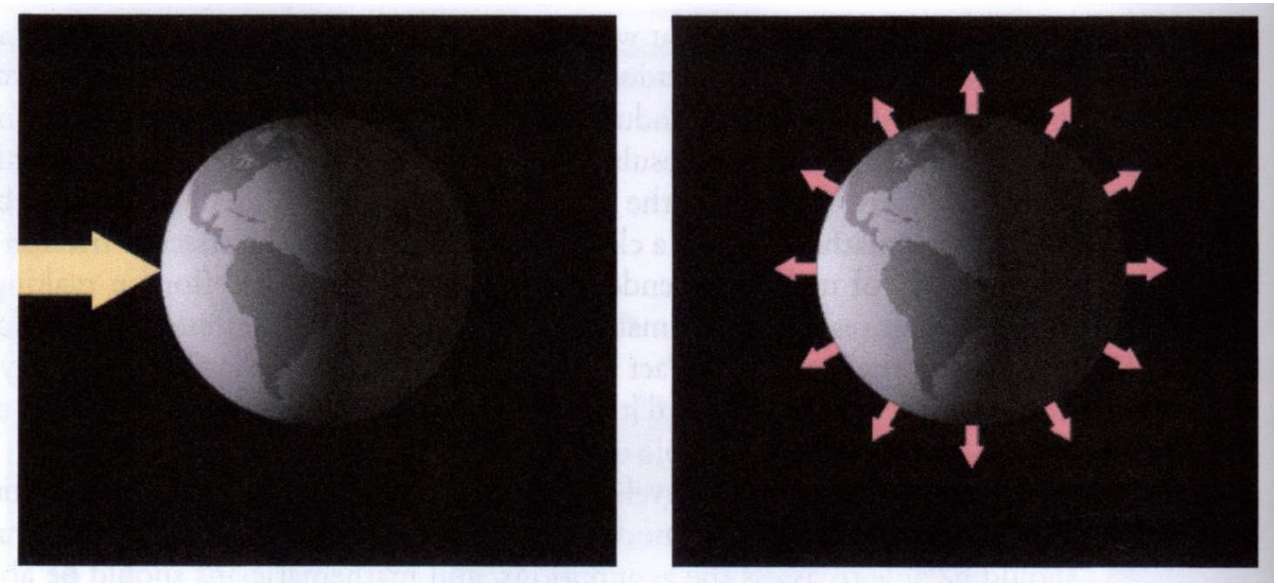
Dec
7
14
21



[redacted]
yahoo!finance
Open 110.85
High 111.96
Low 107.23
Close 108.21
Volume 101.48M
% Change 129.31%
158.63
150.00
125.00
108.21
100.00
75.00
50.00
6.32M
Jul
2016
Jul
2017
Jul
2018
9/4/2018
Jul

Earth's energy balance



E_{in}

incoming shortwave radiation
sunlight

E_{out}

outgoing longwave radiation
heat

if $E_{out} > E_{in}$ planet **cools**

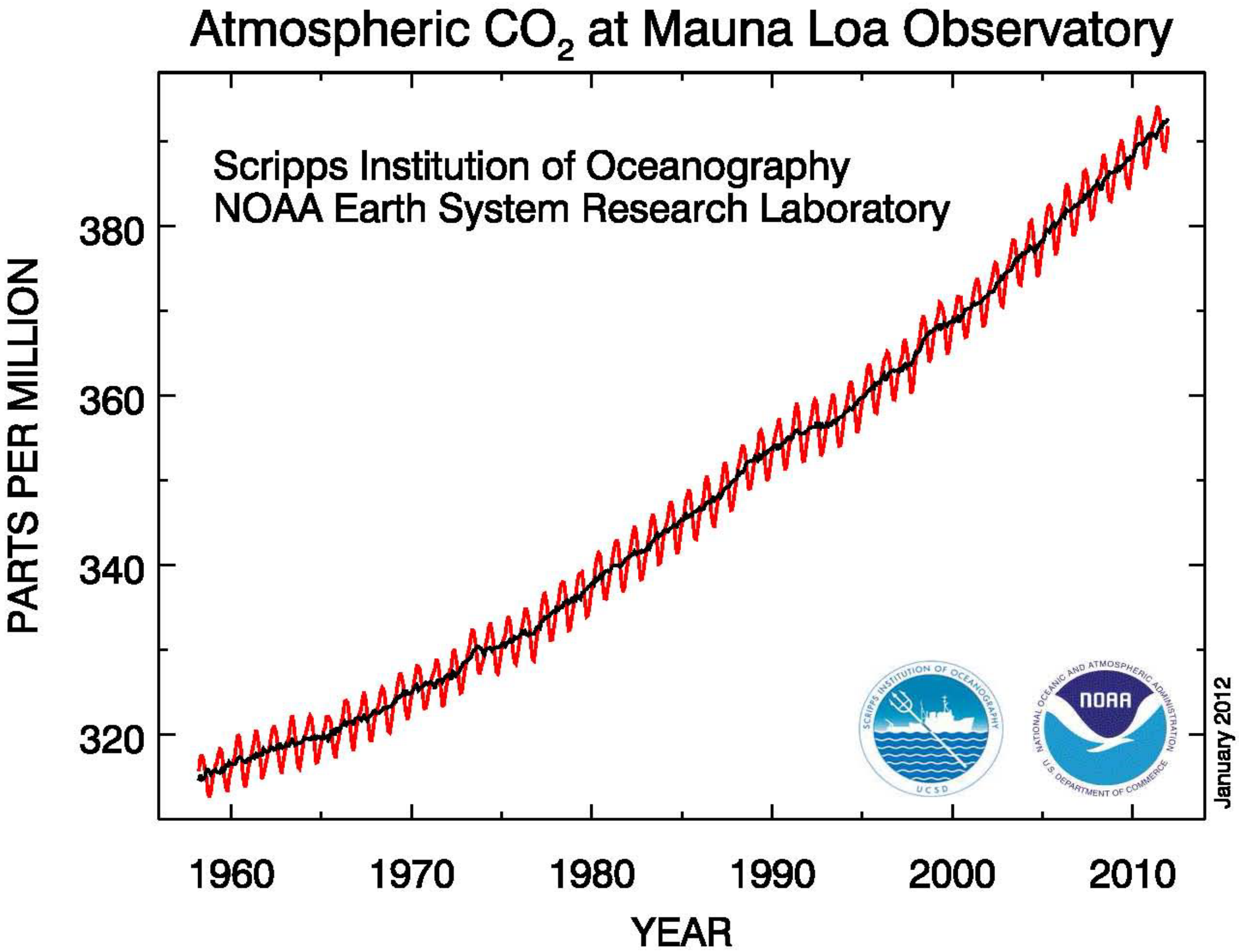
if $E_{out} < E_{in}$ planet **warms**

Basics of global warming date back to the 1800's

Jean Baptiste Joseph Fourier (1768-1830), French mathematician and natural philosopher, did groundbreaking work in mathematics and the theory of heat. He was the first to propose that the Earth's atmosphere acts to raise the planet's temperature.

"As a dam built across a river causes a local deepening of the stream, so our atmosphere, thrown as a barrier across the terrestrial rays, produces a local heightening of the temperature at the Earth's surface."

Thus in 1862 John Tyndall described the key to climate change. He had discovered in his laboratory that certain gases, including water vapor and carbon dioxide (CO_2) are opaque to heat rays. He understood that such gases high in the air help keep our planet warm by interfering with escaping radiation.

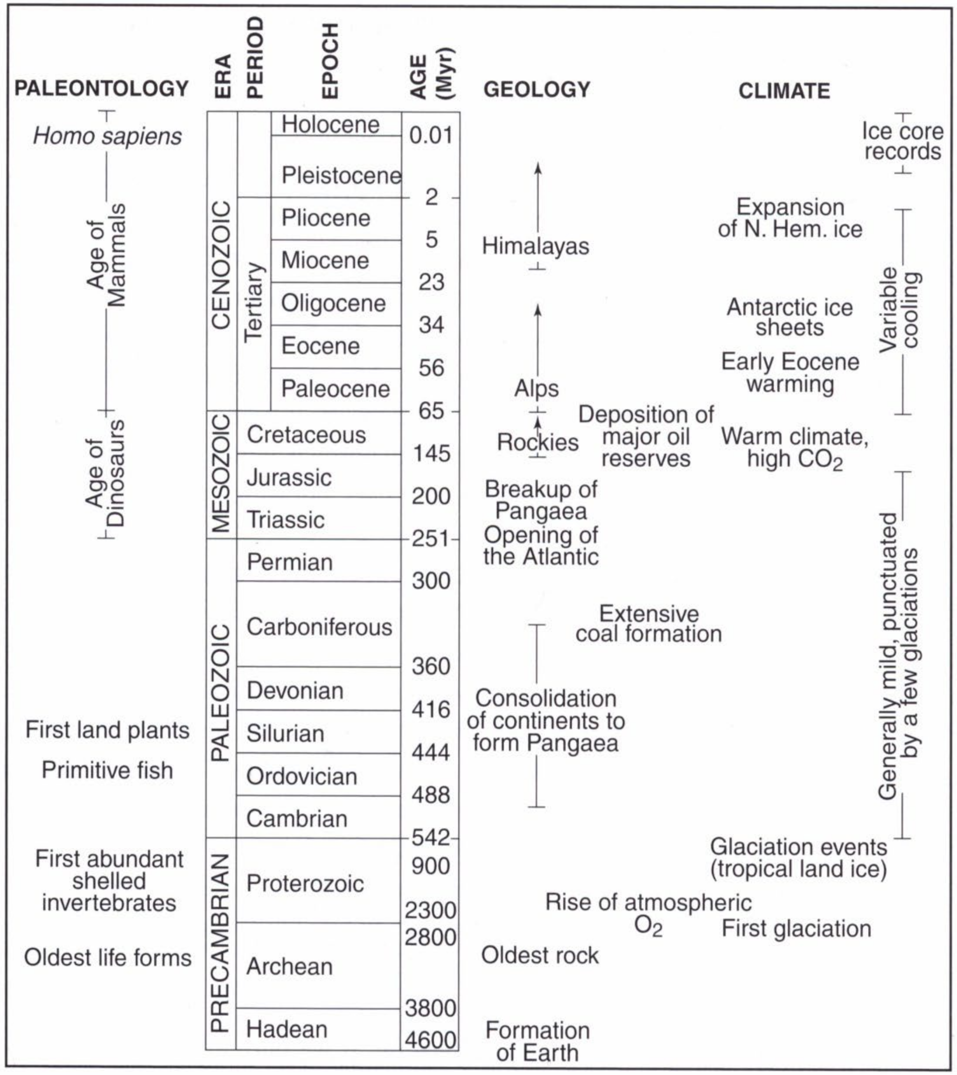


Monthly average carbon dioxide concentration. Seasonal cycle is thought to be driven by the terrestrial biosphere; net consumption of carbon dioxide by biomass in the summer (abundance of light and heat) and net respiration in winter.

Table 1.2 Some events in the history of global warming studies.[4]

1850s	Beginning of the industrial revolution.
1861	John Tyndall notes that H_2O and CO_2 are especially important for infrared absorption and thus potentially for climate. The warming effect of the atmosphere and the analogy to a greenhouse had already been noted by J. B. Fourier in 1827.
1868	Jozef Stefan develops his law for blackbody radiation.
1896–1908	Svante Arrhenius postulates a relation between climate change and CO_2 and that global warming may occur as a result of coal burning.
1917	W. M. Dines estimates a heat balance of the atmosphere that is approximately correct.
1938	G. S. Callendar attempts to quantify warming by CO_2 release by burning of fossil fuels.
late 1950s	Popularization of global warming as a problem, notably by Roger Revelle.
1958	Start of C. D. Keeling's monitoring of CO_2 at Mauna Loa.
1975	First three-dimensional global climate model of CO_2-induced climate change by Suki Manabe.
1979	The "Charney Report" (US National Academy of Sciences).
late 1980s	Seven of eight warmest years of the century to that point.
1990 and 92	Intergovernmental Panel on Climate Change (IPCC) Report and Supplement.

1992	Rio de Janeiro United Nations Conference on the Environment Development; Framework Convention on Climate Change. *"The ultimate objective...is...stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system."*
1995–96	Second Assessment Report of the IPCC: *"The balance of evidence suggests a discernible human influence on global. climate.* [...] *There are still many uncertainties."*
1995	Start of ongoing series of Conferences of the Parties to the Climate Convention: discussion of short term objectives in terms of greenhouse gas emissions by developed countries.
1997	Kyoto Protocol sets targets on greenhouse gas emissions at 5% below 1990 levels by 2008–2125.[5]
2001	Third Assessment Report of the IPCC.
2004	Nine of the ten warmest years since 1856 occurred in past ten years (1995–2004) (1996 was less warm than 1990).
2005	Kyoto Protocol enters into force.
2007	Fourth Assessment Report of the IPCC. Nobel Peace Prize awarded to the few thousand scientists of the IPCC process and one politician.



Geological time scale, names and events in Earth's history, with selected paleoclimate events added in the right hand column. Adapted from Crowley (1983) with added information based on Crowley and North (1991), Zachos *et al.* (2001), Gradstein *et al.* (2004) and Royer (2007). Note that the time scale is expanded towards present, with Myr = millions of years. Major periods of fossil fuel deposition are noted between geology and climate columns, since these provide the source for current anthropogenic CO_2 input to the atmosphere.

climate variability vs. climate change

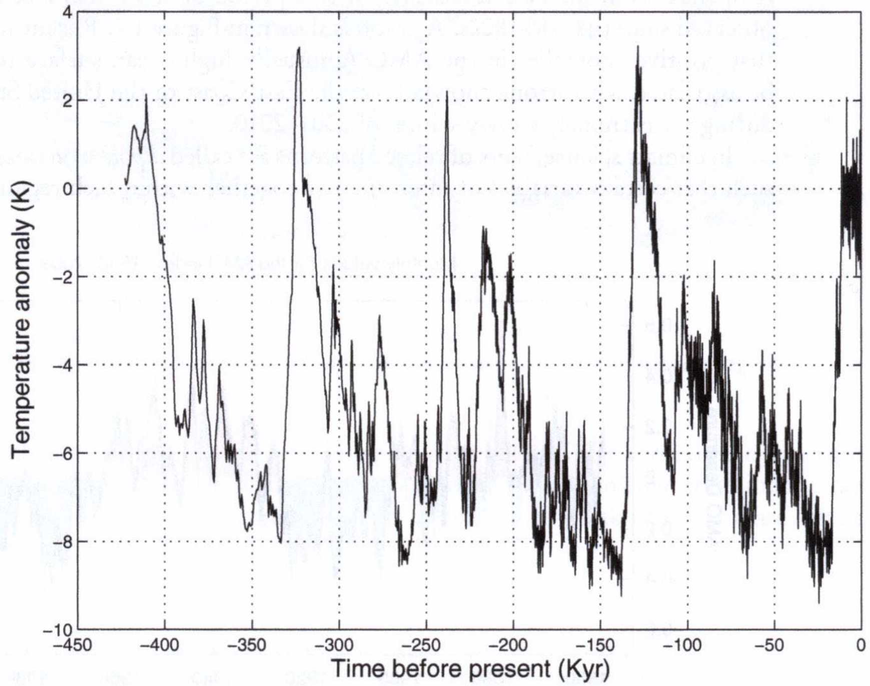


Figure 1.6. *Global mean temperature over the past 420 Kyr. Time increases from left to right. Reprinted courtesy of the U.S. Geological Survey.*

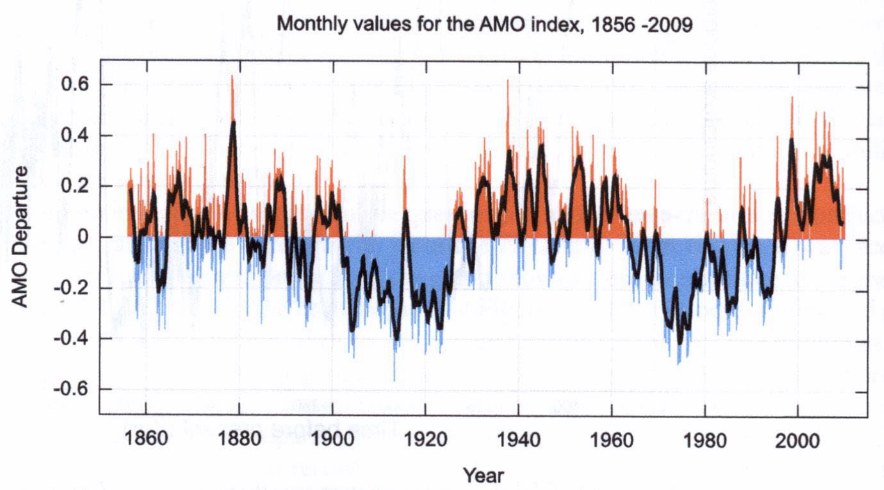


Figure 1.7. *Ocean temperature anomalies for AMO,* 1856–2009 [51].

Sir Gilbert Walker (1868-1958) -- observed years of reduced rainfall over Australia and Indonesia (high air pressure over western Pacific) correlated with low pressure over central and eastern Pacific – SOUTHERN OSCILLATION

His observations vindicated when connected to ocean temperature oscillations observed by fishermen off the coast of South America around Christmas – called "El Nino"

ENSO – El Nino Southern Oscillation
NAO – North Atlantic Oscillation – pressure difference – Icelandic low, Azores high
AMO – Atlantic Multidecadal Oscillation (~ 50 yrs) – sea surface temps in North Atlantic
positive anomalies – high ocean surface temps – associated with strong snowfall in East Coast, as in 2009-2010.

DIPOLE Teleconnections

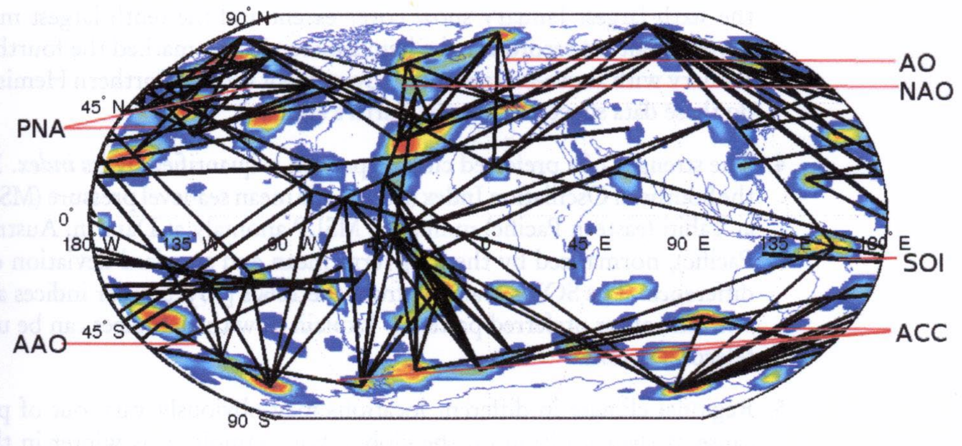


Figure 1.8. *Dipoles in NCEP sea-level data for the period* 1948–1967. *The color background shows the regions of high activity. The edges represent dipole connections between regions.*

Blackbody emission curves for Sun and Earth.

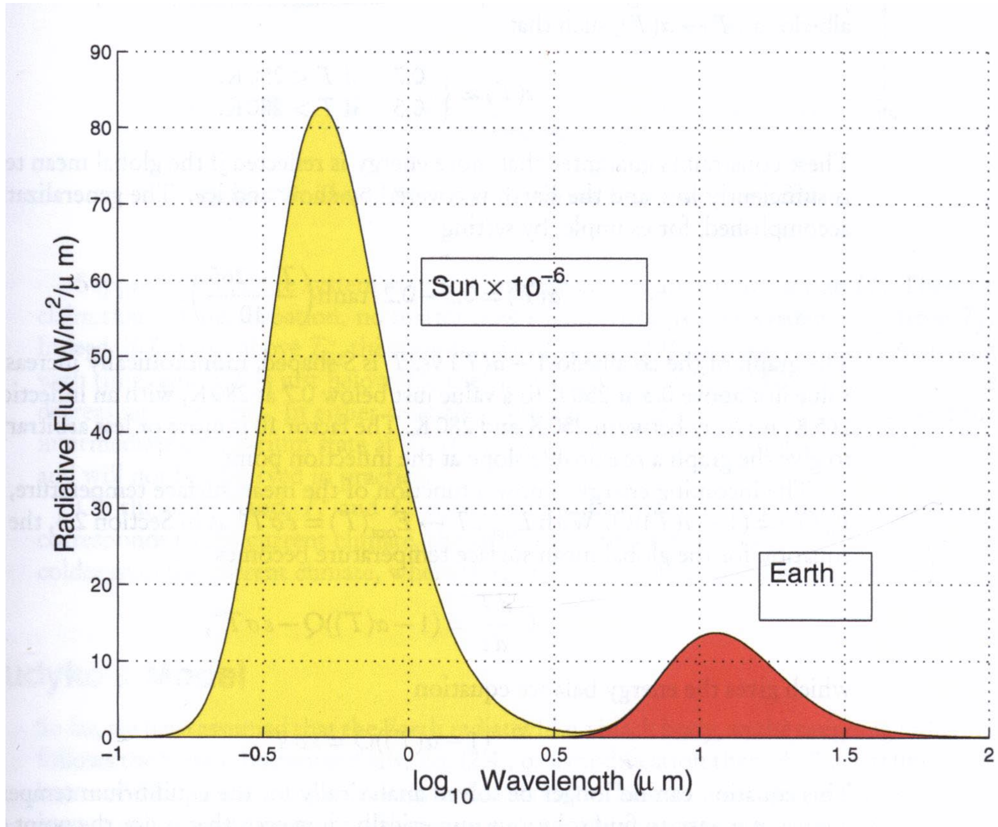


Figure 2.4. *Black-body emission curves of the Sun and the Earth. Reprinted courtesy of NASA.*

A perfect **blackbody** is one that absorbs all incoming light and does not reflect any; it emits electromagnetic radiation (thermal radiative energy) with spectrum that depends on temperature *T* according to Plank's law.

Observed solar spectrum at top of atmosphere and at sea level

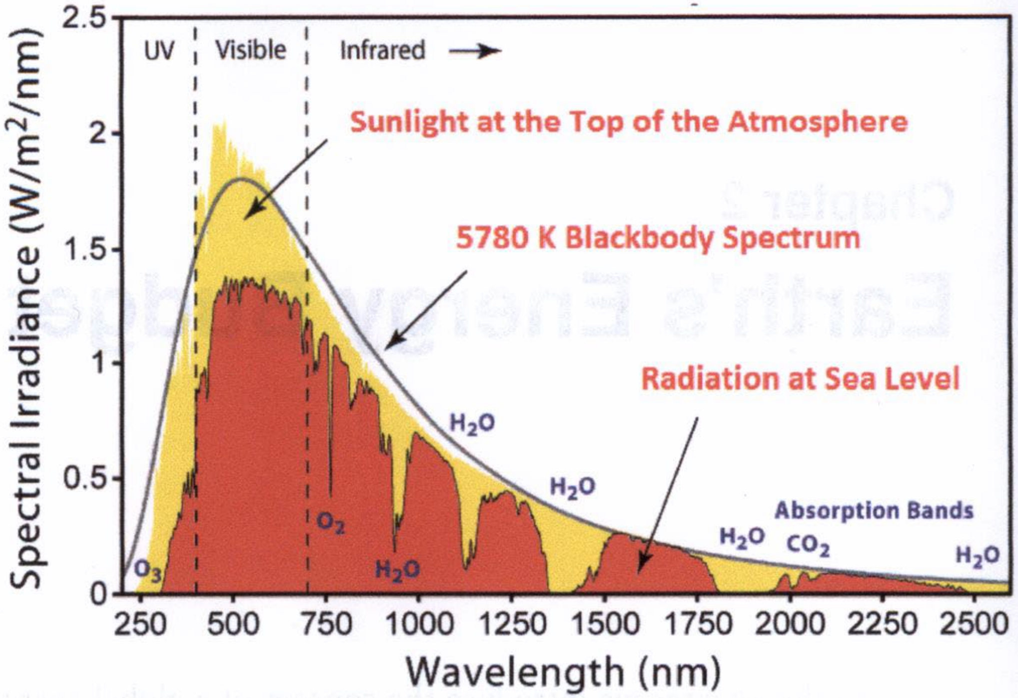


Figure 2.1. *The observed solar spectrum at the top of the atmosphere (yellow) and at sea level on a clear day (red). The solid line (black) is the spectrum the Sun would have if it were a black body at the temperature of* 5,780 *K. Image courtesy of Global Warming Art.*

Surface insolation in Europe.

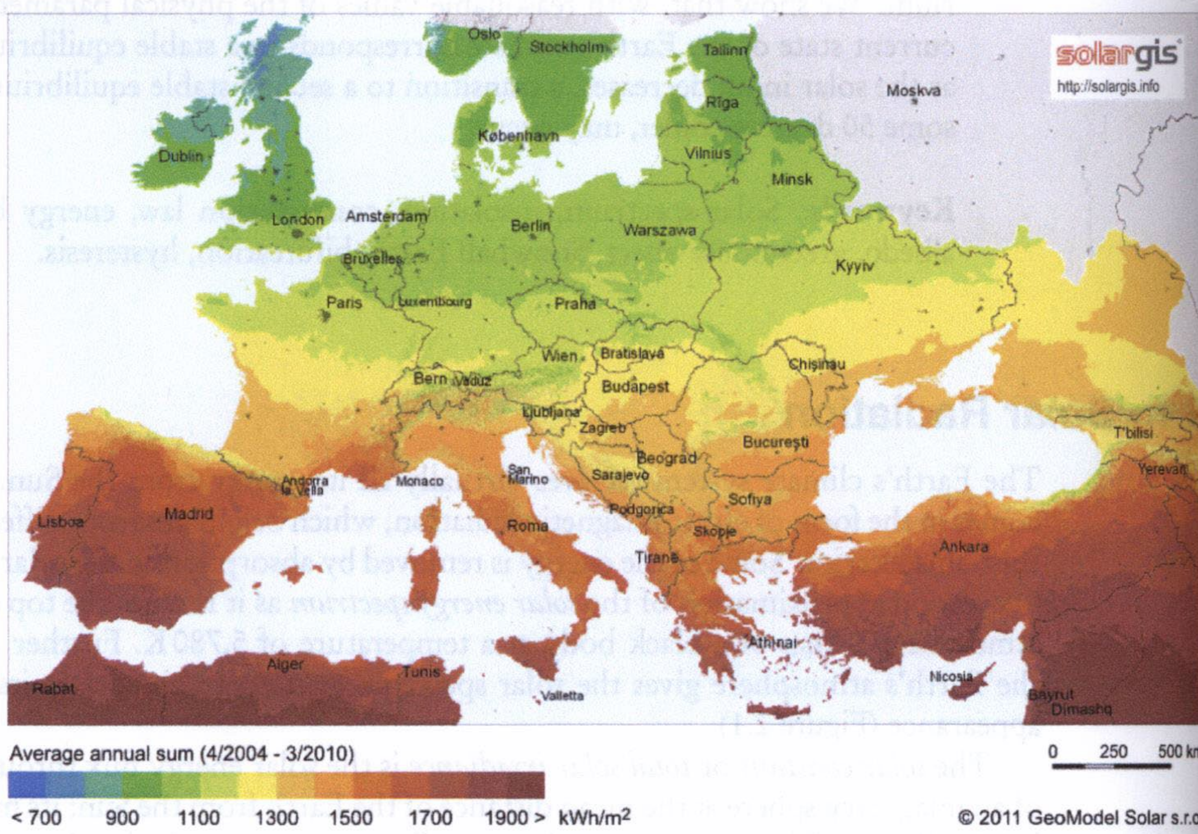


Figure 2.2. *Average annual surface insolation in Europe. Reprinted with permission from SolarGIS.*

The global energy balance

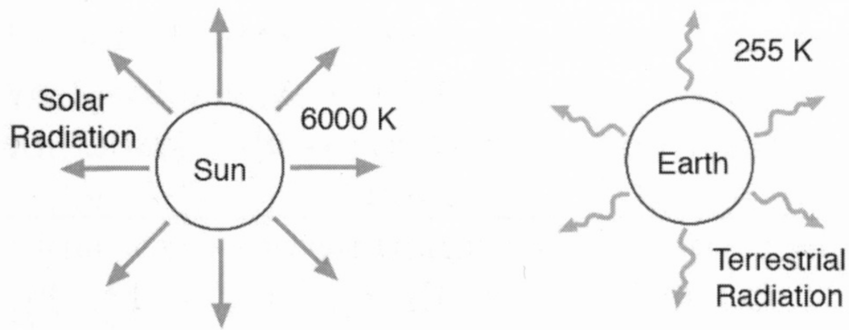


FIGURE 2.1. The Earth radiates energy at the same rate it is received from the Sun. The Earth's emission temperature is 255 K, and that of the Sun is 6000 K. The outgoing terrestrial radiation peaks in the infrared spectrum; the incoming solar radiation peaks at shorter wavelengths, in the visible spectrum.

Consider the radiative equilibrium temperature of Earth. To maintain equilibrium, it warms up (absorbs solar energy) and radiates energy away at same rate it is received. We'll see the emission temperature is 255 K and a body at this temperature radiates primarily in infrared. But atmosphere is strongly absorbing at these wavelengths (H2O and CO2) thus raising surface temperature above emission temperature – ***GREENHOUSE EFFECT***.

radiative energy balance

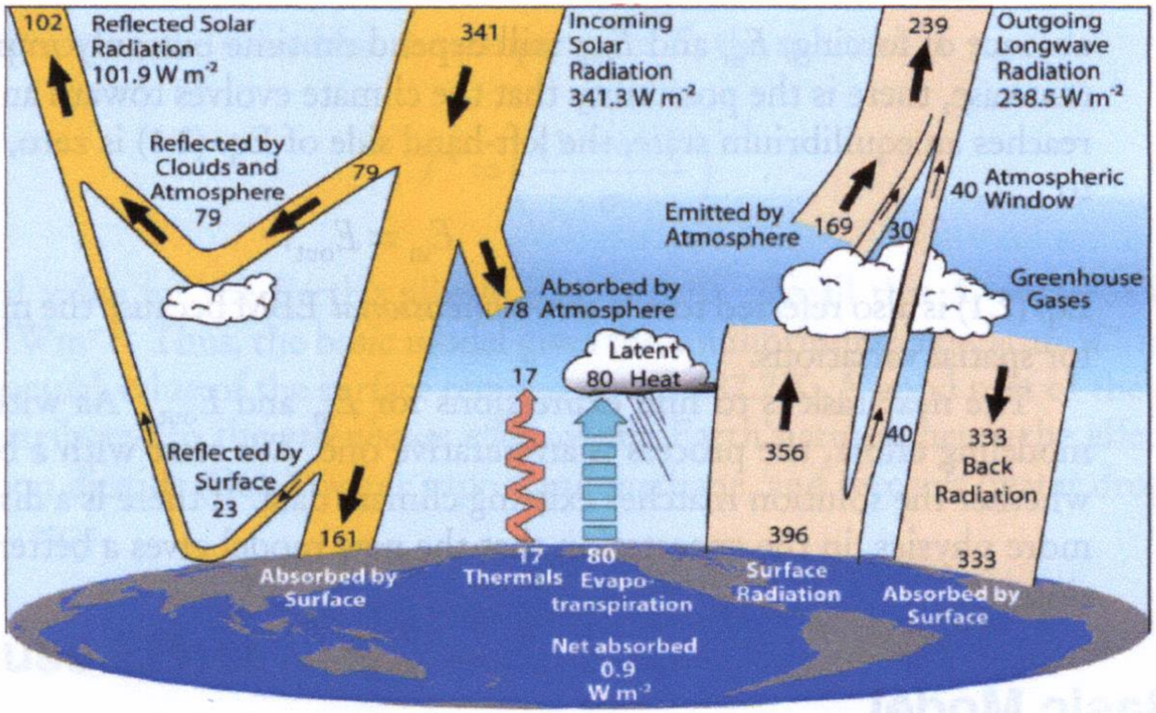


Figure 2.3. *Detailed radiative energy balance* [112].

solar "constant"

- Sun emits energy at rate Q = 3.87 x 10^{26} W

- Flux of solar energy at Earth is called ***solar constant***

$$S_0 = Q / 4\pi r^2 = 1367 \text{ Wm}^{-2}$$

$r =$ distance from Sun to Earth

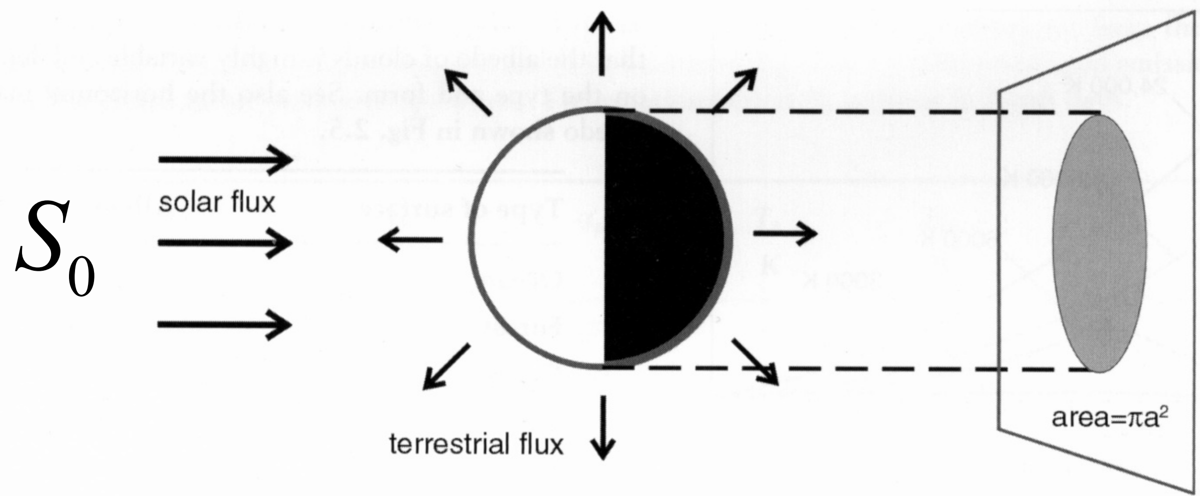


FIGURE 2.4. The spinning Earth is imagined to intercept solar energy over a disk of radius *a* and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

solar power incident on Earth

$$S_0 \pi a^2 = 1.74 \times 10^{17}\,\mathrm{W}$$

Not all radiation absorbed, some reflected

ratio of reflected to incident solar energy = albedo = α

TABLE 2.2. Albedos for different surfaces. Note that the albedo of clouds is highly variable and depends on the type and form. See also the horizontal map of albedo shown in Fig. 2.5.

Type of surface	Albedo (%)
Ocean	2–10
Forest	6–18
Cities	14–18
Grass	7–25
Soil	10–20
Grassland	16–20
Desert (sand)	35–45
Ice	20–70
Cloud (thin, thick stratus)	30, 60–70
Snow (old)	40–60
Snow (fresh)	75–95

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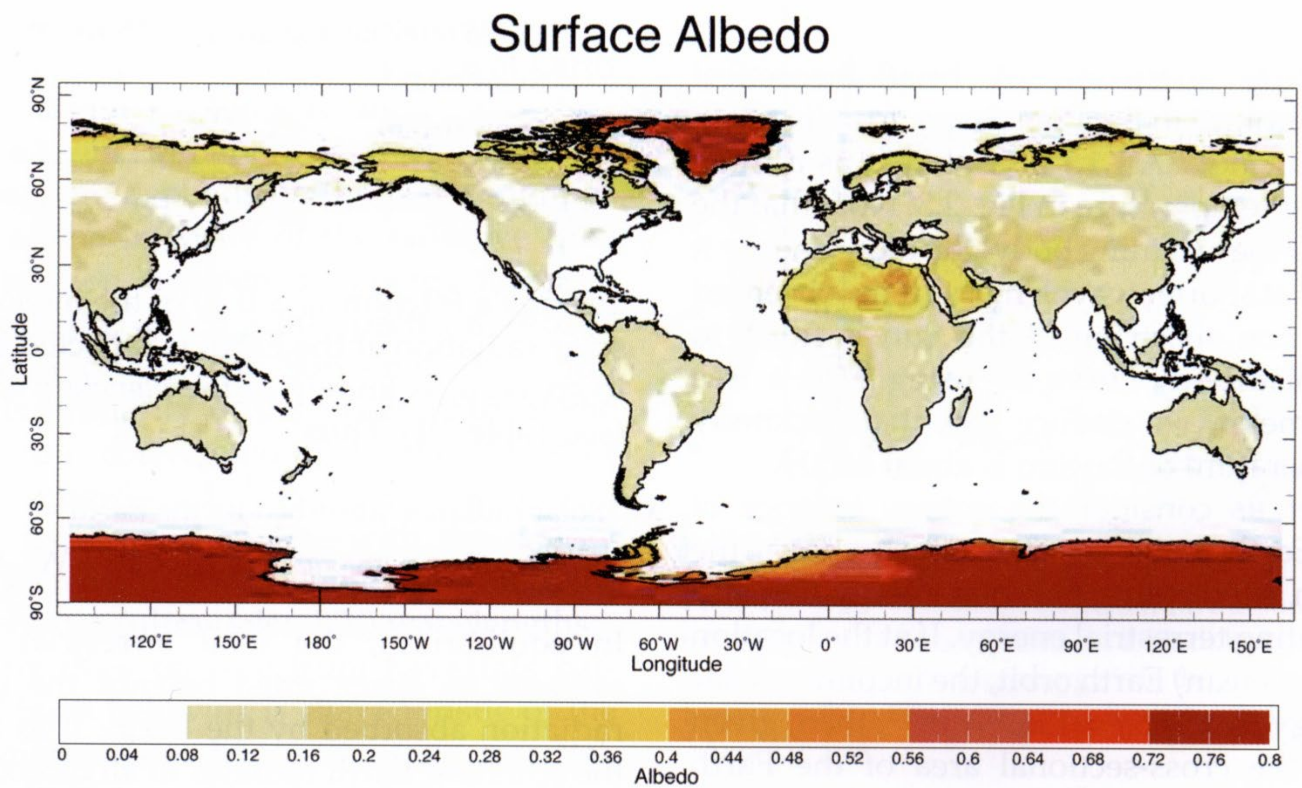


FIGURE 2.5. The albedo of the Earth's surface. Over the ocean the albedo is small (2–10%). It is larger over the land (typically 35–45% over desert regions) and is particularly high over snow and ice (~80%) (see Table 2.2).

Albedo depends on the nature of the reflecting surface, large for clouds, light surfaces like deserts, and especially snow and ice.

Under present terrestrial conditions of cloudiness and snow and ice cover, on average a fraction $\alpha \sim 0.3$ of incoming

Planetary albedo

- Albedo depends on the nature of the reflecting surface, and is large for clouds, light surfaces like deserts, and especially snow and ice.

- Under present terrestrial conditions of cloudiness and snow and ice cover, on average a fraction

$$\alpha_p \sim 0.3$$

of incoming solar radiation is reflected back to space.

- α_p is known as the ***planetary albedo***

Planetary properties

TABLE 2.1. Properties of some of the planets. S_0 is the solar constant at a distance r from the Sun, α_p is the planetary albedo, T_e is the emission temperature computed from Eq. 2-4, T_m is the measured emission temperature, and T_s is the global mean surface temperature. The rotation period, τ, is given in Earth days.

	r 10^9 m	S_0 W m^{-2}	α_p	T_e K	T_m K	T_s K	τ Earth days
Venus	108	2632	0.77	227	230	760	243
Earth	150	1367	0.30	255	250	288	1.00
Mars	228	589	0.24	211	220	230	1.03
Jupiter	780	51	0.51	103	130	134	0.41

Blackbody emission curves for Sun and Earth.

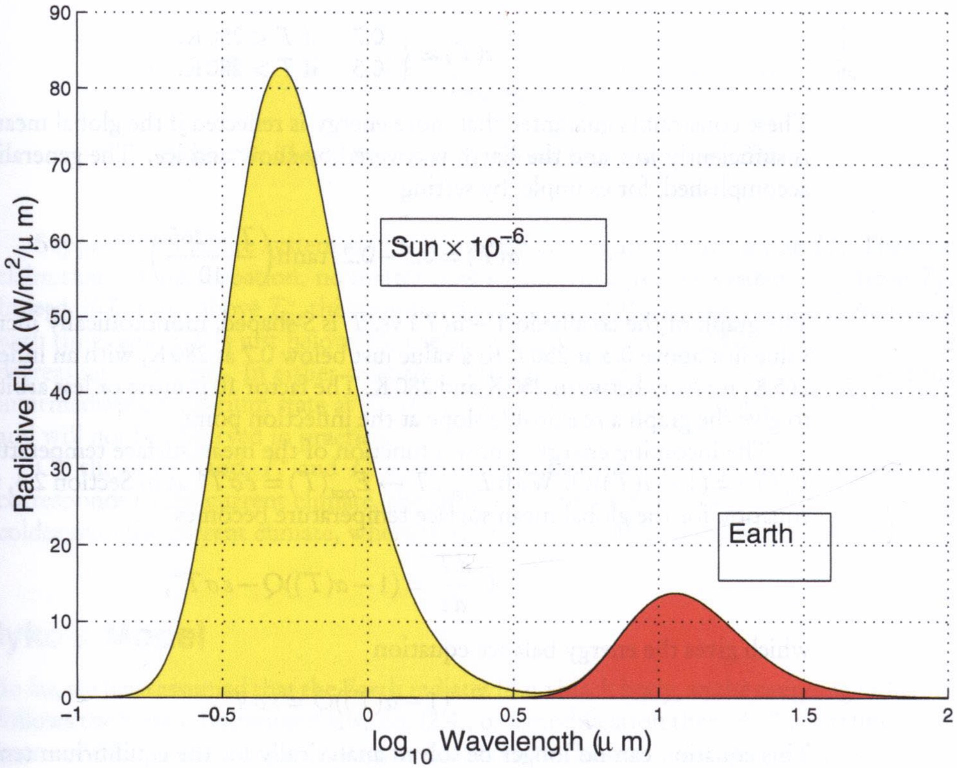
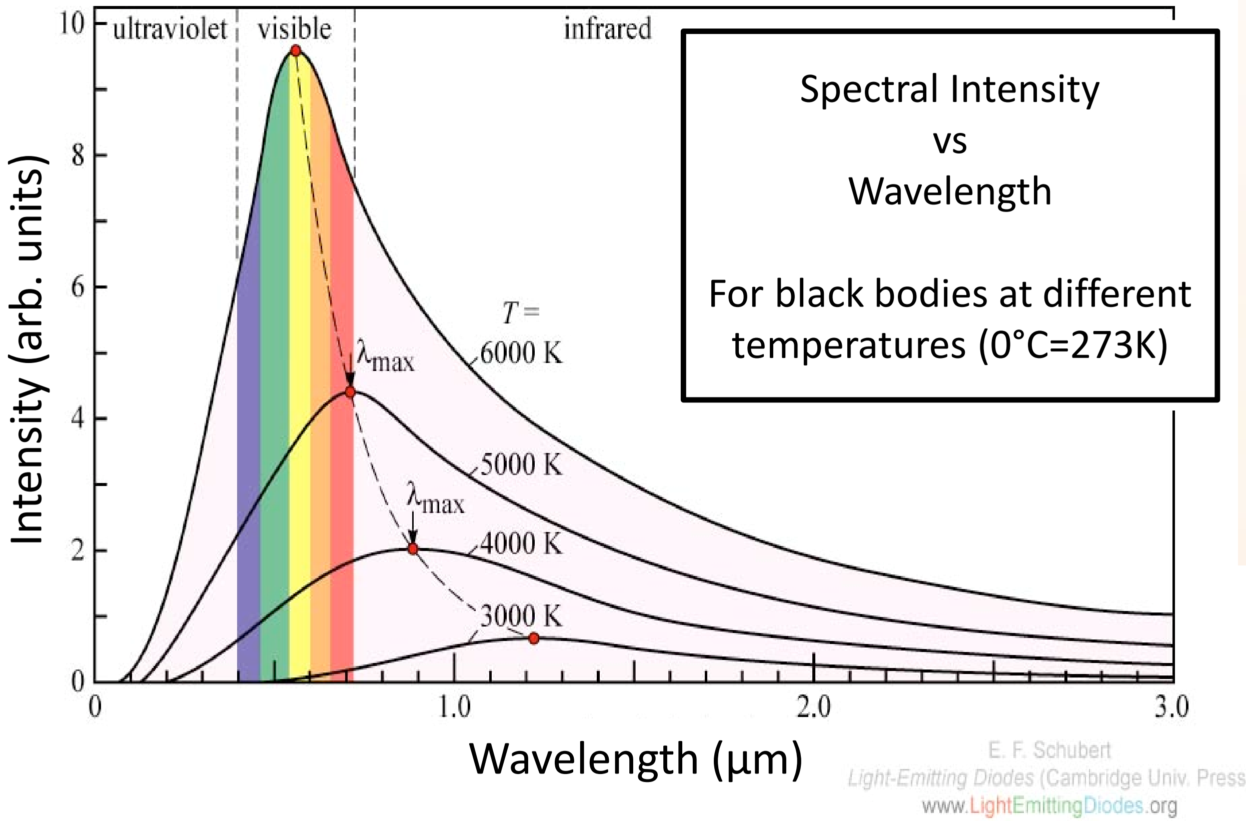
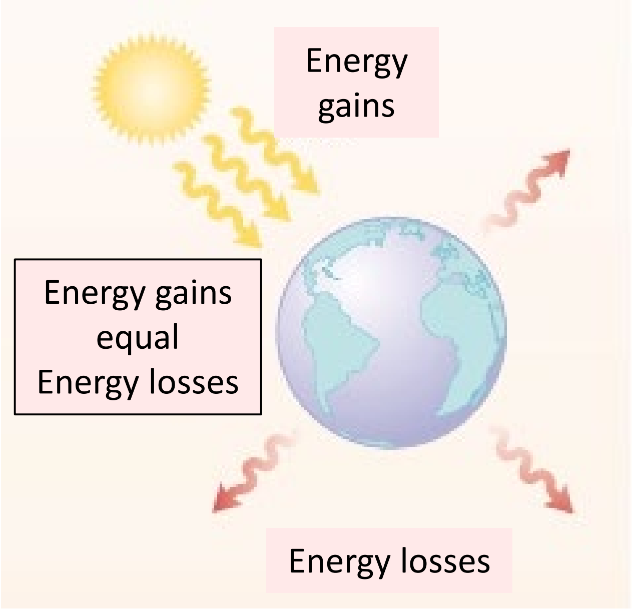


Figure 2.4. *Black-body emission curves of the Sun and the Earth. Reprinted courtesy of NASA.*

Black Body Radiation - Planck





Sun: ~6000K, emits mainly in visible spectrum (shortwave)
Earth: ~300K, emits mainly in IR (longwave)

Integrate over wavelength for temp T to get total emission flux σT^4 -- Stefan-Boltzmann

Radiation emitted by an incandescent material.

Incandescence is the emission of light (visible electromagnetic radiation) from a hot body as a result of its temperature.

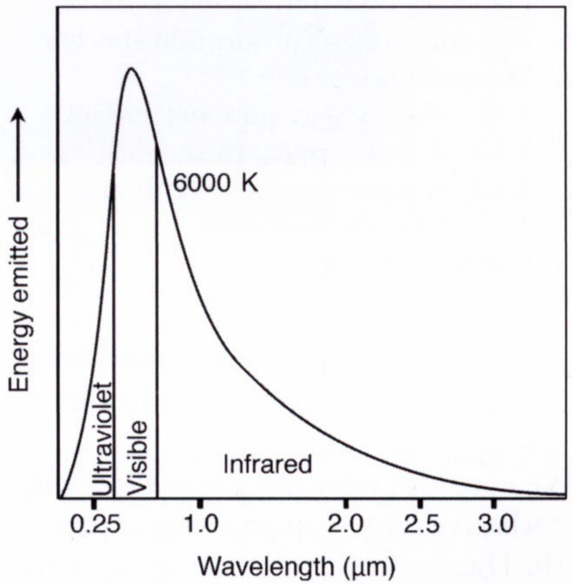


FIGURE 2.2. The energy emitted from the Sun plotted against wavelength based on a black body curve with $T = T_{Sun}$. Most of the energy is in the visible spectrum, and 95% of the total energy lies between 0.25 and 2.5μm (10^{-6}m).

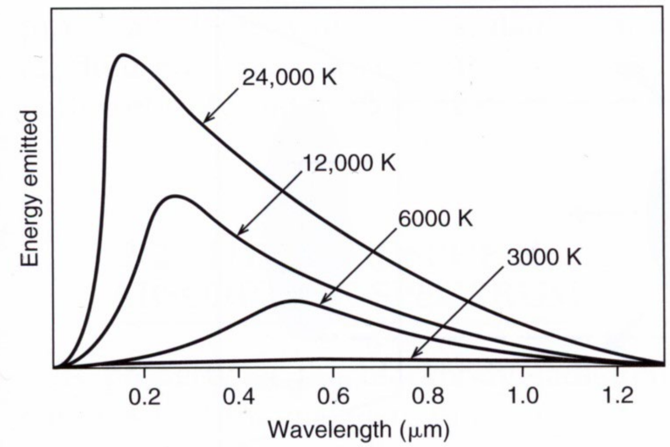


FIGURE 2.3. The energy emitted at different wavelengths for blackbodies at several temperatures. The function $B_\lambda(T)$, Eq. A-1, is plotted.

Planck or blackbody spectrum: wavelength at which the intensity of radiation is a max, and the flux of emitted radiation, depend only on the temperature of the body. The hotter the radiating body, the more energy it emits at shorter wavelengths. If the observed radiation spectrum is fitted to the blackbody curve using T as a free parameter, then the blackbody temp of the sun is 6000 K.

Planck's radiation curve – the Planck function

A blackbody absorbs 100% of incident radiation.
It therefore reflects no radiation and appears perfectly black.

Planck showed the power emitted by a blackbody per unit area, per unit solid angle, per unit wavelength, is given by

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(\exp[\frac{hc}{\lambda kT}]-1)}$$

Figures 2.2 and 2.3 are plots of $B_\lambda(T)$ *vs*. wavelength λ for various T

h is Planck's constant, *c* is the speed of light, *k* is Boltzmann's constant

Integrate $B_\lambda(T)$ over all wavelengths, obtain blackbody radiance

$$\int_0^\infty B_\lambda(T)\,d\lambda = \frac{\sigma}{\pi}T^4$$

$$\sigma = 2\pi^5 k^4 / 15h^3 c^2$$

Stefan-Boltzmann constant

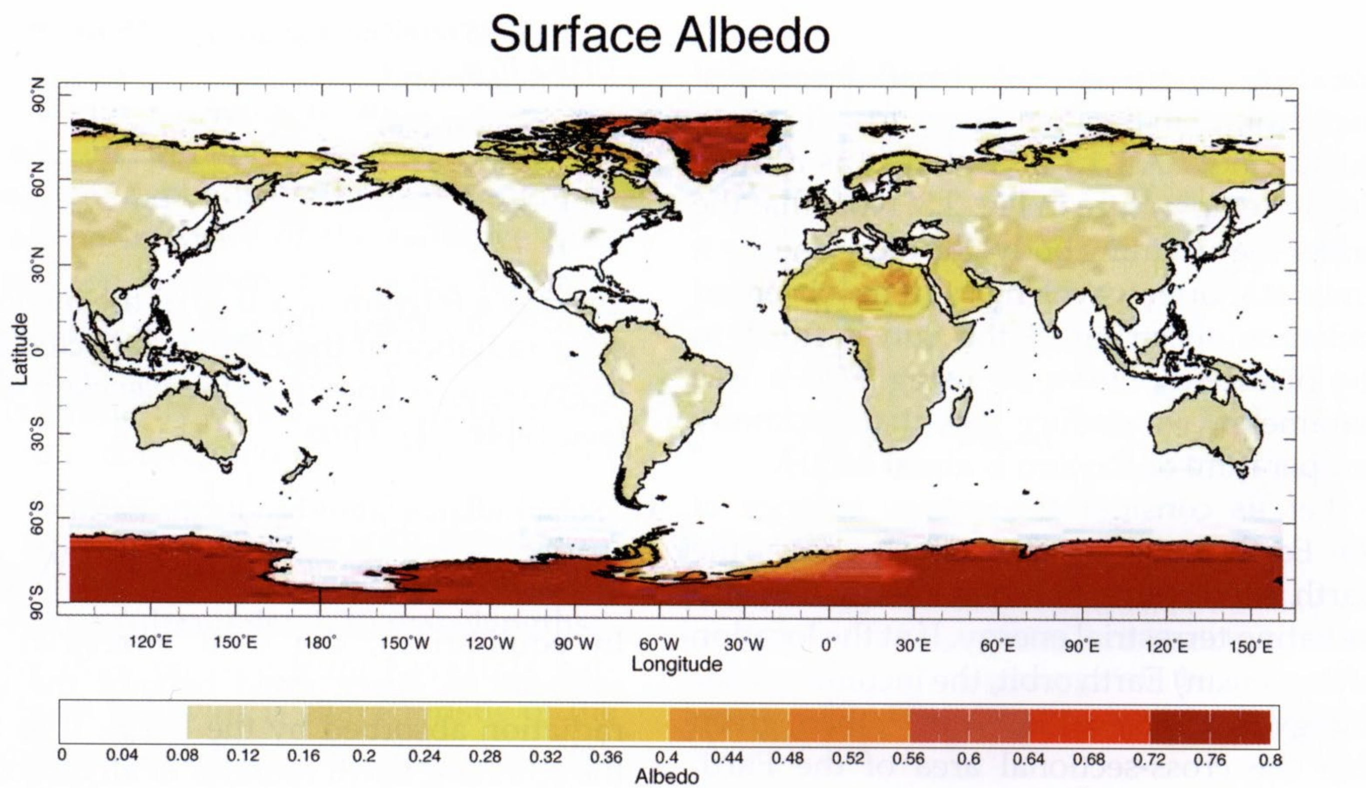


FIGURE 2.5. The albedo of the Earth's surface. Over the ocean the albedo is small (2–10%). It is larger over the land (typically 35–45% over desert regions) and is particularly high over snow and ice (~80%) (see Table 2.2).

Albedo depends on the nature of the reflecting surface, large for clouds, light surfaces like deserts, and especially snow and ice.

Under present terrestrial conditions of cloudiness and snow and ice cover, on average a fraction $\alpha \sim 0.3$ of incoming

Solar radiation absorbed by Earth =

$$(1 - \alpha_p) S_0 \pi a^2 = 1.22 \times 10^{17} \text{ W}$$

In equilibrium, the total terrestrial flux radiated to space must balance the solar radiation absorbed by Earth

If the spinning Earth radiates in all directions like a black body of uniform temperature

T_e known as the ***effective planetary temperature***

or the ***emission temperature of the Earth***

The Stefan-Boltzman law gives
emitted radiation per unit area = σT_e^4

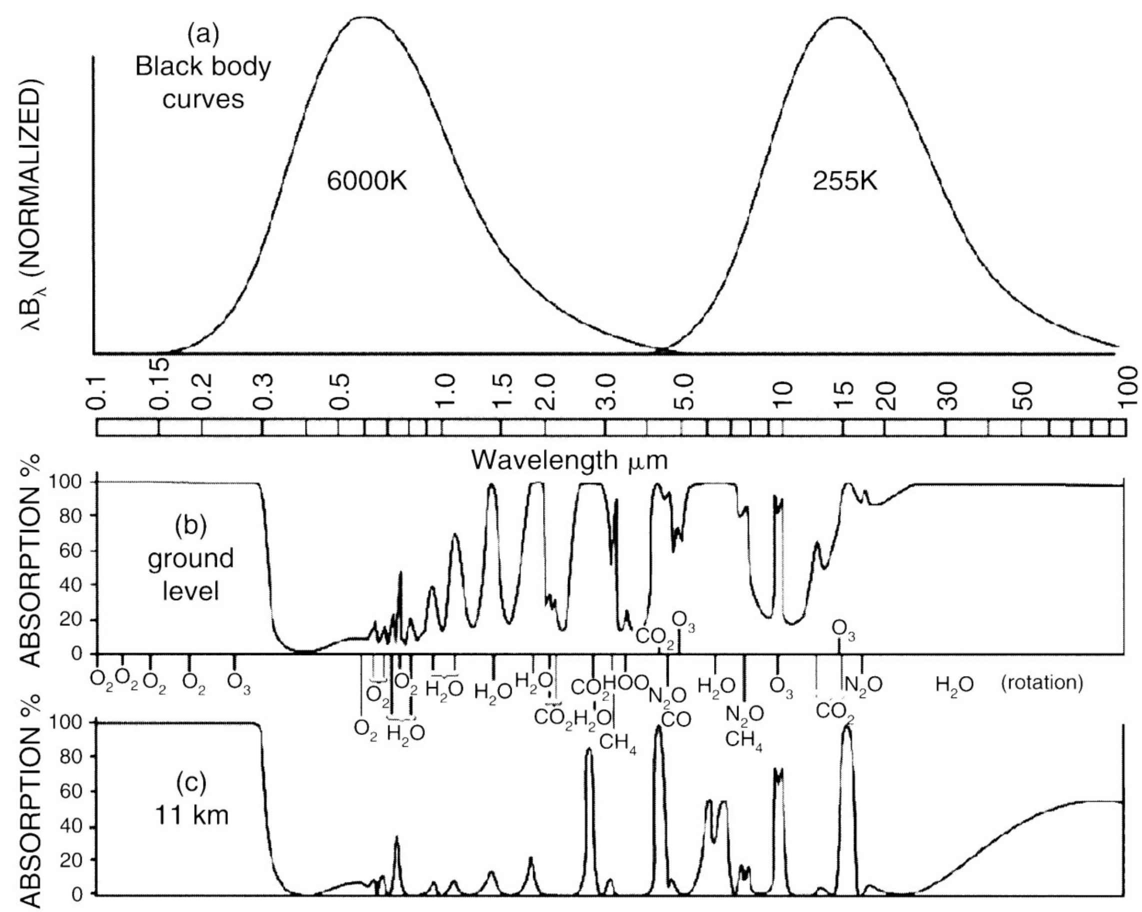
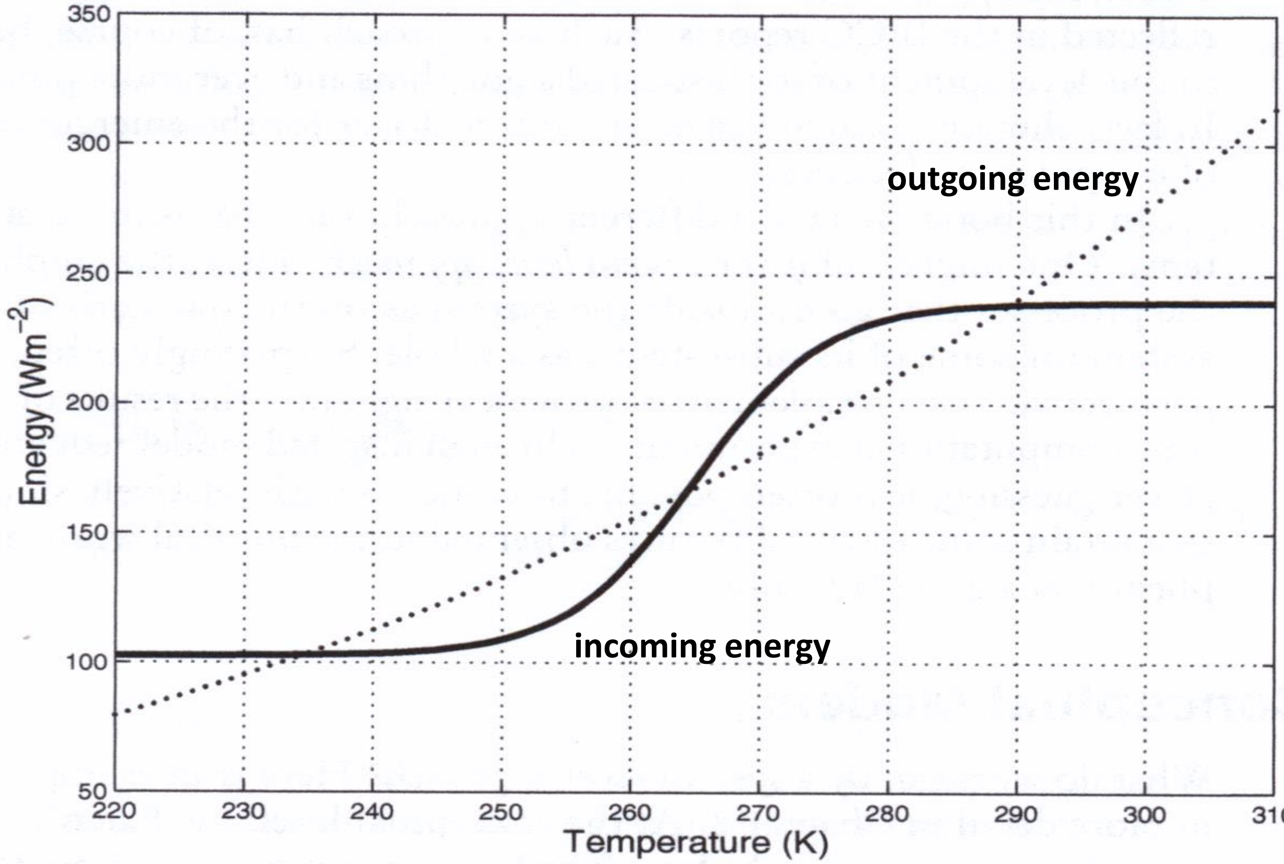


FIGURE 2.6. (a) The normalized blackbody emission spectra, $T^{-4}\lambda B_\lambda$, for the Sun ($T = 6000\,\text{K}$) and Earth ($T = 255\,\text{K}$) as a function of $\ln \lambda$ (top), where B_λ is the blackbody function (see Eq. A-2) and λ is the wavelength (see Appendix A.1.1 for further discussion). (b) The fraction of radiation absorbed while passing from the ground to the top of the atmosphere as a function of wavelength. (c) The fraction of radiation absorbed from the tropopause (typically at a height of 11 km) to the top of the atmosphere as a function of wavelength. The atmospheric molecules contributing the important absorption features at each frequency are also indicated. After Goody and Yung (1989).

Energy balance model with multiple equilibria for Earth's climate.



Outgoing : increasing – a warmer planet radiates more energy.

Incoming : S – shaped from albedo of ice/snow vs. land/ocean.

Bifurcation in surface temperature for forced system.

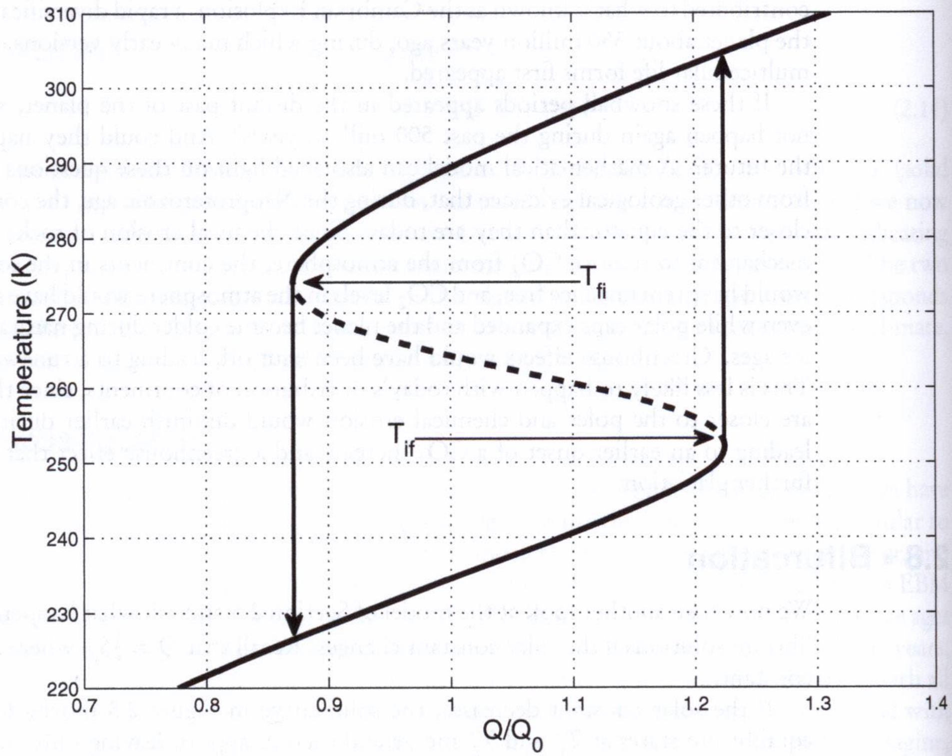


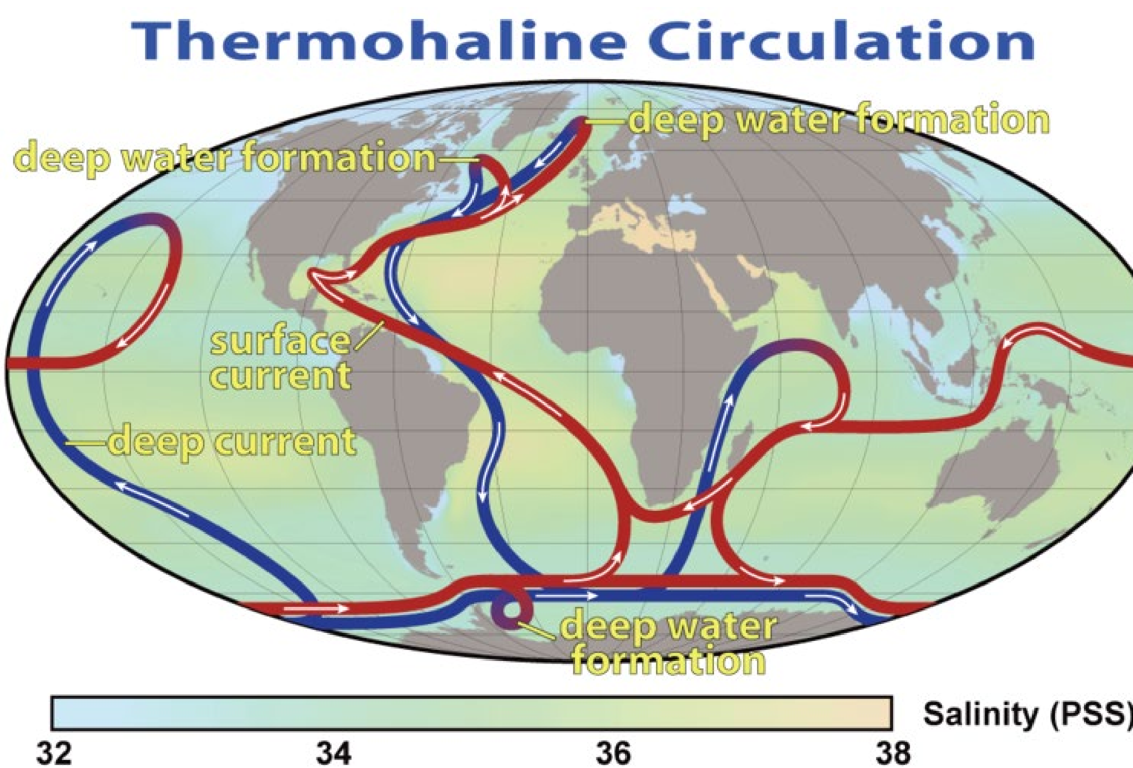
Figure 2.6. *Mean surface temperatures at equilibrium as a function of the solar constant (in units of its present value).*

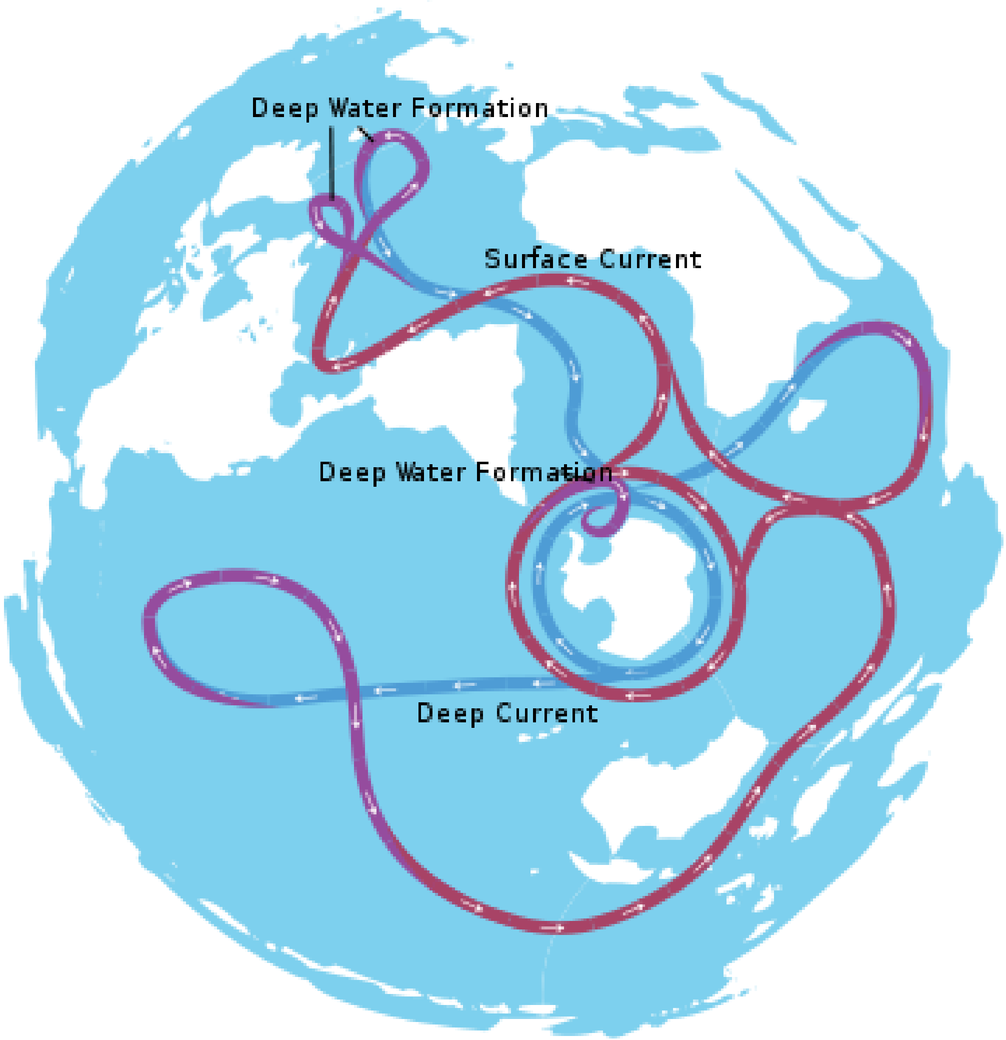
Global Thermohaline Conveyor Belt

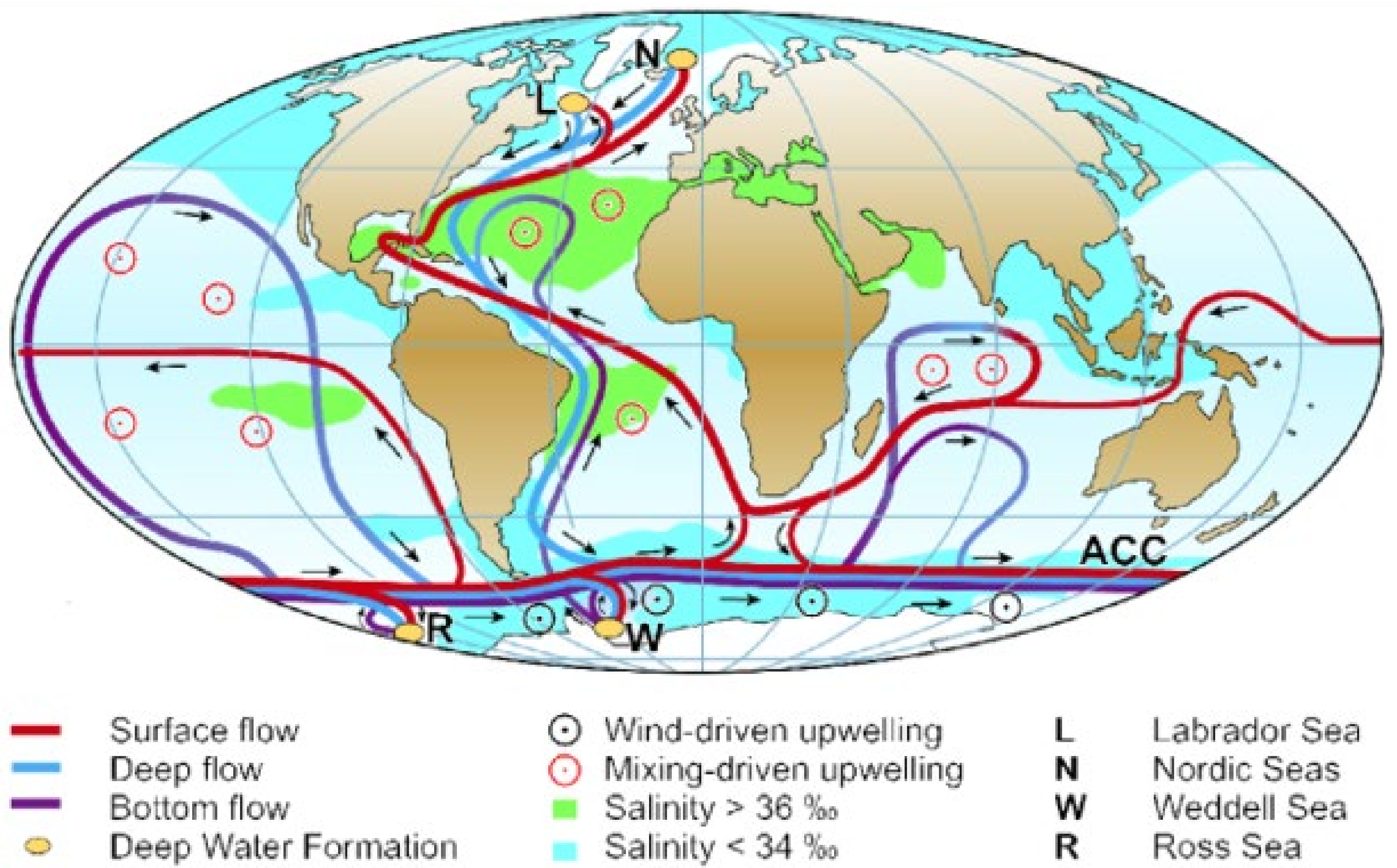
brine expulsion from freezing sea water results in sinking of dense, saltier water to great depths

melting sea ice inputs fresher water into the upper ocean

deep-water formation drives circulation in the world's oceans



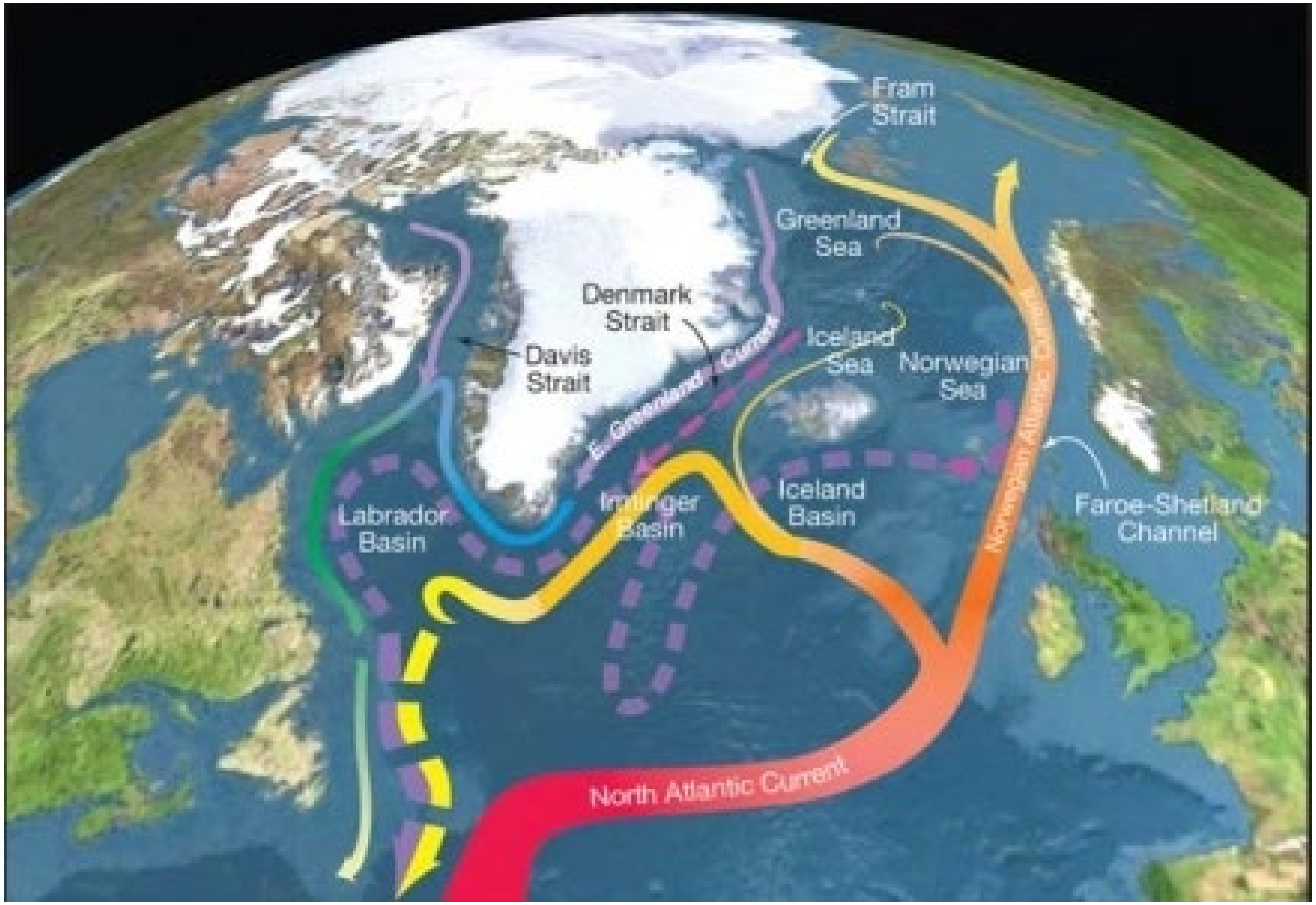

Deep Water Formation
Surface Current
Deep Water Formation
Deep Current

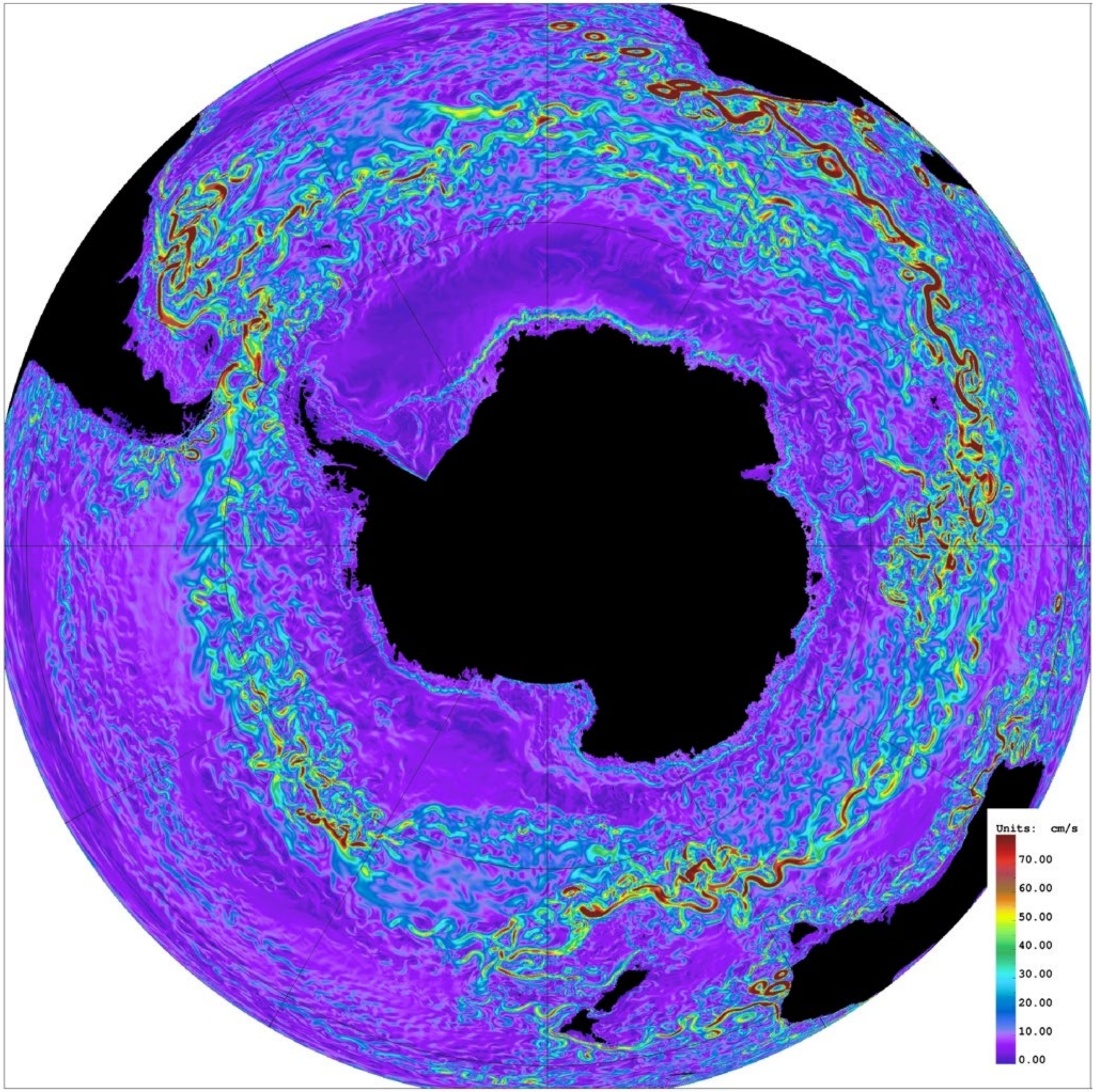

N
L
ACC
R
W
Surface flow
Deep flow
Bottom flow
Deep Water Formation
Wind-driven upwelling
Mixing-driven upwelling
Salinity > 36 ‰
Salinity < 34 ‰
L Labrador Sea
N Nordic Seas
W Weddell Sea
R Ross Sea

Ocean Circulation

- Schematic of the ocean circulation (from Kuhlbrodt et al., 2007) associated with the global Meridional Overturning Circulation (MOC), with special focus on the Atlantic section of the flow (AMOC). The red curves in the Atlantic indicate the northward flow of water in the upper layers. The filled orange circles in the Nordic and Labrador Seas indicate regions where near-surface water cools and becomes denser, causing the water to sink to deeper layers of the Atlantic. This process is referred to as “water mass transformation,” or “deep water formation.” In this process heat is released to the atmosphere. The light blue curve denotes the southward flow of cold water at depth. At the southern end of the Atlantic, the AMOC connects with the Antarctic Circumpolar Current (ACC). Deep water formation sites in the high latitudes of the Southern Ocean are also indicated with filled orange circles. These contribute to the production of Antarctic Bottom Water (AABW), which flows northward near the bottom of the Atlantic (indicated by dark blue lines in the Atlantic). The circles with interior dots indicate regions where water upwells from deeper layers to the upper ocean (see Section 2 for more discussion on where upwelling occurs as part of the global MOC).

http://www.eoearth.org/view/article/156548/





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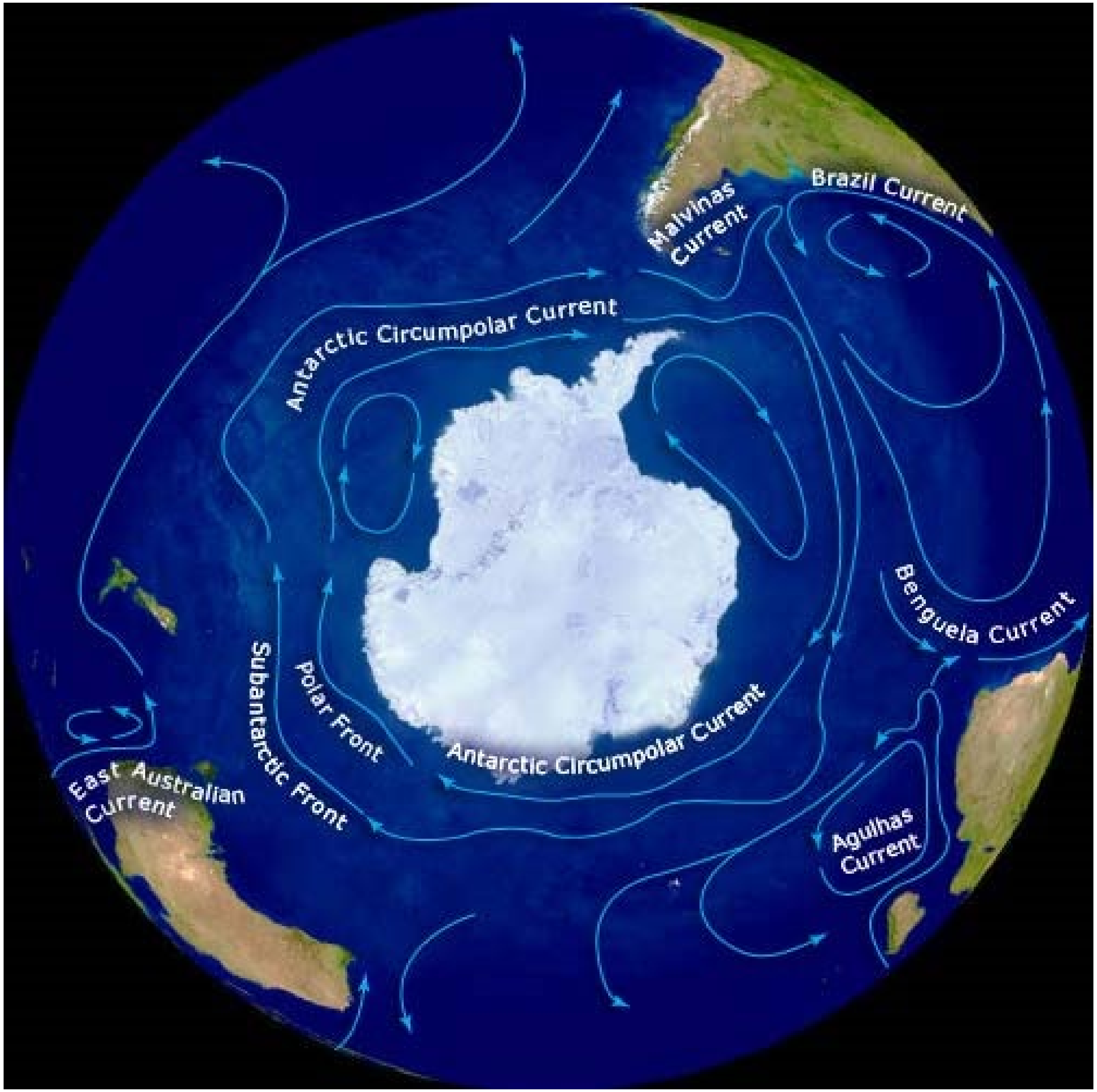

Brazil Current
Malvinas Current
Antarctic Circumpolar Current
Benguela Current
Agulhas Current
Antarctic Circumpolar Current
Polar Front
Subantarctic Front
East Australian Current

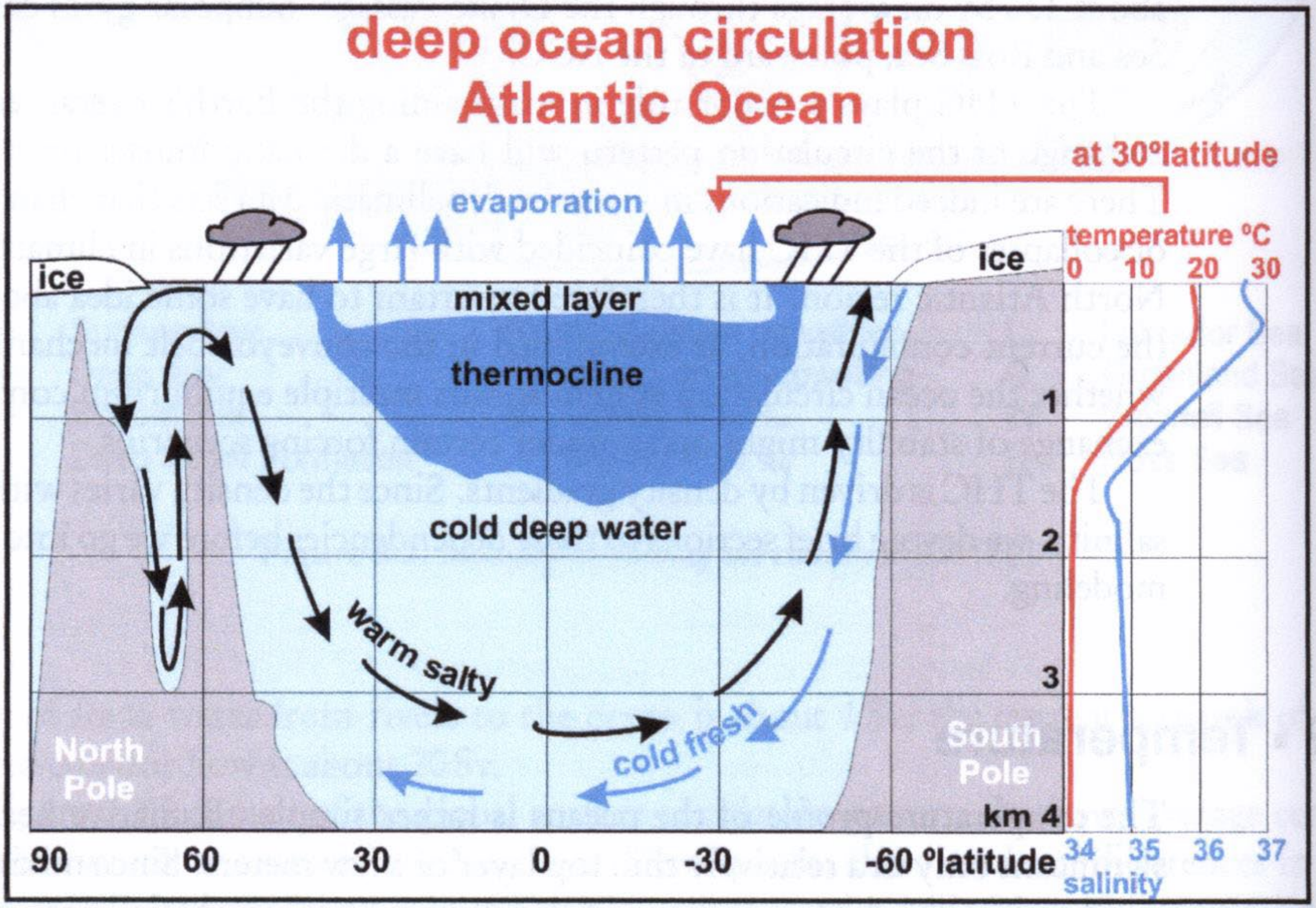

Figure 3.4. *Cross-section of the Atlantic Ocean. Reprinted with permission from Seafriends.*

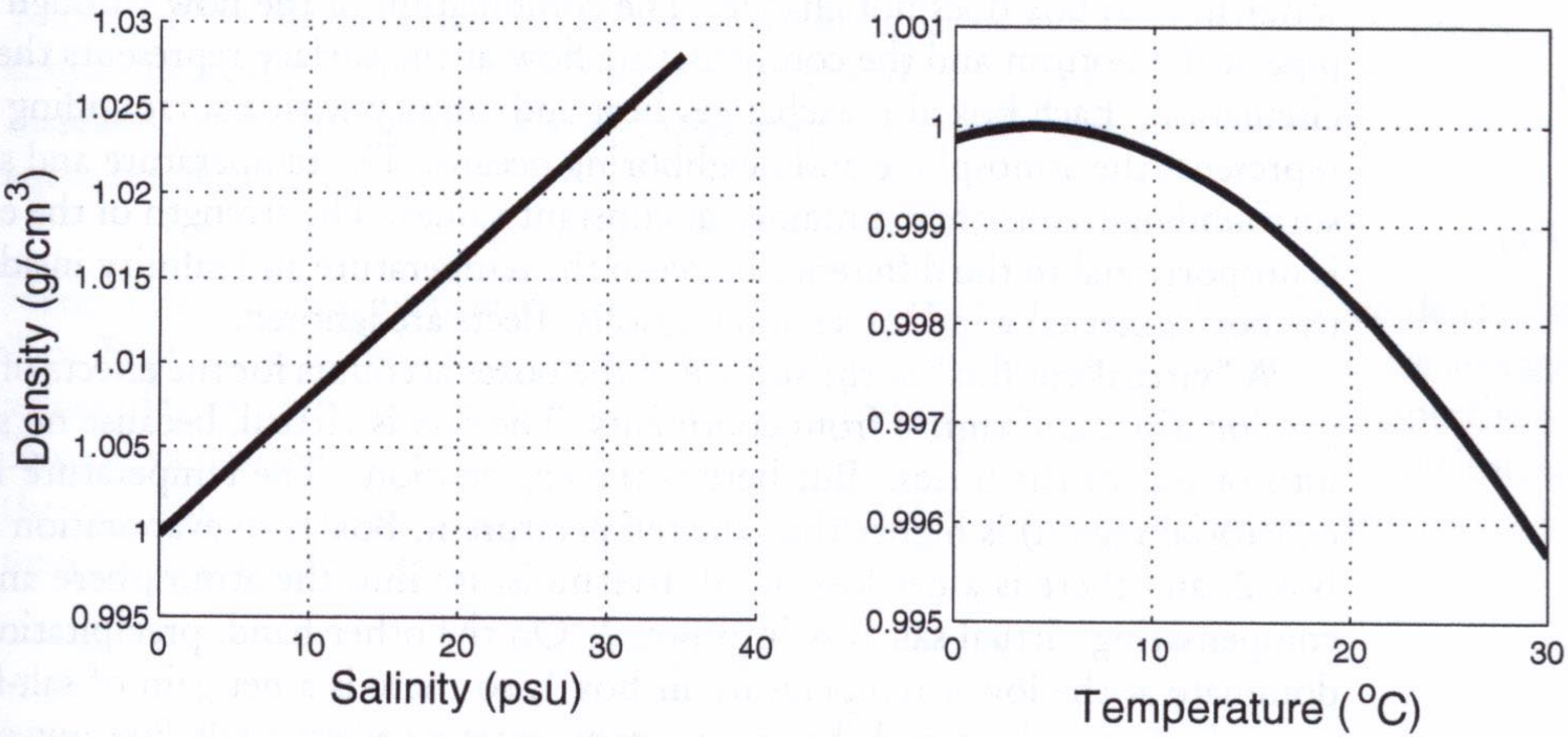

Figure 3.5. *Density as a function of salinity and temperature.*

Oceanic Box Models

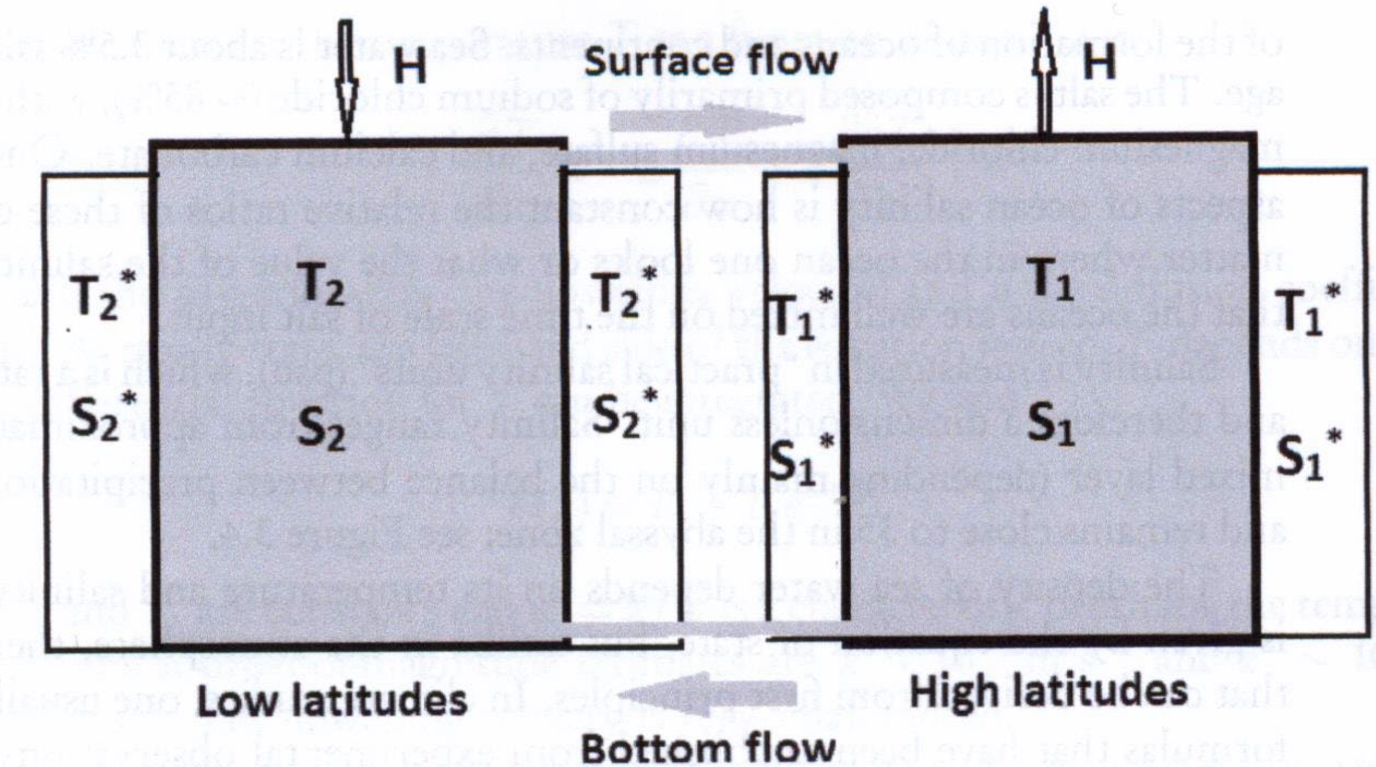

Figure 3.6. *Two-box model of the North Atlantic with evaporation and precipitation.*