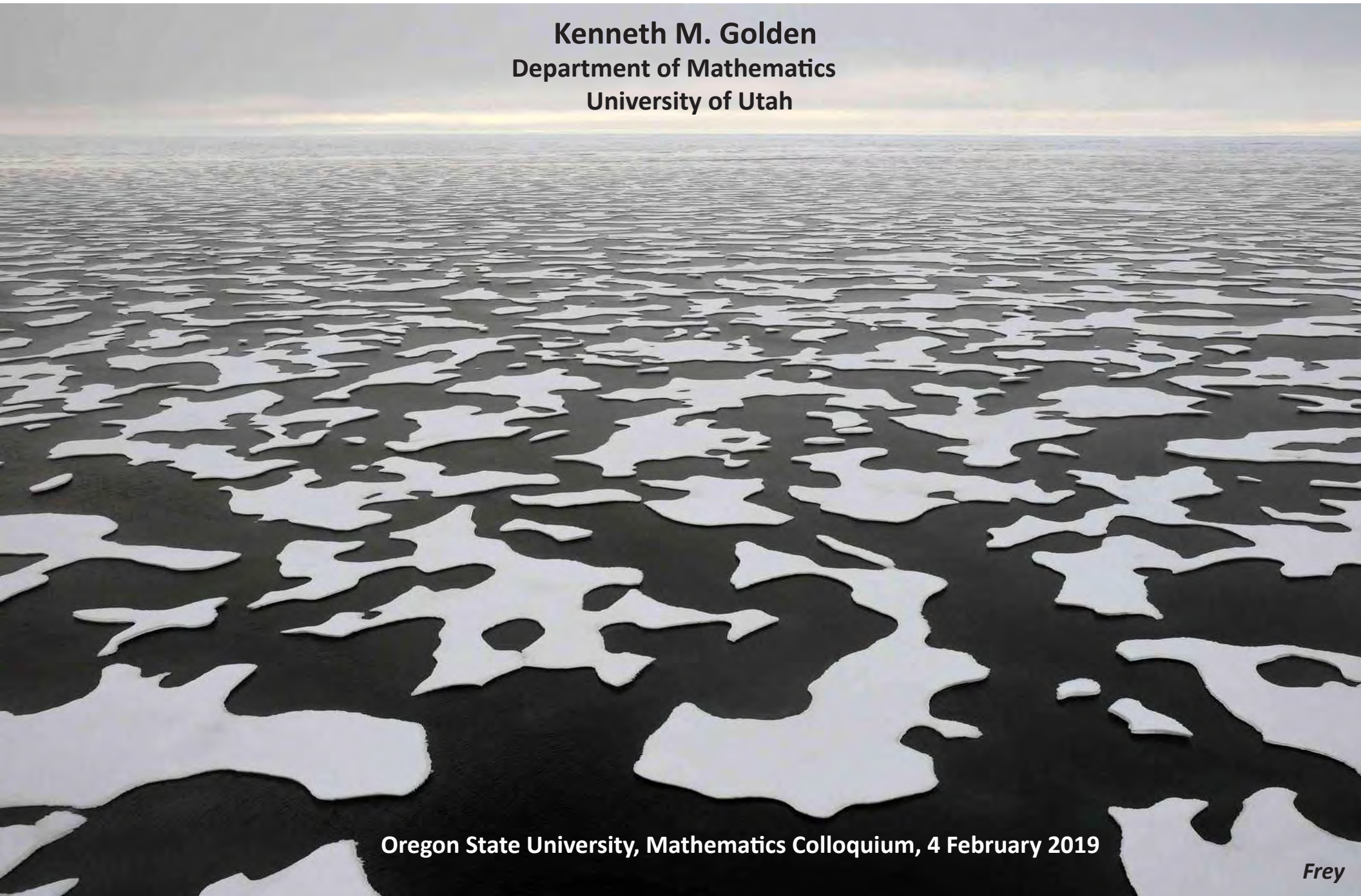


MODELING the MELT:

what math tells us about disappearing polar sea ice

Kenneth M. Golden
Department of Mathematics
University of Utah

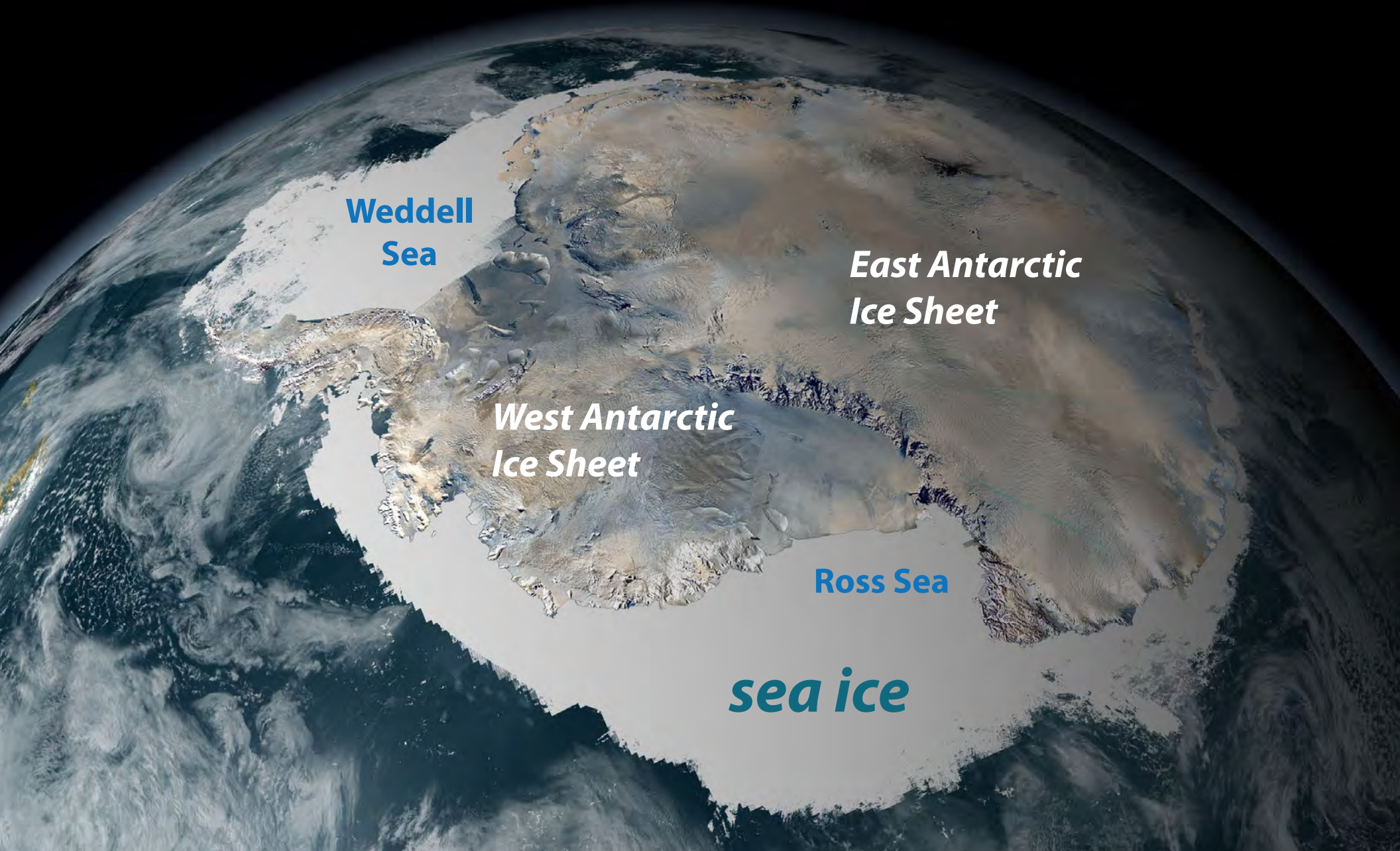


Oregon State University, Mathematics Colloquium, 4 February 2019

Frey

ANTARCTICA

southern cryosphere



**Weddell
Sea**

***East Antarctic
Ice Sheet***

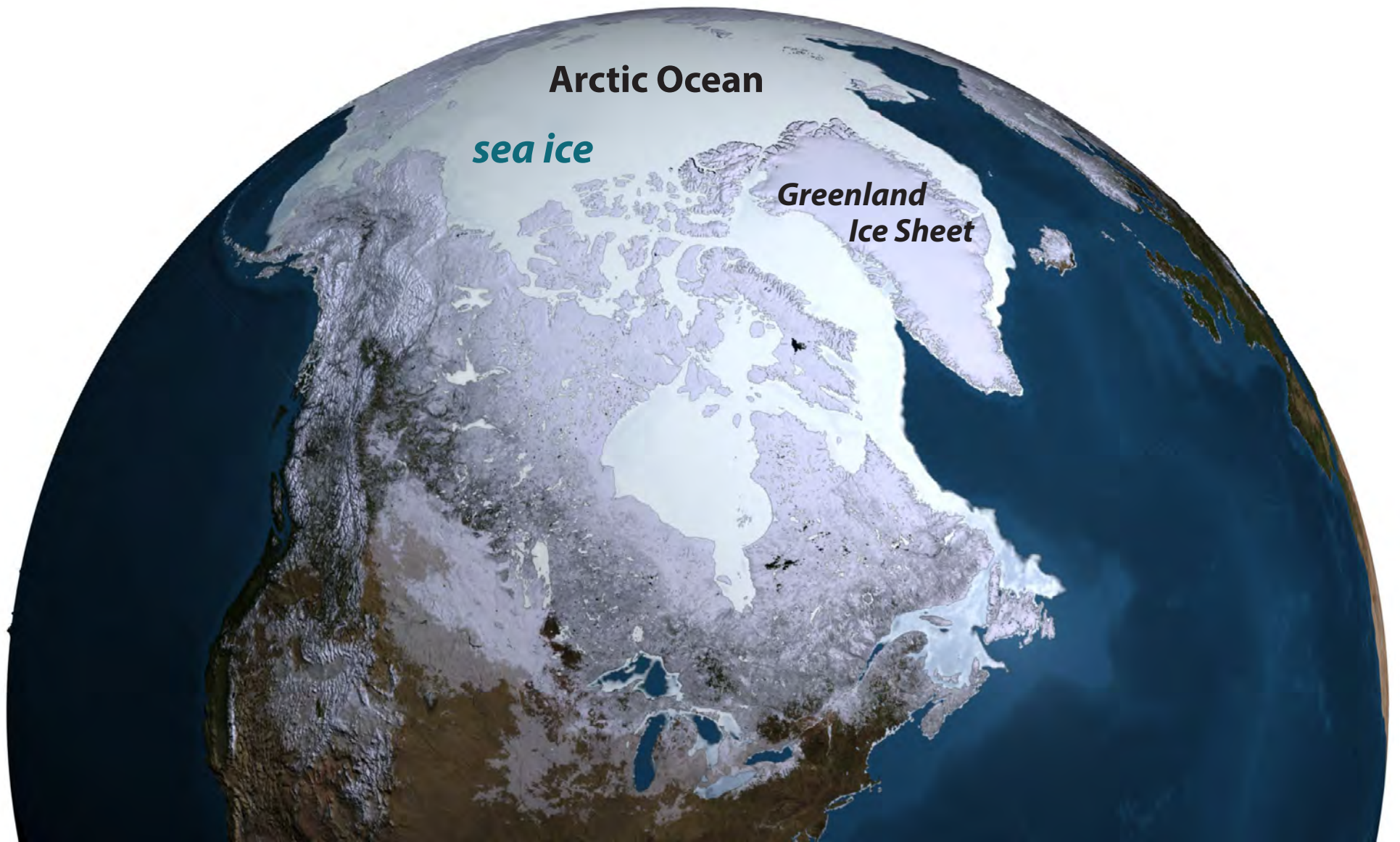
***West Antarctic
Ice Sheet***

Ross Sea

sea ice

THE ARCTIC

northern cryosphere



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of **climate change**



polar ice caps critical to global climate in reflecting incoming solar radiation



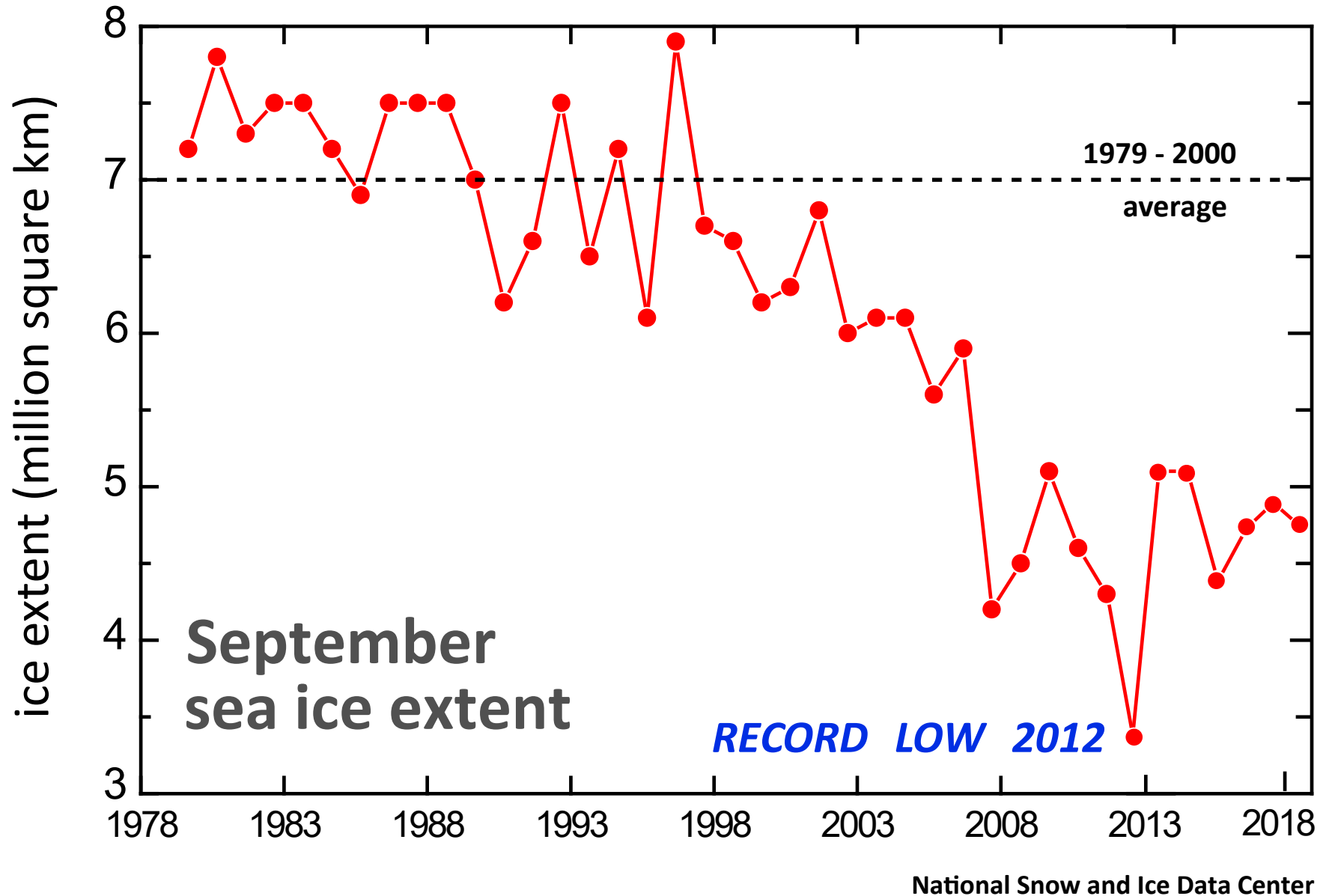
white snow and ice
reflect



dark water and land
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

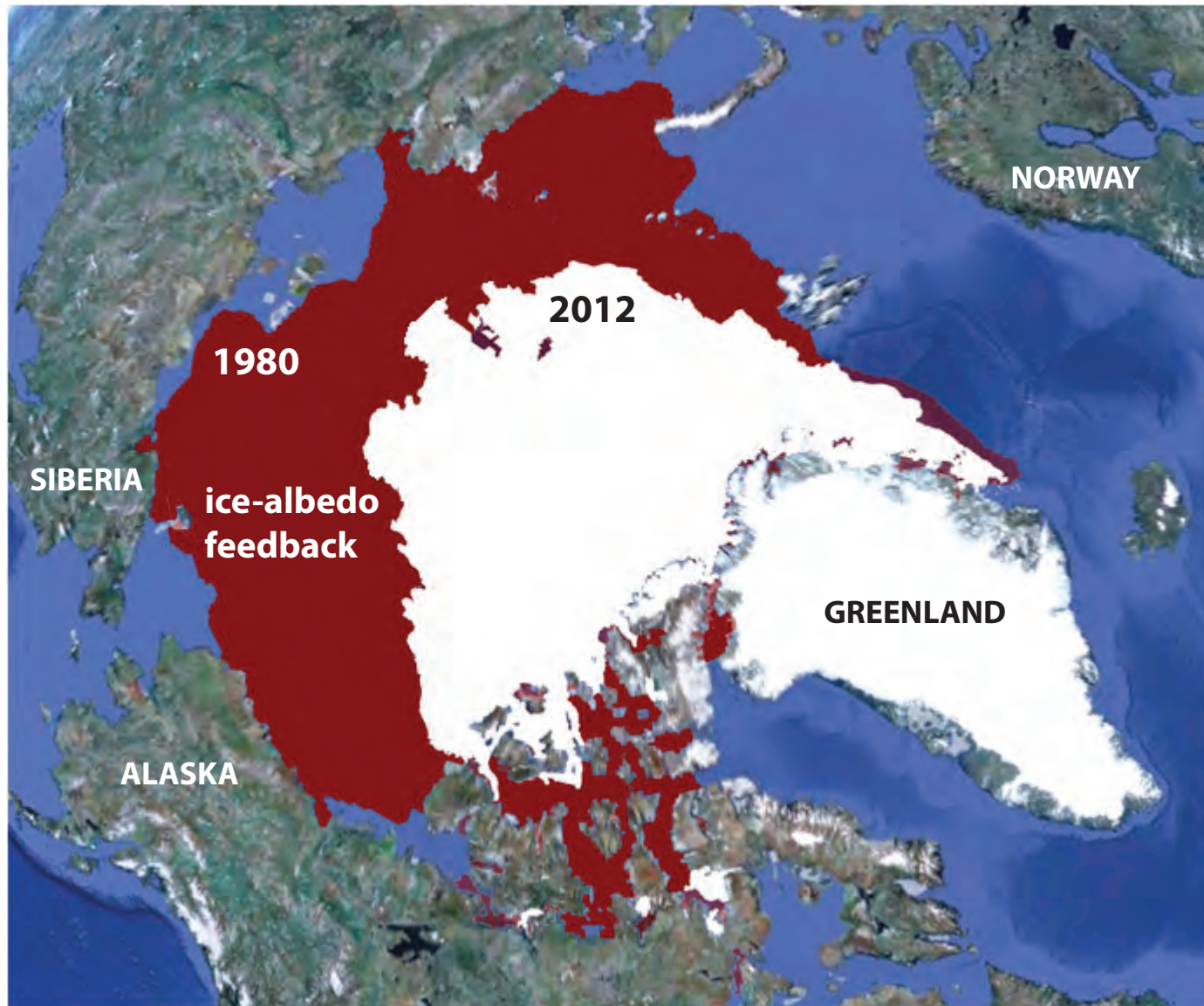
the summer Arctic sea ice pack is melting



Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers

September 2012 -- 3.4 million square kilometers





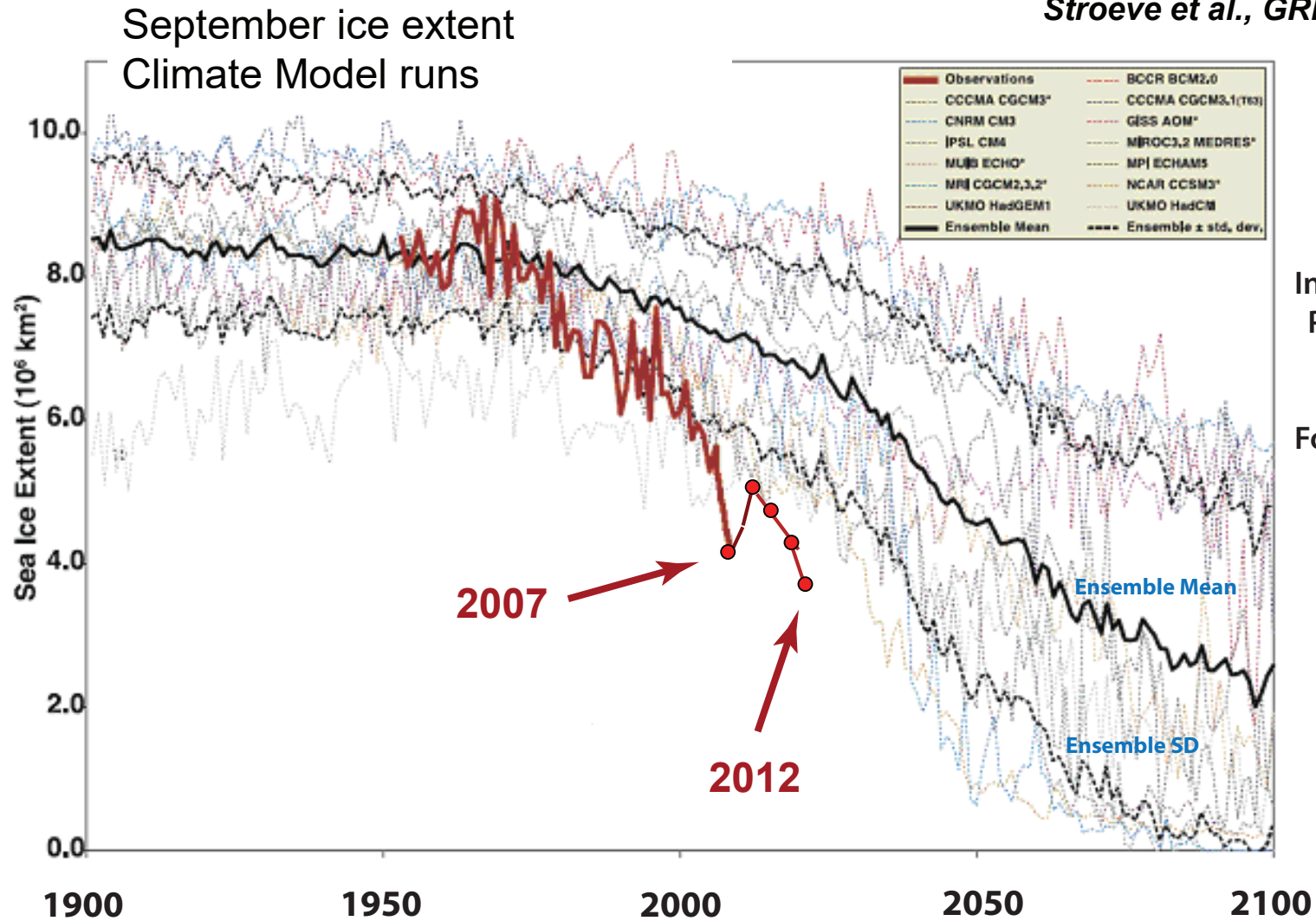
*recent losses
in comparison to
the United States*



Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



**IPCC AR4
Models**

Intergovernmental
Panel on Climate
Change (IPCC)

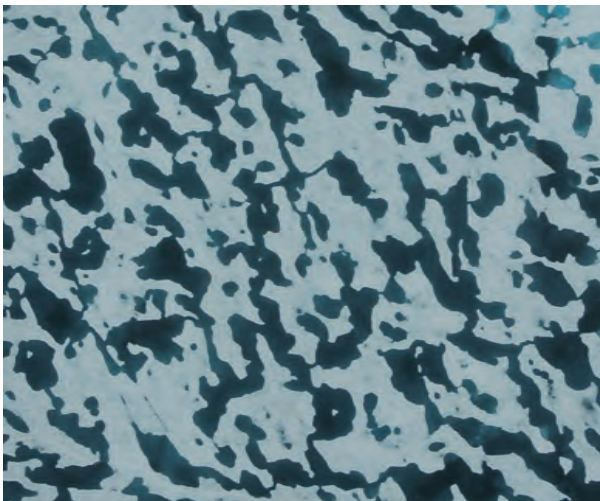
Fourth Assessment
AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



Impact of melt ponds on Arctic sea ice
simulations from 1990 to 2007

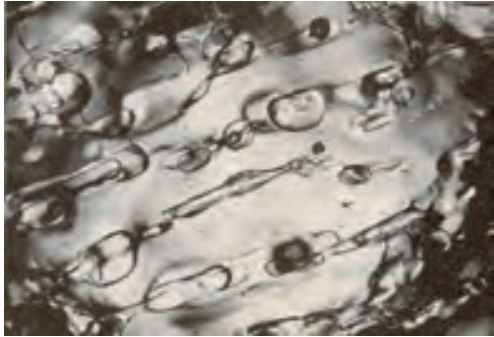
Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

**For simulations with ponds
September ice volume is nearly 40% lower.**

... and other sub-grid scale structures and processes

linkage of scales

sea ice is a multiscale composite



millimeters



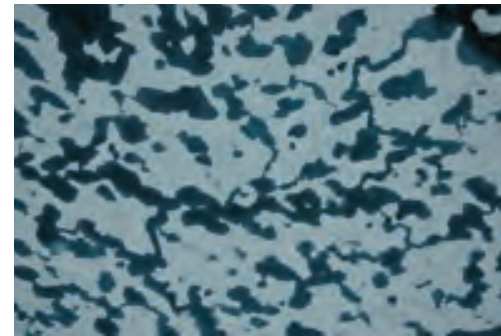
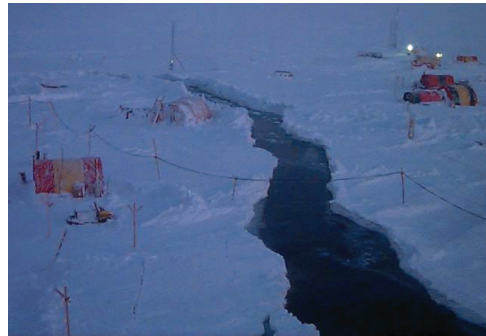
centimeters



meters



meters



kilometers



What is this talk about?

HOMOGENIZATION

Using theories of statistical physics and composite materials to LINK SCALES in the sea ice system ... rigorously compute effective behavior and improve climate models.

1. Sea ice microphysics and fluid transport
2. Stieltjes integral representations for EM properties
3. Extension to polycrystals, advection diffusion, waves in MIZ
4. Fractal geometry of melt pond evolution

critical behavior

cross - pollination

Solving problems in physics and biology of sea ice drives advances in theory of composite materials.

How do scales interact in the sea ice system?



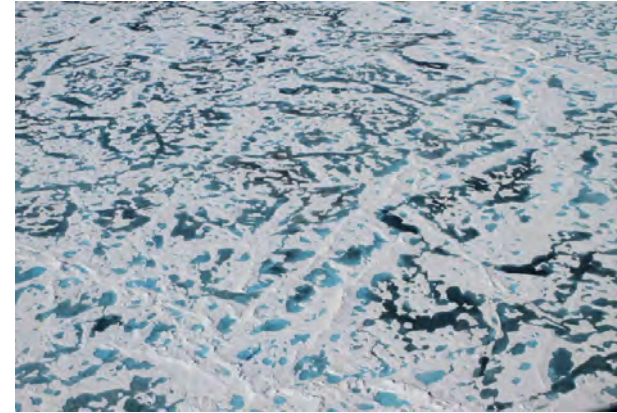
basin scale -
grid scale
albedo

Linking Scales

km
scale
melt
ponds



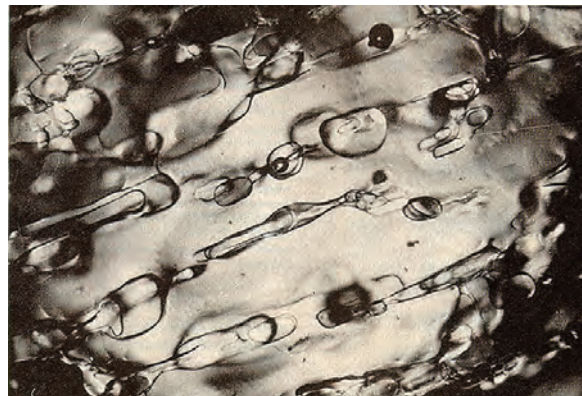
km
scale
melt
ponds



Linking

Scales

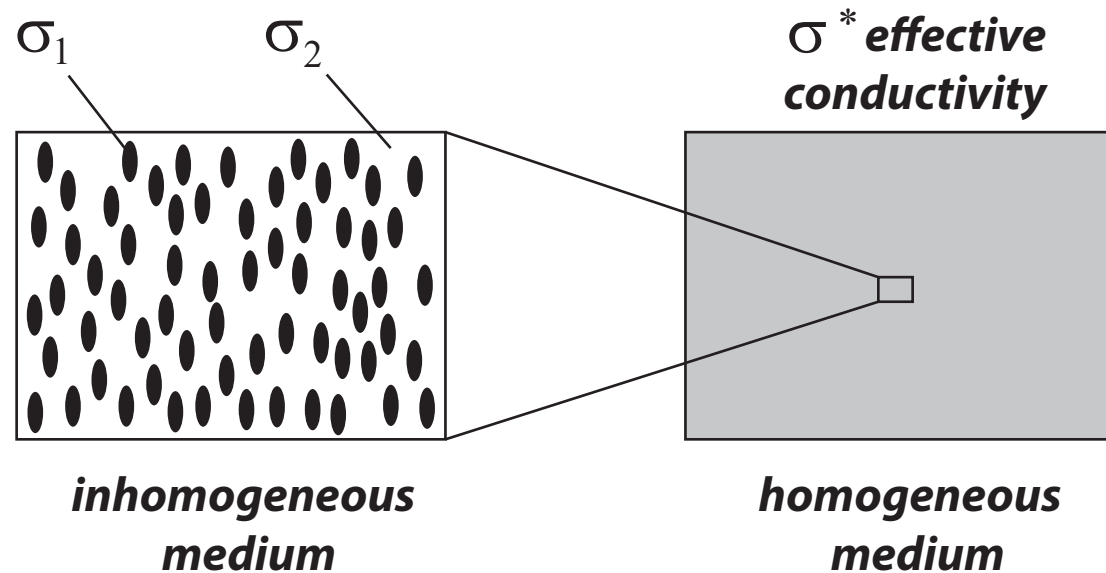
mm
scale
brine
inclusions



meter
scale
snow
topography



HOMOGENIZATION - Linking Scales in Composites



find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres

Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

sea ice microphysics

fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

September
snow-ice
estimates

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO₂*

sea ice ecosystem



sea ice algae
support life in the polar oceans

fluid permeability k of a porous medium

porous
concrete



how much water
gets through the
sample per unit
time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

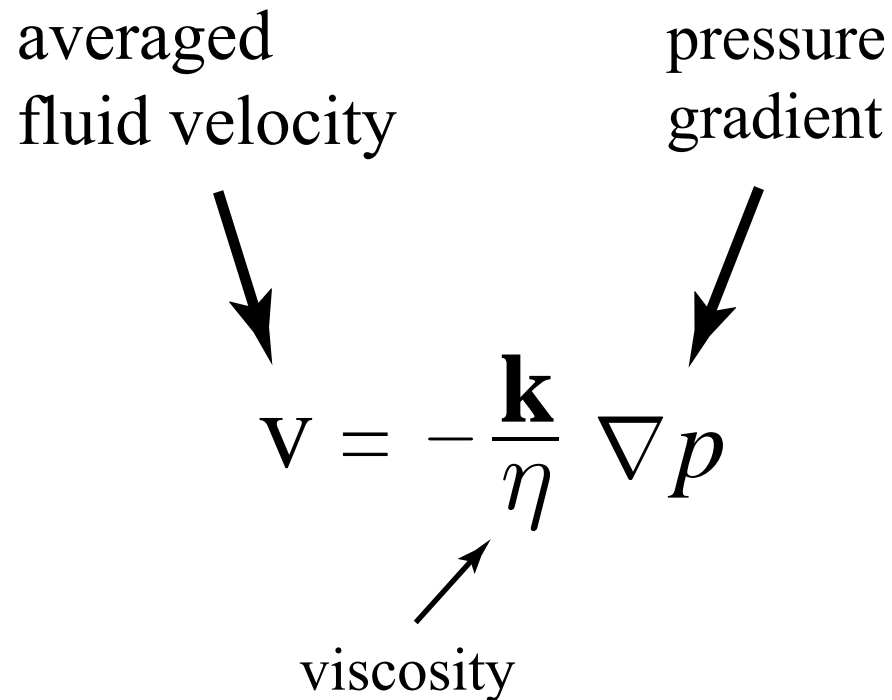
Darcy's Law for slow viscous flow in a porous medium

averaged
fluid velocity

pressure
gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

viscosity

The diagram shows the equation $\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$ centered on the slide. Three labels with arrows point to parts of the equation: 'averaged fluid velocity' points to \mathbf{v} , 'pressure gradient' points to ∇p , and 'viscosity' points to η .

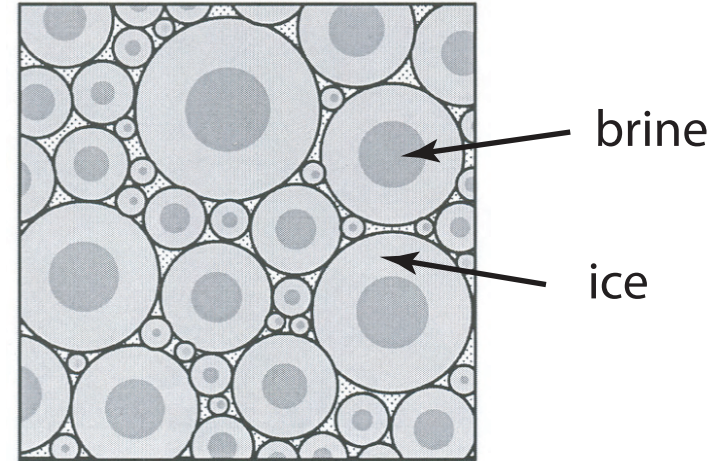
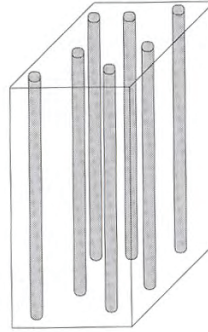
\mathbf{k} = fluid permeability tensor

PIPE BOUNDS on vertical fluid permeability k

Golden, Heaton, Eicken, Lytle, Mech. Materials 2006

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007

vertical pipes
with appropriate radii
maximize k



optimal coated
cylinder geometry

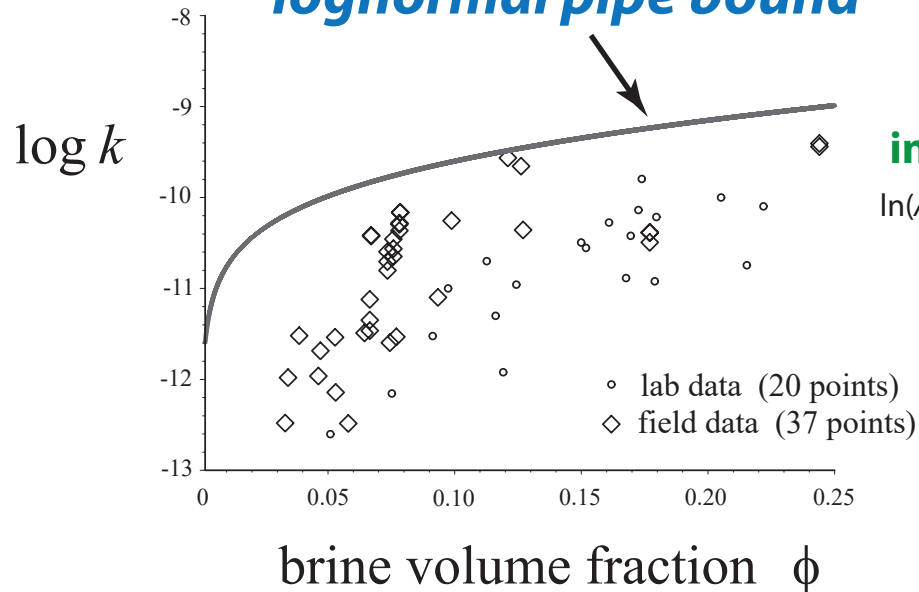
fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)

$$k \leq \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

$\ln(A)$ normally distributed, mean μ (increases with T) variance σ^2 (Gow and Perovich 96)

lognormal pipe bound



Golden et al., Geophys. Res. Lett. 2007

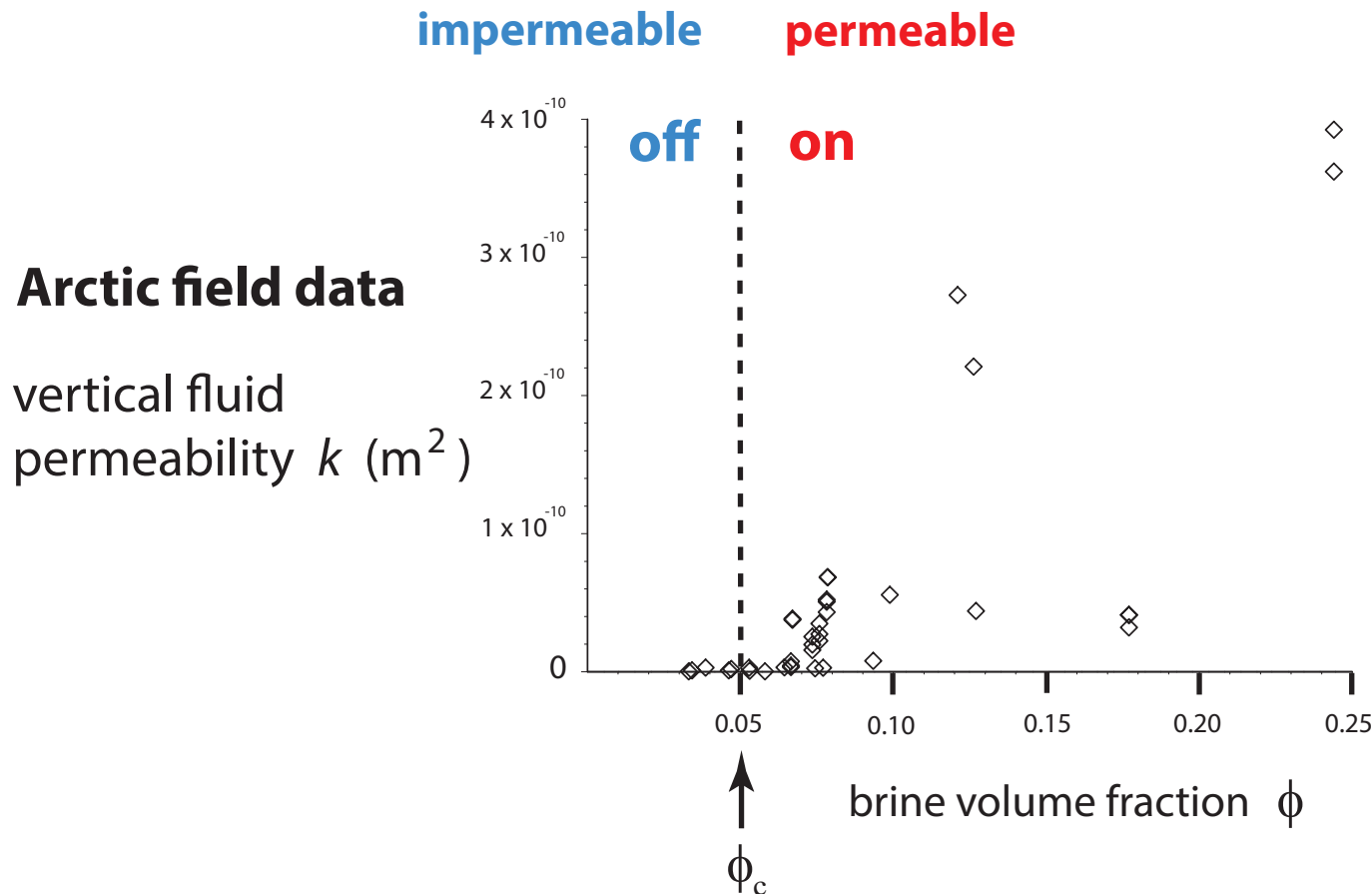
get bounds through variational analysis of
trapping constant γ for diffusion process
in pore space with absorbing BC

Torquato and Pham, PRL 2004

$$\mathbf{k} \leq \gamma^{-1} \mathbf{I}$$

for any ergodic porous medium
(Torquato 2002, 2004)

Critical behavior of fluid transport in sea ice



***“on - off” switch
for fluid flow***

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ \text{C}, S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

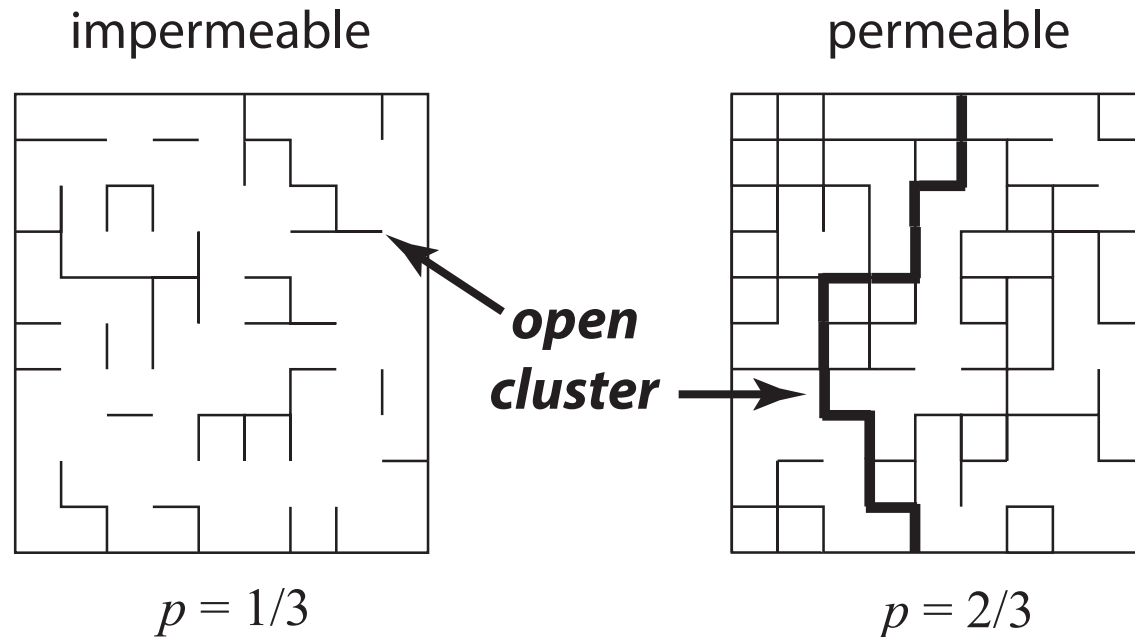
Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond \longrightarrow **open** with probability p
closed with probability $1-p$

percolation threshold

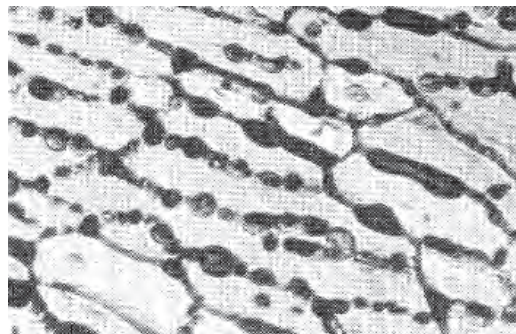
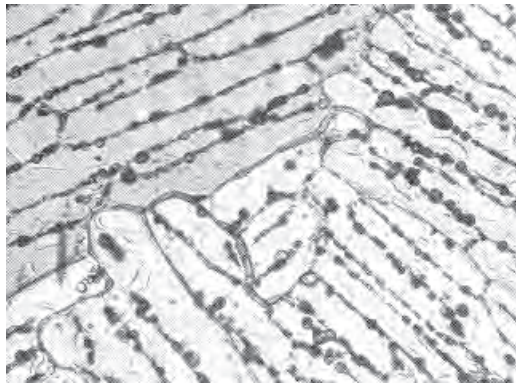
$$p_c = 1/2 \quad \text{for } d = 2$$

smallest p for which there is an infinite open cluster

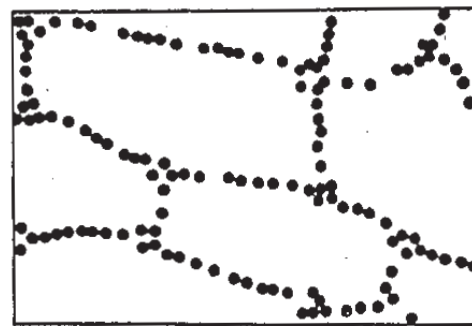
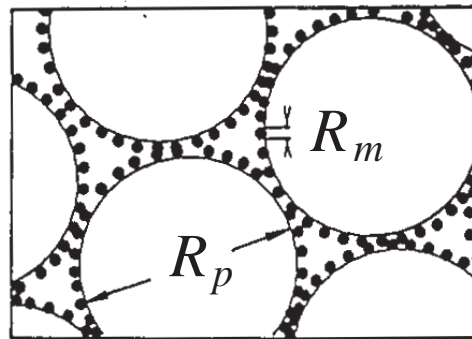
Continuum percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

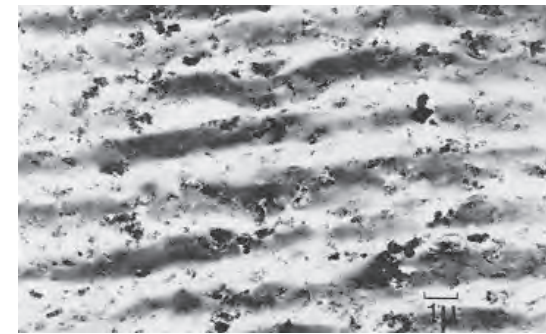
Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed
powder

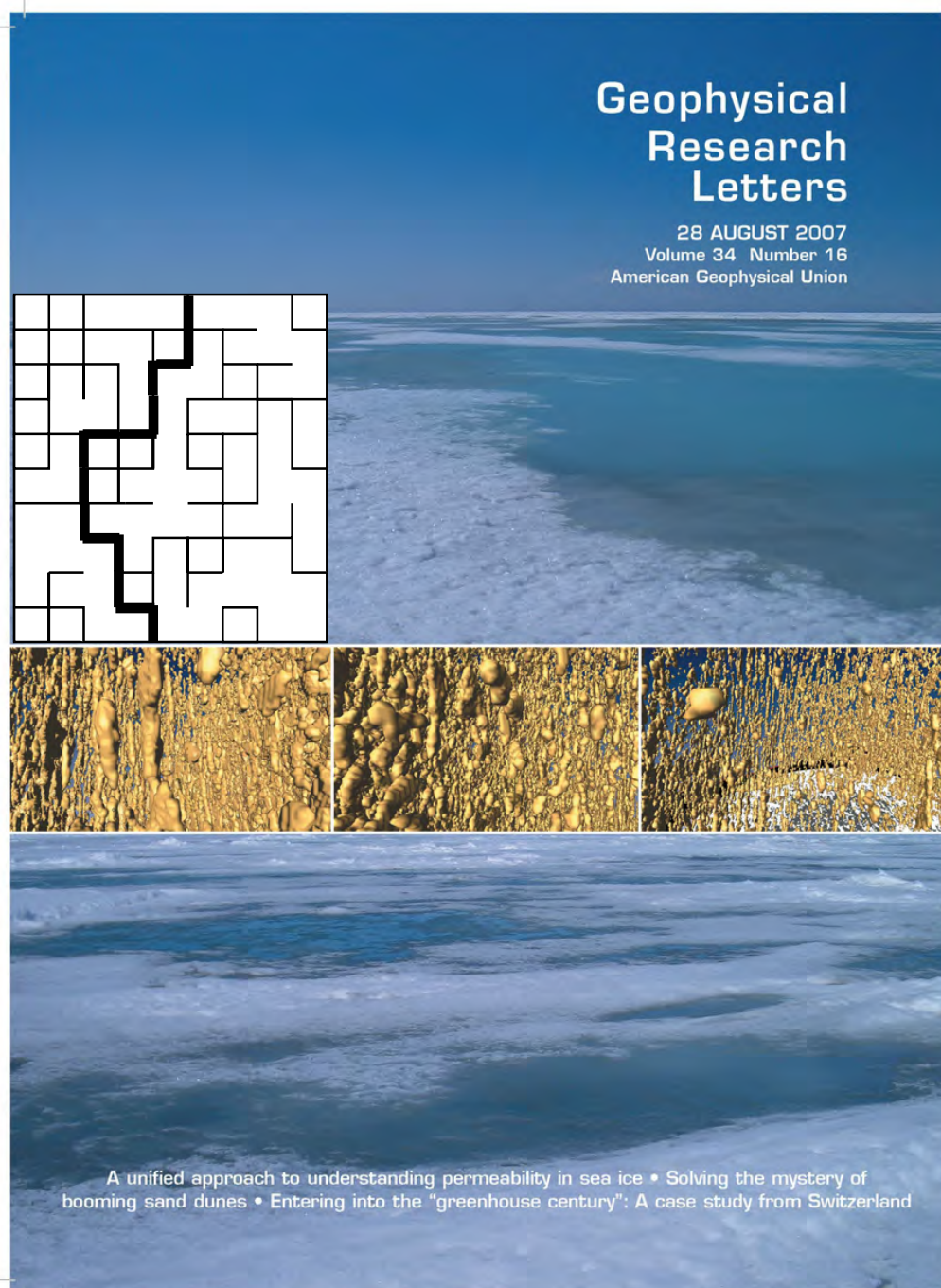


radar absorbing
composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



micro-scale
controls
macro-scale
processes

percolation theory

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical
exponent
t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

***hierarchical model
network model
rigorous bounds***

agree closely
with field data

***X-ray tomography for
brine inclusions***

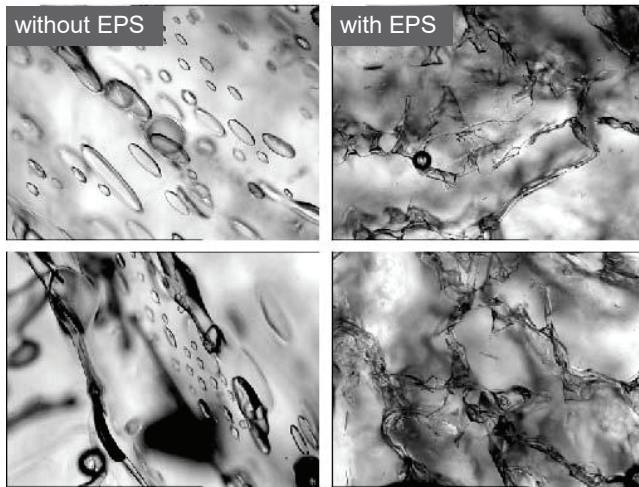
***unprecedented look
at thermal evolution
of brine phase and
its connectivity***

confirms rule of fives

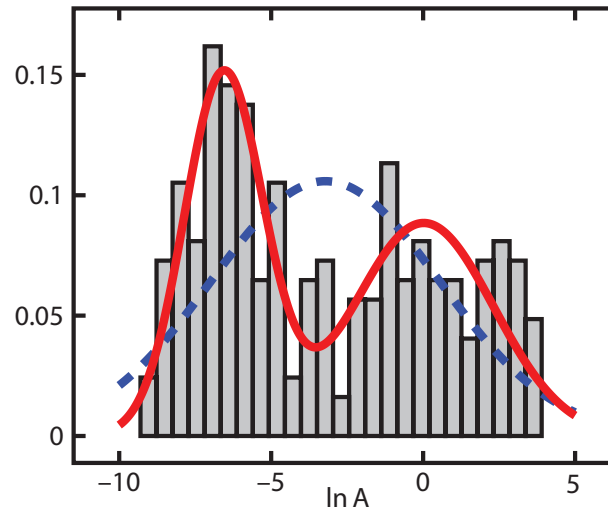
***Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009***

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

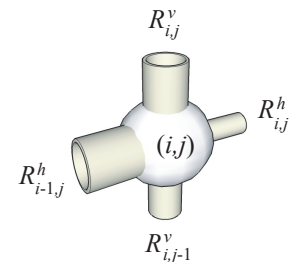
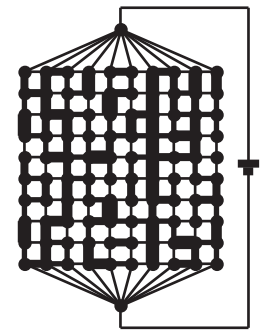
How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k .
- Rigorous bound on k for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden,
Eicken, Morris
Ann. Glac. 2006

How does the biology affect the physics?

Remote sensing of sea ice



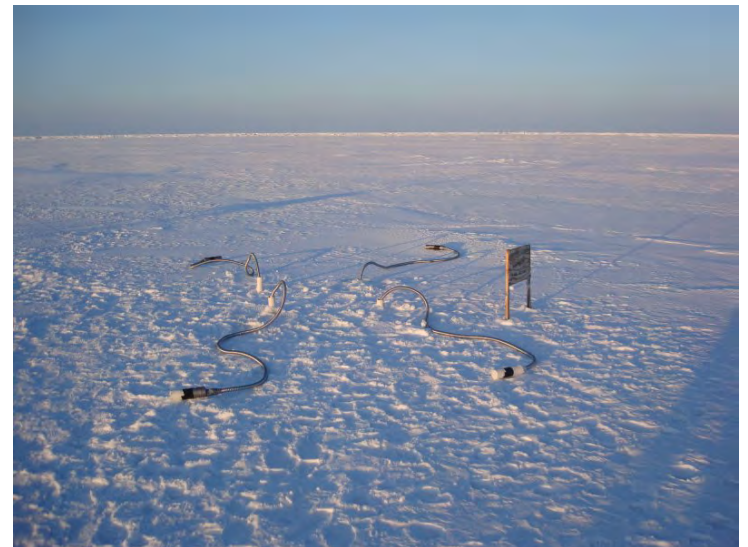
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice
properties from
electromagnetic
(EM) data

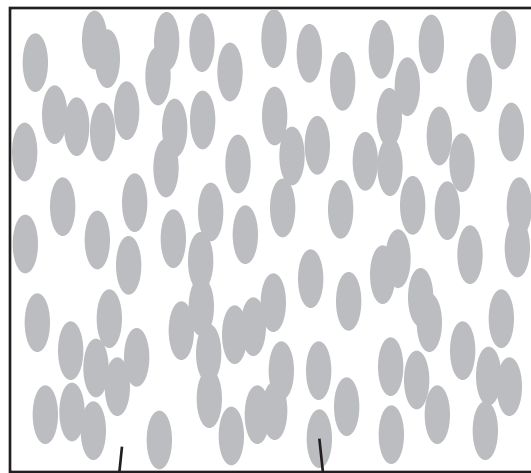
$$\epsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



ϵ_1

ϵ_2



ϵ^*

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1, p_2 = volume fractions of
the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2}, \text{ composite geometry} \right)$$

**What are the effective propagation characteristics
of an EM wave (radar, microwaves) in the medium?**

Analytic Continuation Method

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation
for homogenized parameter

separates geometry
from parameters

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$

← geometry
← material parameters

- μ /
- spectral measure of self adjoint operator $\Gamma\chi$
 - mass = p_1
 - higher moments depend on n -point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla.$$

χ = characteristic function of the brine phase

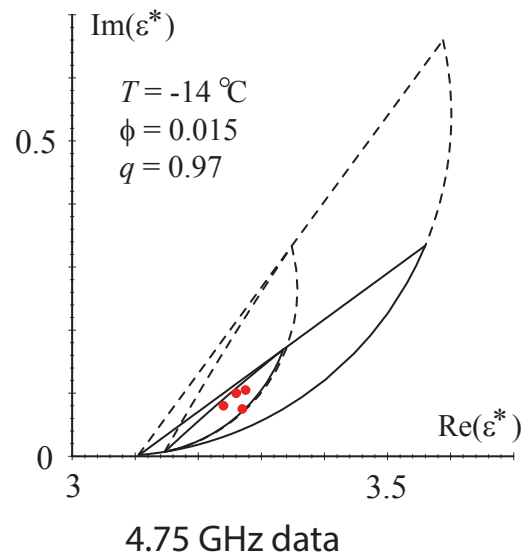
$$E = s (s + \Gamma\chi)^{-1} e_k$$

$\Gamma\chi$: microscale \rightarrow macroscale

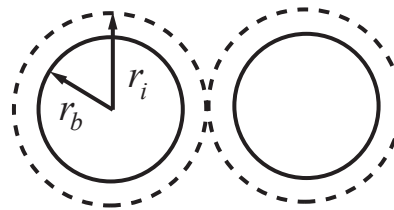
$\Gamma\chi$ *links scales*

forward and inverse bounds on the complex permittivity of sea ice

forward bounds



matrix particle (Bruno 91)



$$q = r_b / r_i$$

$$0 < q < 1$$

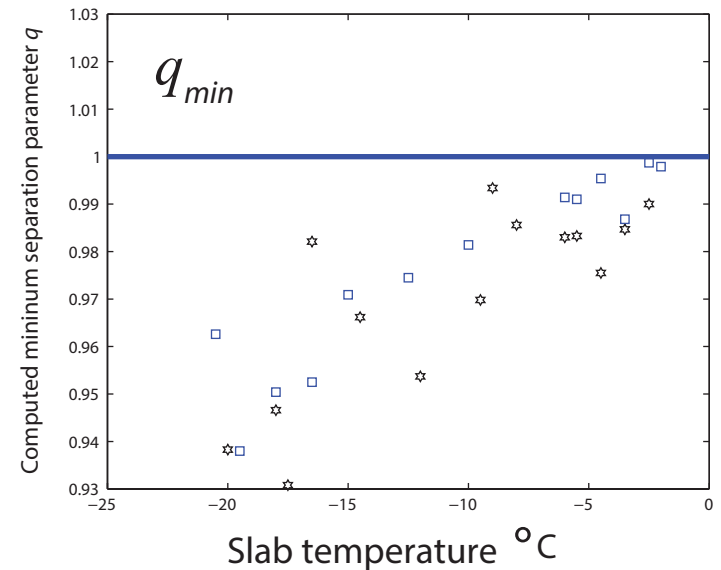
Golden JGR 95, IMA 97

inverse bounds and recovery of brine porosity

Cherkaev and Golden, *Waves in Random Media* 1998

Gully, Backstrom, Eicken, Golden *Physica B*, 2007

inverse bounds



inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ϵ^*

rigorous inverse bound on spectral gap

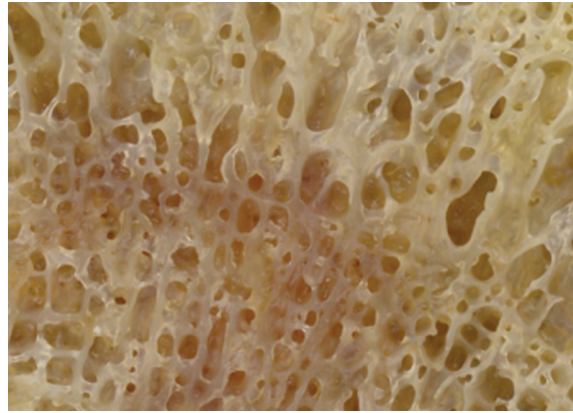
construct algebraic curves which bound admissible region in (p, q) -space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

SEA ICE

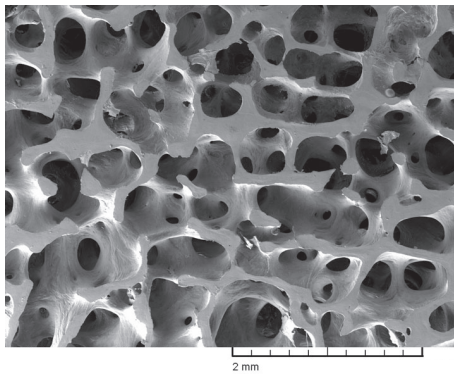


HUMAN BONE

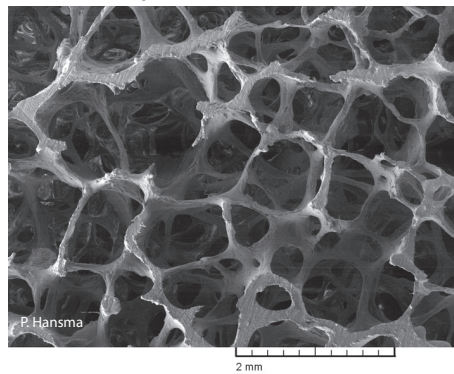


*spectral characterization
of porous microstructures
in human bone*

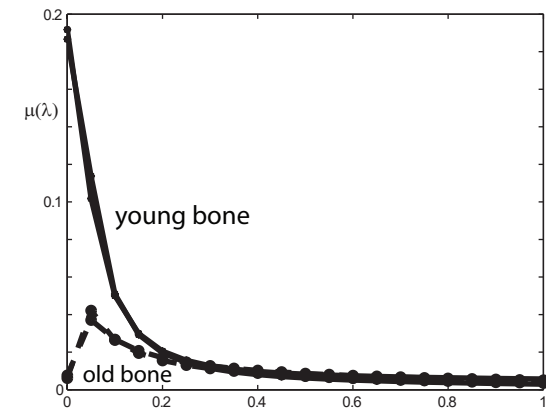
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures
from complex permittivity data



use regularized inversion scheme

*apply spectral measure analysis of brine connectivity and
spectral inversion to electromagnetic monitoring of osteoporosis*

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

direct calculation of spectral measure

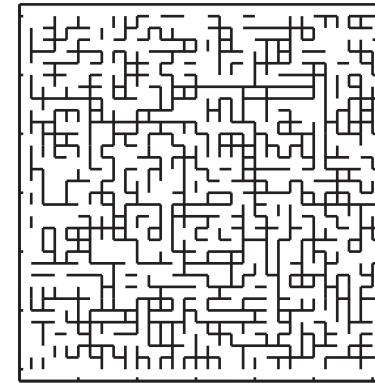
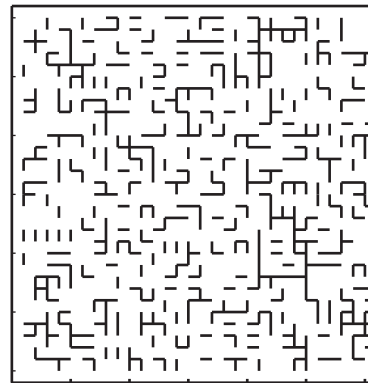
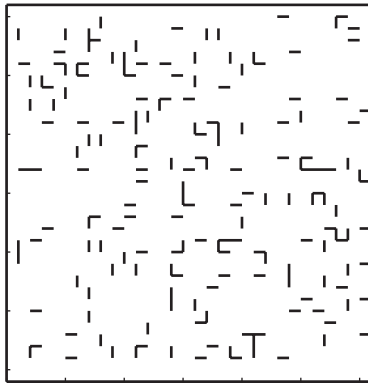
1. Discretization of composite microstructure gives lattice of 1's and 0's (random resistor network).
2. The fundamental operator $\chi\Gamma\chi$ becomes a random matrix depending only on the composite geometry.
3. Compute the eigenvalues λ_i and eigenvectors of $\chi\Gamma\chi$ with inner product weights α_i

$$\mu(\lambda) = \sum_i \alpha_i \delta(\lambda - \lambda_i)$$



Dirac point measure (Dirac delta)

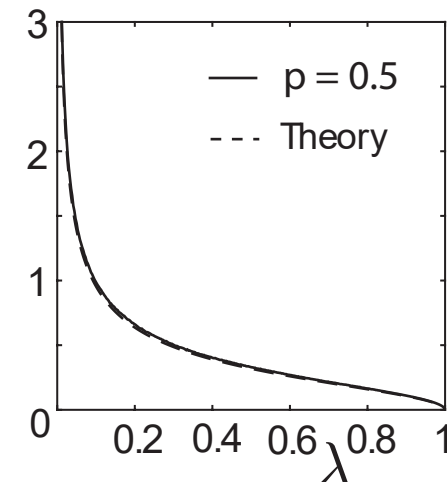
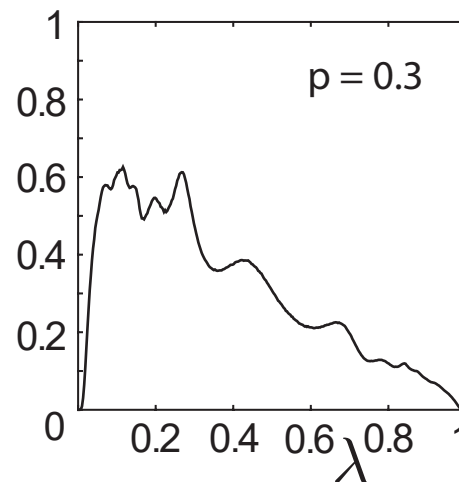
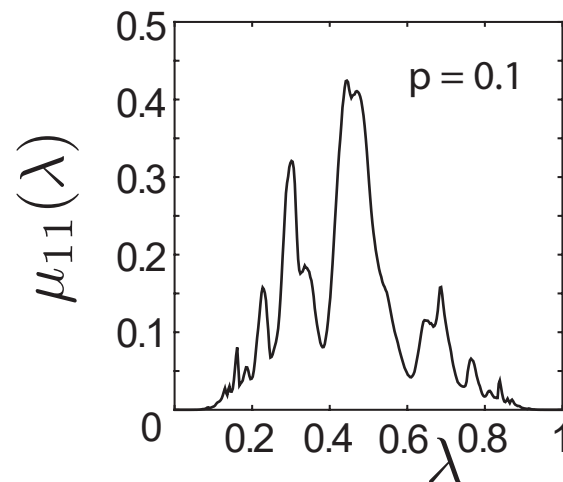
Spectral statistics for 2D random resistor network



Spectral Measures

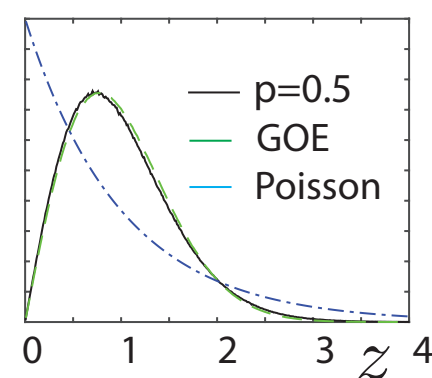
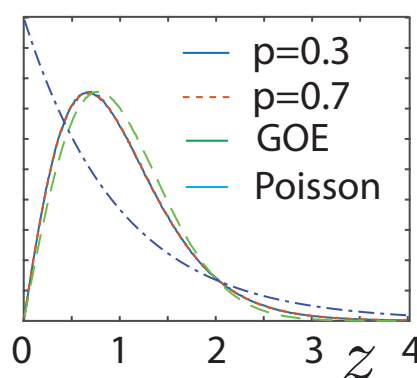
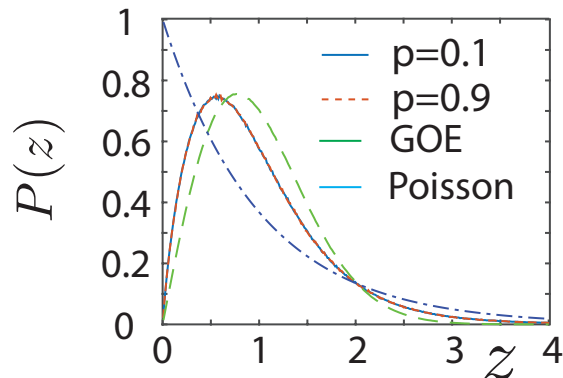
Murphy and Golden, *J. Math. Phys.*, 2012

Murphy et al. *Comm. Math. Sci.*, 2015



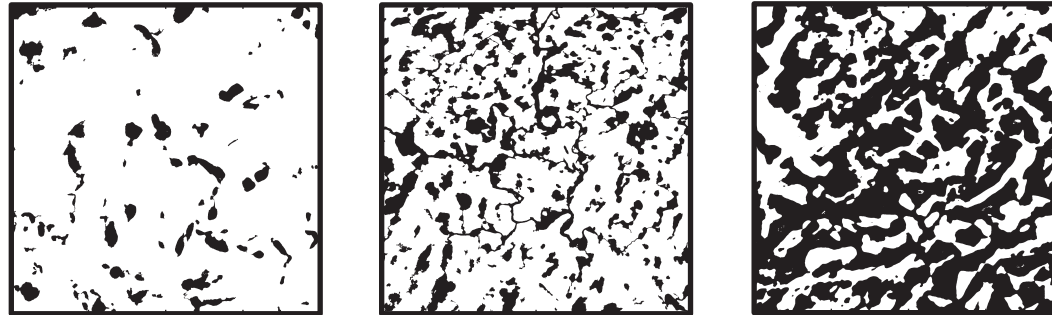
$p_c = 0.5$

Eigenvalue Spacing Distributions

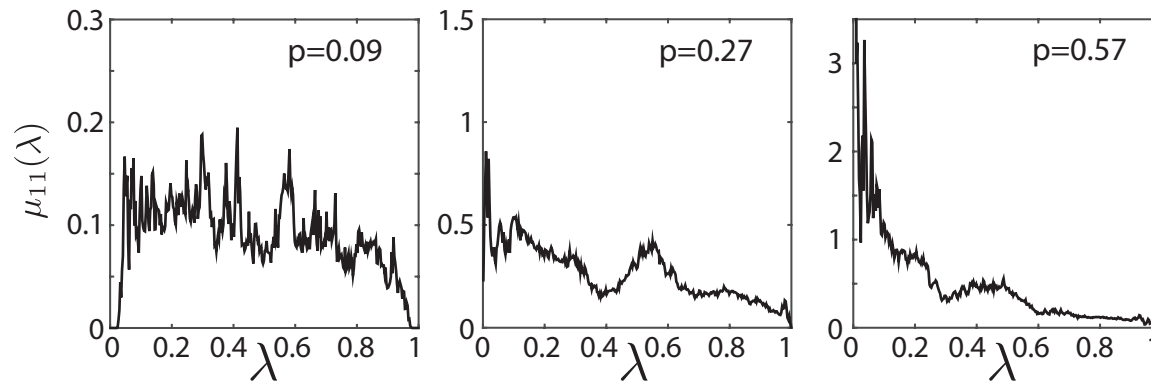


Murphy,
Cherkaev,
Golden,
PRL, 2017

Spectral computations for Arctic melt ponds

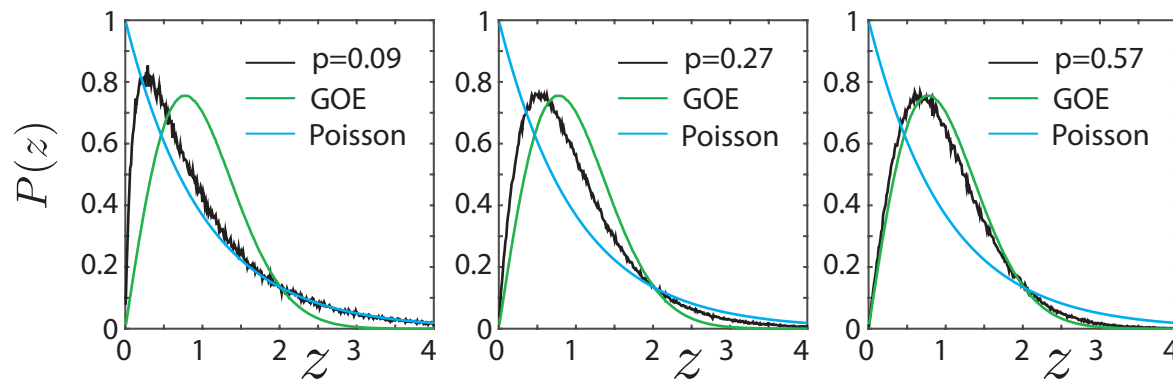


spectral
measures



Ben Murphy
Elena Cherkaev
Ken Golden
2017

eigenvalue
spacing
distributions



uncorrelated



level repulsion

TRANSITION

*eigenvalue statistics
for transport tend
toward the
UNIVERSAL
Wigner-Dyson
distribution
as the “conducting”
phase percolates*

Eigenvalue Statistics of Random Matrix Theory

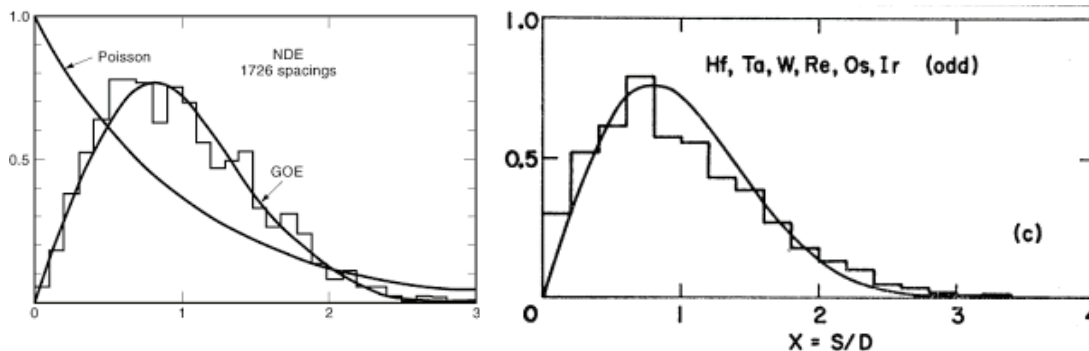
Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

$[N]_{ij} \sim N(0,1), \quad A = (N + N^T)/2 \quad \text{Gaussian orthogonal ensemble (GOE)}$

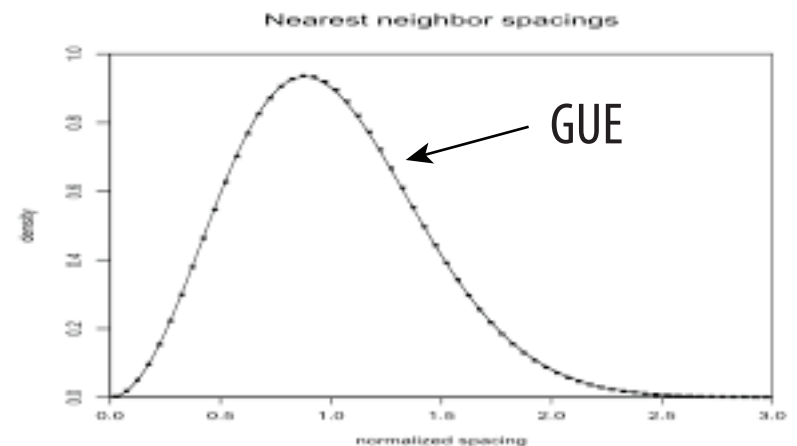
$[N]_{ij} \sim N(0,1) + iN(0,1), \quad A = (N + N^\dagger)/2 \quad \text{Gaussian unitary ensemble (GUE)}$

Short range and long range correlations of eigenvalues are measured by various eigenvalue statistics.

Spacing distributions of energy levels for heavy atomic nuclei

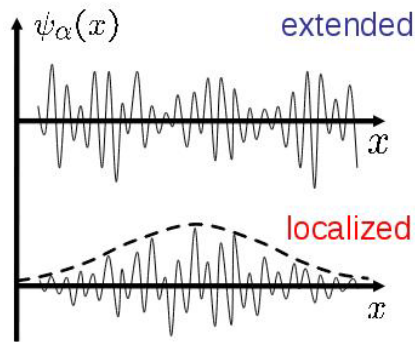


Spacing distributions of the first billion zeros of the Riemann zeta function



RMT used to characterize **disorder-driven transitions** in mesoscopic conductors, neural networks, random graph theory, etc.

Universal eigenvalue statistics arise in a broad range of “unrelated” problems!



metal / insulator transition **localization**

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

**Anderson transition in wave physics:
quantum, optics, acoustics, water waves, ...**

we find a surprising analog

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

**PERCOLATION
TRANSITION**

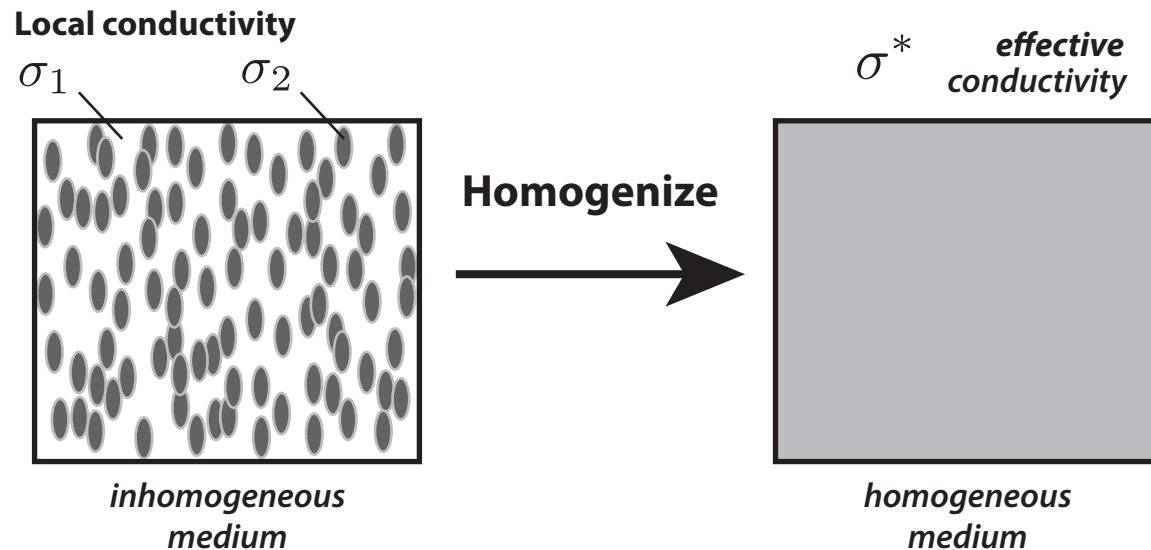


**transition to universal
eigenvalue statistics (GOE)
extended states, mobility edges**

-- but without wave interference or scattering effects ! --

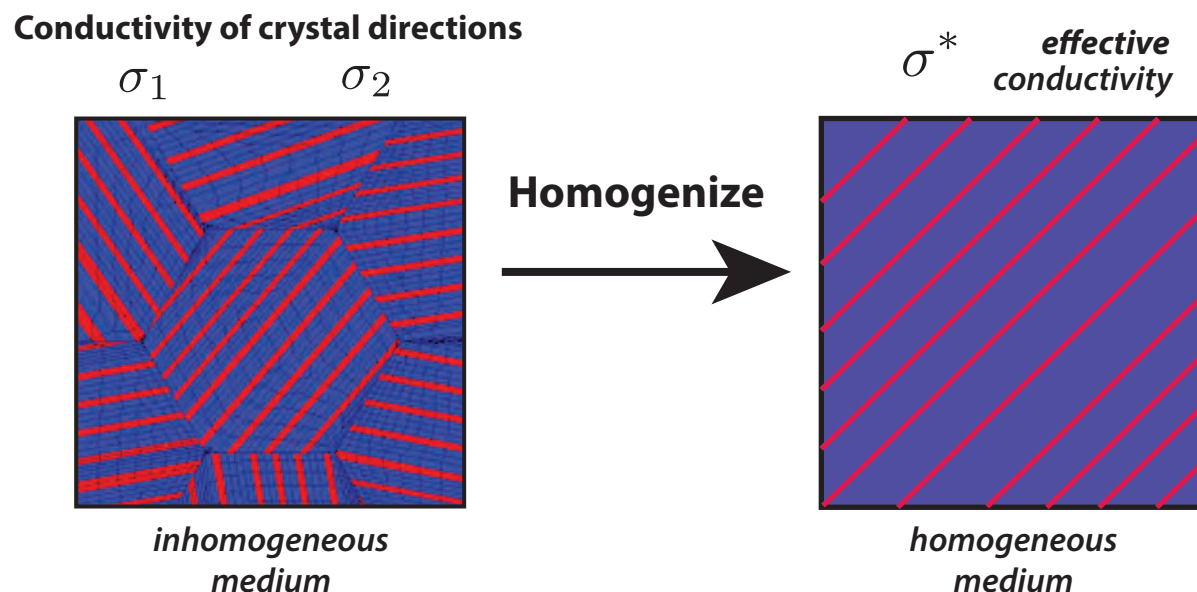
Homogenization for composite materials

**Two-component
composites**



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

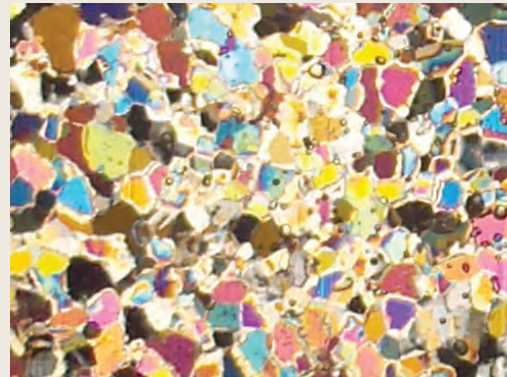
**Polycrystalline
media**



Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**
Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



PROCEEDINGS A

350 YEARS
OF SCIENTIFIC
PUBLISHING

An invited review
commemorating 350 years
of scientific publishing at the
Royal Society

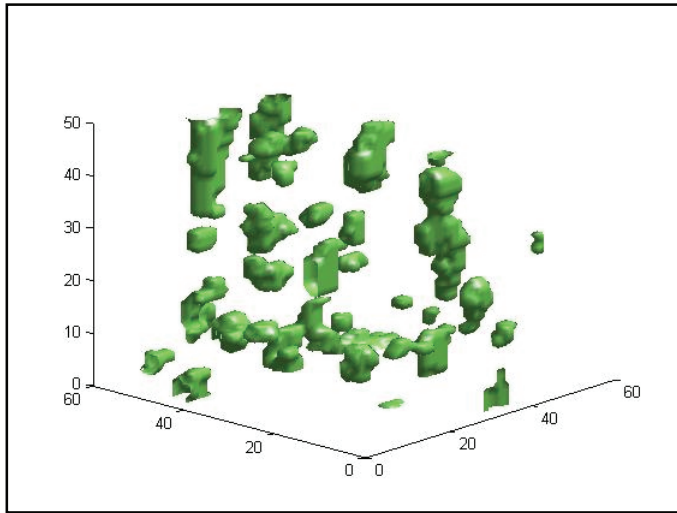
A method to distinguish
between different types
of sea ice using remote
sensing techniques

A computer model to
determine how a human
should walk so as to expend
the least energy

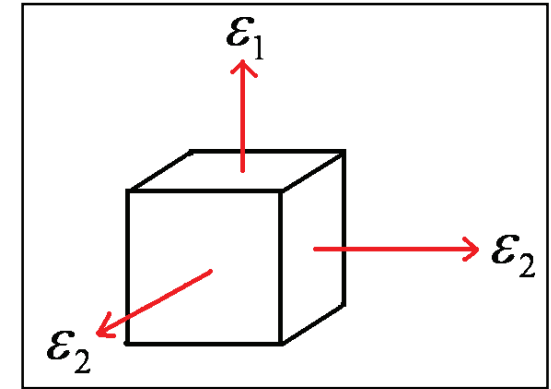


THE
ROYAL
SOCIETY
PUBLISHING

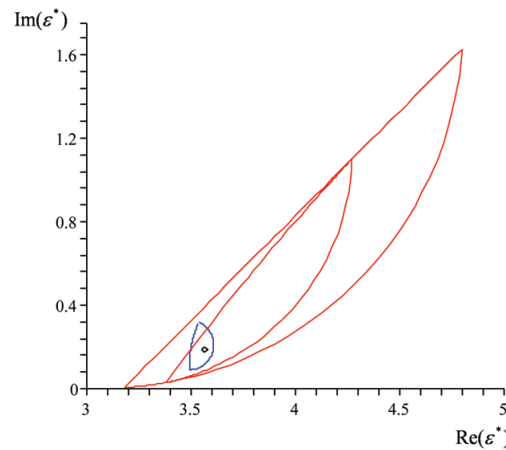
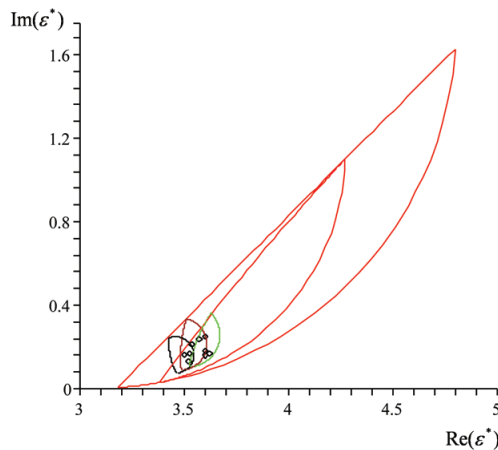
two scale homogenization for polycrystalline sea ice



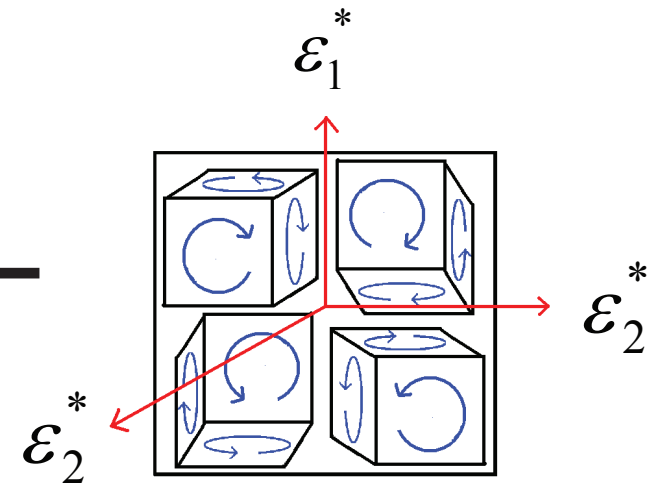
numerical homogenization
for single crystal



analytic continuation
for polycrystals



bounds



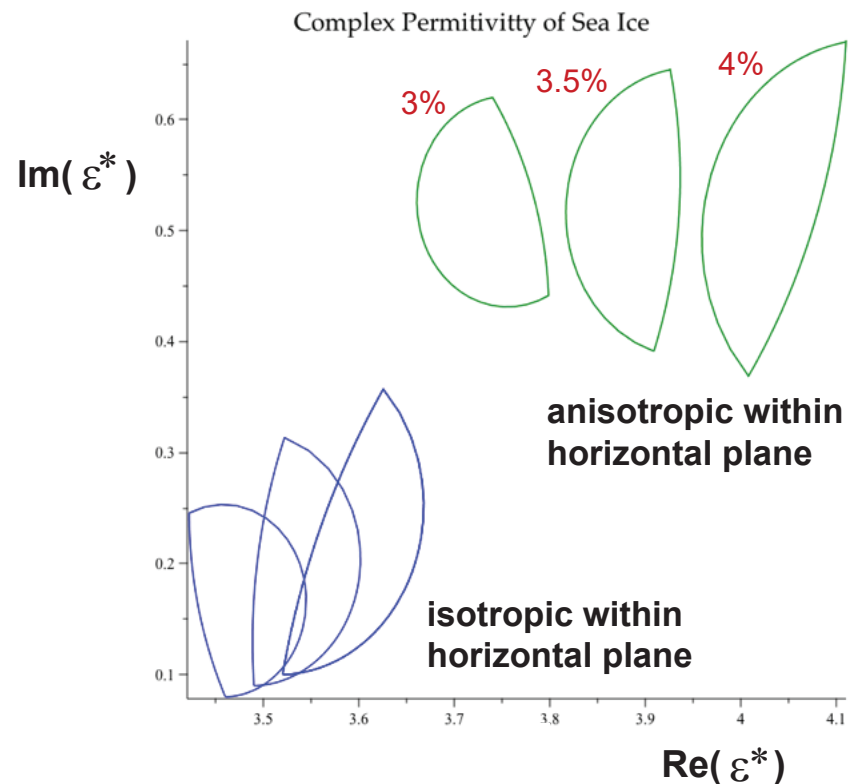
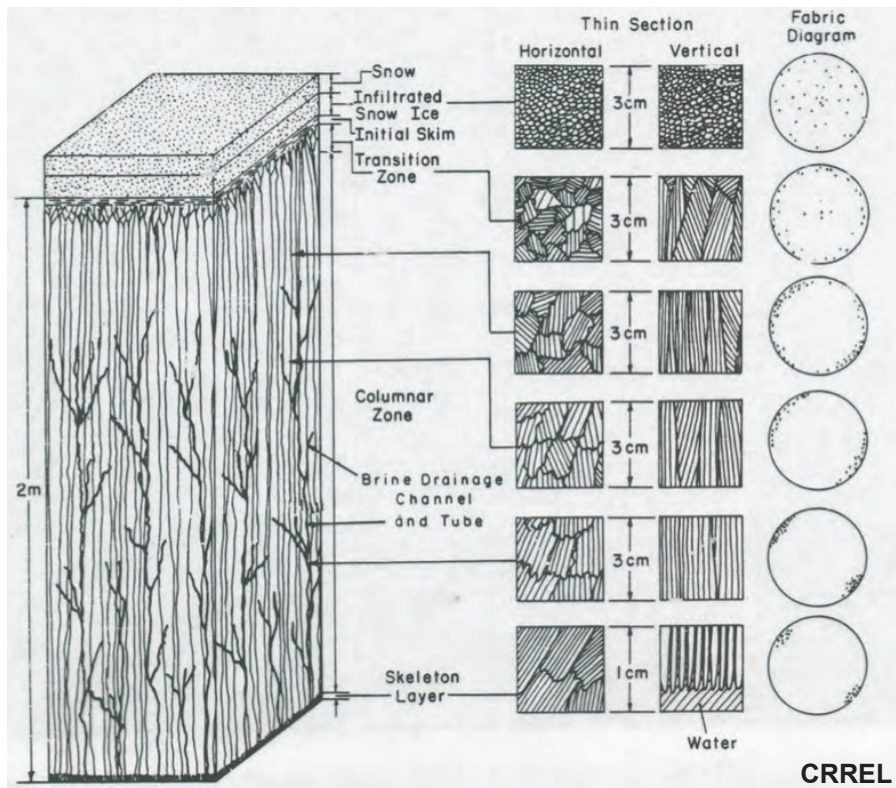
Rigorous bounds on the complex permittivity tensor of sea ice with polycrystalline anisotropy within the horizontal plane

McKenzie McLean, Elena Cherkaev, Ken Golden 2019

motivated by **Weeks and Gow, 1979: c-axis alignment in Arctic fast ice off Barrow**
Golden and Ackley, 1981: radar propagation model in aligned sea ice

input: orientation statistics

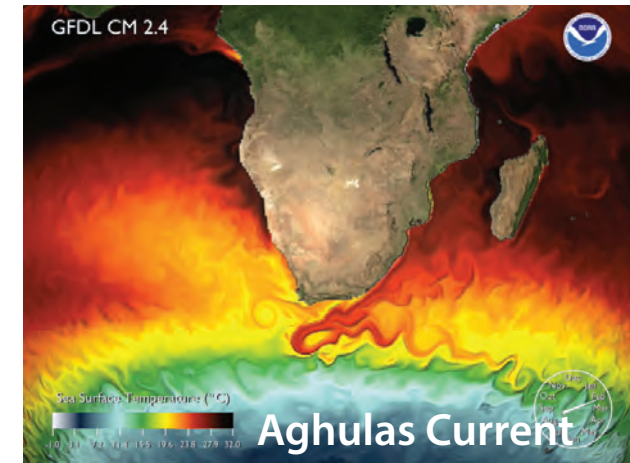
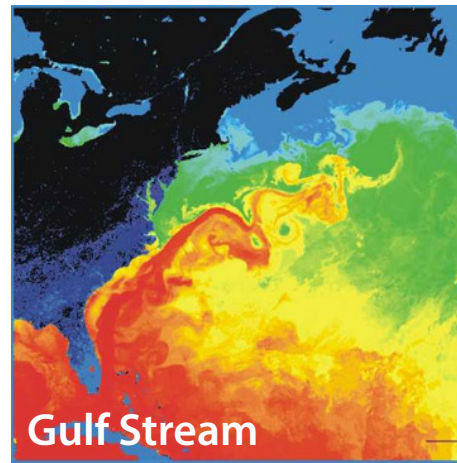
output: bounds



advection enhanced diffusion

effective diffusivity

sea ice floes diffusing in ocean currents
diffusion of pollutants in atmosphere
salt and heat transport in ocean
heat transport in sea ice with convection



advection diffusion equation with a velocity field \vec{u}

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize

$$\frac{\partial \bar{T}}{\partial t} = \kappa^* \Delta \bar{T}$$

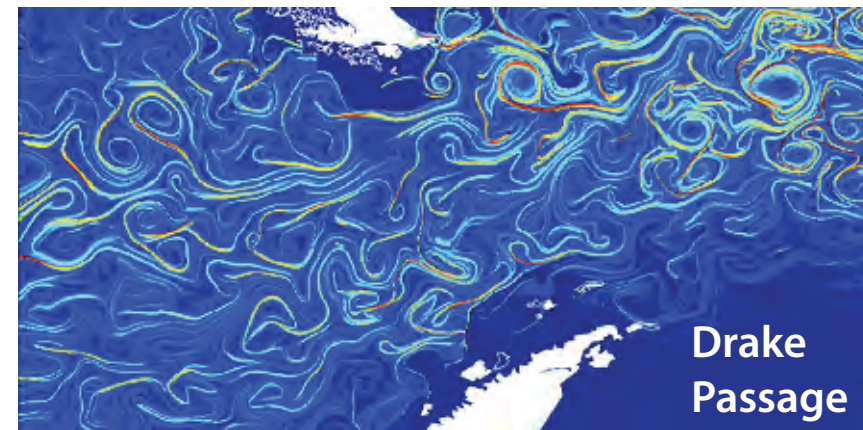
κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math Phys.* 2019



Stieltjes Integral Representation for Advection Diffusion

Murphy, Cherkaev, Zhu, Xin, Golden *J. Math. Phys.* 2019

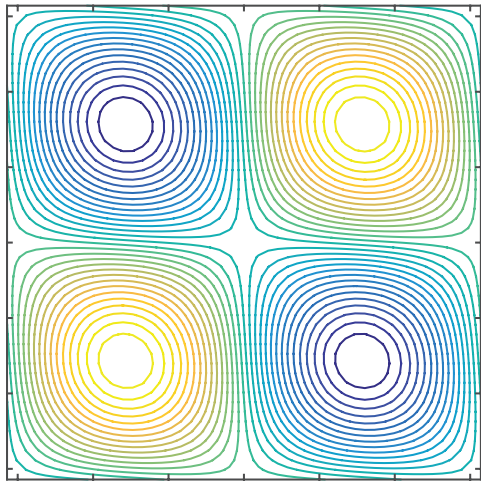
$$\kappa^* = \kappa \left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2} \right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2}$$

- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H\Gamma$
- H = stream matrix , κ = local diffusivity
- $\Gamma := -\nabla(-\Delta)^{-1}\nabla$, Δ is the Laplace operator
- $i\Gamma H\Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ -plane

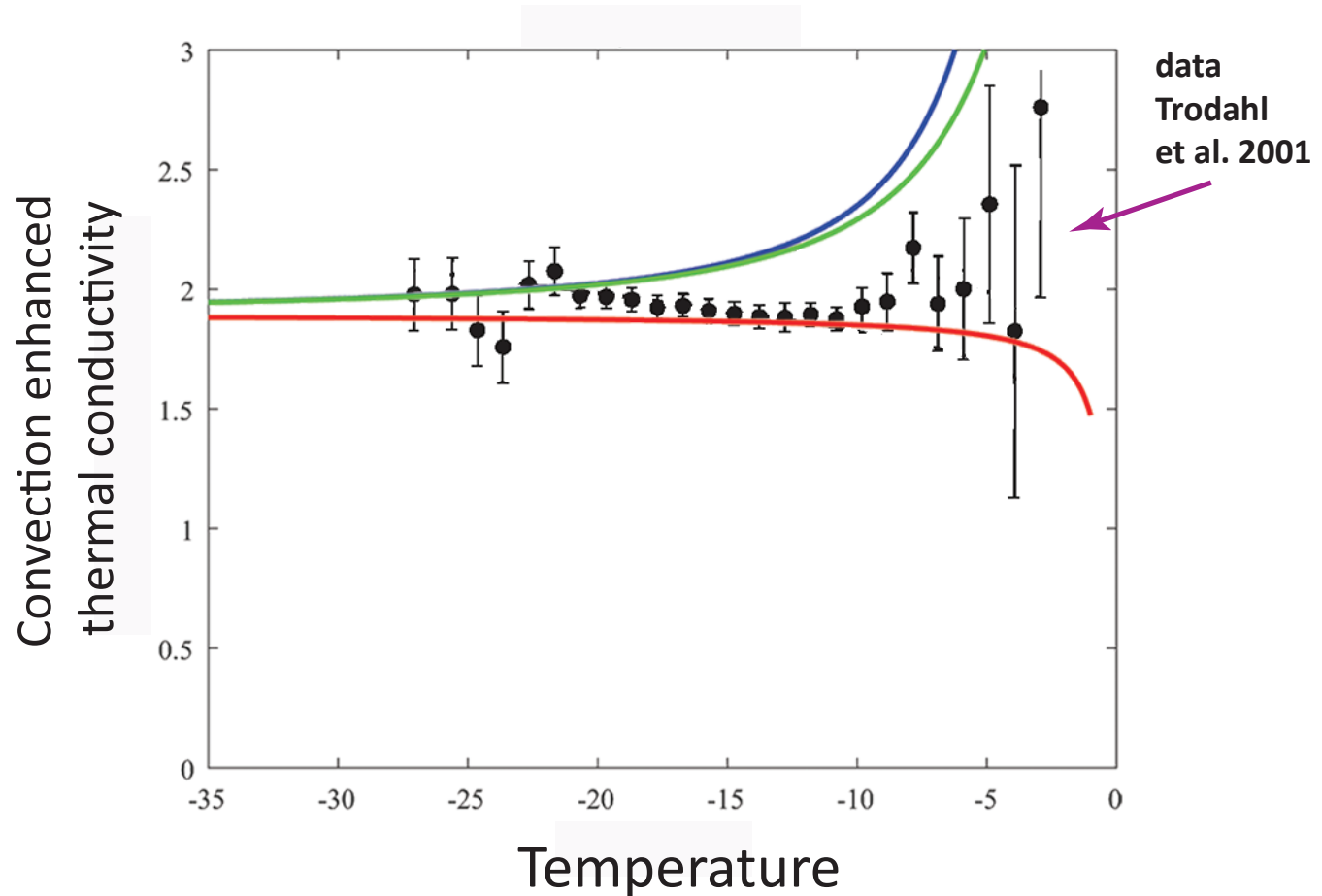
separation of material properties and flow field
spectral measure calculations

Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Dinh, Murphy, Zhu, Cherkaev, Golden 2019



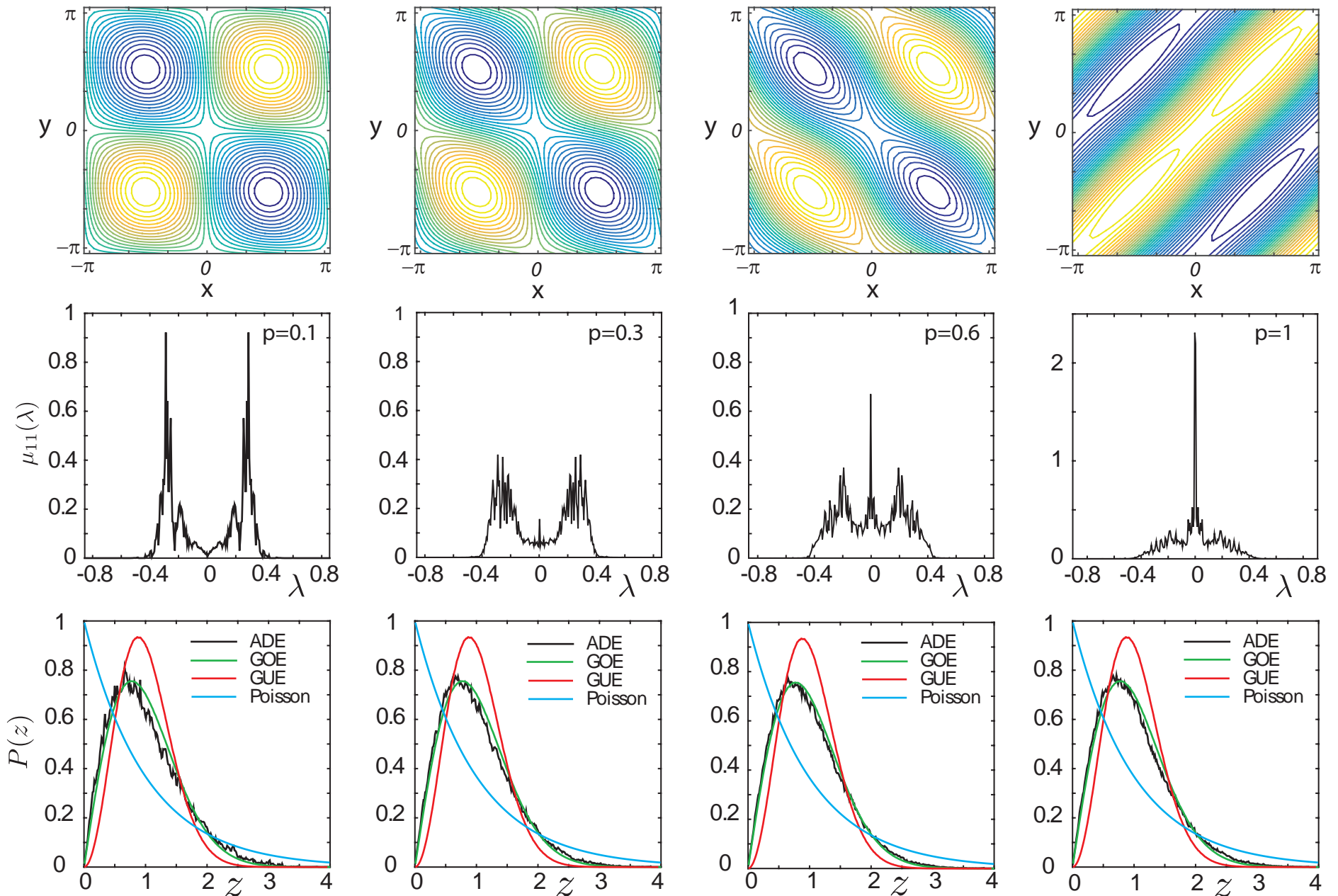
cat's eye flow model for
brine convection cells



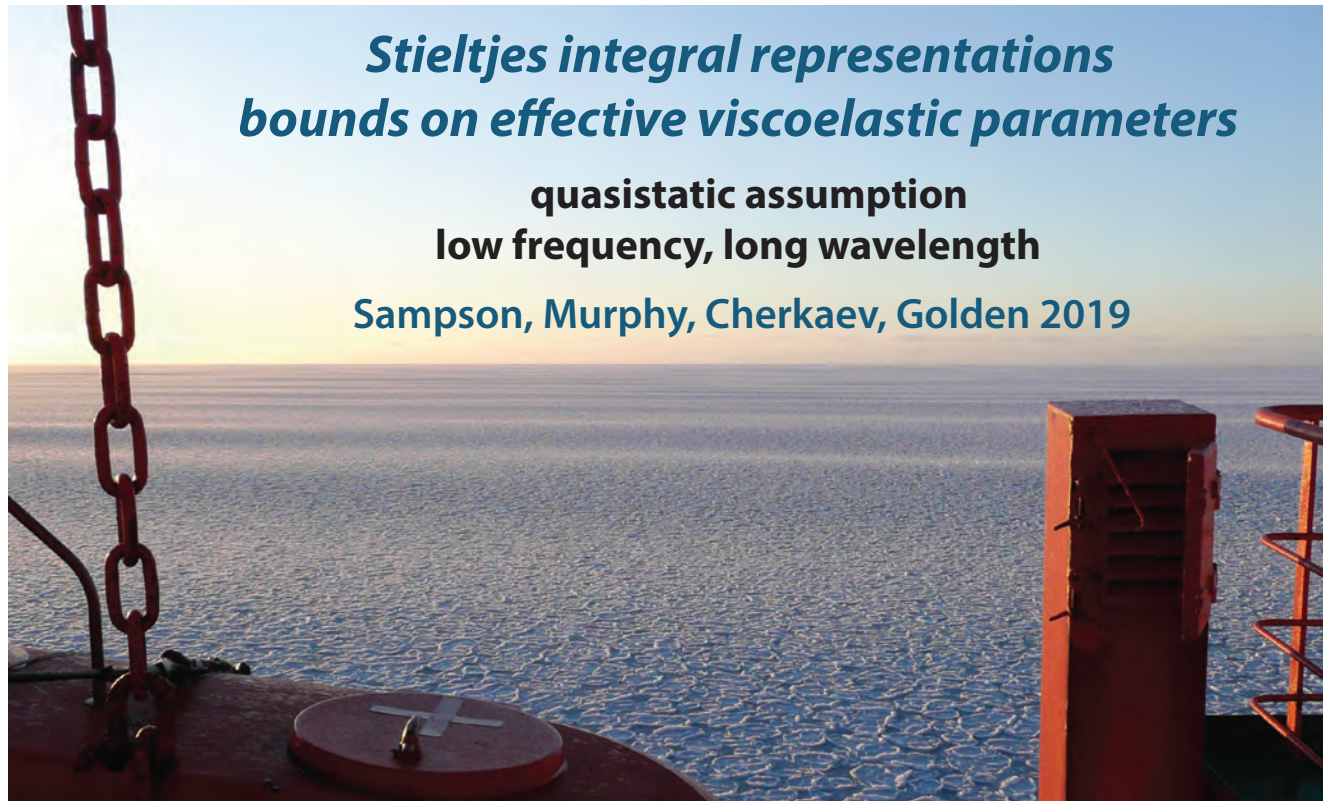
rigorous Padé bounds from Stieltjes integral +
analytical calculations of moments of measure

Spectral measures and eigenvalue spacings for cat's eye flow

$$H(x,y) = \sin(x) \sin(y) + A \cos(x) \cos(y), \quad A \sim U(-p,p)$$



wave propagation in the marginal ice zone



Two Layer Models

Viscous fluid layer (Keller 1998)

Effective Viscosity ν

Viscoelastic fluid layer (Wang-Shen 2010)

Effective Complex Viscosity $\nu_e = \nu + iG/\rho\omega$

Viscoelastic thin beam (Mosig et al. 2015)

Effective Complex Shear Modulus $G_v = G - i\omega\rho\nu$



bounds on the effective complex viscoelasticity

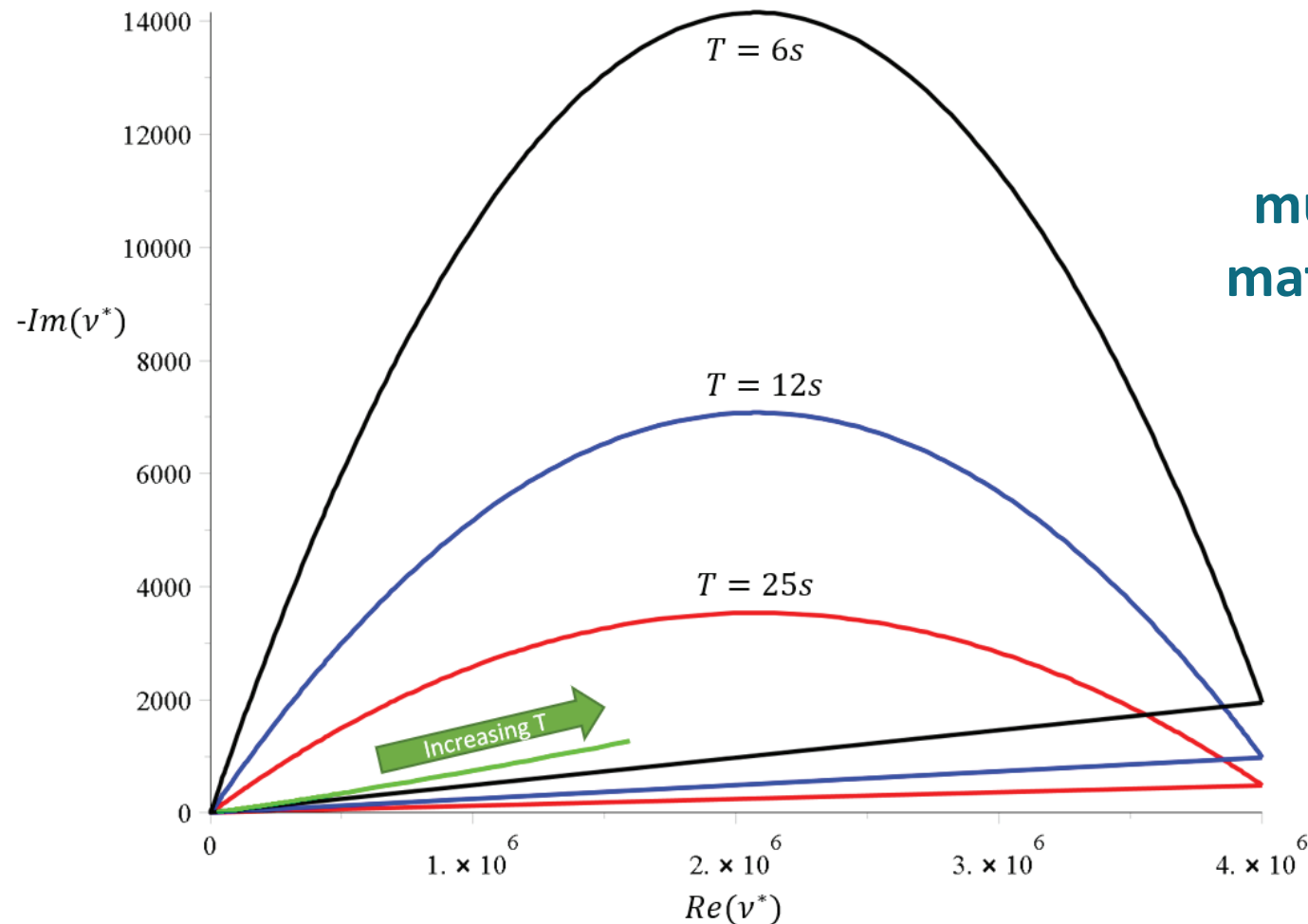
complex elementary bounds
(fixed area fraction of floes)

$$V_1 = 10^7 + i 4875$$

pancake ice

$$V_2 = 5 + i 0.0975$$

slush / frazil



+
much tighter
matrix particle
bounds

Sampson, Murphy, Cherkaev, Golden 2019

Arctic and Antarctic field experiments

*develop electromagnetic methods
of monitoring fluid transport and
microstructural transitions*

extensive measurements of fluid and
electrical transport properties of sea ice:

2007 Antarctic SIPEX

2010 Antarctic McMurdo Sound

2011 Arctic Barrow AK

2012 Arctic Barrow AK

2012 Antarctic SIPEX II

2013 Arctic Barrow AK

2014 Arctic Chukchi Sea



Notices

of the American Mathematical Society

May 2009

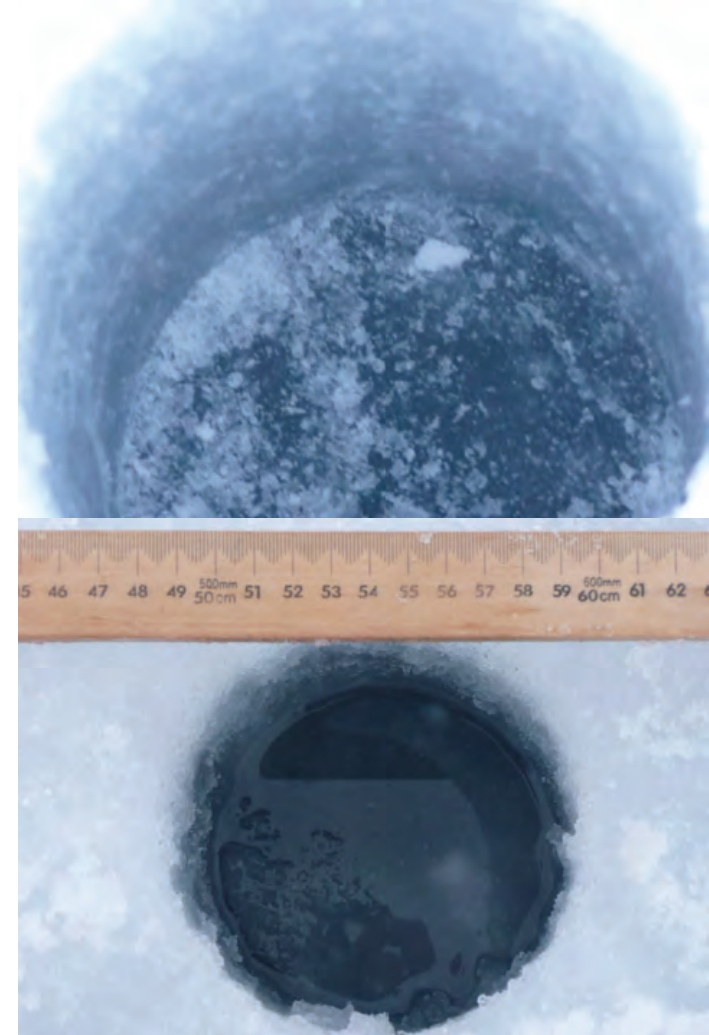
Volume 56, Number 5

Climate Change and
the Mathematics of
Transport in Sea Ice
page 562

Mathematics and the
Internet: A Source of
Enormous Confusion
and Great Potential
page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



***measuring
fluid permeability
of Antarctic sea ice***

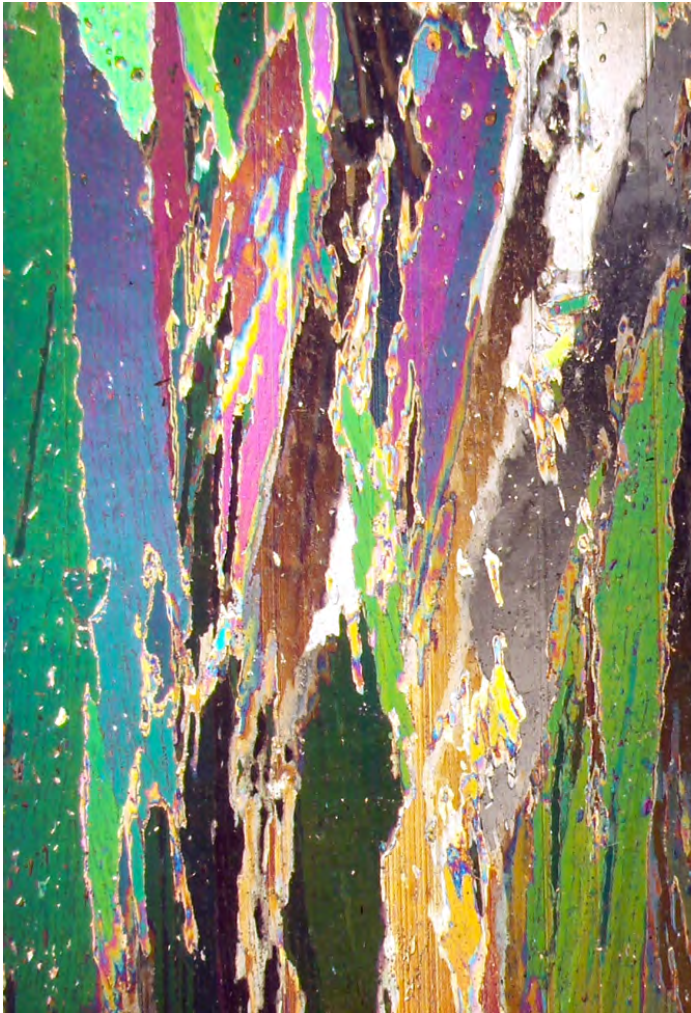
SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

granular

5%



10%

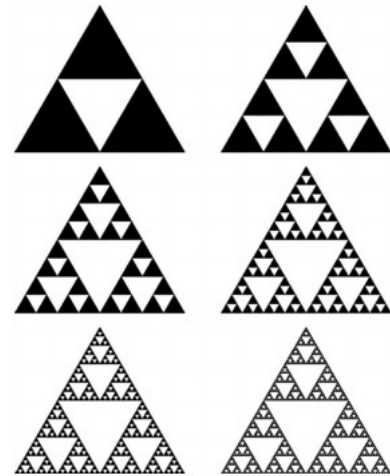


Golden, Sampson, Gully, Lubbers, Tison 2018

tracers flowing through inverted sea ice blocks



fractals and multiscale structure



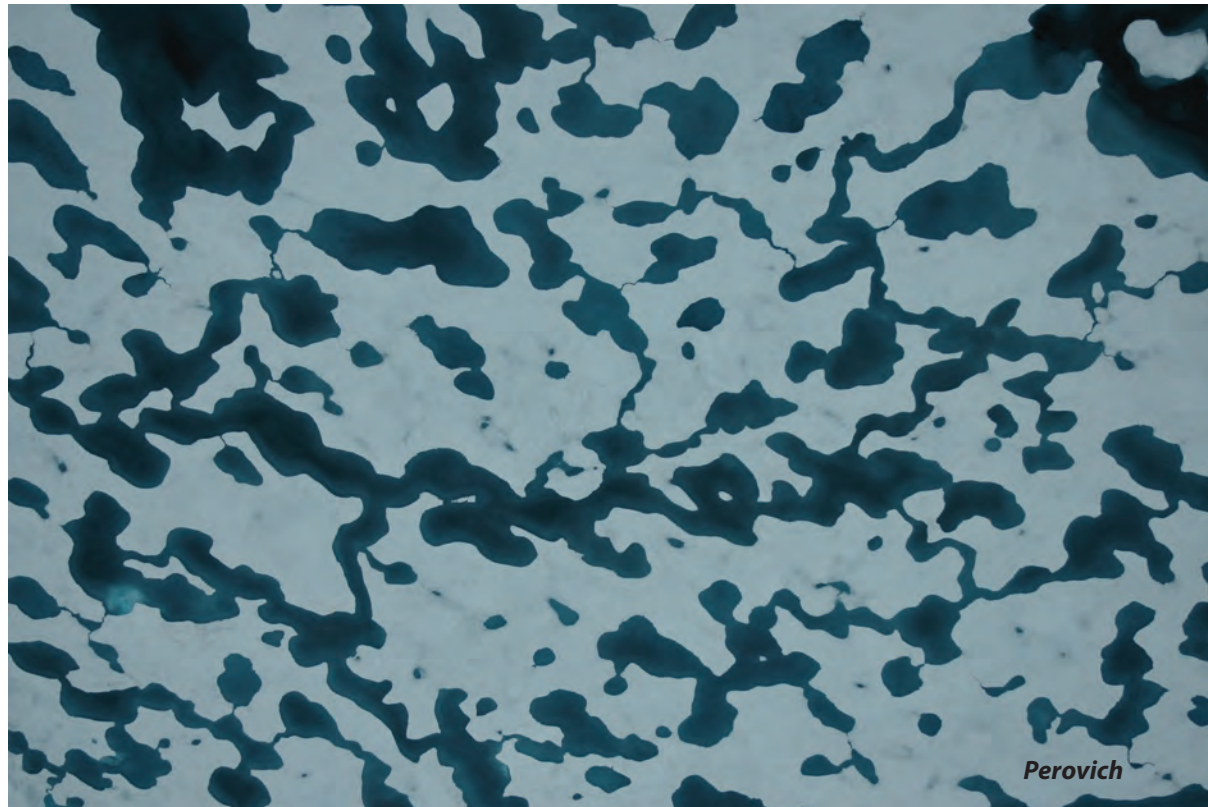
melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006
Flocco, Feltham 2007

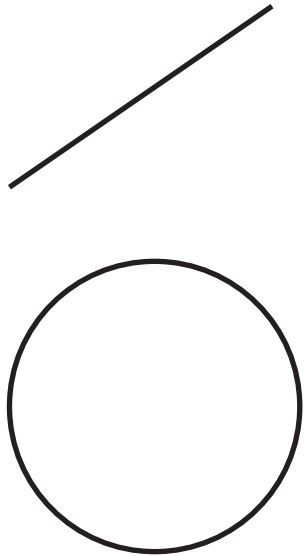
Skyllingstad, Paulson,
Perovich 2009
Flocco, Feltham,
Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

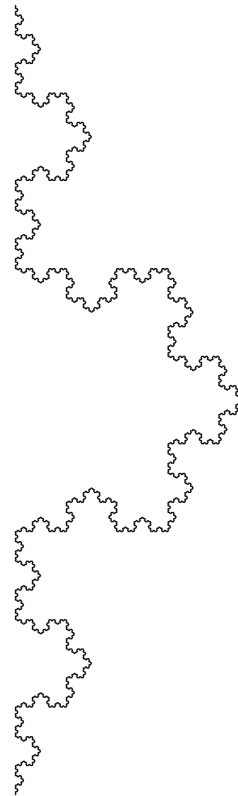
fractal curves in the plane

they wiggle so much that their dimension is >1



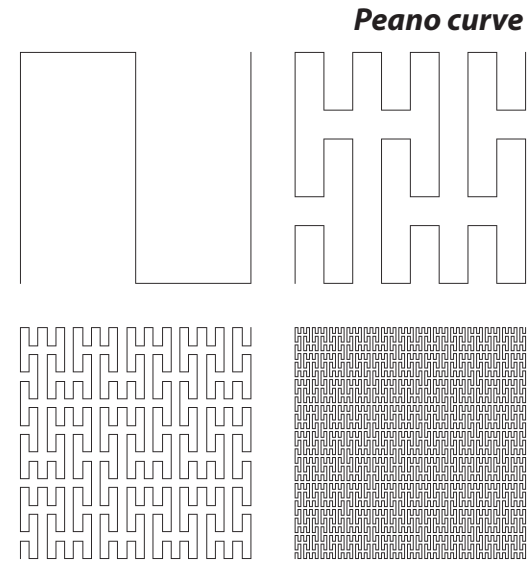
simple curves

$D = 1$



Koch snowflake

$D = 1.26$



Peano curve

Brownian motion

space filling curves

$D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

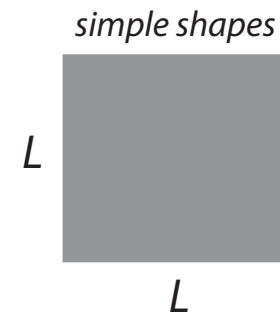
use **perimeter-area** data to find that
cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

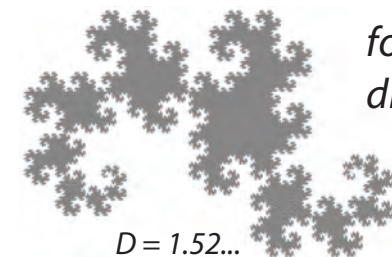


$$P \sim \sqrt{A}$$



$$A = L^2$$
$$P = 4L = 4\sqrt{A}$$

$$P \sim \sqrt{A}^D$$



for fractals with
dimension D

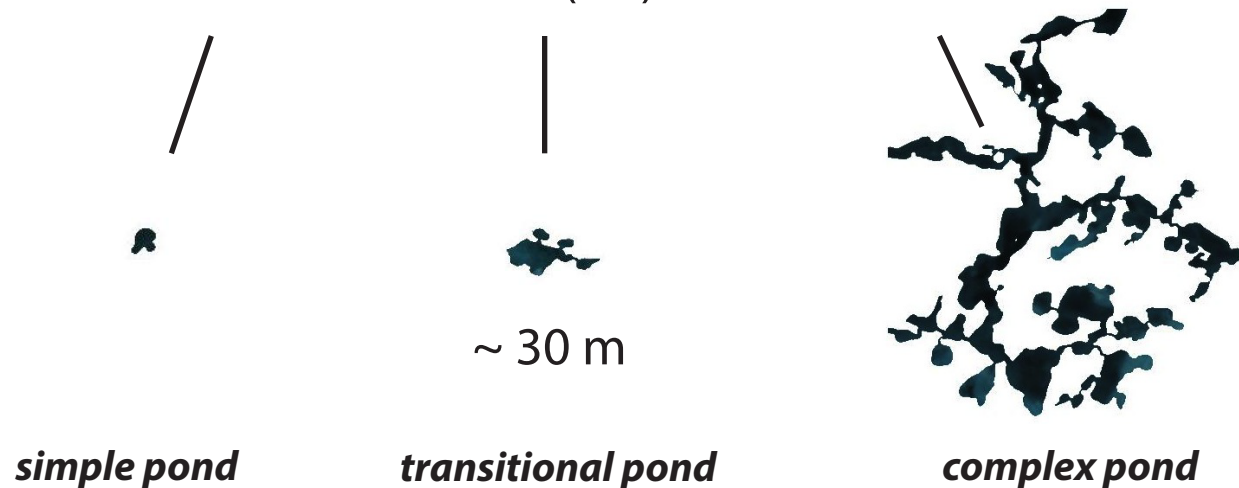
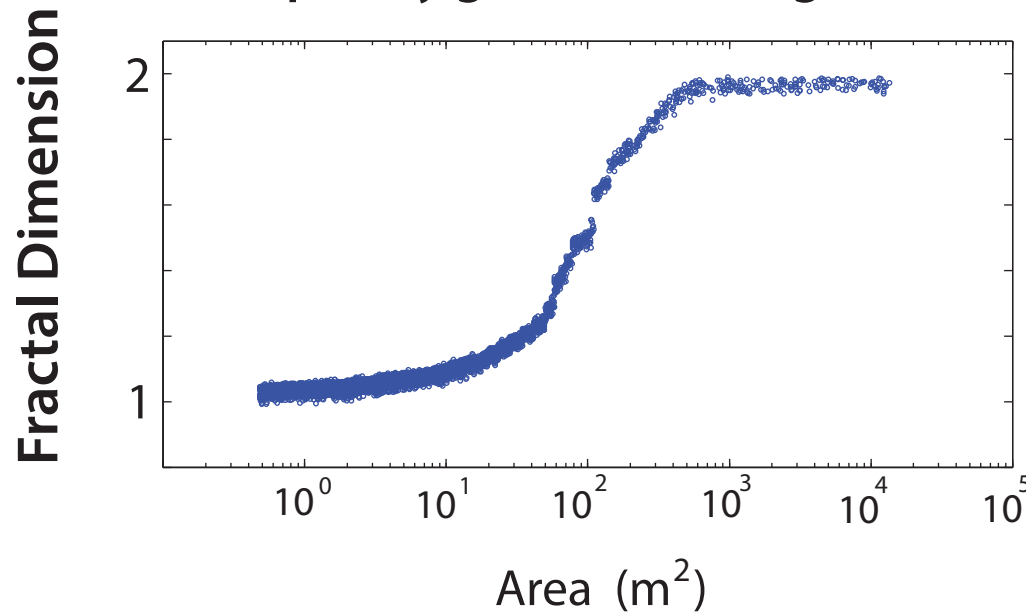
$D = 1.52...$

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

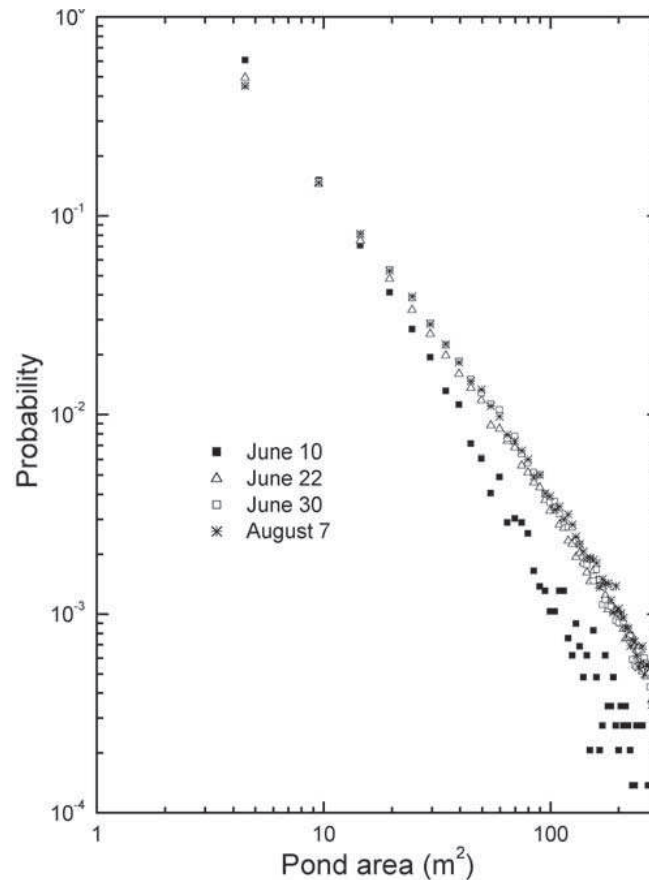
The Cryosphere, 2012

complexity grows with length scale



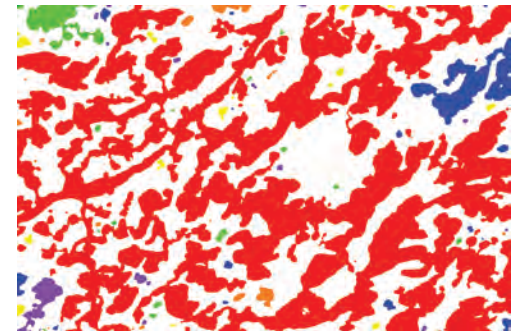
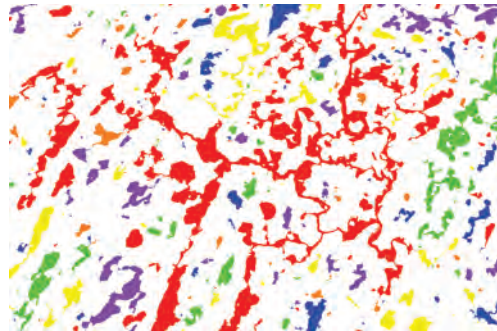
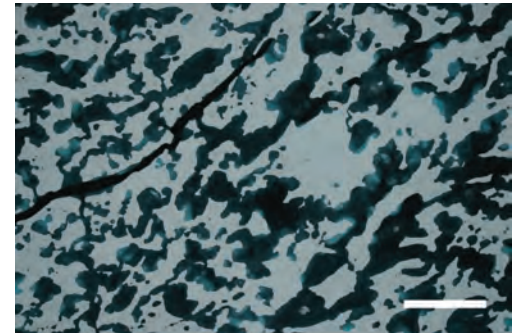
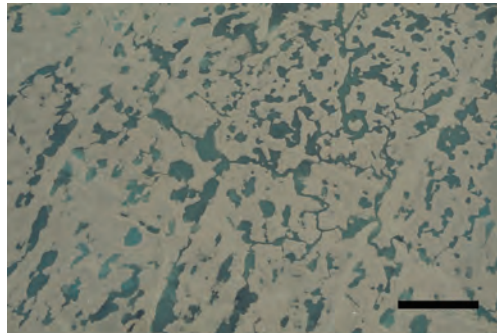
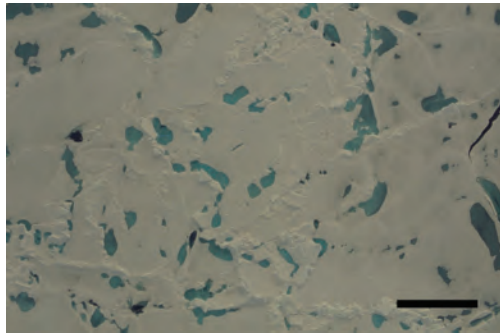
Power law scaling of pond size distribution

Image analysis reveals that the probability distribution of the pond area exhibits power law scaling with exponent about $-3/2$.



[Perovich, Tucker & Ligett, JGR, 2002]

***small simple ponds coalesce to form
large connected structures with complex boundaries***



melt pond percolation

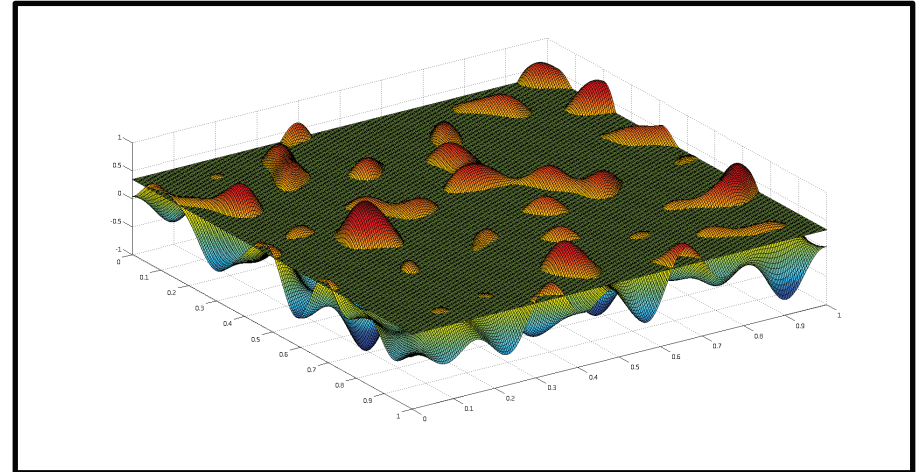
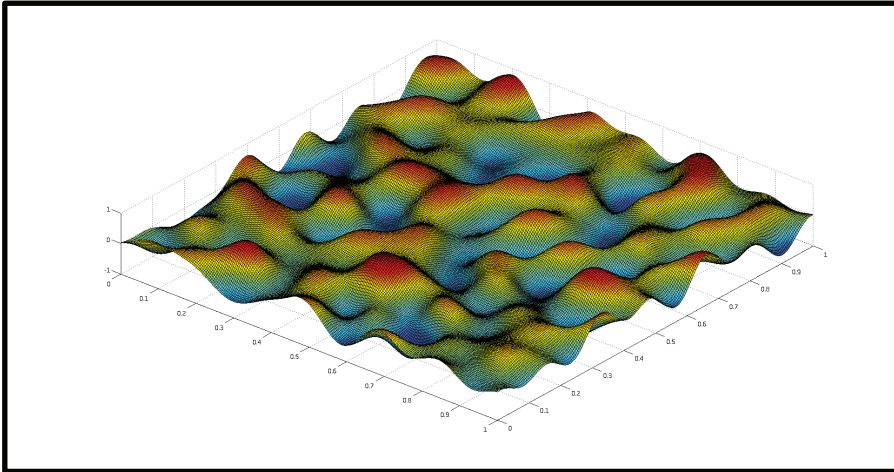
results on percolation threshold, correlation length, cluster behavior

Anthony Cheng (Hillcrest HS), Dylan Webb (Skyline HS), Court Strong, Ken Golden

Continuum percolation model for melt pond evolution

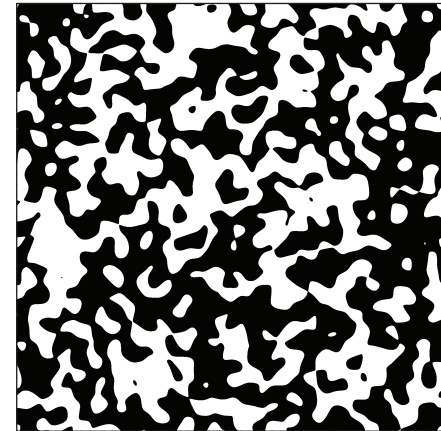
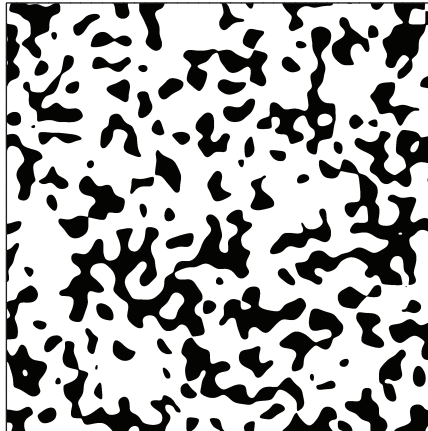
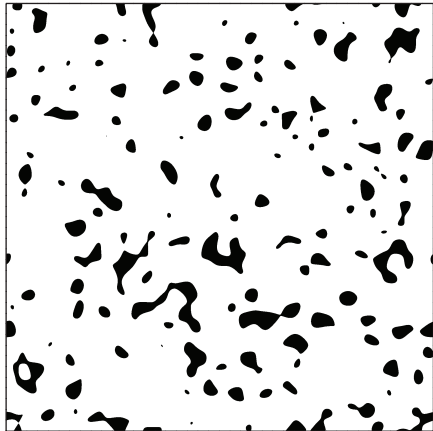
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds

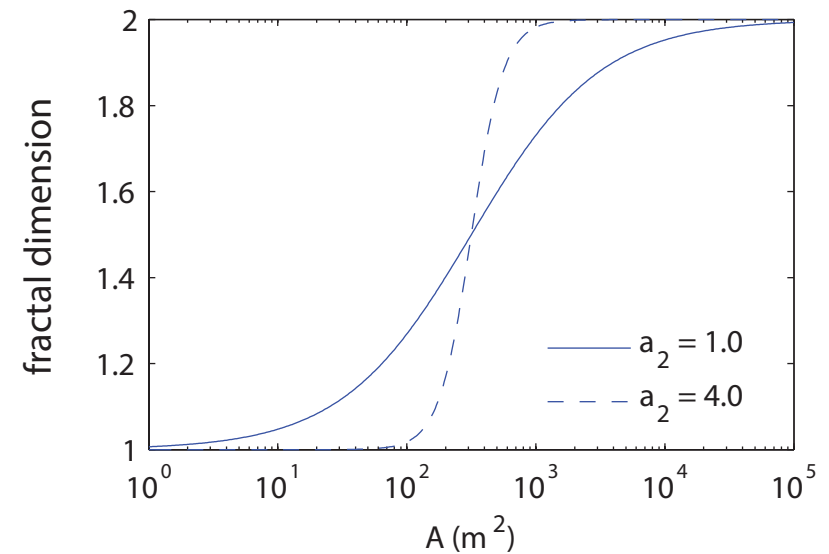
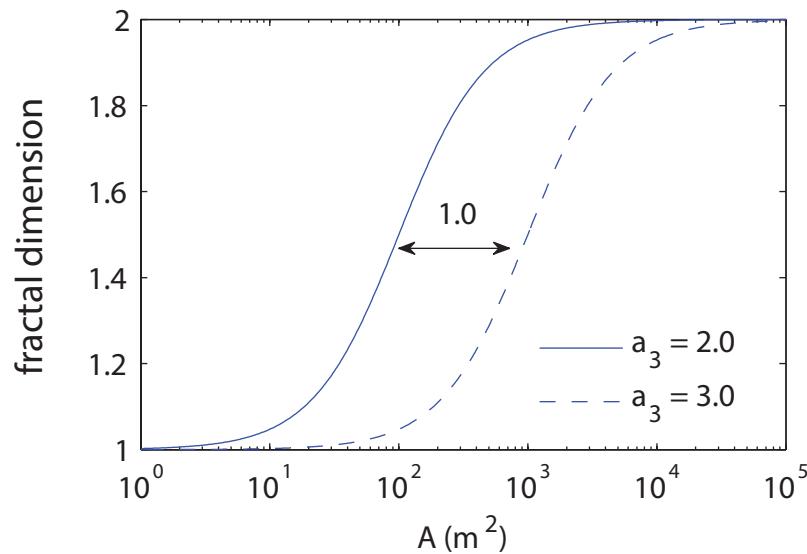


electronic transport in disordered media

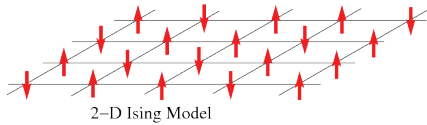
diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

fractal dimension curves depend on statistical parameters defining random surface



Ising model for ferromagnets → Ising model for melt ponds



Ma, Sudakov, Strong, Golden 2019

$$\mathcal{H}_\omega = -J \sum_{\langle i,j \rangle} s_i s_j - \sum_i H_i s_i$$

$$s_i = \begin{cases} \uparrow & +1 \\ \downarrow & -1 \end{cases}$$

water (spin up)

ice (spin down)

random magnetic field
represents snow topography

magnetization $M = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$

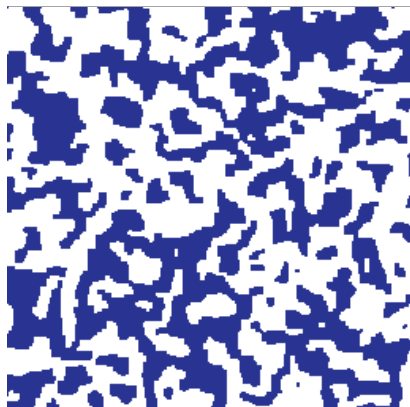
pond coverage $\frac{(M+1)}{2}$
~ albedo

only nearest neighbor
patches interact

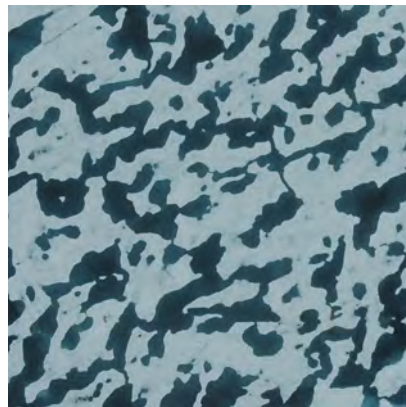
Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

Melt ponds are metastable islands of like spins.

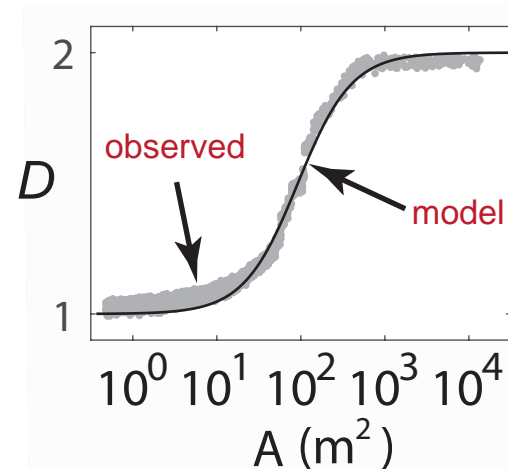
Order from Disorder



Ising
model



melt pond
photo (Perovich)



pond size distribution
exponent

observed -1.5

(Perovich, et al. 2002)

model -1.58

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data



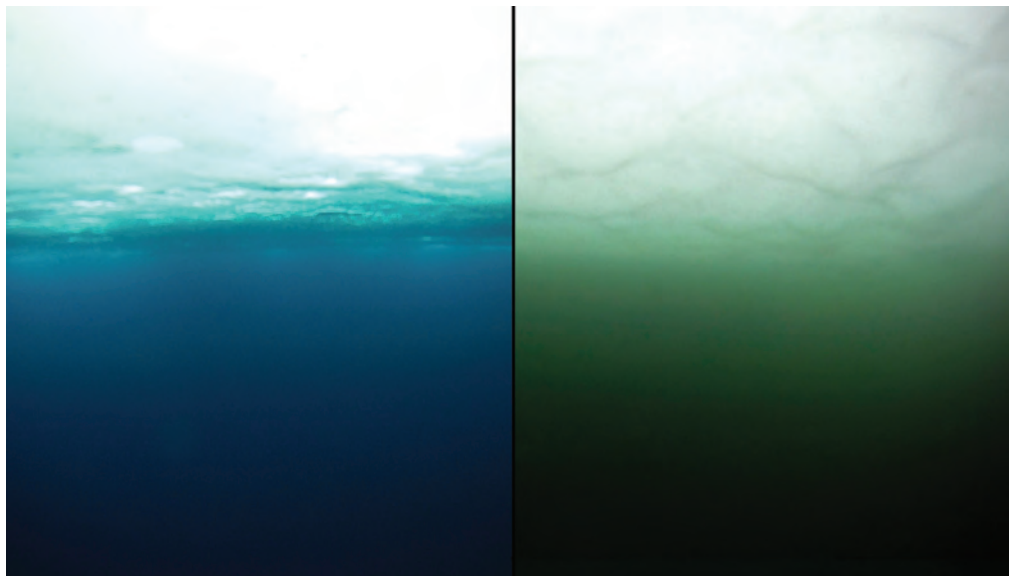
2011 massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light
through sea ice



no bloom

bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice
phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder,
Flocco, Feltham, *Science Advances*, 2017

The distribution of solar energy under
ponded sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, 2019

(2015 AMS MRC)

The Melt Pond Conundrum:

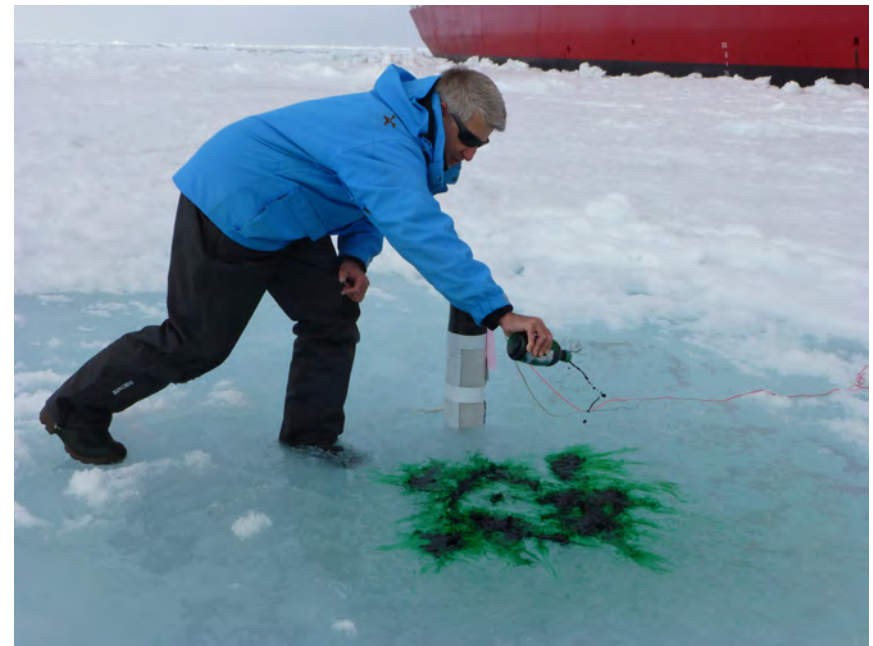
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

*2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy*



Conclusions

1. Summer Arctic sea ice is **melting rapidly**, and **melt ponds** and other processes must be accounted for in order to predict melting rates.
2. **Fluid flow** through sea ice mediates **melt pond evolution** and many processes important to climate change and polar ecosystems.
3. **Statistical physics and homogenization help link scales**, provide rigorous methods for finding effective behavior, and advance how sea ice is represented in climate models.
4. Critical behavior (in many forms) is inherent in the climate system.
5. Field experiments are essential to developing relevant mathematics.
6. Our research will help to **improve projections of climate change**, the fate of Earth's sea ice packs, and the ecosystems they support.

THANK YOU

Office of Naval Research

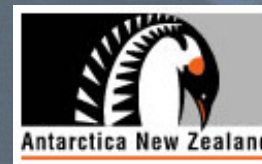
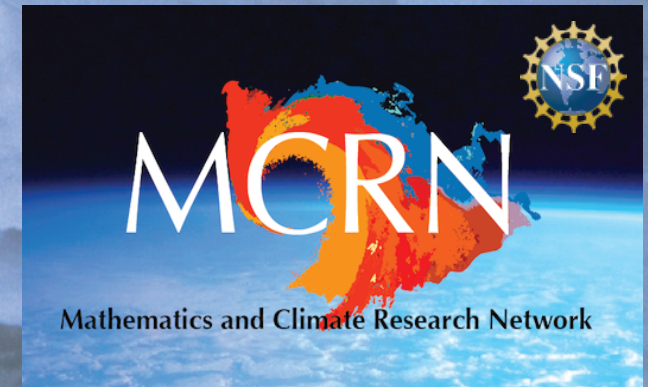
Arctic and Global Prediction Program

Applied and Computational Analysis Program

National Science Foundation

Division of Mathematical Sciences

Division of Polar Programs



Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship *Aurora Australis* without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the *Aurora Australis* in Antarctica.

ision's Hobart office.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The *Aurora Australis* is owned by P&O Australia and chartered by the Antarctic Div-

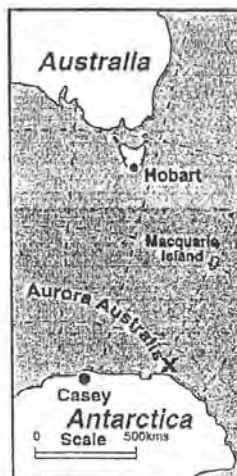
ision for about \$11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire.

Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Ant-

arctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

THE CANBERRA TIMES

Thursday 23 July 1998

Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected *Aurora Australis* would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The *Aurora*, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room"

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats"

Sydney Morning Herald
23 July, 1998

ICEBREAKER BURNS

A pioneering \$2-million Australian scientific voyage to the mid-winter Antarctic polynya is expected to be scrapped following an engine room fire on the *Aurora Australis* yesterday. The 54 people on board were forced on deck in the

