MODELING the MELT: what math tells us about disappearing polar sea ice

Kenneth M. Golden Department of Mathematics University of Utah

250th Anniversary of Dartmouth College A Year of Celebration of the Sciences

ANTARCTICA

southern cryosphere

Weddell Sea

East Antarctic Ice Sheet

West Antarctic Ice Sheet

Ross Sea

sea ice

THE ARCTIC northern cryosphere



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

white snow and ice reflect







dark water and land absorb

albedo
$$\alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers September 2012 -- 3.4 million square kilometers





recent losses in comparison to the United States



Perovich

Arctic sea ice decline: faster than predicted by climate models



challenge

represent sea ice more rigorously in climate models

account for key processes such as melt pond evolution



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes *linkage of scales*

sea ice is a multiscale composite



millimeters

centimeters

meters



meters

kilometers

What is this talk about? HOMOGENIZATION

Using theories of statistical physics and composite materials to LINK SCALES in the sea ice system ... rigorously compute effective behavior and improve climate models.

- **1.** Sea ice microphysics and fluid transport
- 2. Stieltjes integral representations for EM properties
- 3. Extension to polycrystals, advection diffusion, waves in MIZ
- 4. Fractal geometry of melt pond evolution

critical behavior cross - pollination

Solving problems in physics and biology of sea ice drives advances in theory of composite materials.

How do scales interact in the sea ice system?



basin scale grid scale albedo

Linking Scales

km scale melt ponds





km scale melt ponds

Linking

mm scale brine inclusions



Scales



meter scale snow topography

HOMOGENIZATION - Linking Scales in Composites



inhomogeneous medium homogeneous medium

find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

sea ice microphysics

fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







Antarctic surface flooding and snow-ice formation

evolution of salinity profiles
ocean-ice-air exchanges of heat, CO₂

sea ice ecosystem



sea ice algae support life in the polar oceans

fluid permeability k of a porous medium



porous

concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Darcy's Law for slow viscous flow in a porous medium



 $\mathbf{k} =$ fluid permeability tensor

PIPE BOUNDS on vertical fluid permeability k

Golden, Heaton, Eicken, Lytle, Mech. Materials 2006 Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007

> vertical pipes with appropriate radii maximize k





fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)

optimal coated cylinder geometry



$$k \leq \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

In(A) normally distributed, mean μ (increases with T) variance $\sigma^{_2}(\mbox{Gow and Perovich 96})$

get bounds through variational analyis of **trapping constant** γ for diffusion process in pore space with absorbing BC

Torquato and Pham, PRL 2004

 $\mathbf{k} \leq \gamma^{-1} \mathbf{I}$

for any ergodic porous medium (Torquato 2002, 2004)

Critical behavior of fluid transport in sea ice



critical brine volume fraction $\phi_c \approx 5\%$ \checkmark $T_c \approx -5^{\circ}C, S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998 Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007 Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open with probability p closed with probability 1-p*

percolation threshold $p_c = 1/2$ for d = 2

smallest *p* for which there is an infinite open cluster

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

 $\phi_c \approx 5\%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



 $R_{i,j}^{h} \xrightarrow{R_{i,j}^{v}} R_{i,j}^{h}$

Zhu, Jabini, Golden, Eicken, Morris *Ann. Glac*. 2006

- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution;
 Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k.
- Rigorous bound on *k* for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden Multiscale Modeling and Simulation, 2018

How does the biology affect the physics?

Remote sensing of sea ice



sea ice thickness ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

8*

effective complex permittivity (dielectric constant, conductivity)



brine volume fraction brine inclusion connectivity Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



the components

 $\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \right)$, composite geometry

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

Analytic Continuation Method

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)





 $\Gamma \chi$ links scales

Golden and Papanicolaou, Comm. Math. Phys. 1983

forward and inverse bounds on the complex permittivity of sea ice



inverse bounds





inverse bounds and recovery of brine porosity

Cherkaev and Golden, Waves in Random Media 1998 Gully, Backstrom, Eicken, Golden Physica B, 2007 inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ϵ^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden Proc. Roy. Soc. A, 2012

SEA ICE



young healthy trabecular bone



spectral characterization of porous microstructures in human bone

reconstruct spectral measures from complex permittivity data



m





use regularized inversion scheme

apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

Spectral statistics for 2D random resistor network



Spectral computations for Arctic melt ponds



Ben Murphy Elena Cherkaev Ken Golden 2017

eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase percolates

Eigenvalue Statistics of Random Matrix Theory

Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

 $[N]_{ij} \sim N(0,1),$ $A = (N+N^T)/2$ Gaussian orthogonal ensemble (GOE) $[N]_{ij} \sim N(0,1) + iN(0,1),$ $A = (N+N^T)/2$ Gaussian unitary ensemble (GUE)

Short range and long range correlations of eigenvalues are measured by various eigenvalue statistics.



RMT used to characterize disorder-driven transitions in mesoscopic conductors, neural networks, random graph theory, etc.

Universal eigenvalue statistics arise in a broad range of "unrelated" problems!



metal / insulator transition localization

Anderson 1958 Mott 1949 Shklovshii et al 1993 Evangelou 1992

Anderson transition in wave physics: quantum, optics, acoustics, water waves, ...

we find a surprising analog

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017





transition to universal eigenvalue statistics (GOE) extended states, mobility edges

-- but without wave interference or scattering effects ! --

Homogenization for composite materials



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium



Bounds on the complex permittivity of polycrystalline materials by analytic continuation

> Adam Gully, Joyce Lin, Elena Cherkaev, Ken Golden

 Stieltjes integral representation for effective complex permittivity

Milton (1981, 2002), Barabash and Stroud (1999), ...

- Forward and inverse bounds
- Applied to sea ice using two-scale homogenization
- Inverse bounds give method for distinguishing ice types using remote sensing techniques





Proc. Roy. Soc. A 8 Feb 2015

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PROCEEDINGS A



An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques A computer model to determine how a human should walk so as to expend the least energy



advection enhanced diffusion

effective diffusivity

sea ice floes diffusing in ocean currents diffusion of pollutants in atmosphere salt and heat transport in ocean heat transport in sea ice with convection

advection diffusion equation with a velocity field $\,ec u\,$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$
$$\vec{\nabla} \cdot \vec{u} = 0$$
$$homogenize$$
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, Ann. Math. Sci. Appl. 2017 Murphy, Cherkaev, Zhu, Xin, Golden, J. Math Phys. 2018







Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Dinh, Murphy, Zhu, Cherkaev, Golden 2018



rigorous Padé bounds from Stieltjes integral + analytical calculations of moments of measure

Spectral measures and eigenvalue spacings for cat's eye flow

 $H(x,y) = sin(x) sin(y) + A cos(x) cos(y), \quad A \sim U(-p,p)$



Murphy, Cherkaev, Xin, Golden, 2018

wave propagation in the marginal ice zone



Two Layer Models

Viscous fluid layer (Keller 1998) Effective Viscosity

Viscoelastic fluid layer (Wang-Shen 2010) Effective Complex Viscosity $\nu_e = \nu + iG/\rho\omega$

Viscoelastic thin beam (Mosig *et al.* 2015) Effective Complex Shear Modulus $G_v = G - i\omega\rho v_c$



bounds on the effective complex viscoelasticity



Sampson, Murphy, Cherkaev, Golden 2018

Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	Antarctic	SIPEX
2010	Antarctic	McMurdo Sound
2011	Arctic	Barrow AK
2012	Arctic	Barrow AK
2012	Antarctic	SIPEX II
2013	Arctic	Barrow AK
2014	Arctic	Chukchi Sea



Not the American Mathematical Society.

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and the Mathematics of Transport in Sea Ice

page 562

Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



measuring fluid permeability of Antarctic sea ice

SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

5%

granular



10%

Golden, Sampson, Gully, Lubbers, Tison 2018

tracers flowing through inverted sea ice blocks







fractals and multiscale structure



melt pond formation and albedo evolution:

- major drivers in polar climate
- key challenge for global climate models

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006 Flocco, Feltham 2007 Skyllingstad, Paulson, Perovich 2009 Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

fractal curves in the plane

they wiggle so much that their dimension is >1



clouds exhibit fractal behavior from 1 to 1000 km



use *perimeter-area* data to find that cloud and rain boundaries are fractals

 $D \approx 1.35$

S. Lovejoy, Science, 1982

 $P \sim \sqrt{A}$

simple shapes

 $A = L^2$ $P = 4L = 4\sqrt{A}$

 $P \sim \sqrt{A}^{D}$



L

for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012



Power law scaling of pond size distribution

Image analysis reveals that the probability distribution of the pond area exhibits power law scaling with exponent about -3/2.



[Perovich, Tucker & Ligett, JGR, 2002]

small simple ponds coalesce to form large connected structures with complex boundaries



melt pond percolation

results on percolation threshold, correlation length, cluster behavior

Anthony Cheng (Hillcrest HS), Dylan Webb (Skyline HS), Court Strong, Ken Golden

Continuum percolation model for melt pond evolution level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds







electronic transport in disordered media

diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

fractal dimension curves depend on statistical parameters defining random surface



Ising Model for a Ferromagnet



$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

$$\mathcal{H}_{\omega} = -H\sum_{i} s_{i} - J\sum_{\langle i,j \rangle} s_{i}s_{j}$$

nearest neighbor Ising Hamiltonian

for any configuration $\omega \in \Omega = \{-1, 1\}^N$ of the spins

ferromagnetic interaction $J \ge 0$

magnetization



homogenized parameter like effective conductivity



field

M



critical temperature

Ising model for ferromagnets —> Ising model for melt ponds

$$\mathcal{H}_{\omega} = -J \sum_{\langle i,j \rangle}^{N} s_i s_j - \sum_{i}^{N} H_i s_i \qquad s_i = \begin{cases} \uparrow & +1 & \text{water (spin up)} \\ \downarrow & -1 & \text{ice (spin down)} \end{cases}$$

magnetization $M = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_{j} s_{j} \right\rangle$ pond coverage $\frac{(M+1)}{2}$



Melt ponds are metastable islands of like spins in our random field Ising model.

input spin configuration independent binary random variables = +1 with probability F_{in}

Ma, Sudakov, Strong, Golden 2018

Glauber Dynamics (Metropolis at T=0):

if spin flip lowers energy, accept if spin flip raises energy, reject

majority wins, water fills troughs

Metropolis algorithm: if lower accept if raises accept with prob = Gibbs factor

Random initial configuration; as energy is minimized system "flows" toward metastable equilbrium

Order from Disorder

Ising model



melt pond photo (Perovich)

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE)

Ising model results

Minimize Ising Hamiltonian energy

Random magnetic field represents snow topography; interaction term represents horizontal heat transfer.





The lattice constant (1 m) is the length scale above which important spatially correlated fluctuations occur in the power spectrum of snow topography.



2011 massive under-ice algal bloom

Arrigo et al., Science 2012

melt ponds act as *WINDOWS*

allowing light through sea ice



bloom

no bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances*, 2017

The distribution of solar energy under ponded sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, 2018

(2015 AMS MRC)

The Melt Pond Conundrum:

How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy





Conclusions

- 1. Summer Arctic sea ice is melting rapidly, and melt ponds and other processes must be accounted for in order to predict melting rates.
- 2. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.
- 3. Statistical physics and homogenization help *link scales*, provide rigorous methods for finding effective behavior, and advance how sea ice is represented in climate models.
- 4. Critical behavior (in many forms) is inherent in the climate system.
- 5. Field experiments are essential to developing relevant mathematics.
- 6. Our research will help to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

THANK YOU

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Mathematics and Climate Research Network



Australian Government

Department of the Environment and Water Resources Australian Antarctic Division











Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Thursday, July 23, 1998

Australia

Hobart

Macquarie

Island in

Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship Aurora Australis without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast. The extent of the damage is

not known. Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an

inert gas. ... The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known The 25 crew and 54 expedi-

tioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year rut at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the Aurora Australis in Antarctica. ision for about \$11 million year.

P&0

ision's Hobart office. He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available." Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The Aurora Australis is owned by P&O Australia and charted by the Antarctic Div-

director Richard Hein said yes-Casev terday the company was assessing the situation and a number of rescue options were being Scale considered. It was too early to say whether P&O would be liable for the cost of the aborted

Australia managing

mission. The vessel left Hobart last

Wednesday for a seven-week voyage mainly to study a polyn-ya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.

Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

Antarctica

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

THE ADVERTISER (Adelaide) Thurs 23 July 1998

Fire strands Antarctic ship in sea ice

AN engine room fire has Australian Anteretic Div- arctic continent and return disabled the icebreaker Aurora Australis in sea ico, deep in Antarotic waters. Incre were no injuries and

the ship was not in danger after Tuesday night's fire,

Moncur said. But Mr. Moncur said he expected it would have to abandon its

islon director Mr Rex to Hobart for repairs.

Page 14

The cause of the fire was not known but the engines would have to abandon its have been turned off, with pioneering mid-winter voy- the ship 100 neutron miles age to the edge of the Ant- from the Antarctic coast.

THE CANBERRA TIMES Thursday 23 July 1998 Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Austra. lian icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected Aurora Australis would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs. The fire had been extin-

guished and the engines were turned off. leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good. Crew had to wear breathing

The Aurora, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause beavy, sait-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully accessed.





scrahped following an engine-room firscripting. Aurora Australis yesterday. The 54 people on board. Weis (occid on decivin the

2:45 am July 22, 1998

``Please don't be alarmed but we have an uncontrolled fire in the engine room"

about 10 minutes later ...

``Please don't be alarmed but we're lowering the lifeboats"